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Empirical correlations for the compression index of Irish soft soils

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Keywords: compression index, soft clays and silts, water content, liquid limit

Abstract

Numerous correlations have been developed in the literature relating the compression index C_c of soft soils to simple index properties that serve as a useful reality check on oedometer test results. However, many of these empirical correlations are specific to soils of a certain geographic region and/or geological origin and therefore may not be applicable in other contexts. Compression index data and corresponding index properties, specific to a good geographic spread of Irish soft soil sites, have been compiled in this paper. Of all the forms of correlation considered, the relationship between compression ratio C_c and the natural water content is the most fruitful in terms of allowing preliminary predictions of compression index to be made.

1. INTRODUCTION

The compression index C_c, usually determined by oedometer testing, is an important 1-D compressibility parameter with particular relevance to primary settlement calculations for normally consolidated or lightly overconsolidated natural soils. In geotechnical practice, the extent of oedometer testing specified for these softer soils is often insufficient in light of their inherent variability. Furthermore, the reliability of oedometer-derived parameters (including C_c) is heavily dependent upon the appropriateness and quality of the sampling method used. For example, Long (2007) has provided specific recommendations for sampling of estuarine silts. Unfortunately, ideal sample retrieval standards are not always delivered, due to the associated cost and a lack of appreciation of the impact of sample disturbance on laboratory-derived parameters.

Numerous empirical correlations exist between C_c and other properties that are more straight-forward to determine accurately and at significantly less expense. A selection of such correlations is provided in Table 1, from which it can be seen that relationships with the natural water content w_n, liquid limit w_L and initial void ratio e_0 are popular. Disturbed samples are sufficient for these tests. While such correlations should never be used in isolation for detailed design, even in routine projects, they can serve as a useful reality-check on oedometer-derived C_c values. Should suitable agreement be found between the empirically-derived and (generally sparser) measured C_c values, a more complete profile of C_c with depth may be inferred from the empirically-derived C_c values.

Correlations such as those shown in Table 1 are widely referenced by geotechnical designers, although strictly they should not be applied to soils elsewhere without consideration of soil origin and sampling method (i.e. Azzouz *et al.*, 1976). This point is made in Figs. 1a-c in which those correlations between C_c and a single index parameter from Table 1 have been plotted; the correlations with w_L (Fig. 1a) and e_0 (Fig. 1c) vary widely. In light of this, the purpose of this article is to collate published data from soft soil sites with a view to providing local empirical guidance for C_c in Irish fine soils.

2. SETTLEMENT EQUATIONS

Eqns [1] and [2] describe how C_c is used to calculate the 1-D primary settlement ΔH of a soil layer of thickness H_0 subjected to an increase in vertical effective stress of σ'_{v0} to $(\sigma'_{v0} + \Delta\sigma'_v)$ at its centre. The settlement of a normally consolidated soil is given by:

$$\Delta H = \frac{C_c}{1+e_0}\log\left(\frac{\sigma'_{v0}+\Delta\sigma'_v}{\sigma'_{v0}}\right)H_0 \qquad [1]$$

In reality, few soils are truly normally consolidated unless the deposit has been loaded by human activities. The settlement of lightly overconsolidated soils (assuming that they have become normally consolidated by the end of loading) can be computed as:

$$\Delta H = \left[\frac{C_c}{1+e_0}\log\left(\frac{\sigma'_{v0}+\Delta\sigma'_v}{\sigma'_p}\right)+\frac{C_s}{1+e_0}\log\left(\frac{\sigma'_p}{\sigma'_{v0}}\right)\right]H_0 \qquad [2]$$

where σ'_p is the preconsolidation pressure and C_s is the swelling index. For a majority of soils, C_c is typically to 5-10 times larger than C_s (i.e. Budhu 2007) so the C_c term in eqn [2] is often dominant.

3. DEVELOPMENT OF COMPRESSION INDEX DATABASE

A compression index database has been developed from testing of samples from a number of soft soil sites in Ireland (Fig. 2). Approximately three-quarters of the oedometer tests in the database indicate overconsolidation ratios (σ'_p/σ'_{v0}) in the range 1.0 to 1.6. The soil descriptions have been provided in Table 2; the references are mostly theses or published papers derived from academic studies. The interpretation of those data marked with an asterisk has been carried out by the authors of this article. A total of 61 C_c values and corresponding index properties are presented in Table 3.

The criteria used for inclusion of data and the steps taken to ensure consistency in interpretation are summarised below:

C_c values:

(i) Only tests having C_c values derived from standard incremental load oedometer tests on high quality piston or block samples have been included in the database. The ratio $\Delta e/e_0$ (where Δe is the difference between e_0 and the void ratio at the in situ vertical effective stress in the oedometer test) was used at the basis of eliminating some data.

(ii) Laboratory-to-field correction (such as Schmertmann 1953) has not been applied. The reasoning for this is that many earlier correlations do not specify whether this correction has been applied or not, and it is suspected that it was not applied.

(iii) C_c values have been determined over the vertical stress range of σ'_p to $[4\text{-}5]\sigma'_p$.

