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POLYCHROMATIC MODELLING OF THE HUMAN EYE
CONTAINING A GRIN LENS

MARK F. COUGHLAN

UNDER THE SUPERVISION OF

DR. ALEXANDER V. GONCHAROV

Submitted in Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy (Ph.D.)

Applied Optics Group, Physics Department, National University of
Ireland, Galway

July 2017
ABSTRACT

The ingenuity and complexity of the human eye have attracted the interest of many researchers. In this thesis, we take a closer look at the polychromatic nature of the human eye, with special attention paid to the gradient refractive index (GRIN) nature of the human crystalline lens. The first chapter begins by providing an introduction into eye modelling; both schematically and physically, and finishes with information on the chromatic aberrations of the human eye.

The focus of the second and third chapters is modelling chromatic aberration of the eye under various conditions. We first look at the change in longitudinal chromatic aberration (LCA) with ageing and the influence of different GRIN distributions on the transverse chromatic aberration (TCA) of the eye. The effect of choosing different dispersion profiles for the GRIN lens is also investigated, as well as the change in LCA with accommodation. Chapter three considers the built-in GRIN distribution of Zemax and this is used to extend our modelling capabilities. The LCA and TCA across the field are compared to experimental studies and the impact of tilting and decentering the GRIN lens on the TCA is also investigated. Finally, an analytical method to optimise the GRIN distribution and dispersion is shown. The method is then used to create a personalised eye model.

The fourth chapter of the thesis begins by looking at contact lens correction for a personalised eye model. A method is then given for determining the deformable mirror shape of an open-loop, non-pupil conjugated adaptive optics system, which is capable of simulating a contact lens. The quality of the simulation is assessed in both the monochromatic and polychromatic case.

The fifth chapter presents an opto-mechanical artificial eye that can be used for examining multi-wavelength ophthalmic instruments. Standard off-the-shelf lenses and a refractive index matching fluid were used in the creation of the artificial eye. A Hartmann-Shack aberrometer operating at three distinct wavelengths was used in the initial testing. Following this, off-axis chromatic aberrations were analysed by imaging through the artificial eye at two discrete wavelengths.
PUBLICATIONS

Some ideas and figures have appeared previously in the following publications, reproduced wholly or in part with kind permission from The Optical Society of America.

ACKNOWLEDGMENTS

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I want to thank my family for all the support they have given. I thank David and Kate for their constant backing and I thank my parents for everything they have done. I always endeavour to make them proud and I am forever indebted to them.

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<td>Intraocular lens</td>
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<tr>
<td>GRIN</td>
<td>Gradient refractive index</td>
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<tr>
<td>GIGL</td>
<td>Geometry invariant gradient-index lens</td>
</tr>
<tr>
<td>AVOCADO</td>
<td>Accommodating volume-constant age-dependent optical</td>
</tr>
<tr>
<td>LED</td>
<td>Light emitting diode</td>
</tr>
<tr>
<td>RGP</td>
<td>Rigid gas permeable</td>
</tr>
<tr>
<td>LCA</td>
<td>Longitudinal chromatic aberration</td>
</tr>
<tr>
<td>TCA</td>
<td>Transverse chromatic aberration</td>
</tr>
<tr>
<td>BI</td>
<td>Base in</td>
</tr>
<tr>
<td>BO</td>
<td>Base out</td>
</tr>
<tr>
<td>BU</td>
<td>Base up</td>
</tr>
<tr>
<td>BD</td>
<td>Base down</td>
</tr>
<tr>
<td>VF</td>
<td>Visual field</td>
</tr>
<tr>
<td>AOSLO</td>
<td>Adaptive optics scanning laser ophthalmoscope</td>
</tr>
<tr>
<td>FA</td>
<td>Field angle</td>
</tr>
<tr>
<td>CNC</td>
<td>Computer numerical controlled</td>
</tr>
<tr>
<td>HSWA</td>
<td>Hartmann-Shack wavefront aberrometer</td>
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<tr>
<td>SE</td>
<td>Spherical equivalent</td>
</tr>
<tr>
<td>OMDAE</td>
<td>Opto-mechanical dispersive artificial eye</td>
</tr>
<tr>
<td>RMS</td>
<td>Root mean squared</td>
</tr>
<tr>
<td>PMMA</td>
<td>Polymethyl acrylate</td>
</tr>
<tr>
<td>CL</td>
<td>Contact lens</td>
</tr>
<tr>
<td>WF</td>
<td>Wavefront</td>
</tr>
<tr>
<td>DM</td>
<td>Deformable mirror</td>
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INTRODUCTION

1.1 OPTICAL MODELLING OF THE HUMAN EYE

The human visual system consists of two functional parts, namely the eye and certain parts of the brain. An illustration of the human eye is shown in Figure 1, where the important parts are identified. The main outer part of the eye consists of the sclera, a dense non-transparent tissue, mainly responsible for protecting the eye. The choroid is a layer covering the inner side of the sclera and provides oxygen and nourishment to the other parts of the eye. The cornea is a transparent medium, made up of several layers, that begins the process of focusing light onto the back surface of the eye. Behind the cornea is a chamber that holds a watery

Figure 1: The human eye (By National Eye Institute. Used under CC BY 2.0 (http://creativecommons.org/licenses/by/2.0). Original image modified.).
like substance called the aqueous fluid. This chamber also contains the iris of the eye, which is responsible for controlling the amount of light that travels through the eye. Following the iris is the crystalline lens and this part of the eye is responsible for our ability to see near and distant objects. Between the crystalline lens and retina is the vitreous chamber, which contains a clear gel called the vitreous humor. The retina is the inner light-sensitive layer of the eye. Information from the retina, which is gathered by the photoreceptors, is sent along the optical nerve and processed by the brain.

Many researchers have developed schematic models of the human eye, and these models have become essential tools in the field of ophthalmology. For instance, schematic eye models are fundamental in the design and validation of ophthalmic instruments, where they can be used to verify the underlying principal of the instrument. Schematic models can also be used to determine any limitations for the instrument and confirm the instrument specifications. Another important use for schematic models is in the investigation of the benefits arising from optical correction methods, such as spectacle lenses, contact lenses, laser eye surgery and intraocular lenses (IOLs). Extending this, a personalised eye model can be created and used to investigate the benefits of a particular correction method for an individual subject. This is especially beneficial for correction methods that require invasive surgery. Finally, schematic eye models can be used to simply gain a greater understanding of the human eye and aid the training of medical personnel. Different mechanisms, such as the accommodative nature of the human eye or disease development can be investigated with schematic eye models.

Schematic eye models can differ widely in complexity, with some simple schematic eye models designed for a single purpose, while other models attempt to model many features of the human eye, some of which are quite complex. The experimental data upon which schematic eye models are based is usually acquired from a range of different ophthalmic instruments. To acquire data, these instruments can employ many techniques, such as optical coherence tomography, wavefront aberrometry and ultrasound. Experimental data that measures geometrical features can be used to create the schematic eye model, while experimental data on the presence of aberrations can be used to verify the model. Alternatively, aberration data can be used to optimise some unknown features of the eye, which are dif-
difficult to measure directly. In many cases, a large amount of experimental data is gathered from many subjects and the resulting data is usually averaged to define a generic model of the human eye. Some studies attempt to create personalised eye models based on measurements from a single subject.

1.1.1 Paraxial eye models

Paraxial schematic eye models are the simplest models of the human eye, and as the name suggests, are only valid in the paraxial region. The paraxial region is defined as the region in which the replacement of sines of angles by the angles themselves leads to no significant error. If the errors are limited to less than 0.01 per cent, the field angles and entrance pupil diameters are limited to less than 2° and 0.5mm, respectively [1]. Paraxial eye models are generally composed of spherical surfaces that are all aligned on a common axis. The optical mediums are also generally considered to have uniform refractive indices. While paraxial models are unsuitable for certain applications due to their simplicity, they are still a valuable asset in ophthalmology. With paraxial schematic eyes, the location of the paraxial image plane can be obtained along with the positions of the principal planes and nodal points. The position of the entrance and exit pupil can also be determined. The calculation of these planes and points can be employed during the design of ophthalmic instruments. For example, paraxial models can be used to determine the position and image size for Purkinje images in the human eye. These images can be used to determine the radii of curvature for surfaces within the human eye [2]. Paraxial models can also be used to predict the optical power of the human eye, which can be used for selecting the optimal IOL before surgery [3].

Perhaps the most famous paraxial eye model is the model developed by Gullstrand in 1909 [4]. This eye model is known as Gullstrand’s number 1, or exact, eye. To define the geometry of the model, Gullstrand presented the radii of curvature for six spherical surfaces and the distance between each of these surfaces. These values are given in Table 1 along with the refractive index data, which was also presented by Gullstrand. The first column in Table 1 gives the radii of the six spher-
Figure 2: Graphical illustration of Gullstrand number 1 eye.

Gullstrand surfaces defined by Gullstrand. The distance to the next surface is defined by the thickness values directly to the right of the radii values. The refractive index in the space between the two surfaces is given by the value directly to the right of the thickness values. A diagram of the Gullstrand eye model is presented in Figure 2 with all details from Table 1 included. The position of the front principal plane (P), back principal plane (P’), front nodal point (N) and back nodal point (N’) is also illustrated in Figure 2. The distance from the back principal plane to the paraxial focus point is 22.785mm which gives the Gullstrand eye an optical power of 58.64D. The visual axis is defined as a line passing from the fovea through the nodal point of the eye. Interestingly, as can be seen from Table 1, Gullstrand had

<table>
<thead>
<tr>
<th>MEDIUM</th>
<th>RADIUS (MM)</th>
<th>THICKNESS (MM)</th>
<th>REFRACTIVE INDEX</th>
</tr>
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<tr>
<td>Cornea</td>
<td>7.700</td>
<td>0.500</td>
<td>1.376</td>
</tr>
<tr>
<td>Aqueous</td>
<td>6.800</td>
<td>3.100</td>
<td>1.336</td>
</tr>
<tr>
<td>Iris</td>
<td>Infinity</td>
<td>0.0</td>
<td>1.336</td>
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<tr>
<td>Lens cortex</td>
<td>10.000</td>
<td>0.546</td>
<td>1.386</td>
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<td>Lens core</td>
<td>7.911</td>
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Table 1: Geometrical parameters and refractive indices for Gullstrand number 1 eye.
knowledge of the gradient refractive index (GRIN) nature of the human crystalline lens and modelled the lens with a two shell structure. In this shell structure, the outer cortex has a lower refractive index compared to the central nucleus of the lens. The geometry in Table 1 defines the eye model in the unaccommodated state, but Gullstrand also presented the eye model in the accommodated state. Additionally, Gullstrand developed a three surface eye model, whereby the crystalline lens had zero thickness. Similar to the first model, this eye model has two levels of accommodation.

Many other researchers have proposed important paraxial eye models. In 1945, Le Grand published a modified version of Tscherning’s four refracting surface eye model [5]. Similar to Gullstrand, Le Grand presented a simplified version of the model that consisted of a single corneal surface and a crystalline lens with zero thickness. Notably, Le Grand also presented values for the dispersion of the ocular media. While the model of Le Grand is well known, the most utilised dispersive paraxial eye model is possibly the model developed by Thibos et al. [6]. This eye model is a reduced eye model that consists of only a single aspheric refracting surface. An aperture stop is also included in the design. The refractive index of the single medium was chosen such that the overall chromatic aberration of the model matched experimental data and this model is commonly referred to as the chromatic eye. Other paraxial eye models which have proven important in the field of ophthalmology are the Gullstrand-Emsley eye [7], Emsley’s reduced eye [7] and the Bennet and Rabbetts model [8].

1.1.2 Finite eye models

As previously mentioned, the simplicity of paraxial eye models makes them unsuitable for certain applications. To model the aberrations of the eye, more anatomically correct eye models were needed, which led to the development of finite eye models. Rather than being a separate area of research, finite eye modelling builds upon the developments in paraxial eye modelling. Finite eye modelling also requires advanced ophthalmic instruments, which can return accurate measurements on the human eye. At a basic level, finite eye models build upon parax-
Optical modeling of the human eye models by the inclusion of aspheric surfaces. An aspheric surface can be described by the equation for a conic section given by

\[ p^2 = 2rz - (1 + k)z^2, \]

where \( r \) is the radius and \( k \) is the conic constant. We denote \( p \) as giving the position on the vertical axis (equatorial axis) and \( z \) as the position on the horizontal axis (optical axis). Beyond aspheric surfaces, more complex features such as toric surfaces, decentering of elements, tilting of elements and gradient refractive index distributions can also be included in finite eye models. With this increased level of detail, finite eye models are valid for large pupil sizes and can also be used for off-axis modeling.

The first example of a finite eye model was possibly the model proposed by Lotmar in 1971 [10]. This eye model consisted of an aspheric anterior corneal surface and was created for the purpose of predicting off-axis astigmatism. Later, Blaker modified the Gullstrand number 1 eye to create an adaptive schematic eye in 1980 [11]. Importantly, Blaker replaced the shell structure crystalline lens with a two surface gradient refractive index lens. To make the model ‘adaptive’, the lens gradient refractive index, lens surface curvatures, lens thickness and the anterior chamber depth all varied as linear functions of accommodation. Blaker subsequently revised his unaccommodated model to include ageing effects whereby the lens curvatures, lens thickness and anterior chamber depth all varied as a function of age.

<table>
<thead>
<tr>
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<tr>
<td>Retina</td>
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<td>0.0</td>
<td>—</td>
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</table>

Table 2: Geometry for the wide-field model of Escuardo-Sanz and Navarro [9].
1.1 Optical modelling of the human eye

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<td>1.3777</td>
<td>1.376</td>
<td>1.3747</td>
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</tr>
</tbody>
</table>

Table 3: Refractive indices for the wide-field model of Escuardo-Sanz and Navarro [9].

[12]. Other early finite eye models include Kooijman’s model published in 1983 [13] and the adaptive model published in 1985 by Navarro et al. [14]. Following Blaker, Navarro later modified the unaccommodated state of the adaptive model. This modified model, published by Escuardo-Sanz and Navarro was designed to replicate aberrations across a wide field angle and also included dispersion data [9]. Many researchers have used this eye model and it is defined in Table 2 and Table 3, which gives the geometry and refractive index values for the model, respectively. Comparing the wide-field model of Escuardo-Sanz and Navarro to the Gullstrand number 1 eye, we see the inclusion of the conic constants and also that the refractive indices are defined at specific wavelengths. Interestingly, the lens is modelled as a two-surface lens, rather than the four-surface model given by Gullstrand.

Some recent finite eye models have also been used to model the ageing human eye [15, 16]. Studying the ageing human eye is an important exercise because many eye conditions have a strong correlation with age. For example, myopia tends to develop in the young eye, while presbyopia and cataracts tend to evolve in the older eye. Schematic eye models that model the ageing human eye can also be considered the first step towards personalised eye models. While the experimental data is based on averaged values, the values are averaged across different age groups rather than across the entire set of experimental data.
1.1.3 Crystalline lens models

It has been observed that most of the age-related changes in the human eye occur at the crystalline lens [15]. Regarding optical characteristics, the crystalline lens is also the most complex of the ocular components with its GRIN nature. While the presence of a GRIN distribution within the crystalline lens has been known for a long time, the complexity of modelling a GRIN distribution and lack of accurate experimental data has lead to many researchers proposing models for the crystalline lens. In the simplest sense, the crystalline lens can be modelled as a two-surface constant refractive index lens, as shown in the model by Escuardo-Sanz and Navarro [9]. However, to make an eye model more anatomically correct, a shell structure can be used to model the crystalline lens. The simplest case of a shell structure model can be seen in the Gullstrand’s number 1 eye [4]. Another shell structure model is Lotmar’s model, which has seven shells with a 0.005 increment in refractive index between each shell [10]. This number of shells was increased by Pomerantzeff et al. where they modelled the lens with 398 layers of different refractive indices, radii of curvature and thicknesses [17]. Other models using the shell structure for the crystalline lens are the model of Al-Ahdali and El-Messiery [18] and the model of Liu et al. [19]. The advantage of the shell structure over constant refractive index models and GRIN models is that the refractive index distribution approaches the experimentally observed distribution and yet conventional paraxial ray-tracing equations can be used to find the lens power. However, the disadvantage of the shell structure is that optimisation of the refractive index distribution is difficult.

Several researchers have created crystalline lens models containing a continuous distribution of refractive index. This approach to modelling the crystalline lens can be illustrated using the geometry invariant gradient-index lens (GIGL) model proposed by Bahrami and Goncharov [20]. We begin by considering the external surface of the lens. While most optical surfaces are defined in terms of the radius and conic constant, as shown by Equation 1, Bahrami and Goncharov included an additional cubic term. Thus the anterior and posterior surface of the lens can be mathematically defined as
where the $b$ term is a constant that can be used to define a continuous external surface for the crystalline lens. The terms $\rho$, $r$, $k$ and $z$ have the same definition as previously given for Equation 1. The continuous external surface can be achieved by ensuring that the derivative of both the anterior and posterior surface representations are zero at the intersection point, denoted by $z_c$. Thus the continuous surface for the anterior and posterior surface can be defined with an extended version of Equation 2, given by

$$\rho_a^2 = 2r_a(t_a + z) - (1 + k_a)(t_a + z)^2 + b_a(t_a + z)^3$$  \hspace{1cm} (3)$$

$$\rho_p^2 = 2r_p(t_p - z) - (1 + k_p)(t_p - z)^2 + b_p(t_p - z)^3$$  \hspace{1cm} (4)$$

where $\rho_a$ defines a position on the anterior surface and $\rho_p$ defines a position on the posterior surface. The terms $r_a$ and $k_a$ define the radius and conic constant of the anterior surface, respectively, while the terms $r_p$ and $k_p$ define the radius and conic constant of the posterior surface, respectively. The anterior surface is defined from $-t_a$ to $z_c$ and the posterior surface is defined from $z_c$ to $t_p$. Suitable selection of $z_c$, $b_a$ and $b_p$ can be used to satisfy the condition of the external surface having a zero derivative at the intersection point, and subsequently, define a continuous surface. The terms $z_c$, $b_a$ and $b_p$ can be defined by the following equations

$$z_c = \frac{t_aQ_a + t_pQ_p - 2(r_a + r_p) + \sqrt{(t^2Q_aQ_p - 4t(r_aQ_p + R_pQ_a) + 4(r_a + r_p)^2)}}{Q_p - Q_a}$$  \hspace{1cm} (5)$$

$$b_a = \frac{2}{3}\left(\frac{Q_a(t_a + z_c) - r_a}{(t_a + z_c)^2}\right)$$  \hspace{1cm} (6)$$

$$b_p = \frac{2}{3}\left(\frac{Q_p(t_p - z_c) - r_p}{(t_p - z_c)^2}\right)$$  \hspace{1cm} (7)$$
where \( Q_a = 1 + k_a \), \( Q_p = 1 + k_p \), \( t = t_a + t_p \) and all other parameters have been defined previously. The advantage of a continuous representation for the external surface of the lens is that this surface can then be scaled inwards to define lines of constant refractive index, known as iso-indicial contours. Considering only the anterior section, we have the following height for a contour

\[
\rho_a^2 = 2\zeta r_a(\zeta t_a + z) - (1 + k_a)(\zeta t_a + z)^2 + \zeta^{-1}b_a(\zeta t_a + z)^3
\]  

(8)

where \( \zeta \) is defined as \( z/t_a \) and \( z \) goes from \(-t_a\) to 0. Thus \( \zeta \) can be considered the normalised distance along the optical axis. This concept is illustrated in Figure 3, where the external geometry and iso-indicial contours are plotted. With reference to Figure 3, it can be seen that once the refractive index along the optical axis is known, the refractive index at any other point can be obtained by follow-
ing the iso-indicial contours. To define the refractive index along the optical axis, Bahrami and Goncharov used the equation initially proposed by Pierscionek [21] and adopted by Smith and others [22–24]:

\[ n(\zeta) = n_c + (n_s - n_c)(\zeta^2)^p \] (9)

where \( n_c \) and \( n_s \) are the refractive indices at the centre and surface of the lens, respectively, and \( p \) is an age-dependent parameter describing the steepness of the refractive index profile towards the lens periphery. The term \( \zeta \) again defines the normalised distance from the lens centre to the lens periphery. Equation 9 is illustrated in Figure 3 where an example of a refractive index profile along the equatorial and optical axis is plotted. It can be seen from the plot of the refractive index along the optical axis that the anterior section of the crystalline lens is thicker than the posterior section.

