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Modelling stone column installation in an elasto-viscoplastic soil

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Abstract

The majority of numerical studies investigating stone column performance have used ‘wished-in-place’ columns (no installation effects) in conjunction with elasto-plastic soil models (no viscous effects). In this first half of this paper, Cylindrical Cavity Expansion is used in conjunction with the PLAXIS 2D elasto-viscoplastic Soft Soil Creep model to evaluate the effect of creep on column installation. Creep leads to lower post-installation lateral earth pressure coefficients ($K$) than if primary consolidation was considered alone, although the values of $K/K_0$, which decay with distance from the column centre, are nevertheless greater than 1.0. In the second half of this paper, two sets of unit cell analyses have been carried out to investigate the effect of accounting for column installation on subsequent settlement performance. The results indicate that ‘primary’ settlement improvement factors are larger than ‘total’ settlement improvement factors and that the relative differences between them increase when installation is taken into account.

Keywords

Ground Improvement; Stone columns; Finite Element Method; Creep; Installation; Settlement Performance.

List of symbols

$A$ \quad \text{Cross-sectional area of soil unit treated with granular material}

$A_c$ \quad \text{Cross-sectional area of granular column}

$A_c/A$ \quad \text{Area-replacement ratio}
<table>
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<tr>
<td>$D_c$</td>
<td>Column Diameter</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s Modulus</td>
</tr>
<tr>
<td>$E_{50}$</td>
<td>Triaxial Modulus</td>
</tr>
<tr>
<td>$E_{oed}$</td>
<td>Oedometric Modulus</td>
</tr>
<tr>
<td>$E_{ur}$</td>
<td>Unload-reload Modulus</td>
</tr>
<tr>
<td>$E_c/E_s$</td>
<td>Modular Ratio</td>
</tr>
<tr>
<td>$K$</td>
<td>Coefficient of lateral earth pressure (post-installation)</td>
</tr>
<tr>
<td>$K_a$</td>
<td>Coefficient of lateral earth pressure at-rest</td>
</tr>
<tr>
<td>$K_{0^{nc}}$</td>
<td>Coefficient of lateral earth pressure in the normally consolidated condition</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Column Radius</td>
</tr>
<tr>
<td>$a_0$, $a_f$</td>
<td>Initial cavity radius, final cavity radius (pertaining to CCE analyses)</td>
</tr>
<tr>
<td>$c'$, $c_u$</td>
<td>Effective Cohesion, undrained shear strength</td>
</tr>
<tr>
<td>$e_0$</td>
<td>Initial void ratio</td>
</tr>
<tr>
<td>$k$</td>
<td>Constant dependent on column arrangement (square, triangular, or hexagonal)</td>
</tr>
<tr>
<td>$k_x$, $k_y$</td>
<td>Horizontal permeability, vertical permeability</td>
</tr>
<tr>
<td>$m$</td>
<td>Power dictating the stress dependency of soil stiffness</td>
</tr>
<tr>
<td>$n$</td>
<td>Settlement improvement factor, $n = \delta_{untreated}/\delta_{treated}$</td>
</tr>
<tr>
<td>$n_{TOTAL}$</td>
<td>‘Total’ settlement improvement factor (i.e. primary + creep)</td>
</tr>
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</table>
\( n_{\text{PRIMARY}} \) ‘Primary’ settlement improvement factor

\( p, p' \) Mean principal total stress, mean principal effective stress

\( p_{\text{lim}} \) Limit pressure (CCE)

\( p^{\text{ref}} \) Reference pressure

\( r \) Radial distance from the axis of symmetry

\( s \) Column Spacing

\( z \) Depth, Yield Depth

\( \gamma \) Bulk unit weight

\( \kappa^* \) Recompression Index

\( \lambda^* \) Compression Index

\( \mu^* \) Creep Index

\( v, v_{ur} \) Poisson’s ratio, unload-reload Poisson’s ratio

\( \sigma_{xx}, \sigma_{yy}, \sigma_{zz} \) Total radial, vertical (axial) and hoop (tangential) stresses

\( \sigma'_xx, \sigma'_yy, \sigma'_zz \) Effective radial, vertical (axial) and hoop (tangential) stresses

\( \phi' \) Friction Angle

\( \psi \) Dilatancy Angle
1. Introduction and Background

The influence of the installation of vibro-replacement stone columns on the surrounding ground is an area that has received increasing attention in recent times. For instance, ground heave generated by column installation has been investigated as it has implications for nearby structures or buried services. McCabe et al. (2013) have shown that the volume of heaved material displaced corresponds closely to the volume of the column and that the variation of heave magnitude with radial distance from the column agrees well with corresponding measurements for driven piles when normalised by the column radius.