Index testing:

(iv) In a significant number of cases, the water contents were determined from the trimmings of the sample prepared for oedometer testing. Otherwise, the water content reported is from the same depth as the oedometer samples; the same is true of the Atterberg limit values.

(v) The range of soil types is represented on Casagrande chart of plasticity index (I_p) against liquid limit (w_L) in Fig. 3. It should be noted that both the A-line has been extended beyond the range normally presented (i.e. beyond a liquid limit of ≈120%) to accommodate a number of extra data points on the plot. The upper bound of applicability of the chart, known as the U-line, is also included. It is evident from Fig. 3 that, in a few of the references, the soil descriptions given indicate a soil type which is not compatible with the side of the A-line on which they fall. Therefore, in the interests of consistency, these soils are reclassified for subsequent plots as clay or silt according to their position relative to the A-line plot. In addition, one datapoint for marl soils (a lacustrine post-glacial sediment rich in calcium carbonate) is represented under a separate classification.

(vi) Given that C_c values pertain to normally consolidated soils (and those loaded into the normally consolidated region), it was important to impose some level of quality control on the measurements to which they would be correlated, to ensure that these represented soils that were at or relatively close to a normally consolidated state. This process helped eliminate soils that should not be considered soft, in addition to soils that might be soft but with unreliable measurements. The liquidity index (I_L) was used for this purpose, and only data in the range $0.5< I_L <1.5$ were included in the database. Given that the calculated I_L values become increasingly sensitive to small changes in the Atterberg limit values as the plasticity index I_p reduces, a lower limit of I_p=0.15 was imposed as a secondary quality control measure.

(vii) Where values of the initial void ratio e_0 were not provided, they were estimated using eqn [3] with G_s=2.65.

$$e_0 = w_n G_s \qquad [3]$$

4. CORRELATIONS

The following correlations were investigated with a view to finding one or more suitable for practical use:

(i) C_c against w_n (Fig. 4a): this is most rational as both C_c and w_n are influenced by soil composition and structure. In addition, there seems to have been greater consistency in the previous correlations between C_c with w_n (Figure 1b) than with w_L or e_0 (Fig. 1a, 1c). The widely-used Mesri and Ajlouni (2007) correlation is included for reference, and although developed for peat, it is relatively typical of the majority of correlations between C_c and w_n in Fig. 1b.

(ii) C_c against w_L (Fig. 5a): while w_L is not influenced by structure, there have been many attempts to correlate these parameters, because they both pertain to the normally consolidated state (see Table 1). The closest correlations from Figure 1a, i.e. those of Yamagutshi (1959) and Mayne (1980), have been included.

(iii) $C_c/1+e_0$ against w_n (Fig. 6): this normalised version of C_c, referred to as the virgin compression ratio, is sometimes considered as the basis of correlation as it is convenient to use in conjunction with eqns [1] and [2]. However, a linear variation between C_c and e_0 is implicit.

(iv) $C_c/1+e_0$ against w_L (Fig. 7).

5. RESULTS

An examination of Figs. 4 to 7 indicates the following:

(i) Correlations between C_c and w_n or w_L appear more promising than correlations between $C_c/1+e_0$ and w_n or w_L. Interestingly, Long and Boylan (2013), reporting values of C_c and $C_c/1+e_0$ for peats in three countries including Ireland, have also found that C_c relationships with w_n exhibit less scatter than $C_c/1+e_0$ relationships with w_n.

(ii) The Mesri and Ajlouni (2007) relationship represents the data quite well for w_n values in excess of 60% (Fig 4a). Moreover, Long and Boylan (2013) report that $C_c=0.01w_n$ provides a good fit to data on tests from block samples, with $C_c=0.008w_n$ more appropriate for tube samples (C_c in this case has been determined for the stress range σ'_p to σ'_p +50kPa). All peat specimens had w_n values in excess of ≈300%.

(iii) For w_n values less than 60%, the Mesri and Ajlouni (2007) relationship overpredicts compressibility considerably and is not appropriate for use. Therefore, the following best fit equation based on regression analyses, applicable to data having 35% $<w_n$ $<150\%$ is given in eqn [4] and plotted with the data in Fig. 4b (r^2=0.858). The marl datapoint is excluded from the correlation. This equation captures an increase in C_c

with w_n in Irish soils that is sharper than in previous correlations, and the r^2 is at least as good as those found for the few correlations in Table 1 that reported them (e.g. the Azzouz et al. 1976 correlation with w_n was based on n=717 points (Pearson's r=0.79).

$$C_c = 0.014(w_n - 22.7) \quad [4]$$

(iv) The corresponding equation for correlations with w_L (eqn [5]) is plotted on Fig. 5b and has a lower correlation coefficient (r^2=0.809):

$$C_c = 0.0118(w_L - 20.7) \quad [5]$$

(v) The correlation coefficients for the $C_c/1+e_0$ data in Figs. 6 and 7 are typically about 0.6 and less suitable for use than the C_c correlations. This could be related in part to uncertainty in the estimation of e_0 in the absence of G_s measurements.