An advantage of the GIGL distribution is that analytical equations can be defined for paraxial ray tracing. However, it is not possible to alter the GRIN distribution once the external geometry is defined, as the external geometry completely defines the iso-indicial contours. This limits the use of the GIGL model for optimisation and matching of experimental data. For example, it has been observed that the young crystalline lens exhibits a different refractive index profile along the optical and equatorial axis [25]. A recent model which allows the independent adjustment of the optical and equatorial refractive index distributions is the accommodating volume constant age-dependent optical (AVOCADO) model of the GRIN lens proposed by Sheil and Goncharov [26]. In this model, external surfaces are defined in the same manner as the GIGL model, however, with regard to the iso-indicial contours, the radii are scaled non-linearly according to an appropriate power, \( m \). Thus we have the following height for a contour in the anterior section:

\[ \rho_a^2 = 2\zeta^{2m+1}r_a(\zeta t_a + z) - \zeta^{2m}(1 + k_a)(\zeta t_a + z)^2 + \zeta^{2m-1}b_a(\zeta t_a + z)^3 \] (10)

where again, \( \zeta \) is defined as \( z/t_a \) and \( z \) goes from \(-t_a\) to \(0\). This model is an extension of the GIGL model and in fact, reduces to the GIGL model when one
defines $m = 0$. In Figure 4, the external geometry and iso-indicial contours are plotted for a crystalline lens defined with the \textit{AVOCADO} model. The same external geometry as Figure 3 is used, however, there is a noticeable difference in the iso-indicial contours. On the right of Figure 4, it can be seen that the refractive index profile along the optical axis is the same as the \textit{GIGL} model, but there is a clear difference between the refractive index profiles along the equatorial axis.

In addition to the \textit{GIGL} model and \textit{AVOCADO} model, other advanced models of the GRIN crystalline lens that have been published by Blaker [11], Smith et al. [27], Liou and Brennan [28], Goncharov and Dainty [16], Navarro et al. [24], Bahrami et al. [29], Manns, de Castro et al. [30, 31], Diaz et al. [32], Navarro [15] and Polans et al. [33]. It is worth noting that while most of these models require custom programming to implement the GRIN distributions in standard ray-tracing software,
such as Zemax (Radiant Zemax LLC, WA) and Code V (Synopsys Inc., CA), three of the published distributions can be implemented in ray-tracing software using the built-in GRIN distributions [16, 28, 33]. While these distributions do not afford the same flexibility, their reliability and ease of use makes them useful tools, and this is reflected in their wide use. When compared to eye models containing homogeneous crystalline lenses, these models can be used to create more anatomically correct eye models and also offer greater flexibility for optimisation. In other cases, these GRIN lenses can be used for very specific tasks, such as modelling the distortions of the posterior lens surface during optical coherence tomography imaging [34] or reconstructing the optical system of the eye [33, 35]. The Liou and Brennan lens model [28] and the Polans et al. model [33] have fixed distributions whereby the coefficients are defined for a single lens geometry. This differs for the model created by Goncharov and Dainty whereby the coefficients are defined for several lens geometries and the formulae for calculating the coefficients are also given [16]. For the GRIN model published by Goncharov and Dainty, they use the following distribution

\[ n_{\text{ref}}(z,r) = n_0 + n_1 r^2 + n_2 r^4 + n_3 z + n_4 z^2 + n_5 z^3 + n_6 z^4. \quad (11) \]

where \( n_{\text{ref}} \) is the refractive index at the reference wavelength, \( z \) is the axial distance from the anterior lens surface and \( r \) is the radial distance given by \( r^2 = x^2 + y^2 \). The terms \( n_0, n_1, n_2, n_3, n_4, n_5 \) and \( n_6 \) are the refractive index coefficients. These coefficients can be defined in a variety of ways, depending on the constraints placed on the GRIN distribution. Goncharov and Dainty derived expressions for the coefficients such that the iso-indicial contours followed the external geometry of the lens, which requires separate coefficients for the anterior and posterior section of the lens. They also defined formulae for the coefficients such that the lens could be defined as a single component. This requires the condition on the iso-indicial contours being coincident with the external surface of the lens to be relaxed. In this case, the formulae for these coefficients is given by

\[ n_1 = -\Delta n z_m d^2(d - 2z_m)(d - z_m)/m^* \quad (12) \]
Figure 5: 40S lens model. External geometry and iso-indicial contours (left). Refractive index profile along equatorial axis (top, right). Refractive index profile along optical axis (bottom, right).

\[ n_3 = 2\Delta nz_m r_a d^2(d - 2z_m)(d - z_m)/m^* \]  \hspace{1cm} (13)

\[ n_4 = -\Delta nd(d^3 r_a - 3d(3r_a + r_p)z_m^2 + 4(2r_a + r_p)z_m^3)/m^* \]  \hspace{1cm} (14)

\[ n_5 = 2\Delta n(d^3 r_a - d^2(3r_a + r_p)z_m + 2(r_a + r_p)z_m^2)/m^* \]  \hspace{1cm} (15)

\[ n_6 = -\Delta n(d^2 r_a - 2d(2r_a + r_p)z_m + 3(r_a + r_p)z_m^2)/m^* \]  \hspace{1cm} (16)

where

\[ m^* = z_m^2(d - z_m)^2(r_a(d - z_m)^2 + r_p z_m^2) \]  \hspace{1cm} (17)
and $\Delta n = n_{\text{max}} - n_0$. The coefficient $n_0$ gives the refractive index at the lens surface, $n_{\text{max}}$ is the maximum refractive index at $z_m$, the peak plane position, $d$ is the lens thickness, $r_a$ is the anterior lens radius and $r_p$ is the posterior lens radius. This distribution can be implemented in both Zemax and Code V, without the need for custom programming. To illustrate this model, we have plotted the 40S lens given by Goncharov and Dainty in Figure 5.

1.1.4 Personalised eye models

Both paraxial and finite eye models can be modified to create personalised models. As with generic models, the degree of complexity can vary widely between personalised models. To create a personalised eye model, the usual procedure involves collecting a variety of data from a single subject using different ophthalmic instruments. This can include measurements of individual parameters and also measurements of the overall aberrations in the eye. This data is then used to reconstruct the properties of the eye.

To create personalised eye models, Navarro [36] began with a generic eye model and customised the model based on the experimental data collected. This experimental data included both anatomic and optical data. Two different initial eye models were compared; one considered a simple constant refractive index lens, whereas the second model had a GRIN lens. The first step of creating the models involved using the anatomic data, which consisted of the elevation topography of the cornea, pupil centre and axial thickness of the various ocular elements. Next, the optical data, which was the total wave aberration, including second and higher order aberrations was defined and the unknown lens structure that would reproduce the measured wave aberration was obtained through optimisation. Guo et al. [37] followed a similar approach whereby the Gullstrand-Le Grand eye was used as the starting point. This was initially supplemented with individual corneal data, anterior chamber depth and the axial length of the eye. The next step involved optimising the lens shape to match the experimental wavefront aberrations of the
eye. Examples of personalised eye modelling to improve IOL selection can be seen from Canovas and Artal [38], and Rosales and Marcos [39].

1.1.5 Physical eye models

While schematic eye models can be used for theoretical investigations, there is also a need for physical eye models that can be used in experimental studies. Examples of such studies are the testing of vision correction methods and the calibration of ophthalmic instruments. With vision correction methods, artificial eyes have been widely used for examining IOLs. In this area, artificial eyes have been employed to assess the performance of monofocal IOLs [40–49] and also to compare the performance of monofocal IOLs with multifocal IOLs [50–58]. Shown in Figure 6 is an artificial eye for testing IOLs that was developed by Norrby et al. [49]. In addition to IOLs, artificial eyes can be used to evaluate contact lenses [59–61] and spectacle lenses [62–64]. For calibration of ophthalmic instruments, artificial eyes with a known amount of aberrations are used to investigate the performance of instruments that measure aberrations in the eye [65–74]. When performing tests with artificial eyes, the test can be done in either single or double pass. With single pass testing, there are three main methods used. Firstly, a detector that is conjugate to the position of the retina can be used to image a standard visual acuity chart through the artificial eye. An interesting model eye employing this technique was developed by Arianpour et al., in which the curved retinal surface is relayed to a flat detector with a series of optical fibres [50]. Alternatively, an active retina, which generates light that passes through the artificial eye can be used to test instruments such as Hartmann-Shack aberrometers. This technique is employed by Esteve-Taboada et al. with a near-infrared LED behind the retinal surface [71], avoiding unwanted reflections that may occur if a reflection from the retina is used. Finally, an acuity chart can be placed at the back of the artificial eye and an image can be taken through the eye. This method is used by Inoue et al. to test IOLS [43, 53], where a United States Air Force resolution chart was placed on the retinal surface and imaged through the eye. Double pass testing uses a reflection from the retinal surface. This is demonstrated by Fernandez et al. where a rotating diffuser
is used in the position of the retina to break the spatial coherence of the incoming light [66].

Physical eye models can also be categorised in terms of static and dynamic models. The geometry of static physical model eyes is fixed, whereas dynamic artificial eyes have the ability to change certain features. This can simply be the ability to manually translate certain parts of the artificial eye to induce defocus or more advanced techniques using liquid lenses and bimorph mirrors can be employed [75]. In the case of the latter, temporal fluctuations in the aberrations of the eye can be simulated to test adaptive optics systems [66].

Designing and building an anatomically accurate artificial eye presents various difficulties. Firstly, the ocular refractive indices are difficult to replicate. Distilled water is generally used to sufficiently mimic the aqueous and vitreous fluid. However, the refractive indices of the cornea and crystalline lens are not as simple to replicate. Bakaraju et al. used a fluoro-polymer having a refractive index of 1.376 and a Boston rigid gas permeable (RGP) polymer having a refractive index of 1.423 to model the cornea and crystalline lens, respectively [59]. Ji. et al. developed a bio-inspired GRIN human lens [76]. But such materials are not readily available and require custom machining. Another difficulty is replicating the thickness of the cornea. Creating a meniscus lens whose thickness gives the same optical path
length as the average human cornea can run the risk of buckling and inducing unwanted astigmatism. For the above reasons, artificial eyes are generally created to serve their purpose, with certain non-essential features omitted. This simplifies the construction and can make the artificial eye more reliable. A good example of simplification can be seen in the case of testing IOLs, where an achromatic doublet is often used to simulate the cornea [77]. Such a lens has little resemblance to an actual cornea, but it serves the purpose of inducing a converging beam on the IOL, as shown in Figure 6.

1.2 CHROMATIC ABERRATIONS OF THE HUMAN EYE

Like all refractive media, the ocular media of the human eye have a dispersive nature. The simplest way to define the dispersive nature of the ocular media is to define their refractive indices at several important wavelengths. This was shown in Table 3, where the refractive indices for the wide field model of Escuardo-Sanz and Navarro were defined at 458nm, 543nm, 589.3nm and 632.8nm [9]. Defining the refractive indices at wavelengths 486.1nm, 589.3nm and 656.3nm allows the Abbe number to be calculated, which indicates the degree of dispersion for a given medium. The Abbe number can be calculated with the following formula

\[ V_d = \frac{n_d - 1}{n_f - n_c} \]  

(18)

where \( n_f, n_d \) and \( n_c \) are the wavelengths at 486.1nm, 589.3nm and 656.3nm, respectively. If two materials have the same refractive index at \( n_d = 589.3 \text{nm} \), a higher Abbe number will indicate lower dispersion of one medium compared to the other. The Abbe number can be used when describing the difference in power with wavelength for a single refracting surface. For example, if one considers the dispersive paraxial eye model developed by Thibos et al. [6], which consists of a single surface, we can define the power at 589.3nm as

\[ F_d = \frac{n_d - 1}{r} \]  

(19)
where $n_d$ is the refractive index at 589.3nm and $r$ is the radius of the surface.

Now if we consider the difference in optical power between $n_f = 486.1$nm and $n_c = 656.3$nm, we can define the following

$$\Delta F = F_f - F_c = \frac{n_f - n_c}{r}$$  \hspace{1cm} (20)

Finally, considering $r = (n_d - 1)/F_d$, it can be seen that

$$\Delta F = \frac{F_d(n_f - n_c)}{n_d - 1} = \frac{F_d}{V}$$  \hspace{1cm} (21)

where $V$ is the Abbe number of the medium that follows the anterior surface. Thus the difference in optical power between $n_f = 486.1$nm and $n_c = 656.3$nm is directly proportional to the optical power at $n_d = 589.3$nm and inversely proportional to the Abbe number.

While defining the refractive indices at certain discrete wavelengths is sufficient for some applications, a more complete representation can be given by defining a continuous dispersive profile for the medium. This requires a mathematical description of the dispersion such that the refractive index can be determined at any wavelength. Various mathematical equations have been proposed for representing the dispersion of optical media. Some of these mathematical equations have theoretical foundations, while others are chosen based on suitable fits to experimental data. Equations with theoretical foundations can be used to estimate dispersion outside the experimentally measured zone, however caution must be exercised when using equations that do not exhibit a theoretical foundation. To show the wide range of dispersion formulae available, various representations are defined below. These formulae were mainly adapted from a publication by Atchison and Smith, where the dispersion of the various ocular media is investigated [78]. There is Cauchy’s equation [79]:

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \frac{D}{\lambda^6} + ...$$  \hspace{1cm} (22)

The Hartmann equation [80]:
The Cornu dispersion equation [81], which is similar to the Hartmann equation:

\[ n(\lambda) = n_\infty + \frac{K}{(\lambda - \lambda_0)} \]  

The Herzberger equation [82]:

\[ n(\lambda) = A + B\lambda^2 + \frac{C}{(\lambda^2 - \lambda_0^2)} + \frac{D}{(\lambda^2 - \lambda_0^2)^2} \]  

The Conrady equation [83]:

\[ n(\lambda) = n_0 + \frac{A}{\lambda} + \frac{B}{\lambda^{3.5}} \]  

Sellmeier’s dispersion formula [79]:

\[ n^2(\lambda) = 1 + \frac{B_1\lambda^2}{(\lambda^2 - \lambda_{01}^2)} + \frac{B_2\lambda^2}{(\lambda^2 - \lambda_{02}^2)} + \frac{B_3\lambda^2}{(\lambda^2 - \lambda_{03}^2)} + \ldots \]  

And finally, the Schott dispersion equation [84]:

\[ n^2(\lambda) = a_0 + a_1\lambda^2 + \frac{a_2}{\lambda^2} + \frac{a_3}{\lambda^4} + \frac{a_4}{\lambda^6} + \frac{a_5}{\lambda^8} + \ldots \]  

In all these equations, the values and magnitudes of the constants depend on the particular material and the units of \( \lambda \). Atchison and Smith used the Cauchy representation to define the dispersion of the different ocular media [78] and this representation has received widespread use. They defined the dispersion of the various ocular media by combining data from Le Grand [81] and Navarro et al. [14]. It should be noted that the dispersion of the crystalline lens, even though given for a GRIN lens, is based on a scaling of the single lens dispersion given by Le Grand and Navarro. An experimental study measuring the lens dispersion was carried
1.2 CHROMATIC ABERRATIONS OF THE HUMAN EYE

out by Sivak and Mandelmann, but it has been noted that this study has a high associated error [85]. The Cauchy constants for the cornea, aqueous, lens surface, lens centre and vitreous defined by Atchison and Smith are given in Table 4. A plot of the refractive index between 400nm and 700nm for each medium is then shown in Figure 7, where it can be seen that the refractive indices of the various ocular media tend to decrease with an increase in wavelength. Considering this refractive index change with wavelength, it follows that the performance of the human eye changes with wavelength. At a single wavelength, the optics of the human eye can

<table>
<thead>
<tr>
<th>Medium</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<td>5.908450x10¹³</td>
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<tr>
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<td>6.070796x10³</td>
<td>−7.062305x10⁸</td>
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<td>−6.023738x10⁸</td>
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<td>6.521218x10³</td>
<td>−6.110661x10⁸</td>
<td>5.908191x10¹³</td>
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<tr>
<td>Vitreous</td>
<td>1.322357</td>
<td>5.560240x10³</td>
<td>−5.817391x10⁸</td>
<td>5.036810x10¹³</td>
</tr>
</tbody>
</table>

Table 4: Cauchy constants for different ocular media [78].
be evaluated in terms of monochromatic aberrations. These aberrations include
defocus, spherical aberration, coma and astigmatism. The dispersive nature of
the human eye will subsequently cause these aberrations to have a wavelength
dependence, with some aberrations showing a greater wavelength dependence
than others. The two main chromatic aberrations in the human eye are longitudinal
chromatic aberration (LCA) and transverse chromatic aberration (TCA).

1.2.1 Longitudinal chromatic aberration

The LCA can be understood as a change in refractive error with wavelength. Thus
we first derive an expression for the refractive error of the human eye. With ref-
eree to Figure 8, efl is the effective focal length in vitreous space, bfl is the
back focal length, VCD is the vitreous chamber depth, n_v is the vitreous refractive
index and Δz is the distance between the paraxial focus and retina. Given that
F = n_v/efl is the power of the eye and ΔF is the refractive error, it can be seen
that:

\[ F + \Delta F = \frac{n_v}{efl + \Delta z} \]  (29)

\[ \Rightarrow \Delta F = \frac{n_v}{efl + \Delta z} - F \]  (30)

\[ \Rightarrow \Delta F = \frac{n_v}{efl + \Delta z} - \frac{n_v}{efl} \]  (31)

Now, it can be seen that Δz = VCD − bfl. Thus

\[ \Delta F = \frac{n_v}{efl + (VCD - bfl)} - \frac{n_v}{efl} \]  (32)

defines the refractive error. When deriving a formula for the LCA, we consider
the difference in refractive error between two wavelengths, one of which is the
reference wavelength. This is illustrated in Figure 9, where a blue and green ray
is drawn. For our LCA analysis, we assume the reference wavelength to be 589nm
and for simplicity, this ray is shown to be in focus at the retina. With reference to Figure 9, $\Delta P$ is the principle plane separation and other variables are defined similar to Figure 8. The vitreous refractive index at the blue wavelength is given by $n_{\text{blue}}$, while the vitreous refractive index at 589nm is given by $n_{589}$. $F_{\text{blue}}$ is the power of the eye at the blue wavelength. Similar to the derivation of the refractive error, it can be seen that:

$$F_{\text{blue}} + LCA = \frac{n_{\text{blue}}}{efl_{\text{blue}} + \Delta z} \quad (33)$$

$$\Rightarrow LCA = \frac{n_{\text{blue}}}{efl_{\text{blue}} + \Delta z} - F_{\text{blue}} \quad (34)$$

$$\Rightarrow LCA = \frac{n_{\text{blue}}}{efl_{\text{blue}} + \Delta z} - \frac{n_{\text{blue}}}{efl_{\text{blue}}} \quad (35)$$

Now, it can be seen that $\Delta z = bfl_{589} - bfl_{\text{blue}}$. Thus

$$LCA = \frac{n_{\text{blue}}}{efl_{\text{blue}} + (bfl_{589} - bfl_{\text{blue}})} - \frac{n_{\text{blue}}}{efl_{\text{blue}}} \quad (36)$$
This gives the LCA in terms of diopters and it can be considered as a difference in back focal length with wavelength. A variety of methods can be used to measure the LCA of the human eye and these methods can be characterised as either subjective or objective. Bedford and Wyszecki used a stigmatiscope to subjectively measure LCA [86] and this technique was also employed by Wald and Griffin [93]. Gilmartin and Hogan used a Badal optometer [94], Thibos et al. used a vernier alignment technique [6] and Marcos et al. used spatially resolved refrac-

<table>
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<td>Nakajima et al. [92]</td>
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<td>45</td>
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Table 5: LCA measurements from various experimental studies.
1.2 CHROMATIC ABERRATIONS OF THE HUMAN EYE

Chromatic aberrations of the human eye [25].

Figure 10: LCA of the chromatic eye [6], where LCA = 0 at λ = 589nm.