From a geotechnical design standpoint, the lateral soil displacement and remoulding caused by the vibrating poker as columns are installed in cohesive soils, alluded to by Debats et al. (2003), Kirsch (2006), Castro (2007), Guetif et al. (2007) and Castro and Karstunen (2010) is of interest. The result is a favourable increase in the lateral earth pressure coefficient of the in-situ soil from an at-rest value \(K_0\) to a post-installation \(K\) value. Larger \(K\) values and associated increased horizontal stresses in the soil lead to more resistance to lateral bulging of the granular material upon load application, manifesting in enhanced bearing capacity and settlement performance. Moreover, the installation process generates large excess pore pressures which dissipate during consolidation, thus resulting in increased mean effective stresses and consequently an increase in soil stiffness.

The majority of published 2D and 3D numerical studies of stone column settlement performance have assumed the columns to be ‘wished-in-place’, i.e. \(K\) is assumed to be unaffected by the vibratory action of the poker and subsequent compaction of the columns. Some authors have attempted to capture installation effects by using global increased \(K\)
values; e.g. Gäb et al. (2008), based on approaches adopted by Priebe (1976, 1995) and Goughnour and Bayuk (1979) have adopted $K = 1$, while Domingues et al. (2007a,b) have used $K = 0.7$ (between the conservative $K_0 = 1 - \sin \phi'$ for normally consolidated soils (Jaky, 1944) and $K = 1$, where $\phi'$ denotes the friction angle of the soil). However field measurements by Kirsch (2006) have indicated that a constant $K$ with distance from the column centre may not be appropriate.

In this paper, a novel two-stage procedure has been used to account for column installation in a PLAXIS 2D (Brinkgreve et al., 2011) axisymmetric unit cell model. In the first stage, Cylindrical Cavity Expansion (CCE) is used to approximate post-installation $K$ values (and hence a post-installation stress-regime in the ground) from the lateral expansion imposed by the vibrating poker as columns are installed in a soft clay. However, at unit cell radii compatible with typical column spacings, excessive mesh distortions would arise, so remote mesh boundaries are necessary. New $K$ values deduced from the first stage are incorporated into the second stage to determine settlement improvement factors, $n = \frac{\delta_{untreated}}{\delta_{treated}}$ (where $\delta_{untreated}$ and $\delta_{treated}$ denote the settlements of untreated ground and ground treated with stone columns respectively). The modelling captures a variation of $K/K_0$ with radius not usually considered by other methods (e.g. global $K$ increases).

A further original aspect of this work is that the elasto-viscoplastic Soft Soil Creep (SSC) model (Vermeer et al., 1998; Vermeer and Neher, 1999) is used to model the host clay, in acknowledgement that soft clays are often organic in nature and hence predisposed to significant creep settlements. The majority of existing numerical studies, with the exception of Sexton and McCabe (2013), have used either the Mohr Coulomb (MC) or Hardening Soil (HS) models to represent the behaviour of the granular column material and the host clay;
neither of which incorporates viscous effects. For the first stage, a comparison of the SSC
tool model output with and without creep is used to give an indication of the possible effect of
column construction on $K$ values surrounding columns in creep-prone soils. For the second
stage, the impact of considering installation on the settlement performance is investigated for
both the ‘creep’ and ‘no creep’ cases.

2. The Unit Cell Concept

The unit cell technique is commonly used in conjunction with the FE method to represent the
behaviour of an individual column within a large column grid (Figure 1) and is valid except
for columns near the edges of the loaded area (Balaam and Booker, 1981; McKelvey et al.,
2004), which can be assumed to be in the minority for large groups. The amount of host clay
replaced with stone is typically quantified using either the area-replacement ratio, $A_c/A$ (or its
reciprocal version, $A/A_c$), where $A_c$ and $A$ denote the cross-sectional areas of a stone column
and its ‘unit cell’ (hatched in Figure 1) respectively. The area-replacement ratio is related to
the column spacing ($s$) and column diameter ($D_c$) as defined by Equation 1, where $k$ is a
constant dependent on the grid pattern (see Figure 1).

$$\frac{A_c}{A} = \frac{1}{k} \left( \frac{D_c}{s} \right)^2$$ (1)

The majority of vibro-replacement settlement prediction methods (which tend to assume end-
bearing columns) are based on the unit cell assumption (e.g. Balaam and Booker, 1981;
Priebe, 1995; Castro and Sagaseta, 2009; Pulko et al., 2011), with a small number based on
plane strain (e.g. Van Impe and De Beer, 1983) or homogenization techniques (e.g.
Schweiger and Pande, 1986). The analytical methods developed by Castro and Sagaseta (2009) and Pulko et al. (2011) are elastic-plastic extensions of the earlier elastic solution developed by Balaam and Booker (1981). Both of the elastic-plastic methods assume the Mohr-Coulomb failure criterion and account for dilation of the granular column material after yielding. Castro and Sagaseta (2009) have considered an undrained loading situation followed by a consolidation process to allow for the dissipation of excess pore pressures whereas Pulko et al. (2011) have studied the problem under drained conditions. As noted by Castro and Sagaseta (2009), both approaches are considered to be limiting cases of the real situation because load application is not rapid enough to be considered as undrained nor slow enough to be considered as a drained process.