(vi) There is no evidence in Figs. 4 to 7 to suggest any systematic difference in compression behaviour between Irish clays and silts.

Conclusions

In this paper, a potential correlation between the compression index and index properties of Irish soft soils has been investigated for the first time. Only high quality samples were considered and the liquidity index was used as a basis for inclusion in the database.

There is strong evidence of a relationship between the compression index and the natural water content. The widely referenced Mesri and Ajlouni (2007) correlation is suitable for natural water contents above 60% only. A new simple expression (with r^2=0.86) is presented which reflects the steeper relationship between compression index and water content in the range 35%-150%. This expression is appropriate for use with clay and silt soils. Correlations with $C_c/1+e_0$ were also considered but were found to have greater scatter, in keeping with findings in peat soils by Long and Boylan (2013).

The new correlation provides useful guidance for preliminary assessments of the compressibility of Irish soils, but should not be used as a substitute for oedometer tests on high quality samples.

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Table 1. Existing empirical correlations for the compression index C_c

Reference	**Correlation**	**Applicability**
Skempton (1944)	$C_c = 0.009(w_L - 10)$	Remoulded clays
Yamagutshi (1959)	$C_c = 0.013(w_L – 13.5)$	Various clays
Cozzolino (1961)	$C_c = 0.0046(w_L – 9)$	Brazilian clays
Shouka (1964)	$C_c = 0.017(w_L – 20)$	Various clays
Terzaghi and Peck (1967)	$C_c = 0.009(w_L – 10)$	Normally consolidated clays
Schofield and Wroth (1968)	$C_c = 0.0083(w_L – 9)$	Various clays
Azzouz et al. (1976)	$C_c = 0.006(w_L – 9)$	Various clays with $w_L < 100\%$
Mayne (1980)	$C_c = 0.0092(w_L – 13)$	Various clays
Pandian and Nagaraj (1990)	$C_c = 0.003\ w_L\ (1+e_0)$	Various clays
Peck and Reed (1954)	$C_c = 17.66*10^{-5}w_n^2 + 5.93*10^{-3}w_n – 1.35*10^{-1}$	Chicago clays
Moran et al. (1958)	$C_c = 0.0115w_n$	Organic soils
Azzouz et al. (1976)	$C_c = 0.01(w_n – 5)$	Various clays
Azzouz et al. (1976)	$C_c = 0.40(e_0 + 0.001w_n-0.25)$	Various clays
Herrero (1980)	$C_c = 0.01(w_n – 7.549)$	Various clays
Koppula (1981)	$C_c = 0.01w_n$	Various clays
Nagaraj and Murthy (1985)	$C_c = 0.2343w_nG_s$	Various clays
Bowles (1989)	$C_c = 0.0115w_n$	Organic silt and clays
Al Khafaji and Andersland (1992)	$C_c = 0.01w_n$	Various clays
Mesri and Ajlouni (2007)	$C_c = 0.01w_n$	Fibrous peats
Nishida (1956)	$C_c = 0.54(e_0 – 0.35)$	Various clays
Hough (1957)	$C_c = 0.35(e_0 – 0.5)$	Organic soils
Cozzolino (1961)	$C_c = 0.43(e_0 – 0.25)$	Brazilian clays
Sowers (1970)	$C_c = 0.75(e_0 – 0.5)$	Soils with low plasticity

Table 2. Database of Irish soft soils- authors' description

Label	Reference	Site location	Soil description	Test type
1*	Buggy & Peters (2008)	Limerick Tunnel	Silt	Std
2*	Farrell (unpublished data)	Mullingar	Marl	Std
3*	Farrell (unpublished data)	Mullingar	Clay	Std
4*	Joyce (1988)	Letterkenny	Grey clayey silt	Std
5	Kelln et al. (2007)	Limavady Roe Valley	Silty clay	Std
6	Kelln et al. (2007)	Limavady Curly Valley	Silty clay	Std
7*	Long (unpublished data)	Carrick-on-Shannon	Alluvium	Std
8*	Long et al. (2007)	Loughmore	Brown clay	Std
9*	Long (unpublished data)	Sligo	Clay	Std
10*	Long (unpublished data)	Portumna	Clay	Std
11*	McCabe (2002)	Belfast	Estuarine silt	Std
12	O'Kelly (2006)	Carrick-on-Shannon	Grey silt	Std
13	O'Kelly (2006)	Carrick-on-Shannon	Dark grey clay	Std
14	O'Kelly (2006)	Carrickmacross	Grey-brown clay	Std
15	O'Kelly (2006)	Shannon	Black peaty silt	Std
16	O'Kelly (2006)	Waterford	Grey-brown silt	Std