LCA has also been measured objectively by means of reflectometric techniques, such as double-pass retinal images of a slit [95] or of a point source [96] at different wavelengths. Other objective techniques include Hartmann-Shack wavefront sensing [88, 91, 92, 97] and laser ray tracing wavefront sensing [91]. Two studies have measured the LCA across the field and found a very slight increase with eccentricity [96, 98]. Various researchers have also investigated the change in LCA with age and it is generally accepted that little change occurs with ageing. A reduction in LCA with age was observed by Millodot [99] and by Mordi and Adrian [100]. However, this effect was not observed by Howarth et al. [101], Ware [102] and Nakajima et al. [92]. It is often difficult to compare results between different studies due to the variety of wavelength ranges used in the experiments. This can be seen in Table 5, where the measured LCA from some experimental studies are given, along with the wavelength range and number of subjects.

Thibos et al. fitted their LCA results to a Cornu equation and subsequently defined the refractive index of their reduced eye model, known as the chromatic eye, with another Cornu equation [6]. This allows the LCA to be calculated at any wavelength. Using their Cornu equation, it is suggested that the LCA of the average human eye between 400nm and 700nm is 2.1D. A plot of the LCA against wavelength for the chromatic eye is shown in Figure 10. While 2.1D of LCA in the
visible region seems quite large, it is believed that the eye has a natural compensation mechanism, which is based on the non-uniform spectral sensitivity of the eye. This is illustrated in Figure 11, where the luminous efficiency of the eye under photopic conditions (luminance above 3cd/m$^2$) is plotted. It is worth noting that the CIE introduced a photopic eye sensitivity function in 1924, but it is believed that the function underestimates the human eye sensitivity in the blue and violet spectral region [103]. A modified sensitivity function was introduced by Judd and Vos in 1978 [104], where the function has higher values in the spectral region below 460nm. This is the function illustrated in Figure 11, where it can be seen that the eye is most sensitive to wavelengths near 555nm and least sensitive to wavelengths at either end of the visible spectrum. Thibos et al. estimated that when the wavelength with peak sensitivity is in focus, most of the luminance in a white light target is less than 0.25D out of focus [105].

### 1.2.2 Spherochromatism

As mentioned previously, monochromatic aberrations such as spherical aberration, coma and astigmatism will all vary with wavelength. Here we look at the effect of spherochromatism, which is defined as the variation in spherical aberration with
wavelength. Shown in Figure 12, is the overlapped longitudinal spherical aberration of the chromatic eye at 400nm, 589nm and 700nm. The axial length of the chromatic eye was adjusted such that the paraxial focus occurred at the retina for each wavelength and a 6mm entrance pupil diameter was used. It can be seen from Figure 12 that the spherical aberration is larger at 700nm compared to 400nm. Manzanera et al. measured the spherical aberration of three eyes at 440, 488, 532, 633 and 694nm and found a slight increase with wavelength, similar to the chromatic eye [87]. The difference in spherical aberration can be described in diopters, which allows comparison with the LCA and also gives a more intuitive understanding of the significance of spherochromatism. To convert the difference in spherical aberration to diopters, we can use the same formula that was used for calculating the LCA. However, in this case, Δz is defined as the difference in spherical aberration between the chosen wavelength and reference wavelength. This difference in spherical aberration is illustrated for 400nm and 589nm in Figure 12. With the chromatic eye, the magnitude of the spherochromatism at a 3mm ray height, between 400nm and 700nm, was 0.17D. This is more than an order of magnitude
less than the LCA and will become less significant with a reduction in pupil size. Interestingly, for positive spherical aberration, the spherochromatism acts in the opposite direction of the LCA. Thus, when using the formula for the LCA, the LCA with the back focal length defined in terms of the marginal ray will be less than the LCA with the back focal length defined in terms of the paraxial ray.

1.2.3 Transverse chromatic aberration

The second significant chromatic aberration is transverse chromatic aberration. For a centred optical system, TCA will only be present off-axis. With reference to Figure 13, TCA can be understood as a difference in ray height at the retina for chief rays of different wavelengths. Note that the eye model in Figure 13 is a reduced eye model whereby the pupil is located at the cornea and the incident rays are coming from infinity. The presence of the marginal rays in Figure 13 show that LCA is present both on-axis and off-axis. While the TCA can be characterised in terms of the difference in heights for the chief rays, it can also be characterised in terms of the angle separating the chief rays in vitreous space. This is illustrated in the leftmost diagram of Figure 14.

Figure 13: Illustration of TCA.
To measure the TCA of the eye, the measurement must be performed outside the eye. This is illustrated in the rightmost diagram of Figure 14. Here, even though the blue and red ray enter the eye at different angles, they arrive at the same position on the retina. This would lead the subject to believe that the rays were coming from the same position in object space. In addition to the TCA magnitude, it is also important to give the TCA orientation. A simple method to describe the TCA orientation can be defined using a prism, whereby the prism is said to be either base-in (BI), base-out (BO), base-up (BU) or base-down (BD). If the TCA is described as being BI, it means the orientation is the same as would be caused by a BI aligned prism (base towards the nose) in front of the eye. This implies that short wavelengths will end up at the retina more towards the nasal side than long wavelengths. This is shown in Figure 15, along with details on the other orientations.

The TCA of the human eye has been measured by various researchers, with the majority of measurements being made on the foveal TCA. These studies show that the foveal TCA varies across the population [106–111] and the results from some of the studies are given in Table 6. It should be noted that the magnitudes of TCA shown in Table 6 cannot be compared directly, due to the different wavelength ranges used between studies. Regarding peripheral TCA, the theoretical article by Thibos suggests that TCA changes nearly linearly with eccentric viewing angle [112]. Three experimental studies on peripheral TCA have been carried out and all studies confirm that the peripheral TCA changes with viewing angle. The first
1.2 CHROMATIC ABERRATIONS OF THE HUMAN EYE

two studies are subjective studies. Ogboso and Bedell used a subjective alignment method and found the TCA to increase with eccentricity [106]. In the second study, Winter et al. evaluated the effect of prism-induced TCA on foveal and peripheral vision and found that different prisms were needed to optimise peripheral vision compared with central vision [111]. They also observed that the peripheral vision is more sensitive to induced TCA than foveal vision. The main reason for a low number of studies on peripheral TCA is the difficulty in estimating its magnitude by adequate measurement procedures. Recently, the first objective study on the peripheral TCA was carried out by Winter et al. [110], producing much more accurate results. An objective method allows the TCA to be measured without requiring the

![Figure 15: Base-in/base-down (BI/BD) on left and base-out/base-up (BO/BU) on right.](image)

Table 6: Results for experimental TCA measurements at the fovea.
Figure 16: Experimental measurement of TCA in the horizontal visual field (VF) [110]. Negative values for the VF indicate nasal field. A positive sign of horizontal TCA means that the direction of TCA is the same as a BI prism would cause.

Figure 17: Experimental measurement of TCA in the vertical visual field (VF) [110]. Negative values for the VF indicate inferior field. A positive sign of vertical TCA means that the direction of TCA is the same as a BU prism would cause.
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translation of optical information into neural information. The study by Winter et al. used an adaptive optics scanning laser ophthalmoscope (AOSLO) to interleave images of the retina taken at different wavelengths. The contribution to the overall TCA from the AOSLO system was measured and subsequently subtracted from the overall TCA to obtain a measure of the TCA induced by the eye. Winter et al. fitted a line to their experimental results to define the measured TCA across the field. A plot of the average TCA for the four subjects in the horizontal visual field is given in Figure 16, while the TCA in the vertical visual field is given in Figure 17. These plots suggest that the average location of the achromatic axis for the four subjects analysed is about 11 degrees in the temporal visual field and 2.5 degrees in the inferior visual field.

1.2.4 Relationship between LCA and TCA

In this section, we show that it is possible to establish a link between the LCA and TCA of an eye model [113]. To illustrate the relationship, a reduced eye model and two off-axis point sources are shown in Figure 18. Two rays, corresponding to different wavelengths, originate from the point sources and intersect the retina at the same point. It can be seen that the TCA is given by $b - a$, where the angles
b and a are specified relative to the nodal ray of the reference wavelength. From Figure 18, it can also be seen that

\[ a \approx \frac{h}{L_1} \quad \text{and} \quad b \approx \frac{h}{L_2} \quad (37) \]

\[ \Rightarrow \text{TCA} \approx h\left(\frac{1}{L_2} - \frac{1}{L_1}\right) \quad (38) \]

Now, the LCA of the reduced eye model can also be defined as \( 1/L_2 - 1/L_1 \). Therefore

\[ \text{TCA} \approx h\text{LCA} \quad (39) \]

which establishes an approximate linear relationship between the TCA and LCA.

1.2.5 Surface contribution to overall chromatic aberration

When considering the polychromatic performance of the eye or other optical instruments, the overall chromatic aberration gives the best indication of the image quality. However, when one is designing a component or trying to achieve a better understanding of the chromatic aberration, it is important to understand the contribution of each surface to the overall chromatic aberration. In the case of the LCA, this can be achieved with the axial colour coefficient defined as

\[ C_l = n_d y \left( \frac{y}{r} + u \right) \left( \frac{n_d - 1}{n_d V_d} - \frac{n'_d - 1}{n'_d V'_d} \right) \quad (40) \]

where \( n_d \) is the refractive index of the medium at \( \lambda_d = 589.3 \text{nm} \) before the surface, \( y \) and \( u \) are the height and angle of the incident ray at the surface, respectively, \( r \) is the surface radius of curvature and \( V_d \) is the Abbe number of the medium before the surface [114]. The refractive index of the medium after the surface at \( \lambda_d = 589.3 \text{nm} \) is given by \( n'_d \) and the Abbe number of the medium after the surface is defined as \( V'_d \). The \( C_l \) coefficients of all surfaces can be added
together to obtain the total axial colour coefficient, \( C_L \). The total longitudinal axial chromatic aberration, \( \delta_{AX} \), can then be calculated with the following formula

\[
\delta_{AX} = \frac{1}{n_{d_{i1}}u_i^2} C_L
\]  

(41)

where \( n_{d_{i1}} \) is the refractive index at \( \lambda_d = 589.3 \text{nm} \) for the last medium, \( u_i \) is the refracted ray angle at the image plane and \( \delta_{AX} \) is the distance between the paraxial focus at \( \lambda_F = 486.1 \text{nm} \) and \( \lambda_C = 656.3 \text{nm} \). In a similar way, the lateral colour coefficient \( C_t \) is given by

\[
C_t = n_{d_{i1}} \left( \frac{y_c}{r} + u_c \right) \left( \frac{n_d - 1}{n_d V_d} - \frac{n'_d - 1}{n'_d V_d} \right)
\]  

(42)

where \( y_c \) and \( u_c \) are the height and the angle of the incident chief ray at a refractive surface, respectively [114]. Again, the total lateral colour coefficient can be found by simple addition. The total coefficient \( C_T \) can then be used in calculating the transverse lateral chromatic aberration \( \delta_{TLC} \), which corresponds to the vertical distance between the images at \( \lambda_F = 486.1 \text{nm} \) and \( \lambda_C = 656.3 \text{nm} \), using

\[
\delta_{TLC} = \frac{1}{n_{d_{i1}}u_i} C_T
\]  

(43)

where \( n_{d_{i1}} \) and \( u_i \) have the same definition as previous. The advantage of using these methods to calculate both the axial colour and lateral colour is that the contribution of each surface can be investigated by simply analysing the individual coefficients.

1.2.6 Multi-wavelength imaging of the human eye

The optics of the eye are relatively poor, which makes it difficult to carry out high-resolution imaging of the eye. The temporal fluctuations of the eye also add to this difficulty. However, adaptive optics ophthalmoscopy has been successful in overcoming these limitations. Adaptive optics itself was originally proposed in the
1950s as a means of improving astronomical imaging [115]. After successful implementation in astronomy [116], adaptive optics was employed in ophthalmology to obtain high-resolution images of the retina [117]. When imaging the retina, the majority of adaptive optics systems operate in a closed loop setting whereby they measure and subsequently remove both static and dynamic aberrations in real time. A wavefront sensor can be used to measure the aberrations while either a reflective or refractive deformable element can be used to compensate for the aberrations. In such a closed loop system, it is beneficial to conjugate the deformable element plane and the wavefront sensor plane. Furthermore, both these planes should be conjugated to the eye’s pupil in order to achieve the maximum corrected area. This is due to the various aberrations arising at different depths within the eye. The largest angle of correction is called the isoplanatic angle and typical values

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<td>17.0</td>
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Table 7: Geometrical parameters and glass types for the Powell system.
are 1-2 degrees. If a larger isoplanatic angle is required, multi-conjugate adaptive optics can be employed [118].

The majority of adaptive optics systems image the retina at a single wavelength. To image the eye with white light or at multiple wavelengths, the chromatic aberrations of the eye need to be accounted for. Various researchers have created multiple wavelength ophthalmoscopes [119–121]. In these systems, the LCA has been removed by propagating the light from different wavelengths at different vergences. This places an additional constraint on the adaptive optics system whereby it must operate sufficiently at the different vergences. Dubra and Sulai constructed a multi-wavelength system which was diffraction limited over a 3 diopter vergence [120]. If the images acquired at different wavelengths are viewed separately, the TCA does not need to be accounted for. However, to output a single image from a multi-wavelength imaging system, the effect of the TCA must be factored in. Correction of the TCA was achieved by Harmening et al. using an interweaved recording technique [121].

To image the retina with white light, a component which compensates for the LCA of the eye would need to be included in the system. An early example of such a component was proposed by Bedford and Wyszecki [86]. They designed

![Figure 20: LCA of chromatic eye (red) and combined LCA of chromatic eye and Powell system (blue).](image)

Figure 20: LCA of chromatic eye (red) and combined LCA of chromatic eye and Powell system (blue).
a triplet after measuring the LCA of 12 subjects. The disadvantage of their design was that the overall system could only be used for a small field of view since the TCA increases rapidly with incidence angle. A more sophisticated design was created by Powell who designed an air-spaced doublet-triplet system, which had better off-axis performance. A similar system was also proposed by Benny et al. [122]. Fernández et al. created an LCA compensator that was specifically designed for the near infrared [97]. The Powell system [123] is illustrated in Figure 19 with a reduced eye model. The dimensions and materials are given in Table 7. As an investigation into the performance of the Powell system, we modelled the system with the chromatic eye of Thibos et al. [6] and calculated the combined LCA, which is shown in Figure 20. It can be seen that the overall LCA has been reduced from a maximum difference of 2.1D to a maximum difference of 0.09D between 400nm and 700nm.
INFLUENCE OF THE GRIN LENS ON CHROMATIC ABERRATION

2.1 INTRODUCTION

As discussed in Chapter 1, the geometry of the human eye changes throughout ageing, with the majority of changes occurring at the crystalline lens. In spite of these changes, many researchers have not observed a similar change in LCA with ageing. In this chapter, we analyse eye models corresponding to different ages and investigate whether the changing geometry has a significant influence on the LCA. Following this investigation, we create two eye models with identical geometry, but considerably different GRIN distributions and investigate the impact of the GRIN distribution on both the overall LCA and TCA of the eye models. We also compare these results to the LCA and TCA of an eye model with identical geometry, but containing a constant refractive index lens. In addition, we investigate the effect of using an alternative dispersion profile to define the dispersion of the GRIN lens. Finally, we investigate the change in LCA with accommodation for the young adult eye.

2.2 AGE-RELATED CHANGES IN LCA

To examine the age-related changes in LCA, we created eye models corresponding to 20, 30, 40, 50 and 60 years of age. The geometry of these models was determined using a recent publication by Navarro, where the ocular geometries are given as functions of age and accommodation [15]. In this section, we assume that the eye is in the unaccommodated state and we only analyse the horizontal meridian. Also, considering we are currently only interested in looking at the age-related changes in LCA, we presume that the ocular elements of each eye model are centred on
a common axis with no associated tilt. For completeness, the geometries of each eye model are given in Table 8, where the age-related changes can be observed. The corneal model is adapted from a study that fitted a linear regression as a function of age to the main parameters of the anterior and posterior corneal surface topographies [124]. This study used Scheimpflug imaging to determine the corneal shape of 407 normal eyes (211 subjects), with ages ranging from 4 to 79 years. From Table 8, it can be seen that the anterior corneal radius tends to decrease slightly with age. Both the anterior and posterior surfaces show negative conic constants, with the posterior surface being more prolate than the anterior surface. A correlation between the conic constant and ageing was only observed for the anterior surface, and no correlation between corneal thickness and age was

<table>
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Table 8: Geometrical parameters for eye models corresponding to different ages.
Table 9: Additional parameters for the ageing crystalline lenses.

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</tbody>
</table>

found. The external geometry of the lens model is based on a compilation [125] of experimental data by Dubbelman et al. [126] on the changes of the ageing human lens with accommodation. The vitreous chamber depth is chosen such that the total axial length of the eye remains fixed with age. Finally, it is also assumed that the radius of the retinal surface remains constant. When modelling the refractive index distribution within the crystalline lens, rather than using Navarro’s distribution, we used the GIGL distribution published by Bahrami and Goncharov [20], which was defined in Section 1.1.3. The GIGL distribution facilitates paraxial ray tracing and thus our investigation into the LCA could be carried out without the need for ray tracing software or complex numerical techniques. To define the continuous external geometry of the lens, we used the radii and conic constants in Table 8 to calculate $b_a$, $b_p$, and $z_c$. These values are defined in Table 9. To define the refractive index distribution along the optical axis of the crystalline lens, we used the common power-law profile. To determine the $p$ parameter for each of the eye models, we used the equation $p = 2.85 + (1.1 \times 10^{-7}) \text{Age}^4$, which was defined by Navarro [15]. The $p$ values for each of the eye models are also given in Table 9. The geometrical parameters associated with the 20, 40 and 60 year old eye models are graphically illustrated in Figure 21. As can be seen from the overlapping plots, there is little change in the geometry of the cornea, total axial length and retinal shape of the eye. However, a large change occurs at the crystalline lens where the surfaces become more convex and the lens becomes thicker. This thickness increase of the crystalline lens causes a reduction in the aqueous chamber depth and vitreous chamber depth with age.
To define the refractive indices of the cornea, aqueous, lens surface and vitreous, we use the dispersion profiles published by Atchison and Smith [78], as discussed in Section 1.2. The refractive index values at the reference wavelength and Abbe numbers are given in Table 10. To define the dispersion of the lens centre, we use the Atchison and Smith dispersion equation, with one slight modification. At the reference wavelength of 589 nm, we optimised the central refractive index of the crystalline lens such that the paraxial focus occurred on the retina for each eye model. This results in each eye model being emmetropic, even though there is some evidence to suggest that, in general, the younger eye tends to be more myopic than the older eye [127]. Since we were interested in investigating the influence of geometrical changes on the LCA, we omitted this condition. The opti-

<table>
<thead>
<tr>
<th></th>
<th>CORNEA</th>
<th>AQUEOUS</th>
<th>LENS SURFACE</th>
<th>VITREOUS</th>
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<td>1.335</td>
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<td>V_{d}</td>
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<td>50.4</td>
<td>45.6</td>
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Table 10: Refractive indices and Abbe numbers of different ocular media.
2.2 Age-related changes in LCA

Figure 22: Refractive index distribution along the optical axis for the 20 (blue), 40 (red) and 60 (green) year old crystalline lens. (Left). Dispersion through the lens surface and lens centre for the 20 (blue), 40 (red) and 60 (green) year old crystalline lens. Lens surface dispersion is identical for all models. (Right).