3. Cylindrical Cavity Expansion (CCE)

CCE can be used to simulate the lateral expansion of the vibrating poker as columns are installed in a cohesive soil. When using CCE in conjunction with the FE method, the cavity expansion process must begin from a finite cavity radius \( a_0 \) to avoid the development of infinite circumferential strain. Consequently, the final cavity radius \( a_f \) should be obtained by rearranging Equation 2 to observe volume conservation, where \( R_c \) is the radius of the granular column.

\[
a_f^2 - a_0^2 = R_c^2
\]  

(2)

Carter et al. (1979) have reported that doubling the cavity size is sufficient in most cases to ensure that the internal cavity pressure reaches the limit pressure \( p_{lim} \), defined by Gibson and Anderson (1961) as shown in Equation 3, where \( p_0 \) is the original in-situ horizontal total
stress, $E$ is the Young’s Modulus of the soil, $c_u$ is its undrained shear strength and $v$ is its Poisson’s ratio).

$$p_{\text{lim}} = p_0 + c_u \left[1 + \left(\frac{E}{2c_u (1 + v)}\right)\right]$$  \hspace{1cm} (3)

CCE has been used by Debats et al. (2003) and Guetif et al. (2007) to evaluate the improvement to $E$ as a result of column installation in a soft clay in conjunction with the MC model. The authors simulated the installation by expanding a cylindrical hole of 'dummy material' (low Young's modulus) from a radius of 0.25m (vibrating poker radius) to $R_c = 0.55$m. Large excess pore pressures developed at the soil-column interface, but upon dissipation (~11 months after installation), the mean effective stresses increased by an average of 30% in an influence zone of ~6$R_c$ around the column axis. This mean effective stress increase (from $p'_0$ to $p'$) was used to evaluate the increased soil stiffness (from $E_0$ to $E$) using Equation 4, where the power, $m$, dictates the dependence of stiffness on stress level.

The authors used $m = 1$ (logarithmic compression behaviour), which is typical for soft soils, e.g. Brinkgreve et al. (2011).

$$\frac{E}{E_0} = \left(\frac{p'}{p'_0}\right)^m$$  \hspace{1cm} (4)

Castro and Karstunen (2010) have also applied the cavity expansion technique to the undrained installation of a single 10m long 0.8m diameter stone column, confirming that column installation generates excess pore pressures. The authors used the rate independent Modified Cam Clay (MCC, Roscoe and Burland 1968), S-CLAY1 (Wheeler et al., 1997, 2003) and S-CLAY1S (Koskinen et al., 2002) models. The latter models account for
anisotropy and both anisotropy and destructuration respectively. In their study, Castro and Karstunen (2010) evaluated post-installation lateral earth pressure coefficients as detailed in Equation 5, where $\sigma'_{xx}$, $\sigma'_{yy}$ and $\sigma'_{zz}$ denote the effective radial, vertical and hoop (tangential) stresses respectively. In general, the computed values of $K/K_0$ were lower for the S-CLAY1S model than for the S-CLAY1 model (but $K/K_0 > 1$ nevertheless) owing to the destructuration caused by column installation.

$$K = \left( \frac{\sigma'_{xx} + \sigma'_{zz}}{2\sigma'_{yy}} \right)$$  (5)

The analyses carried out in this paper are intended as a development on some of this aforementioned work by (i) incorporating the installation effects into a unit cell model to study the effects of column installation on subsequent settlement performance, and (ii) doing so using an elasto-viscoplastic soil model.

4. Finite Element (FE) Model Parameters

4.1 Soft Clay

The soft clay properties (Table 1) adopted in this study are loosely based upon those of the Bothkennar Carse clay, near Grangemouth in Scotland. A simplified single-layer 5m deep profile (with the stiff crust omitted in the interests of clarity) has been used for the FE model. The clay has been modelled using the Undrained A approach (undrained effective stress analysis with effective stiffness and effective strength parameters). Bothkennar clay is highly structured with an organic content of between 3% and 5% (Paul et al., 1992), an overconsolidation ratio (OCR) of approximately 1.5 (Nash et al., 1992a), and a critical state
friction angle, $\phi' = 34^o$ (Allman and Atkinson, 1992). The high friction angle is attributable to a significant proportion of silt-sized grain particles and the high organic content.