*Data interpreted by authors

Table 3. Compression index database

Label	w_n (%)	w_L (%)	Cc	e_0
1 Limerick Tunnel	47.1	63	0.34	1.20
	69.7	93	0.7	1.85
	57.4	73	0.6	1.54
	59.9	72	0.49	1.51
	93	113	0.61	1.77
	63.6	75	0.48	1.64
	65.2	78	0.6	1.69
	53	60	0.5	1.39
	61.7	70	0.61	1.60
	50	55	0.28	1.28
	49.8	54	0.37	1.32
	132	143	1.55	3.52
	84.8	93	0.79	2.21
	57.6	62	0.39	1.50
	54	57	0.6	1.38
	102	108	1.07	2.93
	57	54	0.39	1.50
	44.9	48	0.23	1.14
	80.2	84	1.06	2.11
	36	32	0.21	0.93
2 Mullingar	244.1	199	2.2	5.98
3 Mullingar	58.9	50	0.64	1.57
	31.7	34	0.23	0.89
4 Letterkenny	40.6	48.5	0.32	1.08
	62.2	75.2	0.502	1.65
	46.5	57.8	0.385	1.23
	67.5	76.5	0.66	1.79
	72.5	81	0.807	1.92
	57.5	63.6	0.48	1.52
	58.6	64	0.52	1.55
	66.1	70.8	0.61	1.75
	93.4	98.8	1.3	2.48
	60	63.2	0.47	1.59
	62.2	65.2	0.51	1.65
	79.4	82.6	0.76	2.10
	80.9	82.2	0.84	2.14

	74.3	72.3	0.59	1.97
5 Limavady Roe Valley	68	73	0.92	1.8
	66	70	0.74	1.75
	70	70	1.01	1.86
6 Limavady Curly Valley	54	56	0.48	1.43
	51	52	0.53	1.35
7 Carrick-on-Shannon	108	106	1.00	3.10
8 Loughmore	40	45	0.24	1.14
	43	47	0.28	1.19
	47	45	0.16	1.18
	49	43	0.25	1.09
9 Sligo	57	61.4	0.31	1.50
10 Portumna	51.7	48	0.25	1.34
	51.9	48	0.28	1.41
11 Belfast	62	71	0.68	1.66
	67.5	77	0.65	1.72
	50	58	0.4	1.34
	63	71	0.62	1.69
	66.3	71	0.5	1.78
12 Carrick-on-Shannon	72	91	0.64	1.91
13 Carrick-on-Shannon	134	143	1.40	3.55
14 Carrickmacross	45	42	0.29	1.19
15 Shannon	143	170	1.77	3.79
16 Waterford	52	58	0.29	1.38
	66	74	0.47	1.75
No. data points	61	61	61	61
Max value	244.1	199.0	2.2	6.0
Mean value	68.1	73.1	0.6	1.8
Min value	31.7	32.0	0.2	0.9
Standard deviation	31.9	30.3	0.4	0.8
Coefficient of variation	0.47	0.41	0.63	0.45