Mised central refractive index values at 589 nm for each eye model could then be used to redefine the $A$ constant of the Cauchy representation given by Atchison and Smith. The optimised $A$ constants, along with the refractive index at 589 nm and Abbe number of the lens centres for each eye model are defined in Table 11. To keep the paraxial focus on the retina for each model, it was found that the central refractive index of the lens had to decrease with age. This reduction in the central refractive index has been proposed as a possible mechanism for the "lens paradox". Briefly, the lens paradox is the measured decrease (or relatively slow increase) in refractive power of the human eye, despite the fact that the lens radii become more highly curved and hence more powerful. The paradoxical nature arises because, although the lens surfaces become more powerful, the lens power decreases. Moffat et al. measured the refractive index with magnetic resonance imaging and found

<table>
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<th>60</th>
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<td>$V_d$ for lens centre</td>
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<td>49.5</td>
<td>49.1</td>
<td>48.7</td>
<td>48.3</td>
</tr>
</tbody>
</table>

Table 11: The $A$ constant, central refractive index values and Abbe number for the lens centre corresponding to different ages.
a decrease with age \cite{128}. However, this explanation for the lens paradox is not widely accepted. They found an average decrease of $3.4 \times 10^{-4}$ per year, which predicts a total change of 0.014 in the lens central refractive index between 20 and 60 years. To keep our eye models emmetropic, the central refractive index changed by 0.013 between 20 and 60 years (1.426 to 1.413). The refractive index distribution along the optical axis for the 20, 40 and 60 year old eye model can be seen in the leftmost diagram in Figure 22. This diagram illustrates the lower central refractive index and increased central plateau with ageing, where the latter is due to the changing $p$ value. The dispersion of the lens surface and lens centre for the 20, 40 and 60 year old eye is shown in the rightmost diagram of Figure 22.

With the geometry and dispersion entirely defined for the models, we then looked at the LCA of each model. This is shown in Figure 23 where it can be seen that the LCA shows little change with age, as observed in many experimental studies. This leads us to conclude that changes in the geometry of the eye and central refractive index of the lens do not have a strong influence on the overall LCA of the eye. In Figure 23, the chromatic eye model of Thibos et al. \cite{6} is also shown as a reference for the average LCA in the human eye.
850nm, the LCA of the 20 year old model was 2.45D, while the LCA of the 60 year old model was 2.60D. Considering the wide spectrum analysed, this is a relatively small change in LCA. Interestingly, the power of the 20 year old model was 62.8D, while the power of the 60 year old model was 62.5D. At first, this seems counter-intuitive because the LCA of the 20 year old model was slightly less than that of the 60 year old model, even though the power was greater. However, when comparing both eye models, it can be seen that the vitreous chamber depth is less for the 60 year old model, simply due to the increase in thickness of the crystalline lens. This causes the back focal length of the 60 year old eye model to be less and this influences the overall LCA. Considering both eye models are emmetropic, the lower power of the 60 year old model means that the back principal plane is located further towards the cornea than for the 20 year old eye model.

2.3 GRIN DISTRIBUTION COMPARISON

Simply dropping the refractive index of the lens centre to offset the increase in power due to the geometrical changes in the ageing human eye is possibly an over simplistic solution to the lens paradox. Evidence suggests that the refractive index distribution changes within the lens [25]. A more recent explanation for the lens paradox has suggested that a change in the refractive index profile along the equatorial axis could compensate for the more convex and thicker crystalline lens [26]. This mechanism suggests that the younger crystalline lens has a more convex central nucleus compared to the older crystalline lens, which tends more towards the GIGL model. This suggestion results in a much different GRIN distribution between the younger and older eye. We were interested in investigating if this affected the chromatic aberrations of the eye, and especially the transverse chromatic aberration considering its tendency to increase with field angle. If it could be shown that the transverse chromatic aberration differed between two GRIN distributions, then experimental studies could be used to constrain either the GRIN distribution or dispersion of the GRIN lens. This would be useful considering the current difficulty in measuring the properties of the GRIN lens.
Figure 24: Crystalline lens comparison. GGL model with $p = 3.13$. (Left). AVOCADO lens model with $p = 3.13$ and $m = 0.74$. (Right).

For our analysis, we set up two eye models. The first model we analysed was the 40 year old eye model as given in the previous section but with two additional features. Since we were analysing the TCA, we included a 3.54 degree tilt on the lens, and a 0.113mm pupil decentre, as defined by Navarro [15]. All other features were kept the same. Since we were interested in isolating the effect of the GRIN structure, we used identical geometry but a different GRIN distribution for the second eye model. The second GRIN model that we used was the AVOCADO model published by Sheil and Goncharov [26] and as defined in Section 1.1.3. This model allows the refractive index profile along the optical and equatorial axis to be defined independently. When setting up this model, we defined the central refractive index 0.01 lower than the previous model and optimised the refractive index distribution along the equatorial axis to make the eye model emmetropic. The nor-
malised refractive index profile along the optical and equatorial axis is given in Figure 24 for each lens model. In the case of the GIGL model, the normalised profile is the same in both the optical and equatorial axis. However, for the AVOCADO model, the refractive index distribution along each of the axes differs.

With the geometry and dispersion of the two eye models completely defined, we first compared the LCA of the models. Unfortunately, the same paraxial equations cannot be used with the AVOCADO model and instead a real ray trace close to the optical axis was carried out. This ray trace was done using a Runge-Kutta method for ray tracing through GRIN structures [26]. Considering that the GIGL model is a special case of the AVOCADO model, we could use the Runge-Kutta method to calculate the LCA of the GIGL model and confirm our previous results. As expected from our previous analysis, there was little difference between the LCA of the eye model containing the GIGL lens and eye model containing the AVOCADO lens. Between 400nm and 700nm, the LCA of the GIGL model was 2.17D, compared to 2.19D for the AVOCADO model. The power of the eye model containing the GIGL lens was 62.71D and the power of the eye model containing the AVOCADO lens was 62.64D.

Next we analysed the transverse chromatic aberration. We were interested in this result because even though the two eye models have similar refractive index distributions along the optical axis, they are quite different when moving away from the optical axis and thus off-axis aberrations could differ. However, we found that this was not the case. Shown in Figure 25 is the TCA of the two eye models with the different GRIN distributions. We calculated the TCA as the angle separating the chief ray at 400nm and 700nm in vitreous space. As can be seen in Figure 25, the magnitude and position of the achromatic axis is very similar. We also calculated the TCA of a constant refractive index lens model and this is plotted in Figure 25. The geometry of this eye model was kept the same as the GRIN eye models and the dispersion of the constant refractive index lens was defined such that the overall LCA was similar for all models. Fitting a linear line to the results, it was found that the TCA of the GIGL model was $-0.394$ arcmins/degree, the AVOCADO model was $-0.399$ arcmins/degree and the constant refractive index model was $-0.397$ arcmins/degree.
Figure 25: Comparison of TCA between GIGL (blue), AVOCADO (red) and constant refractive index (green) lens models. Negative values for the field angle FA indicate nasal field. A positive sign of horizontal TCA means that the direction of TCA is the same as a BI prism would cause.

As previously mentioned, the dispersion profiles for the lens surface and lens centre defined by Atchison and Smith are based on theoretical analysis. In Figure 26, the dispersion profiles of the lens surface and lens centre are plotted for the 20, 40 and 60 year old eye. The dispersion profile of the constant refractive index lens analysed in the previous section is also plotted. Examining these profiles, it can be seen that they appear as almost shifted versions of each other. Considering this might have an impact on the chromatic aberrations, we looked at a different dispersion curve for one of the eye models. The model chosen was the eye model containing the AVOCADO lens. We decided to use the study carried out by Sivak and Mandelmann, where the dispersion of several lenses was measured at the lens surface and centre [85]. It should be noted that this study has a high associated error, but nonetheless, it provides useful data. Using both the lens surface dispersion curve and lens centre dispersion curve from the experimental study causes an
Figure 26: Comparison of dispersion profiles. Blue corresponds to lens surface. Orange, red and green correspond to 20, 40 and 60 years. Black dashed corresponds to constant refractive index model.

Figure 27: Comparison of dispersion profiles. Solid green corresponds to Sivak and Mandelmann surface dispersion. Blue corresponds to Atchison and Smith surface dispersion. Green dashed corresponds to derived central dispersion value.
unrealistic amount of \textit{LCA} in the eye model. Considering the relationship between the \textit{LCA} and \textit{TCA}, this will of course influence the magnitude of \textit{TCA}.

To isolate the effect of the dispersion profile, we assumed the lens surface dispersion equation was correct and derived the lens central dispersion curve such that the eye model was emmetropic and the overall \textit{LCA} matched the previous eye models. The resulting dispersion profiles for the lens surface and lens centre are shown in Figure 27. The dispersion for the lens surface according to the Atchison and Smith publication is also shown. As can be seen, the profiles are no longer shifted versions of each other and the surface dispersion curves differ. The Abbe number for the Atchison and Smith lens surface is 45.6, and the Abbe number for the lens centre is 49.1. In comparison, the Sivak and Mandelmann lens surface Abbe number is 27.2, and the derived lens centre Abbe number is 37.7.

With the new dispersion curves for the lens, we looked at the \textit{TCA} of the eye models. The comparison is shown in Figure 28, where it can be seen that there was little change in the \textit{TCA}. The \textit{TCA} of the AVOCADO model with the Sivak and Mandelmann dispersion definition was $-0.411$ arcmins/degree compared to $-0.399$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure28.png}
\caption{\textit{TCA} comparison between lens with Atchison and Smith dispersion definition (red) and Sivak and Mandelmann dispersion definition (purple). Negative values for the field angle $\text{FA}$ indicate nasal field. A positive sign of horizontal \textit{TCA} means that the direction of \textit{TCA} is the same as a $\text{BI}$ prism would cause.}
\end{figure}
arcmins/degree for the AVOCADO model with the Atchison and Smith dispersion definition.

2.5 CHANGE IN LCA WITH ACCOMMODATION

Since we were interested in the influence of the GRIN lens, we also modelled the change in LCA with accommodation. We chose the geometry of the 20 year old model and defined the accommodation changes from Navarro [15]. We modelled an accommodation change of 4D and calculated the LCA between 400nm and 700nm using the object vergence method. With this method, we calculated the inverse distance of the object to the corneal vertex that caused the paraxial focus to occur on the retina for each wavelength. The inverse distance for both wavelengths could then be used to calculate the LCA.

Figure 29: Change in LCA with accommodation. Unaccommodated lens on left and accommodated (4D) lens on right.
A diagram of the crystalline lens in the unaccommodated and accommodated state is shown in Figure 29. The eye model had a power of 62.8D in the unaccommodated state and a power of 66.3D in the accommodated state. It was found that the LCA changed from 2.11D in the unaccommodated state to 2.31D in the accommodated state.

2.6 CONCLUSION

Our analysis found that the LCA of the eye was stable across different ages, which means that the changing geometry of the eye does not have a significant impact on the LCA. When the LCA was kept constant, we also found that different GRIN distributions did not affect the magnitude of the TCA or the position of the achromatic axis. Following this, it was seen that even replacing the GRIN lens with a constant refractive index lens does not significantly impact the TCA. We also found that the TCA was consistent for different dispersion profiles of the lens surface and lens centre. This analysis leads us to conclude that due to the relationship between the LCA and TCA, they are poor constraints to use together when trying to determine the GRIN distribution or dispersion. Finally, when modelling the change in LCA across the visible spectrum with accommodation, we found a slight increase of 0.2D for a 4D accommodation. However, considering the small magnitude and the spectral sensitivity of the eye, the impact of the changing LCA should be minimal.
POLYCHROMATIC MODELLING OF THE HUMAN EYE

3.1 INTRODUCTION

Our results from Chapter 2 have shown that both the GRIN distribution and dispersion of the distribution do not have a significant influence on the overall chromatic aberrations of the eye. Considering these results, we look at implementing a dispersive eye model within Zemax that uses the built-in Gradient 5 medium. As discussed in Chapter 1, while built-in distributions do not offer the same flexibility as customised distributions, their reliability and ease of use makes them valuable assets. For example, built-in distributions can be used to create anatomically correct eye models, which can then be used to easily replicate experimental studies. In this chapter, an eye model is created with the Gradient 5 medium and subsequently used to replicate an objective study on the TCA across the horizontal and visual field. Using this eye model, we also investigate the impact of tilting and decentering the crystalline lens on the overall TCA, along with the magnitude of LCA across the visual field. Following this, we look at a method for easily defining a polychromatic eye model in Zemax that contains a GRIN lens. This method is then used to create a personalised eye model based on recently acquired experimental data.

3.2 ZEMAX EYE MODEL WITH DISCRETE DISPERSION PROFILE

The GRIN distribution we chose to implement with the Gradient 5 medium was the simplified distribution given by Goncharov and Dainty [16], and as discussed in Section 1.1.3. Regarding the geometry of the eye model, it was kept the same as the 40 year old model defined in Chapter 2. At 589nm, the reference wavelength, we choose $n_0$ and $n_{\text{max}}$ to be the same as in the AVOCADO model. This left one
variable in the definition of the eye model at the reference wavelength, namely $z_m$, the position of the peak plane for the GRIN distribution. This variable is needed to define the GRIN coefficients, which could in turn be used to make the eye model emmetropic. An iterative method was used to find a suitable value for $z_m$. The initial guess was half way across the lens and this was adjusted until the eye model was emmetropic. The final value used for $z_m$ was 1.705mm from the anterior lens surface. The $n_2$ term was then used to optimise the spherical aberration of the lens, and consequently the overall eye model. The spherical aberration of the chromatic eye at 589nm, shown in Section 1.2.2, was used as a guide (see Figure 12). The values for the GRIN distribution coefficients are given in Table 12 and the GRIN distribution is plotted in Figure 30. In Table 12, the coefficients for the reverse orientation of the lens are also given. These coefficients can be found by simply
3.2 ZEMAX EYE MODEL WITH DISCRETE DISPERSION PROFILE

Figure 31: TCA of Zemax model with Gradient 5 lens (blue) and AVOCADO model (red).

Negative values for the field angle FA indicate nasal field. A positive sign of horizontal TCA means that the direction of TCA is the same as a BI prism would cause.

Replacing $z$ in Equation 11 with $t - z$ and rearranging the equation to the original format.

With the eye model containing the simplified GRIN lens, we first looked at the TCA and compared this to the previous values found for the AVOCADO lens eye

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<tr>
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</table>

Table 12: GRIN coefficients for lens in regular orientation and reversed orientation
3.2 Zemax Eye Model with Discrete Dispersion Profile

Figure 32: Vertical TCA of Zemax model with Gradient 5 lens. Negative values for the FA indicate inferior field. A positive sign of vertical TCA means that the direction of TCA is the same as a BU prism would cause. Dashed, black line corresponds to Winter et al. study [110].

To compare the TCA magnitudes, we defined the cornea, aqueous and vitreous refractive index at 400nm and 700nm using the Cauchy constants previously defined in Table 4. We then optimised the $n_0$ value at each of these wavelengths such that the LCA of the Zemax model matched the LCA of the AVOCADO lens eye model. This method of optimisation has the effect of either raising or lowering the dispersion curve for the crystalline lens. The optimised $n_0$ values at 400nm and 700nm are given in Table 13 and a plot of the TCA comparison is shown in Figure 31. As expected, the TCA is very similar with the Zemax model having a TCA of $-0.402$ arcmins/degree and the AVOCADO model having a TCA of $-0.399$ arcmins/degree, in vitreous space.

<table>
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Table 13: $n_0$ coefficients for different wavelengths
3.2 ZEMAX EYE MODEL WITH DISCRETE DISPERSION PROFILE

Figure 33: Horizontal TCA of Zemax model with different tilts and decenters. Dotted, solid and dashed lines correspond to tilts of 1.54, 3.54 and 5.54 degrees, respectively. Blue, red and green correspond to pupil decenters of 0.013, 0.113 and 0.213mm, respectively. Negative values for the field angle FA indicate nasal field. A positive sign of horizontal TCA means that the direction of TCA is the same as a BI prism would cause. Dashed, black line corresponds to Winter et al. study [110].

Next we looked at calculating the TCA outside the eye and comparing the results to a recent experimental study [110]. This study is the first objective measurement of the TCA across the field and the most accurate study carried out to date. For each of the four subjects, the TCA between 543nm and 842nm was measured across both the horizontal and vertical meridian. To replicate this study, it was first required that the Zemax eye model be defined at the same wavelengths used in the experiment. Like before, the refractive indices of the cornea, aqueous and vitreous were defined using the Cauchy representation from Table 4. The $n_0$ value was then optimised such that the overall LCA of the eye model matched the LCA of the AVOCADO model, at the required wavelengths. Again, the optimised $n_0$ values are given in Table 13. To replicate the experimental study, it was next required that the eye model be set up in the reverse orientation, with the retina located in the object space. We decided to analyse the vertical TCA first and chose a vertical meridian that was located 5 degrees in the temporal retinal field. This was chosen...
based on the assumption that the fovea is located 5 degrees temporal of the eye’s optical axis. Across a 30 degree central field, the angle of the chief ray emerging from the eye at 543nm and 842nm was subtracted to define the TCA. The results are shown in Figure 32. The line, which was fit to the calculated TCA, had a slope of 0.25 arcmins/degree. The TCA at the fovea was 0.4 arcmins and the achromatic axis was located 1.5 degrees in the inferior visual field. In Figure 32, the vertical TCA from the objective study of Winter et al. study is also shown [110]. When they averaged their results across three subjects, they found the TCA had a slope of 0.23 arcmins/degree and that the TCA at the fovea was 0.6 arcmins. When comparing the horizontal TCA of the Zemax eye model to the experimental study, we extended the analysis to investigate the effect of tilting and decentering the lens. The chosen tilts were 1.54 degrees, 3.54 degrees and 5.54 degrees. The chosen decentres were 0.013mm, 0.113mm and 0.213mm. These tilt and decentre ranges are consistent with experimental measurements [129]. We chose a horizontal meridian that was located 2 degrees in the inferior retinal field, based on the assumption that the fovea is located 2 degrees below the eye’s optical axis. Shown in Figure 33 is the horizontal TCA of the Zemax eye model with each combination of the defined tilts and decenters. We found that the TCA at the fovea varied between 0.6 arcmins and 1.4 arcmins, while the achromatic axis was located 2.5 degrees to 6 degrees in the temporal visual field. It can be seen that the rate of change of TCA is very similar for all lens orientations. Choosing the 3.54 degree tilt and 0.113mm decentre, the TCA rate of change is −0.23 arcmins/degree. The horizontal TCA from the objective study of Winter et al. study is also shown [110] in Figure 33. When they averaged their results across four subjects, they found the horizontal TCA had a slope of −0.19 arcmins/degree and that the TCA at the fovea was 2.1 arcmins.

3.3 Change in LCA with Visual Field Angle

The variation in LCA with field angle has been studied by two groups of researchers [96, 98]. In the first study, Rynders et al. observed a mean change of approximately 0.6D between the LCA at the fovea and at a 40 degree eccentricity. The wavelength range analysed was 458nm to 632.8nm, and the number of
subjects was four \[96\]. In the second study, Jaeken et al. found a mean change of approximately 0.25D between the \textit{LCA} at the fovea and at a 30 degree eccentricity. The wavelength range used was 473nm to 671nm, and the number of subjects was eleven \[98\]. Of the eleven subjects, five were emmetropes and six were mild myopes. No significant difference was found between the emmetropes and myopes.

To replicate these studies, the \textit{LCA} across the field of the Zemax model with the \textit{Gradient 5} lens was computed using a method similar to the object vergence method. In this method, the inverse distance of the object from the corneal vertex that provides the best focus on the retina is calculated for each wavelength and subtracted. However, in our analysis, a paraxial lens was placed at the corneal vertex and the power of the lens was optimised such that the smallest spot size occurred on the retina. Subtracting the power of the paraxial lens between different wavelengths gave a measure of the \textit{LCA}. The method was the same for both on-axis and off-axis analysis. Shown in Figure 34 is the results of our simulation of the \textit{LCA} across the field. We observed an increase in \textit{LCA} of approximately 0.4D between 0 degrees and 30 degrees eccentricity, for a wavelength range of 400nm to 700nm.
For our previous analysis, the position of the peak refractive index plane was defined using an iterative method. While this method was sufficient, in this section we look at implementing an analytical method for optimising the peak plane location. With a focus on Zemax, we also look at defining a continuous dispersion profile for the various ocular media. This includes a continuous dispersion profile for the GRIN lens defined with the Gradient 5 medium. These steps make it easier to implement eye models within Zemax and this is illustrated at the end of the chapter when a personalised eye model is created.