The default value of the compression index, $\lambda^*$, quoted in Table 1, has been altered to provide a modular ratio, $E_c/E_s = 20$, at $p_{ref} = 100kPa$, where $E_c$ and $E_s$ represent the oedometric moduli ($E_{oed}$) of the granular material and the soft clay respectively and $p_{ref}$ is a reference pressure; $\lambda^*$ and $E_{oed}$ are related as shown in Equation 6. It should be noted that $E_c/E_s = 20$ is only an approximate indicator of the actual value modelled; exact modular ratios can only be quoted for linear elastic models. For elasto-plastic and elasto-viscoplastic models, soil stiffness depends on stress level and OCR and so the quoted modular ratio would only be exact for a normally consolidated soil for which the reference pressures in the soil and column were 100kPa.

The values of the recompression and creep indices have been obtained based on $\lambda^*/\kappa^*$ and $\lambda^*/\mu^*$ ratios of 7 (Allman and Atkinson, 1992) and 25 (Nash et al., 1992b) respectively. The horizontal and vertical permeabilities ($k_x$ and $k_y$) are based on a comprehensive series of in-situ (e.g. pushed-in-place piezometers, self-boring permeameters, BAT system) and laboratory (e.g. oedometer cells, triaxial cells, radial flow cells) tests carried out by Leroueil et al. (1992). A nominal cohesion, $c' = 1kPa$, has been used for numerical stability; the initial void ratio ($e_0$) of 2.0 is based on a testing programme of incremental load (IL) oedometer tests, constant rate of strain (CRS) tests and restricted flow (RF) tests carried out by Nash et al. (1992b).

$$P_{oed}^{ref} = \frac{p_{ref}^{ref}}{\lambda^*}$$ (6)
4.2 Column

The column material (Table 1) has been modelled using the hyperbolic elasto-plastic HS model (Schanz et al., 1999) as a highly permeable drained material with $k_x = k_y = 1.7 \text{m/day}$ (Elshazly et al., 2008). These permeabilities account for infiltration of silt and clay particles into the column during installation. The HS model has two yield surfaces, one to incorporate shear hardening and the other to incorporate compression hardening, the sizes of which are governed by $E_{S0}$ (triaxial modulus) and $E_{oed}$ respectively. The values of $E_{S0}$ and $E_{oed}$ (at $p_{ref} = 100\text{kPa}$) quoted in Table 1 have been obtained from Gäb et al. (2008). The unload-reload modulus ($E_{ur}$) has been calculated as $E_{ur} = 3E_{S0}$, e.g. Brinkgreve et al. (2011). The HS model accounts for the stress dependency of soil stiffness using a power law (see Equation 4); a power of $m = 0.3$ has been selected for the stone, also based on Gäb et al. (2008). The friction and dilatancy angles of $\phi' = 45^\circ$ and $\psi = 15^\circ$ are selected based on Killeen and McCabe (2014), as is the bulk unit weight of $\gamma = 19\text{kN/m}^3$.

For both the clay and the stone, $K_0$ is calculated based on Jaky’s (1944) empirical relationship of $K_0 = 1 - \sin \phi'$. An unload-reload Poisson’s ratio of $\nu_{ur} = 0.2$ is also used for both materials, e.g. Killeen and McCabe (2014).

5. Numerical Modelling - Stage 1

5.1 Analyses Procedure

Firstly, CCE is employed to evaluate post-installation $K$ values. The impact of creep is assessed by performing two separate sets of analyses; one set using the standard value of $\mu^*$
quoted in Table 1 and the other using the minimum allowable value of $\mu^*$ (approximately 5% of the standard value), practically eliminating creep effects. Hereafter, these are referred to as the ‘SSC’ and ‘SSC ($\mu^* \approx 0$)’ cases. PLAXIS does not permit the use of $\mu^* = 0$ with the SSC model. This would be unrealistic as only purely elastic behaviour would be predicted because plastic strains are incorporated in the creep/viscoplastic strain component.

5.2 Analyses Steps

The steps involved in simulating column installation using CCE in conjunction with PLAXIS 2D are as follows:

(i) Generate initial stresses using the $K_0$ procedure. The $K_0$ procedure is described by Brinkgreve et al. (2011). PLAXIS generates vertical stresses that are in equilibrium with the self-weight of the soil and calculates the horizontal stresses based on the specified value of $K_0$ thereafter.