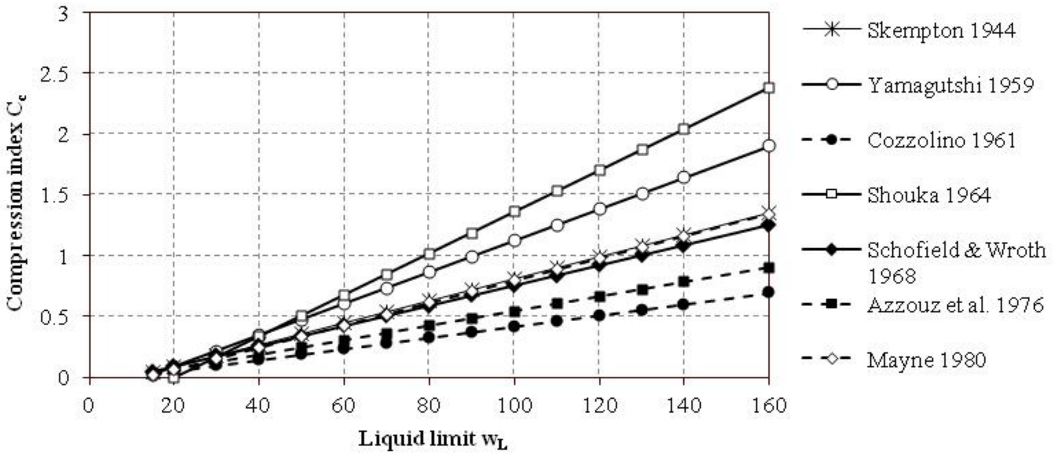

Fig. 1a Existing correlations for C_c Vs w_L

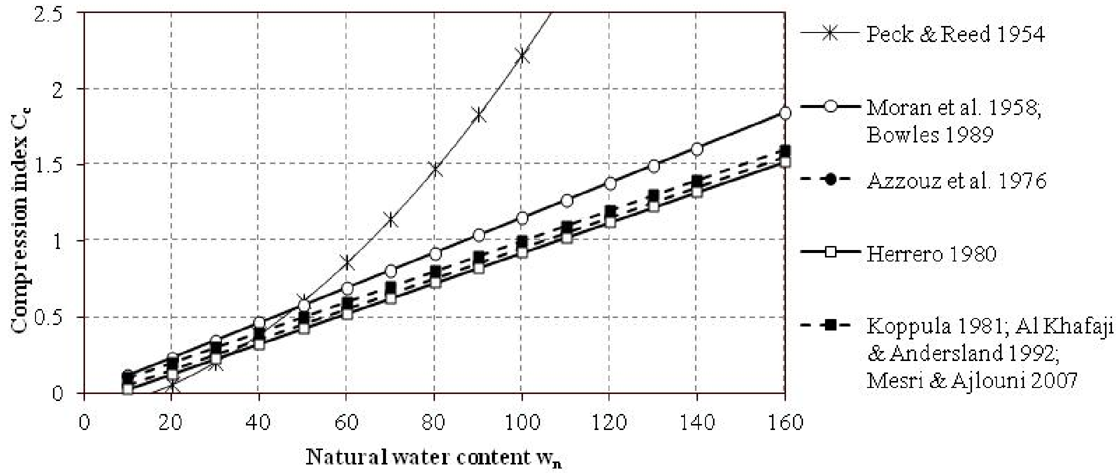

Fig. 1b Existing correlations for C_c Vs w_n

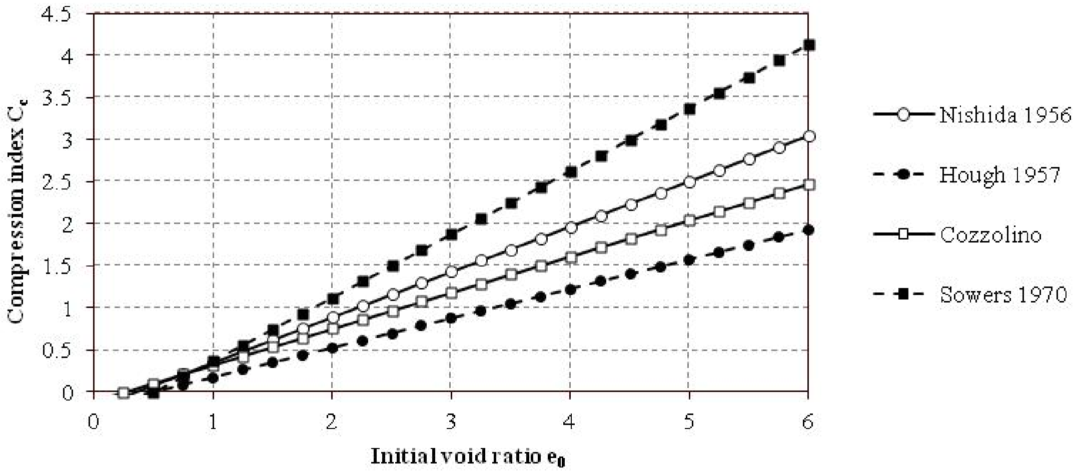