We begin by considering the optimisation of $z_m$, which can be improved upon by deriving expressions for both the effective focal length and back focal length of the eye. This can be achieved by suitable combination of the different elements within the eye. Given the complexity of the GRIN distribution, we begin by analysing the crystalline lens. To calculate the power of the GRIN medium, we assume a parabolic ray path approximation as described by Smith and Atchison [130]. When using this approximation with the GRIN distribution described by Equation 11, the equations for the equivalent and back vertex powers become

$$F_{\text{Eq}} = -\frac{6n_0n_1t}{3n_0 - 2n_1t^2} \quad F_{\text{BV}} = -\frac{6n_0n_1t}{3n_0 + n_1t^2}$$

where $n_0$ and $n_1$ are the refractive index coefficients and $t$ is the thickness of the lens. Using Equation 44, we can define the principal plane location of the GRIN bulk relative to the lens surface as simply $\delta_b = n_0/F_{\text{Eq}} - n_0/F_{\text{BV}}$. For the distribution given by Equation 11, $n_0$ and $n_1$ are independent of direction and hence the estimate for the position of the front principal plane ($H_{b_0}$) and back principal plane ($H'_{b_0}$) is the same, as illustrated in Figure 35.

While the position of the bulk principal planes are calculated assuming the lens to be a single component, to increase the accuracy of the power calculation, we split the lens into the anterior and posterior segment. Equation 44 are applied to each segment individually and the lens is split at the peak plane. This method is chosen based on the refractive index profile, which has a steep increase at the
periphery, but tends to plateau towards the centre of the lens, making the parabolic ray path approximation more applicable for the individual segments. For each segment, the parabolic ray path approximation is applied with the assumption that the rays are travelling towards the peak plane. Therefore, only the thickness differs when Equation 44 is applied to the anterior and posterior segments. After finding the power and principal plane location of each individual segment with Equation 44, the overall power of the GRIN bulk can then be found using the standard thick lens equation

\[ F_B = F_{AB} + F_{PB} - \frac{d_b F_{AB} F_{PB}}{n_{max}}, \]  

where \( F_{AB} \) and \( F_{PB} \) are the powers of the anterior and posterior bulk, respectively. The distance between the principal plane of the anterior bulk (\( H_{ab} \)) and principal plane of the posterior bulk (\( H_{pb} \)) is given by \( d_b \), as illustrated in Figure 35.

With the power and principal plane locations of the GRIN bulk calculated, all that remains is combining the GRIN bulk with the refractive surfaces of the cornea and lens. This can be done in several ways, but we chose a double iteration of the three-component power equation [130] (Equation 46) and its corresponding expression for the back principal plane location of a three component system (Equation 47), which can be easily derived with paraxial ray tracing. These equations are given by

Figure 35: Diagram showing the different principle plane locations and key distances.
\[ F = F_1 + F_2 + F_3 - \frac{d_1 F_1 F_2}{n_2} - \frac{d_2 F_2 F_3}{n_3} - \left( \frac{d_1}{n_2} + \frac{d_2}{n_3} \right) F_1 F_3 + \frac{d_1 d_2 F_1 F_2 F_3}{n_2 n_3} \] (46)

\[ \delta' = n_4 \left( \frac{F_1}{n_2} \frac{d_1 F_1}{n_2} - \frac{d_2 (F_1 + F_2)}{n_3} \right) \left( \frac{d_1 d_2 F_1 F_2}{n_2 n_3} \right) \] (47)

where \( F_1, F_2 \) and \( F_3 \) are the powers of the first, second and third component, respectively and \( \delta' \) is measured with respect to the back surface, as illustrated in Figure 36. The term \( d_1 \) is the distance between the back principal plane of the first component and the front principal plane of the second component. The term \( d_2 \) is the distance between the back principal plane of the second component and front principal plane of the third component. The refractive index \( n_2 \) is the refractive index between the first and second component, while \( n_3 \) is the refractive index between the second and third component. The refractive index \( n_4 \) denotes the refractive index after the third component. For the first iteration, we combine the powers of the anterior cornea (\( F_{ac} \)), posterior cornea (\( F_{pc} \)) and anterior lens surface (\( F_{als} \)) using Equation 46. We denote this value as \( F_{cls} \). The parameters \( d_1 \) and \( d_2 \) become the corneal thickness and anterior chamber depth, respectively. The variables \( n_2 \) and \( n_3 \) become the corneal and aqueous refractive index, respectively. The distance between the anterior lens surface and position of the back principal plane (\( H_{cls} \)) of the cornea and anterior lens surface is also calculated using Equation 47. We denote this \( \delta_{cls} \), where \( n_4 \) becomes \( n_0 \), the lens

Figure 36: Diagram illustrating the combination of three optical systems.
With the second iteration of Equation 46 and Equation 47, we are able to calculate the overall power and principal plane location of the eye model. This is obtained by combining the power of the anterior cornea, posterior cornea and anterior lens surface ($F_{cls}$) with the power of the GRIN bulk ($F_B$) and posterior lens surface ($F_{pl}$). For this calculation, $d_1$ and $d_2$ become $\delta_{cls} + \delta_b$ and $\delta_b$, respectively. Both $n_2$ and $n_3$ become $n_0$, which is the lens surface refractive index, and $n_4$ becomes $n_v$, the vitreous refractive index.

This completes the expressions for the equivalent and back focal length of the overall eye model. The accuracy of these expressions was checked by calculating the power and back focal length of four different models which have the same refractive index distributions as Equation 44. We were then able to compare the calculated values with the exact values obtained from Zemax. The first three models are taken from the publication by Goncharov and Dainty [16] and correspond to simplified eye models of 20, 30 and 40 years. The fourth model is the model of Liou and Brennan [28]. The results are shown in Table 14. It can be seen that the method is accurate to 0.05 D on the power calculation and approximately 30$\mu$m on the back focal length, which is sufficient accuracy for our purpose. Also, it is worth noting that the method underestimates the power by approximately the same amount each time. This makes it particularly suitable for modelling the LCA given that the LCA is a difference in refractive error with wavelength.

With the expression for the back focal length, the parameter $z_m$ appears in the GRIN coefficient $n_1$. Therefore, the formula for $n_1$ can be included in the expression for the back focal length and $z_m$ can be optimised such that the back focal length.
length matches the vitreous chamber depth, making the eye model emmetropic. Alternatively, to define the refractive error of the eye model, the expressions for the effective focal length and back focal length can be used with the expression for the refractive error derived in Chapter 1. This expression can be equated to the required refractive error and the appropriate $z_{m}$ value can be obtained.

We next look at optimising the dispersion of the eye model. In the previous section, the refractive indices were defined at each discrete wavelength, resulting in separate Zemax files at each wavelength. While Zemax allows continuous representations for the dispersion of optical media, the Cauchy representation is not one of the facilitated formulae. In Zemax, the Schott formula, Sellmeier formula, Herzberger formula and Conrady formula are all available as alternatives to the Cauchy formula. To find the formula that gave the best fit, we used the built-in Zemax glass fitting tool and found that the Schott formula gave the best fit between 0.4 µm and 0.85 µm. The Schott formula, which was defined in Chapter 1, is given by

$$n^2(\lambda) = a_0 + a_1\lambda^2 + a_2\lambda^{-2} + a_3\lambda^{-4} + a_4\lambda^{-6} + a_5\lambda^{-8}$$  \hspace{1cm} (48)$$

where $a_0$, $a_1$, $a_2$, $a_3$, $a_4$ and $a_5$ are the dispersion constants. The converted constants are given below in Table 15. Converting the dispersion curves from one format to another introduces a fitting error. To measure the accuracy in convert-

<table>
<thead>
<tr>
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<th>CORNEA</th>
<th>AQUEOUS</th>
<th>VITREOUS</th>
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<tr>
<td>$a_0$</td>
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<td>1.74861824</td>
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<tr>
<td>$a_1$</td>
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<td>4.29001851×10^{-6}</td>
<td>3.19159021×10^{-6}</td>
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<td>1.60637085×10^{-2}</td>
<td>1.47172352×10^{-2}</td>
</tr>
<tr>
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<td>-1.81487057×10^{-3}</td>
<td>-1.84088944×10^{-3}</td>
<td>-1.51524462×10^{-3}</td>
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<td>$a_4$</td>
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<td>1.57909713×10^{-4}</td>
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<tr>
<td>$a_5$</td>
<td>4.24124404×10^{-7}</td>
<td>4.33397565×10^{-7}</td>
<td>3.4687393×10^{-7}</td>
</tr>
</tbody>
</table>

Table 15: Schott Constants for different ocular media.
Figure 37: Subtraction of the eye power with the Cauchy and Schott formulae. Cauchy eye model subtracted from Schott model (solid, blue), Schott cornea only (dashed, red), Schott aqueous only (dashed, purple) and Schott vitreous only (dashed, green).

After determining the dispersion coefficients from the Cauchy to Schott format, we calculated the power of an eye model between 0.4 μm and 0.85 μm with each representation. Because it was only required that the dispersion of the cornea, aqueous and vitreous be redefined, we chose the geometry of the constant refractive index model analysed in Chapter 2. Subtracting the power of the eye model with each representation gave an estimate of the fitting error in terms of diopters, which is a more intuitive measure compared to differences in refractive index values. The solid blue line in Figure 37 shows the result of subtracting the eye models with the Cauchy and Schott representations. The dashed lines show the subtraction of eye models whereby just a single medium’s dispersion representation has been changed to the Schott representation. From Figure 37, it can be seen that maximum deviation between the eye models defined with the Cauchy and Schott dispersion formulae is $1.8 \times 10^{-6}$ D at 400 nm.

With the dispersion of the cornea, aqueous and vitreous defined, we turn to the dispersion of the GRIN lens. In Zemax, the GRIN lens can be given a continuous dispersive profile. When ray tracing through the GRIN medium, the refractive index is first computed at the reference wavelength using Equation 11. The index at
any other wavelength is then obtained using an expression based upon a general expansion of the Sellmeier equation, which is given by

\[ n(z, r, \lambda) = \sqrt{n_{\text{ref}}^2(z, r) + \sum_{i=1}^{3} \frac{K_i \lambda^2 - \lambda_{\text{ref}}^2}{\lambda^2 - L_i}} \]

where

\[ K_i = \sum_{j=1}^{K_{\text{max}}} K_{ij} n_{\text{ref}}^{j-1} \quad \text{and} \quad L_i = \sum_{j=1}^{L_{\text{max}}} L_{ij} n_{\text{ref}}^{j-1} \]

and \( K_{ij} \) and \( L_{ij} \) define the dispersion of the medium. In principle, with the above representation, one could define up to 48 dispersion coefficients. However, here we outline a simple and perhaps the most practical method for coefficient selection where we only use the \( K \) constants associated with \( j = 3 \) and \( L \) constants associated with \( j = 1 \). All other constants are set to zero. Thus Eq. 49 reduces to

\[ n(z, r, \lambda) = n_{\text{ref}}(z, r) \sqrt{1 + K_{13} \frac{\lambda^2 - \lambda_{\text{ref}}^2}{\lambda^2 - L_{11}} + K_{23} \frac{\lambda^2 - \lambda_{\text{ref}}^2}{\lambda^2 - L_{21}} + K_{33} \frac{\lambda^2 - \lambda_{\text{ref}}^2}{\lambda^2 - L_{31}}} \]

where \( n(z, r, \lambda) \) is the refractive index at position \((z, r)\) and wavelength \( \lambda \). The advantage of this coefficient selection is that the polynomial describing the GRIN profile is retained by taking it outside the square root. This can be mathematically defined as \( n(z, r, \lambda) = n_{\text{ref}}(z, r) \alpha(\lambda) \) where \( \alpha(\lambda) \) is the wavelength dependent factor given by

\[ \alpha^2(\lambda) = 1 + K_{13} \frac{\lambda^2 - \lambda_{\text{ref}}^2}{\lambda^2 - L_{11}} + K_{23} \frac{\lambda^2 - \lambda_{\text{ref}}^2}{\lambda^2 - L_{21}} + K_{33} \frac{\lambda^2 - \lambda_{\text{ref}}^2}{\lambda^2 - L_{31}} \]

Now, in the derived expressions for the equivalent and back focal lengths of the eye model, the only GRIN coefficients to appear are \( n_0 \) and \( n_1 \). Thus the desired \( \alpha \) value at each wavelength can be found by determining the factor which,

<table>
<thead>
<tr>
<th>( K_{13} )</th>
<th>( K_{23} )</th>
<th>( K_{33} )</th>
<th>( L_{11} )</th>
<th>( L_{21} )</th>
<th>( L_{23} )</th>
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<td>10.498.107</td>
<td>87763.503</td>
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| Table 16: Dispersion coefficients for GRIN lens |
The TCA of the Zemax model in terms of the chief ray intersection height of the retina. Negative values for the field angle FA indicate nasal field. A positive sign of horizontal TCA means that the direction of TCA is the same as a BI prism would cause.

when multiplied by $n_0$ and $n_1$, gives the required LCA. Therefore, the dispersive nature of the GRIN lens can be used to define the overall LCA of the eye model. This method was used to define the continuous dispersion of the GRIN lens in the Zemax model, such that the overall LCA matched the LCA of the eye model containing the AVOCADO lens. The derived constants are given in Table 16. With the continuous dispersion profile for the eye model, built-in functions in Zemax can be easily used. For example, the TCA can be calculated in terms of the chief ray intersection point on the retina. This analysis was carried out for the Zemax model with no tilt or decentre on the lens and the resultant plot is shown in Figure 38.

### 3.5 PERSONALISED POLychromatic EYE MODEL

The method for optimising $z_m$ and defining the dispersion coefficients was implemented in Mathematica (Wolfram Research, Inc., IL), which made it possible to build a graphical user interface that allows one to perform easy optimisation.
A screenshot of this interface is shown in Figure 39, where data from a recent publication on chromatic aberration and the ageing human eye is used [92]. The Mathematica code behind the user interface is given in Appendix A. The input data is shown in the leftmost column and the returned calculations are given to the right. For completeness, the experimental data that was used is given in Table 17. This experimental data allows us to build a personalised polychromatic eye model. The spherical equivalent, axial length, corneal power and anterior chamber depth are given by SE, AL, KRT and ACD, respectively. The LCA between 840nm and 690nm is given by \( \text{LCA}(840 - 690\text{nm}) \) and the LCA between 690nm and 561nm is given by \( \text{LCA}(690 - 561\text{nm}) \). This LCA data was extended to include the visible spectrum. To convert the corneal power to the individual radii, we assumed the relationship \( r_{cp}/r_{ca} = 0.83 \), where \( r_{cp} \) and \( r_{ca} \) are the posterior and anterior radius.

<table>
<thead>
<tr>
<th>AGE</th>
<th>SE</th>
<th>AL</th>
<th>KRT</th>
<th>ACD</th>
<th>LCA (840-690nm)</th>
<th>LCA (690-561nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>years</td>
<td>D</td>
<td>mm</td>
<td>D</td>
<td>mm</td>
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</table>

Table 17: Experimental data from subject 45 in [92]
respectively [16]. Also, given that the geometry of the lens was not measured, we used the age-related expressions given by Navarro [15] to define the anterior lens radius \( r_{la} \), posterior lens radius \( r_{lp} \) and lens thickness \( t_l \), where the lens was presumed to be in an unaccommodated state.

The first table in the user interface window gives the GRIN coefficients which produce the required refractive error at the reference wavelength. Thus, the user interface can be used to create a monochromatic eye model that contains a GRIN lens with a user defined refractive error. The refractive index distribution along the optical axis is plotted to the right of the GRIN coefficients. Next, the \( \alpha \) constants at each of the specified wavelengths are defined and these \( \alpha \) coefficients must then be fitted to Equation 52 to determine the \( K \) and \( L \) dispersion coefficients. The fitted \( K \) and \( L \) dispersion coefficients are defined in Table 18.

<table>
<thead>
<tr>
<th>( K_{13} )</th>
<th>( K_{23} )</th>
<th>( K_{33} )</th>
<th>( L_{11} )</th>
<th>( L_{21} )</th>
<th>( L_{23} )</th>
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<td>-0.016019855</td>
<td>-1119387.8</td>
<td>19067318</td>
<td>0.035966729</td>
<td>11882.235</td>
<td>202379.01</td>
</tr>
</tbody>
</table>

Table 18: Dispersion coefficients for the personalised GRIN lens
3.5 PERSONALISED POLYCHROMATIC EYE MODEL

Figure 41: TCA in vitreous space of personalised eye model (blue) compared to AVOCADO eye model (red).

The LCA of the personalised eye model is shown in Figure 40, where it is compared to the LCA of the eye model containing the AVOCADO lens that was analysed previously. While it should be noted that the LCA data was extended into the blue spectrum, the subject was chosen due to relatively large LCA between 561nm and 840nm. Using the Cauchy equation fit to the experimental LCA values, the LCA between 400nm and 700nm is 2.67D, compared to 2.19D for the eye model containing the AVOCADO lens. This represents a factor of 1.219 in difference. Next we looked at the TCA of the personalised eye model. This first required that we included conics on the different surfaces. Again, because these were not measured, we assumed the values given by Navarro [15]. We then optimised the $n_2$ coefficient to produce similar spherical aberration to the chromatic eye. We also assumed a 3.54 degree tilt and 0.113mm pupil decentration. If the conics of the cornea were measured and the total spherical aberration of the eye was known, it would be possible to optimise the lens within Zemax to produce the required spherical aberration. The TCA of the personalised model is shown in Figure 41 and it is compared to the TCA of the eye model containing the AVOCADO lens. It was found that the TCA of the personalised model was $-0.483$ arcmins/degree is the horizontal field, compared to $-0.399$ arcmins/degree for the eye model containing the AVOCADO
3.6 conclusion

The built-in Gradient 5 medium of Zemax was used to create an eye model with a GRIN crystalline lens. This eye model had identical geometry to the models previously analysed with the GIGL lens and AVOCADO lens. At first, the refractive indices of all media were defined discretely and the refractive indices for the GRIN lens were chosen such that the LCA was consistent with previous models. Consequently, it was shown that the TCA of the eye model with the Gradient 5 medium was almost identical to the TCA of the eye models with the more complex GRIN distributions. Following this, we looked at replicating some experimental studies. The first study was an objective measurement of the TCA across a 30 degree visual field in both the vertical and horizontal meridian. Compared to this study, our analysis found similar values for the TCA in both meridians. While carrying out this investigation we also investigated the effect of tilting and decentering the lens on TCA. It was observed that this has minimal impact on the magnitude of the TCA slope, but does impact the achromatic axis position and TCA at the fovea. We next looked at the LCA across the visual field. Our analysis found that from 0 to 30 degrees, the LCA between 400nm and 700nm increased by 0.4D. This agreed with the two experimental studies on the change in LCA with visual field angle, which both found a slight increase in LCA with field angle. This work leads us to conclude that if one wants to create an eye model that replicates experimental TCA measurements, the LCA of the eye model should be used to tune the TCA slope, while the tilt and decenters of the components should be used to tune the position of the achromatic axis and magnitude of TCA at the fovea.

While replicating the experimental studies with the Zemax eye model was straightforward, the disadvantages were that optimisation of the GRIN lens had to be carried out iteratively and the refractive indices of the media were defined discretely. To overcome the iterative method, we derived analytical expressions for the effective focal length and back focal length of an eye model created with the Gradient 5 lens. This represents a factor of 1.211 in difference, showing that the relationship between the TCA and LCA can be considered almost linear.
medium. The parameters of the GRIN lens were contained within the expressions for the effective focal length and back focal length and thus could be optimised to produce the required refractive error or LCA. We then redefined the Cauchy representation of the cornea, aqueous and vitreous dispersion to the Schott representation, which can be used in Zemax. We found that the error created from this change in representation was negligible. Following this, we showed a method for defining a continuous dispersion profile of the GRIN lens within Zemax, which involved careful selection of dispersion coefficients.

The methods for optimising and defining the dispersion of the GRIN lens were defined in Mathematica and used to create a graphical user interface. This allowed the parameters of an eye model to be easily derived and the results of a recent experimental study were used to create a personalised eye model. The subject chosen had a large magnitude of LCA, which caused an increase in TCA compared to the other models analysed. This increased LCA supported the analysis that showed the relationship between the LCA and TCA is approximately linear.
4.1 INTRODUCTION

As discussed in Chapter 1, generic eye models can be used to test vision correction methods, with personalised eye models allowing the investigation of customised correction methods. Both generic and personalised eye models can also be used to verify the design of ophthalmic instruments. In this chapter, we investigate the benefit of a custom contact lens for the personalised eye model developed in Section 3.5, along with the design of an instrument for prescribing such a contact lens. In our analysis, we look at a simple aspheric contact lens but the method can be extended to more practical cases, such as prescribing highly aspheric contact lenses for presbyopia patients [131] and hybrid contact lenses for keratoconus patients [132].