(ii) Install ‘dummy material’ over the entire column length to a radial extent, $a_0$. As suggested by Guetif et al. (2007), the cylindrical hole created by the poker is modelled using a ‘dummy material’ with a very low Young’s modulus, $E = 20$ kPa.

(iii) Apply a prescribed displacement (undrained conditions) from an initial radius, $a_0$, to a final radius, $a_f$ (see Figure 2). Three different $a_0$ values of 0.10m, 0.15m and 0.20m have been used for which $a_f$ values have been calculated according to Equation 2 as $a_f = 0.316$m, 0.335m and 0.361m respectively (based on a column radius, $R_c = 0.3$m).

(iv) This cavity expansion phase is followed by a consolidation phase to allow excess pore pressures to dissipate (to a maximum excess pore pressure of 1 kPa) to establish the long-
term stress changes in the ground induced by column installation. Post-installation $K$ values are calculated using Equation 5.

For this stage, the external far boundary of the FE mesh is located 30m away from the axis of symmetry to ensure that results are unaffected by boundary proximity, e.g. Figure 2. For the unit cell radii required for typical $A/A_c$ ratios encountered in practice, the boundary would have been too close, leading to numerical problems. Roller boundaries have been applied to all sides. The mesh consists of approximately 16,000 6-noded triangular elements and has been refined in the region surrounding the cavity (largest strain gradients in this region). The Updated Mesh option (e.g. McMeeking and Rice, 1975), with subsequent updating of the water pressures, has been used to account for the large displacements that occur in this region.

The yield surface of the SSC model is similar to that of the MCC model, but, based on Bjerrum’s (1967) concept, the preconsolidation stress depends on the amount of viscoplastic strain that accumulates with time (Degago et al., 2011), i.e. the yield surface expands due to creep. Accordingly, analyses carried out with the SSC model require a time interval to yield realistic predictions; otherwise the yield cap (which is responsible for plastic deformations) will not be able to move and as a result the model will predict an unrealistically stiff soil response. In this study, a time interval of 0.01 days (~15 minutes, of the same order as the time to install a column in practice) is used for the expansion phase (iii) above. The influence of the duration of the expansion phase on the short-term (after expansion) and long-term (after consolidation) predictions is examined in Section 5.3.3.
5.3 Results

5.3.1 Short-term (after expansion)

The variations of radial and hoop (tangential) total stress with $r/R_c$ ($r$ is the radial distance from the axis of symmetry) at mid-depth (2.5m) after the cavity expansion phase predicted by the SSC model with and without creep are shown in Figures 3 and 4. Also included are predictions obtained using the Gibson and Anderson (1961) undrained analytical solution to provide a frame of reference. The Gibson and Anderson (1961) elastic-plastic method is based on a single soil stiffness whereas the SSC model requires the input of $\lambda^*$ and $\kappa^*$ (which can be related to $E_{50}$ and $E_{ur}$ using Equations 6, 7 and 8; therefore two sets of Gibson and Anderson (1961) predictions are included in Figures 3 and 4 (for $E_{50}$ and $E_{ur}$).

\[
E_{50}^{\text{ref}} = 1.25 E_{\text{oed}}^{\text{ref}} \tag{7}
\]

\[
E_{ur}^{\text{ref}} = \frac{(1 + \nu)(1 - 2\nu) p^{\text{ref}}}{(1 - \nu)\kappa^*} \tag{8}
\]

Examination of Figures 3 and 4 indicates:

- The Gibson and Anderson (1961) predictions of radial total stress ($\sigma_{xx}$) with $r/R_c$ using $E_{ur}$ are in good agreement with the PLAXIS output (Figure 3).
- The SSC and SSC ($\mu^* \approx 0$) predictions are almost identical, as would be expected as viscoplastic strains should be negligible at 0.01 days.
- Using $E_{50}$, Gibson and Anderson (1961) predict that the limit pressure should reach a value of 61.6kPa, whereas using $E_{ur}$, Gibson and Anderson (1961) predict that the limit
pressure should reach a value of 73.6kPa. For the SSC and SSC ($\mu^* \approx 0$) analyses, the internal cavity pressures reach 71.9kPa and 71.7kPa respectively (Figure 3).

- The variation of hoop (tangential) total stress ($\sigma_{zz}$) with $r/R_c$ predicted by the SSC and SSC ($\mu^* \approx 0$) analyses is smoother than that predicted using the simplified elastic-plastic analytical formulation (Figure 4).