Fig. 1c Existing correlations for C_c Vs e_0

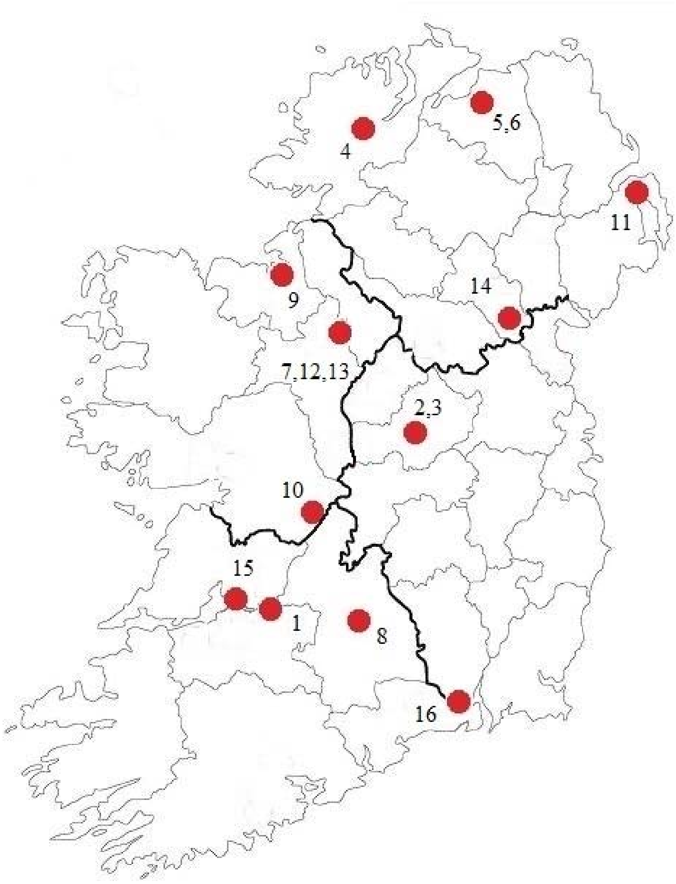

Fig. 2 Site locations

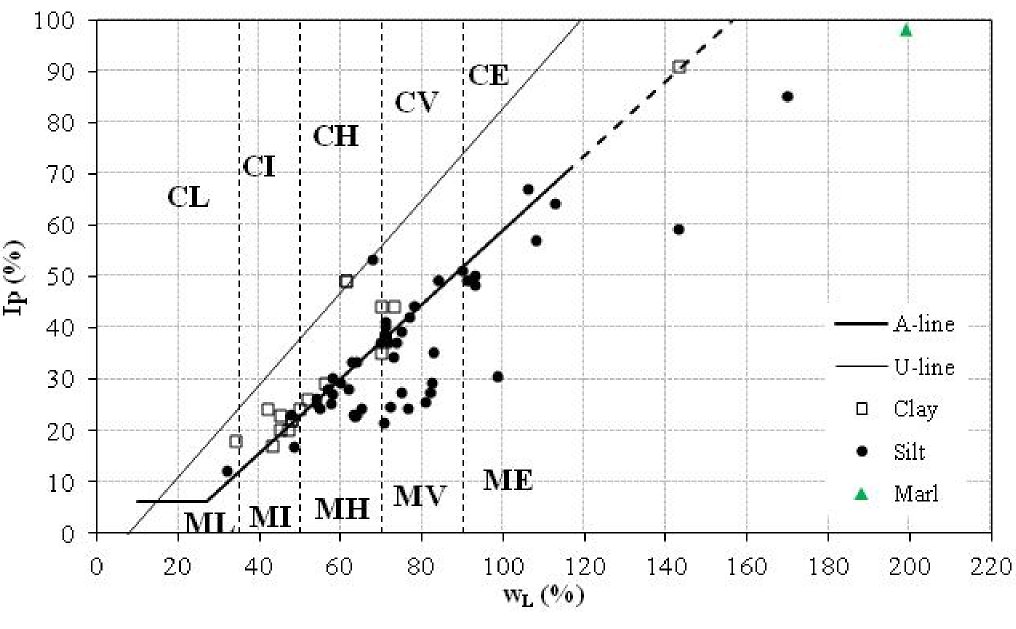

Fig. 3 Casagrande chart: plasticity index against liquid limit

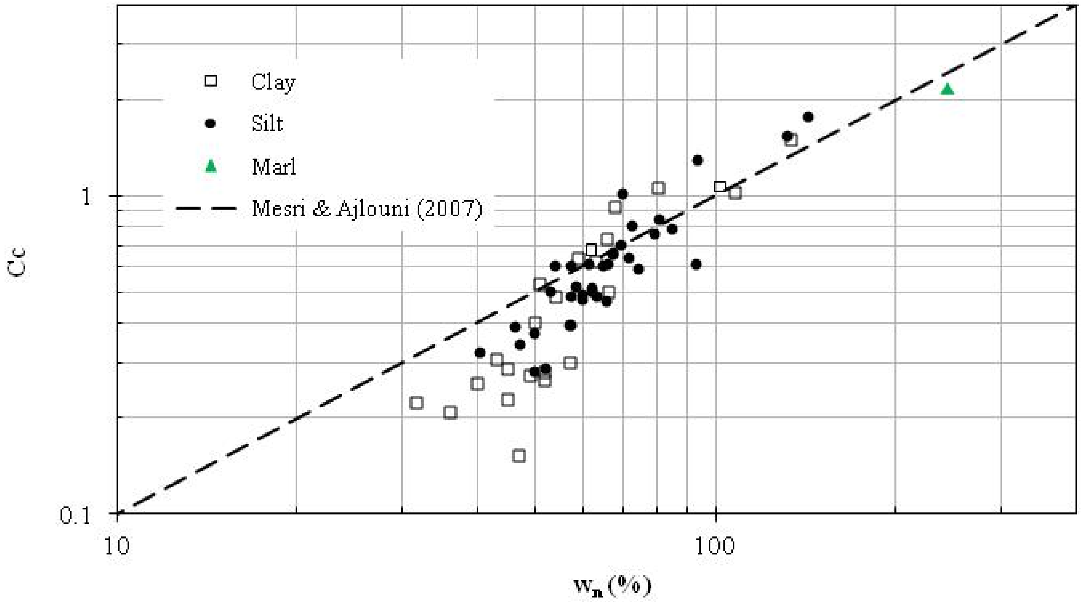