4.2 PERSONALISED ASPHERIC CONTACT LENS

The first step of our analysis was to set up the personalised eye model with the best-fit spherical contact lens. The metric chosen for this optimisation was the minimisation of the on-axis RMS spot size at the reference wavelength. In this chapter, we choose the reference wavelength as 555nm because we wish to later use the spectral sensitivity of the eye in our analysis. The resultant contact lens, defined in Table 19, corrected the −1.1D refractive error present in the eye model. When defining the contact lens, a few assumptions were made. First, the contact lens thickness was chosen to be 0.2mm and the posterior surface was chosen such that it conformed to the anterior surface of the cornea. The material chosen for the contact lens was PMMA, due to the wide availability of optical data. The optical characteristics of the more common hydrogel contact lenses are difficult to acquire.
Figure 42: RMS wavefront error vs horizontal field for personalised Zemax model with optimised spherical (dashed, green) and aspheric (green) contact lens.

Figure 43: RMS wavefront error vs vertical field for personalised Zemax model with optimised spherical (dashed, green) and aspheric (green) contact lens.
Table 19: Parameters for spherical and aspheric contact lens (CL).

<table>
<thead>
<tr>
<th></th>
<th>Radius</th>
<th>Conic</th>
<th>Thickness</th>
<th>Material</th>
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<td>0.2mm</td>
<td>PMMA</td>
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<tr>
<td>Aspherical</td>
<td>7.397mm</td>
<td>−0.81</td>
<td>0.2mm</td>
<td>PMMA</td>
<td>−1.21D</td>
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</tbody>
</table>

due to the challenges in testing such materials. Following the optimisation of the spherical contact lens, the same procedure was carried out for the aspheric contact lens, except the anterior radius and anterior conic constant of the contact lens were set to be variables. The optimised variables for the aspheric contact lens are given in Table 19. To compare the correction provided by the spherical and aspheric contact lens, we looked at the RMS wavefront error for a 20 degree field in both the horizontal (Figure 42) and vertical (Figure 43) directions. It can be seen from these plots that an improvement in vision is achievable with the aspheric contact lens, with the biggest improvement occurring at the centre of the field.

The difficulty with a personalised contact lens is that prescribing the lens is not straightforward. While a spectacle lens remains relatively fixed in position, a contact lens is subject to constant movement on the eye, making it difficult to determine the best correction. The best correction derived from a personalised eye model might not be completely satisfactory in the on-eye setting due to the contact lens decentration. It would therefore be useful if the correction derived from a personalised model could be combined with an iterative subjective method. Ideally, this method would not require manufacturing of several personalised contact lenses, as this would be both time consuming and expensive. A more attractive option could be the simulation of a personalised contact lens using an adaptive optics system. This would allow a trial and error method to be used in the prescribing of the contact lens, without the need for manufacturing the lens.

### 4.3 Contact Lens Simulation with Adaptive Optics

Adaptive optics was discussed in Chapter 1 as a means of removing both static and dynamic aberrations in the human eye to improve the resolution of systems.
imaging the retina. While removal of aberrations is usually desired, adaptive optics systems can also be employed to induce aberrations. For example, Piers et al. used an adaptive optics visual simulator to alter the amount of spherical aberration observed by a subject and subsequently measured their visual acuity [133]. The goal of their study was to investigate the benefit of an aspherical IOL design. The obvious advantage of such a method is that subjective feedback can be obtained, without the need for invasive surgery. Several other researchers have also used an adaptive optics visual simulator to investigate IOL correction [134–139]. Madrid-Costa et al. used an adaptive optics visual simulator to compare Ortho-k and LASIK vision correction [140]. Many of these studies used the Crx1 Visual Simulator available from Imagine Eyes (Imagine Eyes, Orsay, France). Apart from avoiding invasive surgery, another advantage of employing a visual simulator is that a range of aberrations can be simulated in quick succession. This is beneficial for assessing customised contact lens designs.

In this section, we look at a method for deriving the shape of a deformable mirror in an open-loop, non-pupil conjugated visual simulator. We wish to derive an expression for the deformable mirror shape without using a ray tracing software, as this serves as a proof-of-concept exercise. To illustrate the method, we define the deformable mirror shape to simulate the aspheric contact lens defined in Table 19. We initially assume that the deformable mirror is located at the anterior cornea vertex, as illustrated in Figure 44. While this is unrealistic in practical terms, relay optics can be used to allow the mirror to act in this plane. To achieve a wide field simulation it is beneficial to have the mirror conjugate to this plane, as this is the plane in which the contact lens acts. Figure 44 shows the contact lens positioned on the cornea of the eye with an incoming plane wavefront (WF). As a first step, two rays are traced through the system. The first ray is traced along the path ABC, where A is a point on the incoming wavefront, B is the intersection point between the ray and contact lens, and C is the point on the contact lens/cornea boundary. The second ray is traced along the optical axis such that:

\[ AB + BC \cdot n_{\text{contact}} = FG \cdot n_{\text{contact}} + GH \cdot n_{\text{cornea}} \]  

(53)
where point H is defined solely to satisfy the condition. The ray at point H is then reversed and traced to point G. Point G corresponds to the deformable mirror plane, which coincides with the anterior corneal vertex. The ray is then traced to point I, which is the position of the final reference plane. The position of this final reference plane is arbitrary and it is assumed that the ray travels through air to reach the final reference plane. Next, the ray at point C is reversed. However, it is assumed that the medium is now air rather than the contact lens medium. This results in a different angle of refraction at point C. This is illustrated by the red and green lines in Figure 44. The point D, which lies on the refracted ray, is then determined by satisfying two conditions. The first condition is that:

\[ HG \cdot n_{\text{cornea}} + GI = CD + DE \]  

(54)
which preserves the optical path length. The second condition is that the ray DE is parallel to the optical axis. Point D then defines a required discrete reflection point on the deformable mirror. The success of the method can be understood by now tracing a ray backwards through the system. It can be seen that the ray EDC will follow the same path in the cornea as the ray ABC and both rays begin parallel to the optical axis. Thus, for this ray, the optical performance of the contact lens and deformable mirror are identical. The next step is to move from discrete points on the deformable mirror (DM) and instead determine the continuous surface profile. In wavefront space, the only aberration that the wavefront entering and reversing from the corneal medium could have picked up is defocus and spherical aberration (all surfaces are considered to be rotationally symmetric). Therefore the following mirror shape can be used as a means of producing a plane wavefront at the final WF reference plane:

\[ z = \frac{c}{2} r^2 + \alpha_1 r^4 + \alpha_2 r^6 \]  

(55)

where \( z \) is the sag, \( c \) is the curvature, \( r \) is the radial coordinate and \( \alpha_1 \) and \( \alpha_2 \) are coefficients. Given that three variables are defined, a single ray can be traced at three different entrance heights to find the required D points on the deformable mirror surface. The three points can then be used to find the three required coefficients, namely \( c \), \( \alpha_1 \) and \( \alpha_2 \). In our example, we traced a ray at a height of 1.33mm, 2.66mm and 4.00mm. The points on the deformable mirror surface were then fit to Equation 55. The resulting coefficients are given in Table 20. This analysis was carried out in Mathematica and the system was then modelled in Zemax to verify the results, where the deformable mirror was modelled as an even asphere. When calculating the equivalent conic constant of the deformable mirror, the value obtained is much larger than unity. Thus, while the anterior surface of the contact lens was a prolate ellipse, the surface of the deformable mirror can be considered an oblate ellipse.

The derivation of the deformable mirror shape was carried out using only an on-axis beam. Therefore, it can be expected that the simulation will deteriorate with large off-axis angles. To investigate the quality of the off-axis simulation, we
Figure 45: RMS wavefront error vs. horizontal field at 555nm for aspheric CL and DM. Grey line gives λ/14 deviation from CL RMS wavefront error.

Figure 46: RMS wavefront error vs. vertical field at 555nm for aspheric CL and DM. Grey line gives λ/14 deviation from CL RMS wavefront error.
compared the RMS wavefront error across the field for the personalised eye model with the contact lens and the personalised eye model with the deformable mirror simulating the contact lens. The analysis was carried out across a 20 degree field in both the horizontal and vertical directions, at the reference wavelength of 555nm. The RMS wavefront errors across the horizontal field are plotted in Figure 45 and the RMS wavefront errors across the vertical field are plotted in Figure 46. In these plots, we also illustrate a λ/14 deviation (Maréchal criterion) from the personalised eye model and contact lens RMS wavefront error using a grey line. This can be used as a metric for assessing the overall quality of the contact lens simulation. The analysis shows that the quality of simulation is essentially diffraction limited over the central 10 degree field, with a slight drop-off occurring outside this.

### 4.4 Polychromatic Simulation Quality

Our analysis shows that the deformable mirror is capable of simulating the optical performance of the contact lens for a monochromatic target. However, this might not be the case for a polychromatic target considering the mirror is a reflective element, while the contact lens is a refractive element. To assess the quality of the polychromatic simulation, we use the polychromatic RMS wavefront error function in Zemax. Twelve wavelengths between 400nm and 700nm were defined. Rather than consider the contribution of each wavelength equally, we weighted them using the photopic luminous efficiency curve of the human eye, as defined in Section 1.2. The RMS error vs. horizontal field is given in Figure 47 and the RMS error vs. vertical field is given in Figure 48. Like before, we plotted a λ/14 deviation from the wavefront error of the personalised eye model and contact lens, where λ is 555nm.

<table>
<thead>
<tr>
<th>c (mm⁻¹)</th>
<th>α₁ (mm)</th>
<th>α₂ (mm)</th>
<th>Power (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM shape</td>
<td>−6.25245x10⁻⁴</td>
<td>−3.042x10⁻⁵</td>
<td>−2.814x10⁻⁷</td>
</tr>
</tbody>
</table>
Figure 47: Polychromatic RMS wavefront error vs. horizontal field for aspheric CL (solid) and DM (dashed). Grey line gives $\lambda/14$ deviation from CL RMS wavefront error.

Figure 48: Polychromatic RMS wavefront error vs. vertical field for aspheric CL (solid) and DM (dashed). Grey line gives $\lambda/14$ deviation from CL RMS wavefront error.
Interestingly, it can be seen that the polychromatic simulation quality is very similar to the monochromatic case, which suggests that the reflective nature of the deformable mirror has little impact on the simulation quality. To understand why the monochromatic and polychromatic simulation quality are so similar, we compare the LCA and TCA for three specific cases; eye model only, eye model plus contact lens and finally eye model plus deformable mirror. The LCA comparison is shown in Figure 49 and the TCA comparison is shown in Figure 50. From these figures, it can be seen that both the contact lens and deformable mirror have little impact on the LCA and TCA. To understand this further, we look at the impact of the individual surfaces on the overall LCA. This can be achieved using the Seidel analysis described in Section 1.2.5, with the axial colour coefficient given by

\[
C_l = n_d y \left( \frac{y}{r} + u \right) \left( \frac{n_d - 1}{n_d V_d} - \frac{n'_d - 1}{n'_d V'_d} \right)
\]

where \(n_d\) is the refractive index of the medium at \(\lambda_d = 589.3\)nm before the surface, \(y\) and \(u\) are the height and angle of the incident ray at the surface, respectively, \(r\) is the surface radius of curvature and \(V_d\) is the Abbe number of the medium before the surface. The refractive index of the medium after the surface at \(\lambda_d = 589.3\)nm is given by \(n'_d\) and the Abbe number of the medium after the surface is defined as \(V'_d\). The \(C_l\) coefficients of all surfaces can be added together to obtain the total axial colour coefficient, \(C_L\). The total longitudinal axial chromatic aberration, \(\delta_{AX}\), can then be calculated with the following formula

\[
\delta_{AX} = \frac{1}{n_{di} u_i^2} C_L
\]

where \(n_{di}\) is the refractive index at \(\lambda_d = 589.3\)nm for the vitreous fluid, \(u_i\) is the refracted ray angle at the image plane and \(\delta_{AX}\) is the distance between the paraxial focus at \(\lambda_F = 486.1\)nm and \(\lambda_C = 656.3\)nm. Similar to before, this analysis was applied to three specific cases; cornea of the eye model, cornea plus contact lens and finally the cornea plus deformable mirror. The results are given in Table 21. It can be seen that the anterior surface of the contact lens induces more chromatic aberration than the anterior cornea. However, with the contact lens placed on
Figure 49: LCA of eye model (blue), eye model with contact lens (green) and eye model with deformable mirror (red).

Figure 50: TCA of eye model (blue), eye model with contact lens (green) and eye model with deformable mirror (red).
the eye, the anterior cornea becomes a negative surface and subsequently offsets the increase in chromatic aberration due to the anterior surface of the contact lens. In the case of the deformable mirror, even though the mirror surface induces no chromatic aberration, it alters the angle of the incoming rays, which slightly increases the chromatic aberration contribution of the anterior corneal surface.

Table 21: Individual surface contribution to overall chromatic aberration. S₁ refers to the anterior contact lens surface, S₂ refers to the anterior corneal surface and S₃ refers to the posterior corneal surface.

<table>
<thead>
<tr>
<th></th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
<th>LCA(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eye</td>
<td>—</td>
<td>−6.82151x10⁻⁴</td>
<td>−1.11031x10⁻⁵</td>
<td>−0.44</td>
</tr>
<tr>
<td>Eye + CL</td>
<td>−7.75879x10⁻⁴</td>
<td>1.14171x10⁻⁴</td>
<td>−1.10290x10⁻⁵</td>
<td>−0.47</td>
</tr>
<tr>
<td>Eye + DM</td>
<td>—</td>
<td>−6.88293x10⁻⁴</td>
<td>−1.12301x10⁻⁵</td>
<td>−0.48</td>
</tr>
</tbody>
</table>

The main purpose of this chapter was to define a method for deriving the deformable mirror shape to simulate a contact lens. The personalised eye model created in Chapter 3 was used in the analysis and the benefit of a customised aspheric contact lens was shown. The method to derive the mirror shape was then defined by means of example and it was shown that only the anterior corneal shape and refractive index of the cornea are required to define the mirror shape. It was found that the simulation quality was essentially diffraction limited across the central 10 degree visual field, with a small drop off occurring outside this central field. It was also observed that the polychromatic simulation quality produced similar accuracies in simulating the contact lens compared to the monochromatic case. This was further investigated by showing that the LCA and TCA of the eye model with the contact lens and deformable mirror were almost indistinguishable. Following this, Seidel analysis of the individual surface contributions was also carried out. This analysis showed that the anterior surface of the contact lens induced more chromatic aberration than the cornea. However, with the contact lens placed on
the eye, the cornea becomes a negative surface and offsets the impact of increased chromatic aberration due to the contact lens.

While we showed that it was possible to derive the deformable mirror shape without ray-tracing software, to simulate more complex contact lenses, it would be easiest to define the method within a ray-tracing system. This is also the case for simulating the decentration of simple contact lenses. In our analysis, we presumed that the deformable mirror was located at the anterior corneal vertex. Of course this is not practical in a laboratory setting and a real system would require relay lenses along with other optical components. Further analysis would require the consideration of the tear film and the demand placed on the deformable mirror. Details on extending the system are given in the future work section of the final chapter.
AN OPTO-MECHANICAL ARTIFICIAL EYE

5.1 INTRODUCTION

In this chapter, we look at the creation of an opto-mechanical artificial eye designed to operate over a wide central field, and which replicates the chromatic aberrations of the human eye. Our goal was to build an artificial eye that was of simple construction, but yet produced similar aberrations to those found in the human eye. This required that off-the-shelf lenses and minimum machining would be used in the construction of the artificial eye. With special emphasis placed on reproducing chromatic aberrations and axial optical path length, the proposed model eye would be beneficial for designing and testing many ophthalmic instruments that operate at multiple wavelengths; in particular instruments using low-coherence interferometry [141] and adaptive optics for retinal imaging [120].

5.2 ARTIFICIAL EYE DESIGN

When designing the artificial eye, our goal was to keep the design as anatomically correct as possible, without introducing too much unnecessary complexity. We used the schematic eye of Escudero-Sanz and Navarro, which was defined in Section 1.1.2, as a useful guide [9]. This schematic eye model has a constant refractive index crystalline lens and replicates wide-field aberrations, making it a suitable choice. In the human eye, the posterior cornea has a relatively small contribution to the overall performance, essentially due to the small difference in refractive index between the aqueous fluid and cornea. Considering this and the difficulty in replicating the posterior cornea, we applied a simplification to the design and chose a convex-plano lens for modelling the cornea. This lens (Stock No. #45-694), selected from Edmund Optics Inc. (NJ, USA), features a relatively low refractive in-
dex due to the availability of fused silica ($n = 1.458$ at $\lambda = 0.589\mu m$). The chosen radius of curvature of 8.25mm is appropriate to simulate the anterior corneal surface. While this radius has a slightly larger value than the average radius of the anterior cornea ($r = 7.72\text{mm}$ [9]), it allows one to compensate partly for the increase in optical power due to the lack of a negative posterior corneal surface and the higher refractive index of fused silica compared to that of the cornea ($n = 1.376$ at $\lambda = 0.589\mu m$ [9]). Considering the higher refractive indices of the materials used in the artificial eye, a higher power on the cornea is needed to reduce the axial length of the eye and thus keep the optical path length in line with realistic values. The full geometry of the lens is given in Table 22 and the overall design is illustrated in Figure 51. The plano surface of the convex-plano lens defines the iris plane, where an aperture stop of 5mm in diameter is placed. The chosen aperture stop gives an entrance pupil diameter of approximately 6mm, similar to a dilated pupil. While the plano surface of the lens provides useful support for the iris, it also produces an unrealistic interface. To avoid unwanted reflections from this surface, we used a fused silica refractive index matching fluid to model the aqueous fluid. This refractive index matching fluid (Code 50350) is specifically designed to match the dispersion of fused silica and it was sourced from Cargille-Sacher Laboratories Inc. (NJ, USA). To our knowledge, this is the first instance of using index matching
Table 22: Geometrical parameters and refractive indices at 589nm for the artificial eye.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Radius (mm)</th>
<th>Thickness (mm)</th>
<th>Refractive Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.25</td>
<td>3.0</td>
<td>1.458413</td>
</tr>
<tr>
<td>2</td>
<td>Infinity</td>
<td>2.0</td>
<td>1.458714</td>
</tr>
<tr>
<td>3</td>
<td>11.7</td>
<td>3.6</td>
<td>1.516740</td>
</tr>
<tr>
<td>4</td>
<td>-11.7</td>
<td>15.496</td>
<td>1.458714</td>
</tr>
</tbody>
</table>

fluids in the creation of an artificial eye. The thickness of the fluid chamber was chosen such that the thickness of the convex-plano lens and fluid chamber produced a similar optical path length to the optical path length of the cornea and aqueous chamber of the schematic eye [9].

To model the crystalline lens, we chose the LB1494 lens from Thorlabs Inc. (NJ, USA), which is an N-BK7 bi-convex lens. N-BK7 glass was a suitable choice due to the higher refractive index compared to fused silica, which agrees with the higher refractive index of the human crystalline lens compared to other media. The radii of curvature of the bi-convex lens was kept similar to the average anterior radius of the human crystalline lens and the thickness produced a similar optical path length compared to the average human eye. To model the vitreous fluid, the fused silica matching fluid was used again. The vitreous depth in Table 22 gives the distance for the paraxial focus occurring at the final surface of the artificial eye at a wavelength of 589nm. Finally, the retina was modelled with a semi-transparent disc and field stop. The dimensions in Table 22 give an overall optical power of 62.9D. This power is comparable with a typical 60D unaccommodated power for the human eye. Therefore, introducing a spectacle lens in front of the artificial eye will result in a similar refractive error correction as in the case of a real eye.

With this design, the bi-convex lens is submerged in an index-matching liquid on both sides making it possible to easily translate the bi-convex lens along the optical axis to induce defocus in the system. Additionally, the refractive index difference between the N-BK7 lens and fused silica matching liquid is only 0.058 at 589nm, which helps to achieve more realistic levels of intensity for Purkinje reflections from the lens (P3 and P4) [2], compared to a commonly used water-
glass interface. Also, the overall optical path length in the artificial eye is 35.3 compared to 32.3 in the schematic eye [9]. This is about 9% higher, yet within a realistic range in view of inter-subject variability. The design of the retinal surface offers two modes of operation. In the first mode, the semi-transparent disc allows a reflection to be obtained, which makes the artificial eye compatible with a Hartmann-Shack aberrometer. In the second mode, light can be input from the reverse-side of the artificial eye to illuminate the semi-transparent disc, which acts as a scattering medium. The field stop can then be imaged through the system to investigate off-axis chromatic aberrations.