- Examination of the distributions of $\sigma_{zz}$ with $r/R_c$ (Figure 4) indicates that the radii of influence predicted by Gibson and Anderson (1961) are $\sim 5R_c$ and $\sim 10R_c$ using $E_{50}$ and $E_{ur}$ respectively; the latter is in good agreement with the PLAXIS influence radii of $\sim 10R_c$, despite the different shape. The shape of the distribution of $\sigma_{zz}$ with $r/R_c$ that would be obtained using the linear elastic perfectly plastic MC model, for example, would be similar to that obtained using Gibson and Anderson's (1961) method because the MC model is also based on elastic-plastic, and not elasto-plastic theory.

**5.3.2 Long-term (after consolidation)**

The variations of $K/K_0$ with $r/R_c$ calculated using the SSC and SSC ($\mu^* \approx 0$) analyses after consolidation are compared in Figure 5. In both cases, the $K/K_0$ values revert to unity at $\sim 15R_c$. The predictions indicate that creep (viscoplastic strain) marginally lowers the values of $K/K_0$ (the short-term predictions in Figures 3 and 4 were identical). The predictions in Figure 5 pertain to an initial radius, $a_0 = 0.15$m, although it has been verified that the $K/K_0$ values are independent of the amount of lateral expansion and depth.

Also included in Figure 5 are field values of $K/K_0$ reported by Kirsch (2006) mostly beyond $r = 5R_c$. The ‘Field 1’ data pertain to a group of 25 no. 9m long, 0.8m diameter stone columns installed in a silt layer ($K_0 = 0.91$) while the ‘Field 2’ data were obtained for a group of 25 no.
6m long, 0.8m diameter columns installed in a sandy silt \((K_0 = 0.57)\). The columns were installed in a square grid in both cases with measurements obtained using earth pressure cells and pore water pressure cells. The ‘Field 2’ data are more comparable to the situation modelled. The SSC and SSC \((\mu^* \approx 0)\) predictions are in moderate agreement with the field data for \(r/R_c > 8\). The low \(K/K_0\) values close to the column in the field (e.g. where the field values are below those predicted using PLAXIS) were attributed to remoulding and dynamic effects. The purpose of the field comparisons is simply to indicate that the PLAXIS 2D \(K/K_0\) values (after consolidation) obtained in this study are in reasonable agreement with field predictions.

### 5.3.3 Influence of expansion phase duration

As noted in Section 5.2, the predictions were obtained using a time interval of 0.01 days for the cavity expansion phase. The influence of the expansion phase duration on the short-term (undrained) and long-term (after consolidation) predictions for the case with a standard creep coefficient is examined in this section. This is a worthwhile exercise because the influence of creep (using the SSC model or any other soil model) on post-installation soil properties, with the exception of Castro et al. (2012), has received very little investigation to date.

The variations of radial and hoop (tangential) total stress with \(r/R_c\) after the cavity expansion phase for different time intervals are shown in Figures 6 and 7. The 0.01 day, 0.1 day, 1 day and 10 day predictions are very close to one another. The predictions using a zero-time interval (unrealistic for the SSC model because the yield cap will not be able to move, e.g. Section 5.2) are much higher. In general, the cavity pressure decreases marginally as the time interval increases (attributable to the influence of viscoplastic strain), although the effect is
almost negligible apart from the zero time interval case for which the cavity pressure appears overestimated; intuitively, it would be expected that the SSC model standard creep coefficient cavity pressure (using a zero time interval, Figure 6) would be comparable with the SSC ($\mu^* \approx 0$) cavity pressure (0.01 day time interval, Figure 3).

Figure 8 shows that the influence of the duration of the expansion phase on the long-term $K/K_0$ predictions is minor. The predictions for 0.01 days, 0.1 days, 1 day and 10 days collapse on one another, although the $K/K_0$ values using a zero time interval are out of kilter with the other predictions, owing to the initially-overestimated cavity pressure.

6. Numerical Modelling - Stage 2

6.1 Analyses Procedure

Stage 2 involves using the output $K$ values from Stage 1 to generate an input $K$ profile for a unit cell model to study the influence of installation on settlement reduction. The 5m long unit cell model is shown in Figure 9. The boundary conditions applied to the unit cell reflect oedometric conditions; roller boundaries are applied vertically while the base is fixed in all directions. The influence of installation is studied by comparing $n$ values for a scenario with $K/K_0 = 1$ (Case A) against $n$ values derived based on the modified initial $K$ profile from the CCE analyses (Case B, $K/K_0 > 1$). The procedure employed to obtain the $n$ values for Case B is novel because the FE mesh following the CCE stage is too distorted to use as a realistic starting point to study settlement behaviour. Additionally, the influence of creep is examined by comparing $n$ values for the two scenarios, ‘SSC’ and ‘SSC ($\mu^* \approx 0$)’, subsequently
referred to as ‘total’ ($n_{TOTAL}$) and ‘primary’ ($n_{PRIMARY}$) settlement improvement factors respectively. These analyses are performed for the range $3 < A/A_c < 10$.