Fig. 4a Plot of C_c against w_n

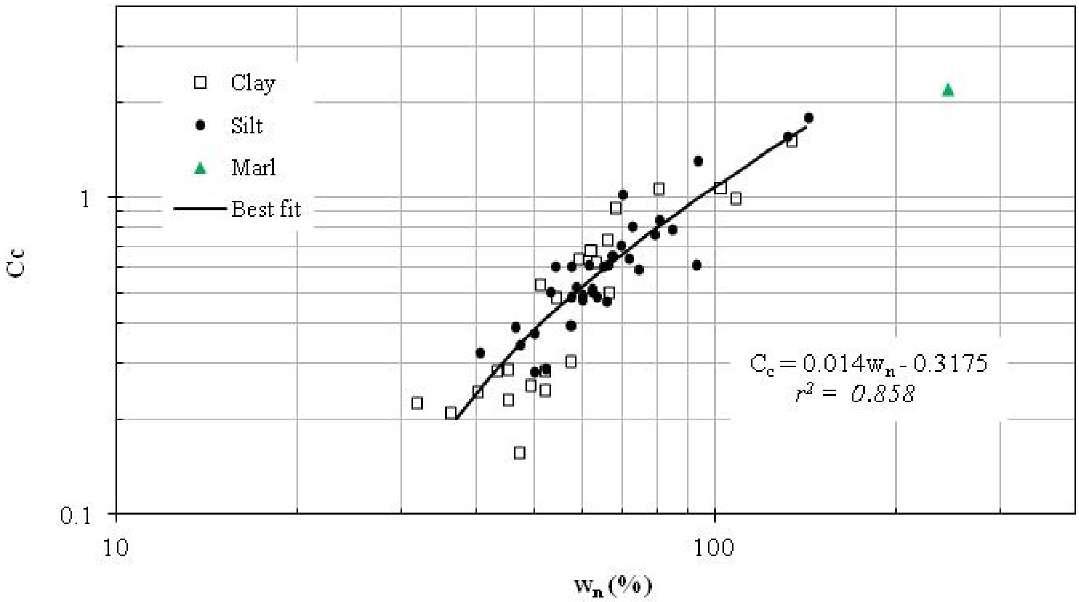

Fig. 4b Present correlation between C_c and w_n

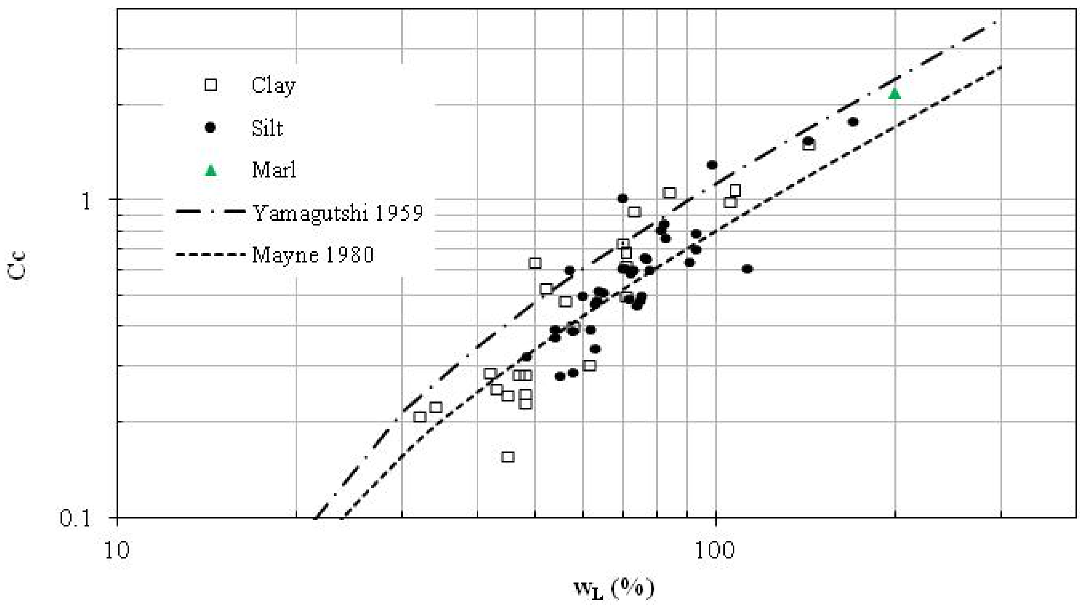

Fig. 5a Plot of C_c against w_L