A mid-section through the mechanical design of the artificial eye is shown in Figure 52. The mechanical design was created in Autodesk Inventor (Autodesk Inc., CA, USA). Where possible, off-the-shelf components were used and machining was kept to a minimum. However, due to space limitations and to avoid vignetting, custom machining was carried out to create the corneal lens holder, shown as 1 in Figure 52. This machining was carried out on a computer numerical controlled (CNC) lathe to produce maximum accuracy. The aperture stop and field stop were also both laser-cut from anodised aluminium foil. The majority of parts were sourced from Thorlabs. The convex-plano lens, bi-convex lens and semi-transparent disc are shown in Figure 52 as 2, 3 and 4, respectively. The front and back cage plate, shown respectively as 5 and 6, are fixed in position. Rubber seals, shown at 8 and 9, are placed between the cage plates and the acrylic tube. While the front and back cage plates are fixed, the central cage plate, shown as 7, is free to move along the optical axis. This movement is precisely controlled by the micrometer screw gauge shown at 10. When the central cage plate is shifted along the optical axis, the total axial length of the system remains fixed and consequently the overall volume remains constant. This eliminates the need for a secondary storage reservoir and makes the sealing process easier.

With the chosen mechanical design, the artificial eye can be kept in a horizontal position without leakage of the index matching fluid. This allows the artificial eye to be tested without the need for a folding mirror. Also, since no adhesives are used, each component can be replaced simply. Therefore, if needed, the aperture stop can be replaced with a different diameter aperture stop and the initial axial length can be varied to allow different levels of initial refractive error to be intro-
Figure 52: Mechanical design of the artificial eye. 1-corneal lens holder. 2-corneal lens. 3-crystalline lens. 4-retinal disc. 5, 6, 7-cage plates. 8, 9-rubber seals. 10-micrometer screw gauge.

duced. Overall, the mechanical design is compact, which allows easy mounting and integrating with other ophthalmic instruments.

As mentioned previously, our goal was to keep the design as anatomically correct as possible, without introducing too much complexity. This is advantageous when trying to create similar properties between artificial eyes and schematic eyes. Here we compare the RMS wavefront error of the artificial eye and the schematic eye of Escudero-Sanz and Navarro [9] between -10 and 10 degrees. To carry out
the comparison across the field, we replaced the flat surface at the rear of the artificial eye with a curved surface (radius of curvature = -9.0mm). The results are shown in Figure 53. The wavelength used in the analysis was 589nm and the pupil diameter was 5mm for both the artificial eye and schematic eye. The difference in wavefront error can mainly be attributed to excessive spherical aberration in the artificial eye, due to the presence of spherical-only surfaces.

Next we compared the spot diagrams of the artificial eye and schematic eye, as shown in Figure 54. The spot diagrams were analysed on-axis and at 10 degrees off-axis. Again, we replaced the flat surface at the rear of the artificial eye with a curved surface (radius of curvature = -9.0mm). The analysis was carried out at 458nm, 589nm and 633nm for a 5mm pupil diameter on both the artificial eye and schematic eye. The schematic eye is well-defined at these wavelengths and they are suitably located for our experimental analysis. It can be seen from Figure 54 that the artificial eye has less chromatic aberration, but more spherical aberration than the schematic eye.

In addition to the RMS wavefront error and spot diagrams, we also looked at individual aberrations for different positions of the crystalline lens within the ar-

Figure 53: RMS wavefront error vs. field angle at 589nm. Schematic eye (green) and artificial eye (blue).
Figure 54: Spot diagram comparison for artificial eye (left column) and schematic eye (right column). Blue, green and red correspond to 458, 589 and 633nm.

Artificial eye. This is shown in Table 23, where the change in individual zernikes for different geometries are given. The term $Z(0, 2)$ gives the defocus, $Z(0, 4)$ gives primary spherical and $Z(-1, 3)$ gives the vertical coma. The units are waves and the wavelength analysed was 589nm. We also calculated the power of the artificial eye for different lens positions. From Table 23, it can be seen that the primary spherical aberration and vertical coma remain relatively constant with lens position. However, as expected, the defocus and power of the artificial eye show a change with lens position.
Table 23: Optical characteristics of the artificial eye. The term $Z(0,2)$ gives the defocus, $Z(0,4)$ gives primary spherical and $Z(-1,3)$ gives the vertical coma.

<table>
<thead>
<tr>
<th>$d_2$ (mm)</th>
<th>$Z(0,2)$</th>
<th>$Z(0,4)$</th>
<th>$Z(-1,3)$</th>
<th>POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>4.174</td>
<td>0.450</td>
<td>—</td>
<td>63.7D</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>4.103</td>
<td>0.474</td>
<td>0.849</td>
<td></td>
</tr>
<tr>
<td>$2^\circ$</td>
<td>1.703</td>
<td>0.441</td>
<td>—</td>
<td>62.9D</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>1.600</td>
<td>0.461</td>
<td>0.873</td>
<td></td>
</tr>
<tr>
<td>$3^\circ$</td>
<td>$-0.547$</td>
<td>0.431</td>
<td>—</td>
<td>62.1D</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>$-0.667$</td>
<td>0.451</td>
<td>0.884</td>
<td></td>
</tr>
</tbody>
</table>

Table 24: Comparison of several artificial eyes. AC-anterior cornea modelled. PC-posterior cornea modelled. SP-single pass testing. DP-double pass testing. AG-adjustable geometry. AP-adjustable pupil. DD-dispersive design. OMDAE-Opto-mechanical dispersive artificial eye described in this chapter.
Finally, we compare some features of our artificial eye design with artificial eyes created by other researchers. To show the comparison, we generated Table 24. We selected the artificial eyes that can be used for assessing ophthalmic instruments [65–72, 74, 110], and for which sufficient information on their construction is available. The first feature analysed was whether the anterior cornea was modelled, considering that many artificial eyes model the cornea and crystalline lens with a single positive lens or mirror. Following this, we checked the presence of a posterior corneal surface in the design of the artificial eyes. To analyse the testing methods, we considered whether the artificial eyes employed single or double pass testing. We also looked at the adjustable features of the eye, which were adjustable pupils and adjustable geometry. We define an adjustable pupil as a pupil that can be easily modified to give a continuous range of diameters, rather than replacement with a different pupil. We define adjustable geometry as the ability to change the dimensions in the artificial eye, which includes movement of components and deformations of mirrors. Finally, we considered artificial eyes that included dispersive features in their design.

5.3 EXPERIMENTAL TESTING

There was two main tests carried out on the artificial eye. The first test was used to measure the LCA of the artificial eye and also to investigate the possibility of inducing defocus by movement of the bi-convex lens. The second test was used to assess whether the artificial eye could be used to examine systems that correct both off-axis LCA and TCA.

Test 1: On-axis testing

The first test was carried out with a Hartmann-Shack wavefront aberrometer (HSWA) that operates at three distinct wavelengths (561nm, 690nm and 840nm). This HSWA has been described in detail elsewhere [92]. An image of the artificial eye in front of the HSWA is shown in Figure 55, where the artificial eye was assembled such that it was approximately emmetropic at 589nm. The HSWA was used to determine the
second-order Zernike defocus coefficient (C₀²) of the artificial eye at 561nm, 690nm, and 840nm. This coefficient, given in microns, could then be used to calculate the spherical equivalent (SE) of the artificial eye at each of the wavelengths using

\[
SE = -\frac{4\sqrt{3}C₀²}{r²}
\]  

(58)

where \(r\) is the pupil radius and the \(SE\) is given in dipters \([142]\). The \(SE\) was measured three times for each wavelength and the average value was taken. The ambient temperature at the time of testing was approximately 22°C. The averaged \(SE\) at each wavelength was then subtracted to determine the \(LCA\) of the artificial eye. These \(LCA\) values, given in Table 25, were then fit to a 3-term Cauchy equation and the resulting equation was offset such that the point of zero \(LCA\) occurred at 589nm. A comparison of this offset curve and the \(LCA\) curve of the chromatic eye is shown in Figure 56.

<table>
<thead>
<tr>
<th>LCA (840 – 690nm)</th>
<th>LCA (690 – 561nm)</th>
<th>LCA (840 – 561nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.29D</td>
<td>0.37D</td>
<td>0.66D</td>
</tr>
</tbody>
</table>

Table 25: LCA of the artificial eye.
In a separate test, the bi-convex lens was translated along the optical axis using the micrometer screwgauge to investigate the possibility of inducing a refractive error (defocus). This test did not change the overall optical path length of the eye. The second-order Zernike defocus coefficient \( (C_0^2) \) at 840nm was recorded before and after the translation. These values were then converted to diopters using Equation 58. The change in SE was 1.9D after a 5mm translation of the lens.

The first part of the testing showed that the LCA of the artificial eye is similar to that of the chromatic eye. In another study, Llorente et al. measured the LCA of 36 eyes between 543nm and 787nm, which is a comparable spectrum to our measurements [91]. With their fitting equation, they found the focus shift between infrared and green was 0.72D. Using the Cauchy equation that we fit to the experimental data from the artificial eye, the LCA of our model eye is 0.63D over the same wavelength range. Comparing the measured LCA to the amount predicted using the theoretical dispersion curves, it was found that the measured LCA was 0.08D larger over the 561nm - 840nm spectrum. This difference probably arises due to a combination of factors. Aside from errors associated with the Hartmann-Shack aberrometer, other sources of error may include temperature fluctuations and a small difference in the fluid dispersion compared to the theoretical dispersion curve published by the company. Temperature fluctuations are particu-
larly important with respect to optical fluids given their relatively large temperature coefficients. For the fused silica matching fluid used in the artificial eye, the temperature coefficient is \( \frac{dn}{D}{/}dt = -0.000386 \) for temperatures in the range of 15-35°C.

The second part of the on-axis testing showed that the mechanics of the artificial eye could be used to induce defocus in the system and change intraocular distances. For systems measuring axial distances, lens displacement by a certain amount (set by the micrometer screwgauge) enables one to know values for the change in aqueous and vitreous length simultaneously. Regarding the magnitude of defocus induced, the 1.9D is probably a small change in refractive error given that 5mm is a large movement of the lens. Thus, the method for moving the lens might be more suitable for simulating small changes in refractive error. If a larger amplitude of defocus is required, for example to calibrate a badal system [35], the more traditional approach of modifying the vitreous chamber length could be used.

**Test 2: Off-axis testing**

The second test was used to analyse the possibility of using the artificial eye for testing a multi-wavelength imaging system. The experimental setup for Test 2 is shown in **Figure 57**. A white light source was combined with a collimating lens.

![Figure 57: Experimental setup for test 2.](Image)
and bandpass filter to illuminate the semi-transparent disc at the rear of the artificial eye. This disc then became a scattering medium and the field stop could be imaged through the eye. A simple CMOS camera and infinity-corrected microscope objective were used as the imaging system. The edge of the field stop corresponded to a half-field of approximately 9.2° in object space. This field angle, chosen due to manufacturing constraints, was close to our initial 20° full-field goal for a wide central field artificial eye design. The field stop was imaged with two different bandpass filters in place. These filters had central wavelength values of 450nm and 650nm. Shown in Figure 58 on the left is the raw image of the field stop when the semi-transparent disc is illuminated with the blue light. Using an image processing software, we binarised the raw image and applied an edge-detection algorithm to define the edge of the field stop. The processed image is shown on the right in Figure 58, where the edge is defined with a blue colour to help distinguish which image was being analysed. After processing both the blue and red images, we then overlapped the images for comparison. This is shown in Figure 59, where it is clear that the field stop appears larger when it is imaged with the blue light illuminating the semi-transparent disc. This apparent change in diameter of the field stop is purely due to chromatic aberrations considering that the absolute size of the field stop is fixed and the images were taken in quick succession without modifying the optical setup.
Considering the imaging system that we used was a refractive system, the overall difference is a combination of the artificial eye and imaging system. This will be the situation for any refractive system that images the eye. In such a case, the overall chromatic aberration must be corrected rather than just the chromatic aberration of the eye. With test 2, complete correction of the chromatic aberrations would result in the image of the field stop at each wavelength overlapping exactly. With knowledge of the pixel size, we were able to measure the difference in size for the field stop imaged with blue and red light illumination. To do this, we took the image shown in Figure 59 and measured the distance between the blue and red curve at sixteen equally spaced points around the circumference of the field stop image. We averaged these values and found that the mean difference was
44\,\mu m. Modeling the microscope objective as a paraxial lens having the same focal length as the objective and no chromatic aberration, we simulated the test in Zemax and found that the difference between the blue and red curve should be 20\mu m. Unfortunately, the apo correction in the microscope objective is valid only within a 1\,mm field, while our imaging test used approximately 6\,mm. Thus the chromatic aberration of the objective at such a large field angle adds to the predicted value of 20\mu m. To compare the theoretical TCA of the artificial eye with the expected amount for the human eye, we compared the TCA of the Zemax model defined in Chapter 3 and the TCA of the artificial eye. To carry out this comparison, we again replaced the flat surface at the rear of the artificial eye with a curved surface (radius of curvature = -9.0\,mm) and measured the TCA in terms of the difference between the chief ray intersection point at the retina between 450\,nm and 650\,nm. The results are shown in Figure 60, where it can be seen that the predicted TCA of the artificial eye is similar to that of the Zemax model.
A simple design for an opto-mechanical artificial eye was presented in this chapter. The artificial eye was designed to function over a wide central field, with similar chromatic aberrations to those found in the human eye. The theoretical RMS wave-front error across the field and spot diagrams were compared for the artificial eye and a wide-field schematic eye. The artificial eye showed larger amounts of spherical aberration, due to the spherical surfaces, and slightly less chromatic aberration. The mechanical design of the artificial eye used mostly off-the-shelf components and minimum machining, which allowed for easy transportation and integrating with ophthalmic devices.

The first test on the artificial eye used a Hartmann-Shack wavefront sensor that operated at three distinct wavelengths. This setup was used to measure the LCA of the artificial eye. As expected from our theoretical analysis, the LCA was less than that of the chromatic eye, but still sufficiently large for testing of ophthalmic instruments. Following the LCA measurement, we investigated the possibility of inducing defocus in the artificial eye by translation of the lens simulating the crystalline lens. A 5mm translation of the lens produced a refractive error change of 1.9D. Given the overall dimensions of the artificial eye, this is a large translation of the lens. While movement of the lens can be used to simulate small refractive errors, larger refractive errors could be simulated by adjustment of the overall axial length.

The second part of the testing was used to investigate the possibility of using the artificial eye for testing multi-wavelength imaging systems. In this setup, the scattering medium behind the field stop of the artificial eye was illuminated with light of two different wavelengths. At each wavelength, images of the field stop were recorded and input into an image processing software. The edges of the field stop were detected and overlapped to reveal the presence of chromatic aberrations. In this method, complete correction of the off-axis chromatic aberrations by an ophthalmic instrument would result in each of the images overlapping exactly.

Overall the artificial eye performed well in testing. One major disadvantage of the design was that testing could only be done at a single off-axis angle. To test
many different angles, several field stops would be required. Improvements to this part of the design and other parts of the artificial eye are discussed in the future work section of the final chapter.
6.1 CONCLUSIONS

The conclusions from this thesis can be divided into three main parts. The first part relates to the results from the modelling carried out in Chapter 2 and Chapter 3. This began with the creation of several eye models corresponding to different ages. When analysing the LCA of the different eye models, we observed little change in the LCA with age. This leads us to believe that the geometrical changes associated with the ageing human eye do not have a significant influence on the LCA. When we considered different GRIN distributions, we found that this had little impact on both the LCA and TCA, including the position of the achromatic axis. This result was strengthened by the fact that we could produce very similar results with an eye model containing a constant refractive index lens. We also looked at the effect of using an alternative dispersion profile for the GRIN lens. This alternative dispersion profile was constrained such that the overall LCA of the eye model remained consistent with previous eye models. As predicted by our analysis in Chapter 1 on the relationship between the LCA and TCA, the TCA of the eye model with the alternative dispersion profile was consistent with previous results. With our interest in the GRIN lens, the final part of Chapter 2 looked at the change in LCA with accommodation and found a 9% increase in LCA for a 4D accommodation. However, given the relatively small increase and the non-uniform spectral sensitivity of the human eye, we believe this increase would have a very small impact on visual quality.

With our initial analysis showing that the GRIN distribution did not have a significant impact on the chromatic aberrations, we looked at creating an eye model with the built-in Gradient 5 medium in Zemax. Even though the built-in distribution in Zemax does not facilitate the implementation of the GIGL or AVOCADO model, it does provide a GRIN model that reasonably approximates the profile ob-
served in the human eye. The eye model we created within Zemax had identical geometry to the the eye models previously analysed with the GIGL and AVOCADO lenses. This allowed us to compare the chromatic aberrations for the various lens models, and it was found that there was little difference. Following this, we looked at replicating some experimental studies. The first study was an objective measurement of the TCA across a 30 degree visual field in both the vertical and horizontal meridian. Compared to this study, our analysis found similar values for the TCA in both meridians. While carrying out this investigation we also analysed the effect of tilting and decentering the lens on TCA. It was observed that this has minimal impact on the magnitude of the TCA slope, but does affect the achromatic axis location and TCA at the fovea. This work lead us to conclude that if one wants to create an eye model that replicates experimental TCA measurements, the LCA of the eye model should be used to optimise the TCA slope, while the tilt and decenters of the components should be used to tune the position of the achromatic axis and magnitude of TCA at the fovea. After investigating the impact of tilting and decentering the lens, we next looked at the LCA across the visual field. Our analysis found that LCA increased slightly with field angle. This agreed with the two experimental studies on the change in LCA with visual field angle, which both found a slight increase in LCA with field angle.

While replicating the experimental studies with the Zemax eye model was straightforward, the disadvantages were that optimisation of the GRIN lens had to be carried out iteratively and the refractive indices of the media were defined discretely. To overcome the iterative method, we derived analytical expressions for the effective focal length and back focal length of an eye model created with the Gradient 5 medium. The parameters of the GRIN lens were contained within the expressions for the effective focal length and back focal length, and thus could be optimised to produce the required refractive error or LCA. We then redefined the Cauchy representation of the cornea, aqueous and vitreous dispersion to the Schott representation, which can be used in Zemax. We found that the error due to the change in representation was negligible. Following this, we showed a method for defining a continuous dispersion profile of the GRIN lens within Zemax, which involved careful selection of dispersion coefficients.
The methods for optimising and defining the dispersion of the GRIN lens were
defined in Mathematica and used to create a graphical user interface. This allowed
the parameters of an eye model to be easily derived and the results of a recent
experimental study were used to create a personalised eye model. The subject chosen
had a large magnitude of LCA, which caused an increase in TCA compared to
the other models analysed. The factor by which the TCA increased was very simi-
lar to the ratio of the personalised model LCA and LCA of the other models. This
validated the approximate linear relationship between the LCA and TCA, derived
in Chapter 1.

The next part of the thesis related to the method for deriving the shape of a
deformable mirror to simulate a contact lens. The personalised eye model created
in Chapter 3 was used in the analysis and the benefit of a customised aspheric
contact lens was first shown. The method to derive the mirror shape was then
deﬁned by means of example and it was shown that only the anterior corneal
shape and refractive index of the cornea are required to deﬁne the mirror shape.
It was found that the simulation quality was essentially diffraction limited across
the central 10 degree visual ﬁeld, with a small drop off occurring outside this ﬁeld
range. It was also observed that the polychromatic simulation quality produced
similar accuracies compared to the monochromatic case. This was further inves-
tigated by showing that the LCA and TCA of the eye model with the contact lens
and deformable mirror were quite similar. Following this, Seidel analysis of the
individual surface contributions was also carried out. This analysis showed that
the anterior surface of the contact lens induced more chromatic aberration than
the cornea. However, with the contact lens placed on the eye, the cornea becomes
a negative surface and offsets the impact of increased chromatic aberration due to
the contact lens.