6.2 Establishing an input $K$ profile for the Case B unit cell models

The $K/K_0$ decay with $r/R_c$ for both the ‘SSC ($\mu^* \approx 0$)’ and ‘SSC’ cases (Figure 5) is approximated by a stepwise reduction as shown in Figures 10a and 10b; average $K/K_0$ values have been interpreted at intervals of 0.1m representing concentric rings around the axis of symmetry. The $K/K_0$ values are used as input parameters for the unit cell (Case B) models; a different initial $K/K_0$ profile has been used for the SSC ($\mu^* \approx 0$) and SSC analyses (Figure 10a versus Figure 10b). Also demarcated on Figure 10 is the radial extent ($r/R_c = 3.16$) relevant to the largest unit cell modelled in this paper ($A/A_c = 10$).

6.3 Analyses Steps

The steps in each analysis are outlined in (i)-(vi) below.

(i) Generate initial stresses using the $K_0$ procedure. Initial stress generation is different for Cases A and B; initial stresses in the clay for Case A are generated using a uniform $K = 1 - \sin \phi' = 0.44$, while for Case B, initial stresses are generated based on the modified initial $K$ profiles derived in Section 6.2. The mean effective stresses ($p'$) are plotted in Figures 11a (Case A, same for ‘SSC’ and ‘SSC($\mu^* \approx 0$)’), 11b (Case B, ‘SSC($\mu^* \approx 0$)’) and 11c (Case B, ‘SSC’).

(ii) ‘Install’ the columns using the ‘wished-in-place’ technique.

(iii) Apply a plastic nil-step phase with a small time interval to restore any out-of-equilibrium stresses generated by the ‘wished-in-place’ installation.
(iv) Place a plate over the surface of the unit cell; the plate acts as a loading platform and prevents differential settlements between the surfaces of the column and the surrounding soil.

(v) Apply a 100kPa load in undrained conditions.

(vi) Allow a consolidation phase. The pore pressures generated by load application will dissipate; settlements practically cease for the ‘SSC (μ* ≈ 0)’ case thereafter.

For these analyses, fine meshes consisting of approximately 2,000 6-noded triangular elements have been used. Mesh refinement studies have been carried out to verify that the predictions of maximum displacement at the unit cell surface (same in soil and column) and average mean effective stresses in the column and soil at the end of the final phase of analysis are unaffected by any further mesh density increases thereafter.

6.4 Results

6.4.1 Settlement Improvement Factors: Case A

Firstly, \( n_{\text{PRIMARY}} \) and \( n_{\text{TOTAL}} \) values for Case A are compared in Figure 12 to establish the influence of creep (without considering installation effects), if any. In both cases, the relevant \( n \) values have been derived after the end-of-primary (EOP) consolidation for both the untreated (no columns) and treated cases. Granular columns significantly accelerate the consolidation process (e.g. Figure 13, Case A) and so a true appraisal of \( n \) can only be made after EOP in both cases. For the untreated case, primary consolidation takes approximately 500 days whereas at \( A/A_c = 10 \), the EOP time is approximately 50 days and decreases thereafter with increasing stone replacement.
At all values of $A/A_c$ in Figure 12, the $n_{TOTAL}$ values are less than their $n_{PRIMARY}$ counterparts. This suggests that incorporating creep leads to lower settlement improvement factors than would be obtained for primary consolidation alone. The extent to which creep reduces the $n_{PRIMARY}$ values is dependent on the relative proportions of primary and creep settlement in the untreated profile (~80% primary and ~20% creep after 100 years in this case).

Also included in Figure 12 are analytical predictions of $n_{PRIMARY}$ obtained using Castro and Sagaseta (2009) and Pulko et al. (2011) to provide context for the FE-derived $n_{PRIMARY}$ values. Examination of Figure 12 indicates that the FE-derived $n_{PRIMARY}$ values are in good agreement with these analytical predictions. However, the analytical methods overpredict $n_{TOTAL}$ for the scenario modelled, and so it seems appropriate to use a lower settlement improvement factor if creep is present.

6.4.2 Settlement Improvement Factors: Case A versus Case B

Settlement improvement factors for Cases A and B are compared in Figures 14a and 14b for the SSC ($\mu^* \approx 0$) and SSC analyses respectively. The findings are as follows:

- For both the SSC and SSC ($\mu^* \approx 0$) analyses, accounting for column installation (larger $K$ values) leads to a larger $n$ at all values of $A/A_c$, i.e. larger $K$ values indicate increased horizontal stresses in the soil; these larger horizontal stresses provide more resistance to lateral bulging of the granular material.
- The $n_{PRIMARY}$ values (Figure 14a) increase more than the $n_{TOTAL}$ values (Figure 14b), as can be seen by comparing the hatched ‘areas’ between the Case A and B curves (the hatching is used for optical purposes to help highlight differences between the Case A and
Case B \( n \) values for the two scenarios). This is to be expected because lower post-installation \( K \) values are used for the case with creep (see Figures 5 and 10), i.e. a different starting point.