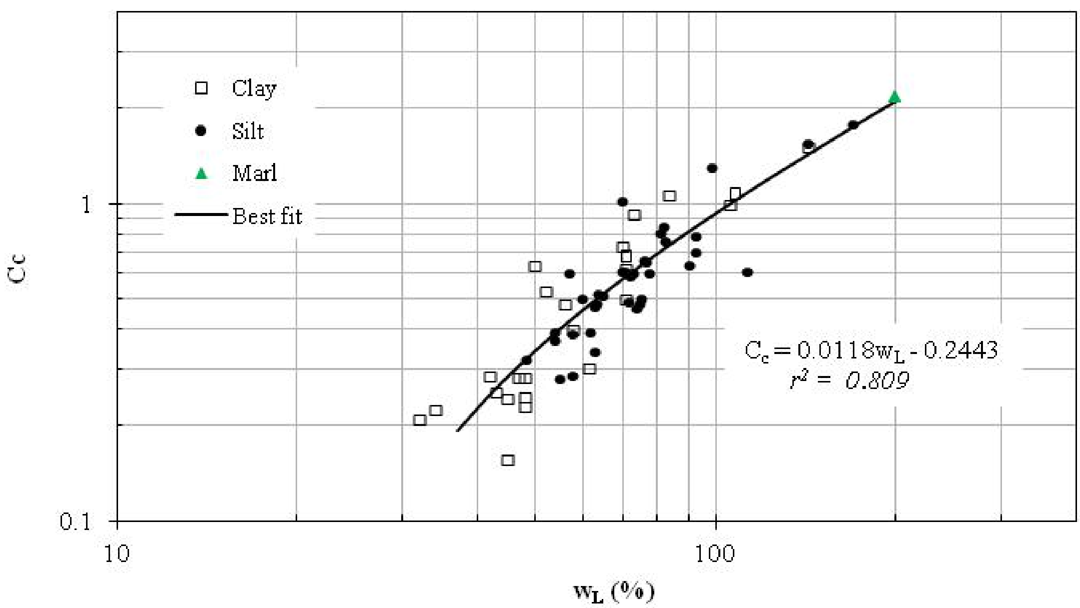

Fig. 5b Present correlation between C_c and w_L

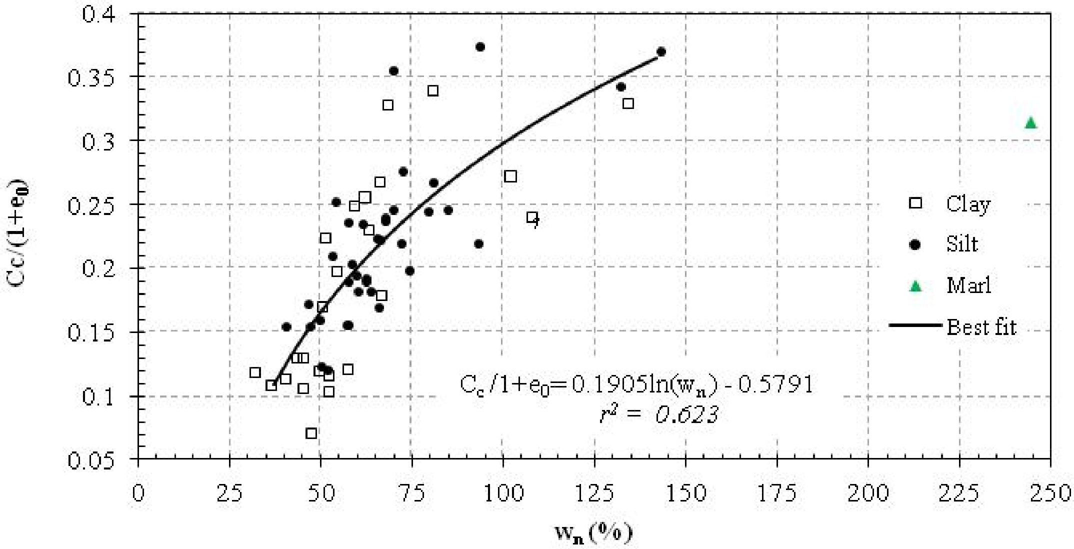

Fig. 6 Plot of $C_c/1+e_0$ against w_n

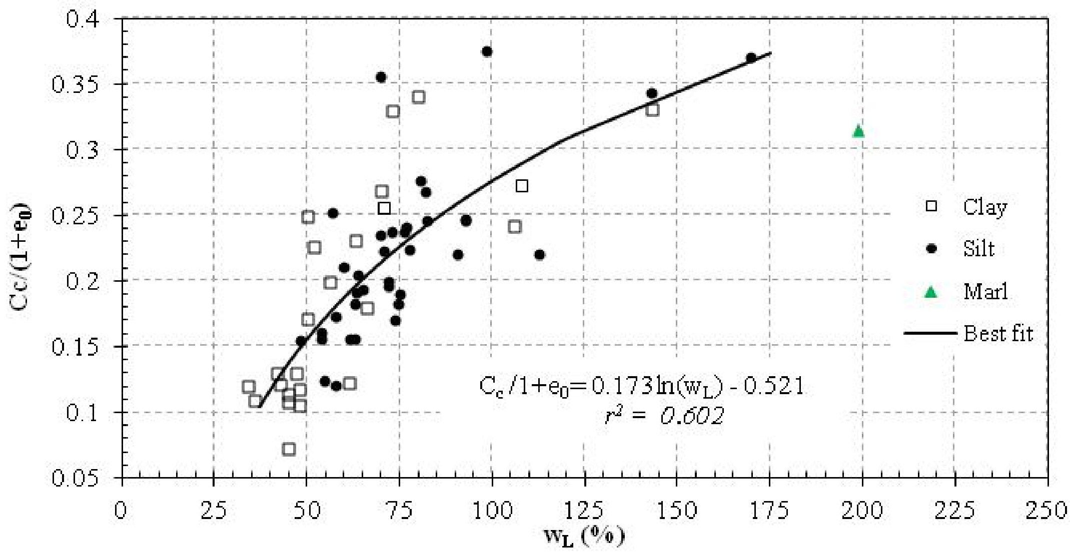

Fig. 7 Plot of $C_c/1+e_0$ against w_L