The final part of the thesis related to the opto-mechanical artificial eye. The artifi-
cial eye was designed to function over a wide central ﬁeld, with similar chromatic
aberrations to those found in the human eye. The first test on the artificial eye used
a Hartmann-Shack wavefront sensor that operated at three distinct wavelengths.
This setup was used to measure the LCA of the artificial eye and even though the
LCA was less than that of the chromatic eye, it was still sufﬁciently large for testing
of ophthalmic instruments. Following the LCA measurement, we investigated the
6.2 future work

Similar to the conclusions, we can divide the future work into three main areas. Given the emphasis on the GRIN crystalline lens in Chapter 2 and Chapter 3, there is potential for a polychromatic comparison of the GRIN crystalline lens and IOLs. Some researchers have studied the chromatic properties of IOLs and observed some interesting results. For example, Siedlecki et al. investigated the chromatic dispersion of several IOLs and reported that the chromatic dispersion of the IOLs changed greatly when the base materials were doped with UV-blocking compounds [143]. They observed an Abbe number of 52.80 for an acrylic IOL, followed by an Abbe number of 27.30 for a doped-acrylic IOL. Zhao et al. measured the refractive indices and the Abbe numbers of various acrylic and silicone IOLs using an Abbe refractometer [144]. The reported refractive index values and Abbe numbers were respectively: 1.55 and 37 for Alcon’s acrylic IOLs, 1.47 and 55 for AMO’s acrylic IOLs, 1.46 and 42 for AMO’s silicon IOLs, and 1.51 and 43 for HOYA’s acrylic IOLs. Their results suggest that chromatic dispersion varies between manufacturers, even when the base materials are the same. Nakajima et al. measured the LCA of eyes implanted with IOLs from three different manufacturers and compared the possibility of inducing defocus in the artificial eye by translation of the lens simulating the crystalline lens. We found that this method was suitable for simulating small refractive error changes, but a more common approach of lengthening the vitreous chamber could be used to simulate large refractive errors.

The second part of the testing was used to investigate the possibility of using the artificial eye for testing multi-wavelength imaging systems. In this setup, the scattering medium behind the field stop of the artificial eye was illuminated with light of two different wavelengths. At each wavelength, images of the field stop were recorded and input into an image processing software. The edges of the field stop were detected and overlapped to reveal the presence of chromatic aberrations. The aberrations were due to both the artificial eye and imaging system. Correction of the chromatic aberrations by an ophthalmic instrument would result in the images of the field stop overlapping exactly.
results to the LCA observed in normal eyes [145]. For eyes implanted with IOLs from two of the manufacturers, they observed the same LCA as in normal eyes. However, they observed lower levels of LCA in eyes implanted with IOLs from one of the manufacturers. Aside from a polychromatic comparison between the GRIN crystalline lens and IOLs, there is also potential to use chromatic aberration measurements for constraining certain features of the eye. For example, given that the anterior section of the eye is well defined, the LCA could be used to constrain the posterior surface of the lens, since it’s optical power is linked to the LCA of the eye. Knowledge of the posterior lens surface radius of curvature is very beneficial for IOL surgery.

Our chromatic aberration modelling in Chapter 2 and Chapter 3 can also be applied to the study of myopia. It has been suggested that the emmetropization of the eye is linked to the LCA of the eye [146]. This theory is attractive because the LCA gives the visual system a method of determining whether the eye is myopic or hyperopic. This is not the case for defocus at a single wavelength, as the eye would be unable to distinguish whether the focus point was in front or behind the retina. It has also been suggested that the peripheral vision of the eye has an important role in the development of myopia [147]. In this thesis, we showed that both the LCA and TCA increase with visual angle. Further investigations into peripheral vision could be carried out with our eye models. Presently, there is certainly no well-defined theory on the cause of the current myopia epidemic. Therefore, it is important to have accurate polychromatic models for designing experiments and developing new corrective devices.

The work in Chapter 4 provided a valuable proof-of-concept for the simulation of a contact lens with an adaptive optics system. However, to completely verify the concept, many areas need to be further explored. Firstly, our modelling presumed that the deformable mirror was located at the corneal vertex. Of course, this would not be the situation in an ophthalmic instrument and instead a series of relay optics would be required. An example of a possible design for a more complete system is shown in Figure 61. In this system, lens 1 (L1) and lens 2 (L2) define the badal system, which can be used to simulate the optical power of the contact lens. This can be achieved by altering the separation of the lenses. Lens 3 (L3) is used to place the screen at infinity.
Figure 61: Possible visual simulator design. L1, L2, L3 are lens 1, 2 and 3. BS stands for beam splitter.
While we showed that the polychromatic simulation quality was satisfactory, introducing refractive relay optics will induce chromatic aberrations and these must be considered in the overall design. With the inclusion of relay optics, rather than having the mirror acting in a plane located at the corneal vertex, it is possible that the mirror could almost be conjugated to the curved surface of the cornea. This would improve the off-axis simulation quality. In the design shown in Figure 61, it can be seen that the pupil of the eye is not conjugated to the deformable mirror plane.

The overall design would need to be tested to determine any limitations. For example, while the badal system might reduce the requirements on the deformable mirror, aberrations induced by the refocusing movement of the badal system would need to be considered during the simulation. The possibility of simulating more complex contact lenses, such as contact lenses for correcting keratoconus would also need to be investigated.

The design of the artificial eye in Chapter 5 can be extended to create a more customisable artificial eye. In the current design, a refractive index matching liquid was used, which was important for disguising the plano surface of the convex-plano lens. However, if the convex-plano lens was replaced with a meniscus lens, a custom liquid with a specific dispersion profile could be used. This would allow greater control over the chromatic aberrations of the artificial eye. Manufacturing the meniscus lens with an aspheric surface would also allow greater control over the spherical aberration. Another change could be made to the semi-transparent disc at the rear of the artificial eye. While the semi-transparent disc and field stop produced a successful testing target, increased information could be obtained by replacing them with an opaque surface having a curvature matching the field curvature of the artificial eye. On this curved surface, laser etching could be used to create a series of white concentric rings, which would allow many off-axis angles to be analysed simultaneously. White spots placed both on-axis and off-axis would allow the LCA to be observed by examining the size of the spots after being imaged through the eye. These spots could also be used with a Hartmann-Shack aberrometer. The design for the proposed target is shown in Figure 62. The illumination for the concentric rings and spots could be delivered by optical fibres, which would prevent any unwanted reflections. These changes would improve
the similarity between the artificial eye and the average human eye, along with allowing for improved off-axis testing capabilities.
APPENDIX

The following section illustrates Mathematica code that can be used to calculate the GRIN coefficients for an eye model in Zemax that contains a GRIN lens. The code also allows the alpha values to be calculated for a given set of LCA values. These alpha values can be used to define a continuous dispersion of the GRIN lens.

A.1 Mathematica Code

Listing 1: Mathematica code

(* all variables are cleared *)

ClearAll["Global\'*"]

(* all variables are defined *)

dF = 0.0;
rc = 7.646;
rcp = 6.2;
rla = 10.38;
rlp = -5.38;
tc = 0.55;
ACD = 3.47;
tl = 3.874;
VCD = 15.571;
wr = 0.589;
n0 = 1.384406;
nmax = 1.409180;
w1 = 0.4;
w2 = 0.45;
w3 = 0.5;
w4 = 0.55;
w5 = 0.6;
w6 = 0.7;
LCA1 = -1.722;
LCA2 = -1.000;
LCA3 = -0.530;
LCA4 = -0.2006;
LCA5 = 0.04372;
LCA6 = 0.3815;

(* the wavelengths and LCA values are grouped together *)

w = {w2, w3, w4, w5, w6};
LCA = {LCA1, LCA2, LCA3, LCA4, LCA5, LCA6};

(* the refractive index at the required wavelengths is calculated *)

(* first an empty list is defined *)

n = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}};

(* the required coefficients for the Schott formula are defined *)

a0c = 1.85356200;
a1c = 2.03728606*^-004;
a2c = 1.66297314*^-002;
a3c = -1.89484614*^-003;
a4c = 1.68474069*^-004;
a5c = -2.66795232*^-007;

a0a = 1.74663622;
a1a = 3.11984080*^-005;
a2a = 1.61116386*^-002;
a3a = -1.85849603*^-003;
a4a = 1.60853137*^-004;

a5a = 2.52312136^-007;

a0v = 1.74893106;
a1v = -1.77732549^-004;
a2v = 1.45218434^-002;
a3v = -1.45963763^-003;
a4v = 1.22152199^-004;
a5v = 7.07418502^-007;

(* the refractive indices at each wavelength are calculated *)

Do[

n[[i, 1]] = Sqrt[a0c + a1c w[[i]]^2 + a5c/w[[i]]^8 + a4c/w[[i]]^6 + a3c/w[[i]]^4 + a2c/w[[i]]^2];

n[[i, 2]] = Sqrt[a0a + a1a w[[i]]^2 + a5a/w[[i]]^8 + a4a/w[[i]]^6 + a3a/w[[i]]^4 + a2a/w[[i]]^2];

n[[i, 3]] = Sqrt[a0v + a1v w[[i]]^2 + a5v/w[[i]]^8 + a4v/w[[i]]^6 + a3v/w[[i]]^4 + a2v/w[[i]]^2];

, {i, 1, 7}]

(* expressions for the optical power and back principal plane position are defined *)

F[ncr_, nar_, nvr_, a_] := (-a n0 + nvr)/rlp + 1/(nar ncr rca rcp rla) (-ACD (a n0 - nar (nar (rcr (rcp - tc) + tc) - ncr (rcp + ncr (rcr - rcp - tc) + tc)) + nar (a n0 rcp (nrc (rcr - tc) + tc) - ncr (rcr - rla) (nrc (rcr - tc) + tc) - ncr rla (rcp + ncr (rcr - rcp - tc) + tc)))) + 1/(a n0 nar ncr (rcr rcp rla rlp) (a n0 - nvr) (ACD (nar ncr (rcr - tc) + tc) - ncr (rcp + ncr (rcr - rcp - tc) + tc)) (a n0 (rla - tl) + nar tl) + nar ((-nar (rcp - rla) (nrc (rcr - tc) + tc) - ncr rla (rcp + ncr (rcr - rcp - tc) + tc) + rla tc + a n0 rcp ((-1 + ncr) rla tc + (nrc (rcr - tc) + tc) tl))) - (6 a n0}
n1 tl (3 n0 + n1 (tl - zm) zm))/(3 n0 - 2 n1 (tl - zm)^2) - (3 n1 (a n0 - nvr) tl^2 (3 n0 + n1 (tl - zm) zm))/(rlp (3 n0 - 2 n1 (tl - zm)^2) (3 n0 - 2 n1 zm^2)) - (3 n1 tl (-ACD (nar (ncr (rca - tc) + tc) - ncr (rcp + ncr (rca - rcp - tc) + tc))) (a n0 (2 rla - tl) + nar tl) + nar ((nar (rcp - rla) (ncr (rca - tc) + tc) + ncr rla (rcp + ncr (rca - rcp - tc) + tc))) tl - a n0 rcp (2 (-1 + ncr) rla tc + (ncr (rca - tc) + tc) tl)))(3 a n0 + a n1 (tl - zm) zm))/((3 n0 - 2 n1 (tl - zm)^2) (3 n0 - 2 n1 zm^2)) - (3 n1 (a n0 - nvr) tl^2 (3 n0 + n1 (tl - zm) zm))/(rlp (3 n0 - 2 n1 (tl - zm)^2) (3 n0 - 2 n1 zm^2)) - (3 n1 tl (-ACD (nar (ncr (rca - tc) + tc) - ncr (rcp + ncr (rca - rcp - tc) + tc))) (a n0 (2 rla - tl) + nar tl) + nar ((nar (rcp - rla) (ncr (rca - tc) + tc) + ncr rla (rcp + ncr (rca - rcp - tc) + tc))) tl - a n0 rcp (2 (-1 + ncr) rla tc + (ncr (rca - tc) + tc) tl)))(3 a n0 + a n1 (tl - zm) zm))/((3 n0 - 2 n1 (tl - zm)^2) (3 n0 - 2 n1 zm^2)) - (3 n1 (a n0 - nvr) tl^2 (3 n0 + n1 (tl - zm) zm))/(rlp (3 n0 - 2 n1 (tl - zm)^2) (3 n0 - 2 n1 zm^2));

d[nocr, nar, nvr, a, _] := (nvr (1/(nar ncr rca rcp rla) (-ACD (nar (ncr (rca - tc) + tc) - ncr (rcp + ncr (rca - rcp - tc) + tc))) (a n0 (2 rla - tl) + nar tl) + nar ((nar (rcp - rla) (ncr (rca - tc) + tc) + ncr rla (rcp + ncr (rca - rcp - tc) + tc))) tl - a n0 rcp (2 (-1 + ncr) rla tc + (ncr (rca - tc) + tc) tl)))(3 a n0 + a n1 (tl - zm) zm))/((3 n0 - 2 n1 (tl - zm)^2) (3 n0 - 2 n1 zm^2)) - (3 n1 tl (-ACD (nar (ncr (rca - tc) + tc) - ncr (rcp + ncr (rca - rcp - tc) + tc))) (a n0 (2 rla - tl) + nar tl) + nar ((nar (rcp - rla) (ncr (rca - tc) + tc) + ncr rla (rcp + ncr (rca - rcp - tc) + tc))) tl - a n0 rcp (2 (-1 + ncr) rla tc + (ncr (rca - tc) + tc) tl)))(3 a n0 + a n1 (tl - zm) zm))/((3 n0 - 2 n1 (tl - zm)^2) (3 n0 - 2 n1 zm^2)))/(2 a n0 ((-a n0 + nvr)/rlp + 1/(nar ncr rca rcp rla) (-ACD (a n0 - nar) (nar (ncr (rca - tc) + tc) - ncr (rcp + ncr (rca - rcp - tc) + tc))) + nar (a n0 rcp (ncr (rca - tc) + tc) - nar (rcp - rla) (ncr (rca - tc) + tc) - ncr rla (rcp + ncr (rca - rcp - tc) + tc))) + 1/(a n0 nar ncr rca rcp rla rlp) (a n0 - nvr) (ACD (nar (ncr (rca - tc) + tc) - ncr (rcp + ncr (rca - rcp - tc) + tc))) (a n0 (rla - tl) + nar tl) + nar ((-nar (rcp - rla) (ncr (rca - tc) + tc) - ncr rla (rcp + ncr (rca - rcp - tc) + tc)) tl - a n0 rcp ((-1 + ncr) rla tc + (ncr (rca - tc) + tc) tl)))(3 a n0 + a n1 (tl - zm) zm))/((3 n0 - 2 n1 (tl - zm)^2) (3 n0 - 2 n1 zm^2))));
zm) zm)/((3 \(n_0 - 2\) \(n_1 (tl - zm)^2\)) (3 \(n_0 - 2\) \(n_1 zm^2\)) - (3 \(n_1 (a n_0 - nvr) tl^2\) (3 \(n_0 + n_1 (tl - zm) zm) (3 \(n_0 - 2\) \(n_1 zm^2\)) - (3 \(n_1 tl (-ACD (nar (ncr (rca - tc) + tc) - ncr (rcp + ncr (rca - rcp - tc) + tc)) (a n_0 (2 rla - tl) + nar tl) + nar ((nar (rcp - rla) (ncr (rca - tc) + tc) + ncr rla (rcp + ncr (rca - rcp - tc) + tc)) tl - a n_0 rcp (2 (-1 + ncr) rla tc + (ncr (rca - tc) + tc) tl)) (3 a n_0 + a n_1 (tl - zm) zm))/((tl - zm) zm (rla (tl - zm)^2 + rlp zm^2)) + (3 \(n_1 (a n_0 - nvr) tl^2\) (ACD (nar (ncr (rca - tc) + tc) - ncr (rcp + ncr (rca - rcp - tc) + tc)) (2 a n_0 rla - a n_0 tl + nar tl) + nar ((-nar (rcp - rla) (ncr (rca - tc) + tc) - ncr rla (rcp + ncr (rca - rcp - tc) + tc)) tl + a n_0 rcp (2 (-1 + ncr) rla tc + (ncr rca + tc - ncr tc) tl)) (3 a n_0 + a n_1 (tl - zm) zm))/((tl - zm) zm (rla (tl - zm)^2 + rlp zm^2))));

(* an expression for the \(n_1\) GRIN coefficient is defined *)

n1 = ((n_0 - nmax) tl^2 (tl - 2 zm))/((tl - zm) zm (rla (tl - zm)^2 + rlp zm^2));

(* the zm which gives the required refractive error is defined *)

efleyer = n[[1, 3]]/F[n[[1, 1]], n[[1, 2]], n[[1, 3]], 1];

bfleyer = efleyer + d[n[[1, 1]], n[[1, 2]], n[[1, 3]], 1];

root = FindRoot[F[n[[1, 1]], n[[1, 2]], n[[1, 3]], 1] + dF/1000 - n[[1, 3]]/(efleyer + (VCD - bfleyer)) == 0, {zm, tl/2.1, 0, tl}];

zm = zm /. root;

(* the zm value is used to calculate the GRIN coefficients *)

n1 = ((n_0 - nmax) tl^2 (tl - 2 zm))/((tl - zm) zm (rla (tl - zm)^2 + rlp zm^2));
\[ n_3 = \frac{(2 \cdot (-n_0 + n_{max}) \cdot r_{la} \cdot t_l^2 \cdot (t_l - 2 \cdot z_m))}{(t_l - z_m) \cdot z_m \cdot (r_{la} \cdot (t_l - z_m)^2 + r_{lp} \cdot z_m^2)}; \]

\[ n_4 = \frac{-((-n_0 + n_{max}) \cdot t_l \cdot (r_{la} \cdot t_l^3 - 3 \cdot (3 \cdot r_{la} + r_{lp}) \cdot t_l \cdot z_m^2 + 4 \cdot (2 \cdot r_{la} + r_{lp}) \cdot z_m^3))}{((t_l - z_m)^2 \cdot z_m^2 \cdot (r_{la} \cdot (t_l - z_m)^2 + r_{lp} \cdot z_m^2))}; \]

\[ n_5 = \frac{(2 \cdot (-n_0 + n_{max}) \cdot (r_{la} \cdot t_l^3 - (3 \cdot r_{la} + r_{lp}) \cdot t_l^2 \cdot z_m + 2 \cdot (r_{la} + r_{lp}) \cdot z_m^3))}{((t_l - z_m)^2 \cdot z_m^2 \cdot (r_{la} \cdot (t_l - z_m)^2 + r_{lp} \cdot z_m^2))}; \]

\[ n_6 = \frac{-((-n_0 + n_{max}) \cdot (r_{la} \cdot t_l^2 - 2 \cdot (2 \cdot r_{la} + r_{lp}) \cdot t_l \cdot z_m + 3 \cdot (r_{la} + r_{lp}) \cdot z_m^2))}{((t_l - z_m)^2 \cdot z_m^2 \cdot (r_{la} \cdot (t_l - z_m)^2 + r_{lp} \cdot z_m^2))}; \]

\[ \text{grin} = \{n_0, n_1, n_3, n_4, n_5, n_6\}; \]

\[ (* \text{the alpha values are determined} *) \]

\[ \text{alpha} = \{0, 0, 0, 0, 0, 0\}; \]

\[ \text{Do[} \]

\[ \text{efleye} = n[[i, 3]]/F[n[[i, 1]], n[[i, 2]], n[[i, 3]], a]; \]
\[ \text{bfleye} = \text{efleye} + d[n[[i, 1]], n[[i, 2]], n[[i, 3]], a]; \]

\[ \text{root} = \text{FindRoot}[L_C[[i - 1]]/1000 - n[[i, 3]]/(\text{efleye} + (\text{bfleye} - \text{bfleye})) + n[[i, 3]]/\text{efleye} == 0, \{a, 1.0, 0.9, 1.1\}]; \]

\[ \text{alpha[[i - 1]]} = a \/. \text{root}; \]

\[ , \{i, 2, 7\} \]

\[ (* \text{the GRIN coefficients and alpha values are printed} *) \]

\[ \text{Print["GRIN coefficients : ", grin}] \]
\[ \text{Print["Alpha values : ", alpha]} \]


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