- Accordingly, the percentage differences between the \( n_{\text{PRIMARY}} \) and \( n_{\text{TOTAL}} \) values (relative to \( n_{\text{PRIMARY}} \), see Figure 15) for Case B are larger than those for Case A but the conclusions are consistent: \( n \) values are lower when creep is considered.

7. Conclusions

In the first half of this paper, CCE has been implemented in finite element analyses to calculate cavity pressures (short-term) and post-installation \( K \) values (long-term) following stone column installation in a soft clay. The influence of creep has been investigated by comparing the SSC model output with and without creep (\( \mu^* \approx 0 \)). The results indicate that:

(i) The internal cavity pressure after the initial cavity expansion phase is unaffected by creep (based on a direct comparison of the SSC model output with and without creep), as would be expected for the short time interval used (a negligible creep effect would be expected).

(ii) However, creep (viscoplastic strains) results in lower \( K/K_0 \) after the consolidation phase (although the values of \( K/K_0 \) are greater than 1.0).

In the second half of the paper, \( K/K_0 \) profiles generated using the CCE procedure for both the ‘creep’ and ‘no creep’ cases have been imported successfully as input profiles for axisymmetric unit cell models. Settlement performance for this ‘Case B’ has been analysed and compared to a scenario with \( K/K_0 = 1 \) (Case A). The approach used in Case B is novel; the prescribed displacements in the CCE stage would cause both excessive heave and
numerical problems (due to boundary proximity) for the size of unit cell required to give typical $A/A_e$ ratios ($<< 100R_e$). The modelling approach, which captures a variation of $K/K_0$ with radius not usually considered by other methods (e.g. global $K$ increases), can be considered as an improvement upon the conventional ‘wished-in-place’ column installation technique.

For Case A ($K/K_0 = 1$), ‘total’ (i.e. primary plus creep) settlement improvement factors are lower than their ‘primary’ counterparts. This indicates that creep should not be ignored in vibro-replacement design. For Case B ($K/K_0$ as predicted by CCE), i.e. when installation (increased $K$) is taken into account, both the ‘total’ and ‘primary’ settlement improvement factors increase. The ‘primary’ settlement improvement factors increase more than the ‘total’ settlement improvement factors and so the differences between them increase as a result.

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References


Table 1. Finite Element (FE) Model Parameters

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Table 1. Finite Element (FE) Model Parameters
Figure 1. Unit cells; (a) triangular (b) square (c) hexagonal

Figure 2. FE Mesh for CCE stage
Figure 3. Radial total stress ($\sigma_{xx}$) versus $r/R_c$ after cavity expansion

Figure 4. Hoop (tangential) total stress ($\sigma_{zz}$) versus $r/R_c$ after cavity expansion

SCC and SSC ($\mu^* \approx 0$) are almost coincident.
Figure 5. $K/K_0$ versus $r/R_c$ after consolidation (influence of creep and comparison of predictions with field data).

Figure 6. Radial total stress ($\sigma_{xx}$) versus $r/R_c$ after cavity expansion (influence of expansion phase duration).
Figure 7. Hoop (tangential) total stress $(\sigma_{zz})$ versus $r/R_c$ after cavity expansion (influence of expansion phase duration)

Figure 8. $K/K_0$ versus $r/R_c$ after consolidation (influence of expansion phase duration)
Figure 9. 5m unit cell model

Figure 10. Adopted $K/K_0$ Profiles; (a) SSC ($\mu^* = 0$) (b) SSC
Figure 11. (a) $p'$ with depth for Case A (b) $p'$ with depth for Case B - SSC ($\mu^* \approx 0$) (c) $p'$ with depth for Case B - SSC ($A/A_c = 10$ in all cases)

Figure 12. Comparison of FE-derived $n_{\text{PRIMARY}}$ and $n_{\text{TOTAL}}$ values for Case A with analytical solutions
Figure 13. Settlement vs. log(time) for Case A; (a) SSC ($\mu^* = 0$) (b) SSC

Figure 14. $n$ versus $A/A_c$ for Cases A and B; (a) SSC ($\mu^* = 0$) (b) SSC

Figure 15. Percentage differences between $n_{\text{PRIM}}$ and $n_{\text{TOTAL}}$ for Cases A and B