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A UNIFIED VISCOPLASTIC MODEL FOR HIGH TEMPERATURE LOW CYCLE FATIGUE OF SERVICE-AGED P91 STEEL

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ABSTRACT

The finite element (FE) implementation of a hyperbolic sine unified cyclic viscoplasticity model is presented. The hyperbolic sine flow rule facilitates the identification of strain-rate independent material parameters for high temperature applications. This is important for the thermo-mechanical fatigue of power plant where a significant stress range is experienced during operational cycles and at stress concentration features, such as welds and branch connections. The material model is successfully applied to the characterisation of the high temperature low cycle fatigue behaviour of a service-aged P91 material, including isotropic (cyclic) softening and non-linear kinematic hardening effects, across a range of temperatures and strain-rates.

KEYWORDS: service-aged P91, strain-rate independence, unified viscoplasticity, high temperature low cycle fatigue, material Jacobian.

INTRODUCTION

Grade P91 steel, a 9Cr martensitic steel developed at Oakridge National Laboratory (ORNL) [1], is used extensively in fossil-fuel based power plant due to its high creep strength and low coefficient of thermal expansion. However, modern and next generation power plants are subjected to thermomechanical fatigue (TMF) due to a rapid rise in the number of start-up cycles to accommodate renewable sources of energy. This results in premature failure of plant components, including plant header and piping systems. Thus, there is a greater requirement to be able to characterise the advanced materials of modern and next generation plants under realistic loading conditions and accurately predict failure at the component level.

The high strength of such 9Cr steels may be attributed to its complex microstructure, consisting of prior austenite grains, packets and blocks in a hierarchical format [2]. The blocks contain long martensitic laths and more equi-axed subgrains [2, 3]. The martensitic laths contain MX type precipitates which resist motion of mobile dislocations and $M_{23}C_6$ carbonitrides are dispersed along boundaries to increase the resistance of the material to creep deformation [4]. Although 9Cr steels have a high creep strength, fatigue loading results in a coarsening of this microstructure, leading to the cyclic softening phenomenon observed in such materials [2, 3, 5] and to a reduction in creep strength.

Corresponding Author: Email: r.barrett2@nuigalway.ie Tel: +353 91 493020 Fax: +353 91 563991 To account for the micro-structural evolution of 9Cr steels at the macroscopic level, a number of material models have been developed, including the power law based unified model of Chaboche [6-9] and the two-layer viscoplasticity model by Farragher et al. [10, 11]. However, to account for the strain-rate effects observed in 9Cr steels and to allow for a varying creep exponent to be defined, a hyperbolic sine constitutive law, such as [12-15], proves more effective. This enables more accurate extrapolation to a wider range of (realistic) loading conditions (see Fig. 1) as observed in plant [12, 16]. Due to the lengthy test times, these realistic plant conditions are not easily achieved in laboratory experiments and thus, reliable extrapolation techniques are vital to obtain results to a high degree of accuracy at the component level.

The current study presents the FE implementation, in implicit form including the consistent tangent stiffness matrix, for a hyperbolic sine, cyclic viscoplastic material model [12]. Applications to a service-aged P91 (SA-P91) martensitic steel are presented. The SA-P91 steel was obtained from a superheater outlet header which was operated under subcritical conditions for 35,186 hrs [17]. The chemical composition of the SA-P91 steel prior to industrial service is contained in Table 1 and details on the heat treatment process, as well as plant start-up and shut-downs, can be found in [17]. Test specimens were manufactured from this material, including both welded and unwelded specimens. In this work, attention is focused on the unwelded specimen high temperature, low cycle fatigue (HTLCF) behaviour for temperatures of 20°C, 400°C and 500°C under strain control conditions, across a range of strain-rates [17, 18].

The material model is calibrated for the SA-P91 steel and then validated against results for a range of uniaxial tests under different loading conditions, at temperatures of 20 °C, 400 °C and 500 °C. The development and implementation of the material Jacobian for efficient FE simulation of the high temperature response of this material is also presented.

NOMENCLATURE

- $b_{\rm i}$ Isotropic hardening decay rate (-).
- *C*_i Hardening modulus (MPa).
- **D** Material Jacobian matrix (MPa).
- *E* Young's Modulus (MPa).
- *f* Viscoplasticity function (MPa).
- G Shear modulus (MPa).
- I Identity matrix (-).
- *k* Initial cyclic yield stress (MPa).
- **n** Normal tensor (-).
- *p* Accumulated effective plastic strain (-).
- *Q* Saturated value of isotropic hardening (MPa).
- *R* Isotropic hardening (MPa).
- **s** Deviatoric stress tensor (MPa).
- **s**^{tr} Deviatoric trial stress tensor (MPa).
- T Temperature (°C).
- **x** Deviatoric kinematic hardening tensor (MPa).
- Z_i Material constants (-).
- α Cyclic viscoplastic material parameter (/s).
- α_{CR} Viscoplastic material parameter (/s).

- α_{COE} Coefficient of thermal expansion (°C/s).
- β Cyclic viscoplastic material parameter (/MPa).
- γ_i Kinematic hardening parameter (-).
- ε Strain tensor (-).
- $\boldsymbol{\epsilon}^{el}$ Elastic strain tensor (-).
- $\boldsymbol{\epsilon}^{\mathrm{pl}}$ Plastic strain tensor (-).
- ϵ^{th} Thermal strain tensor (-).
- σ Stress tensor (MPa).
- $\sigma_{\rm e}$ von Mises stress (MPa).
- $\sigma_{\rm e}^{\rm tr}$ von Mises trial stress (MPa).
- σ^{exp} Experimental stress (MPa).
- σ^{th} Theoretical stress (MPa).
- σ_v Viscous stress (MPa).
- *v* Poisson's ratio (-).
- χ Kinematic hardening tensor (MPa).
- Λ Elasticity matrix (MPa).

MATERIAL MODEL

The material model is based on a hyperbolic sine constitutive equation [12]. This model represents a variation of the Chaboche unified power law [6, 7], by enabling strain-rate independence of the material parameters. For the hyperbolic sine model, the constitutive equation describing the accumulated effective plastic strain-rate is defined as [12]:

$$\dot{p} = \alpha \sinh\beta f \tag{1}$$

where the function *f* is given by:

$$f = J_2(\boldsymbol{\sigma} - \boldsymbol{\chi}) - R - k \tag{2}$$

In equation (2), the back stress tensor, χ , accounts for the Bauschinger effect and the isotropic softening term, *R*, simulates the cyclic softening behaviour observed in 9Cr steels. The increment in stress, $\Delta \sigma$, is defined using the multi-axial form of Hooke's law:

$$\Delta \boldsymbol{\sigma} = \boldsymbol{\Lambda} : \Delta \boldsymbol{\varepsilon}^{\text{el}} = \boldsymbol{\Lambda} : \left(\Delta \boldsymbol{\varepsilon} - \Delta \boldsymbol{\varepsilon}^{\text{pl}} - \Delta \boldsymbol{\varepsilon}^{\text{th}} \right)$$
(3)

where the increment in plastic strain is determined using the hyperbolic sine flow rule. The flow rule, which is derived from a thermodynamic framework [12], is defined as:

$$\dot{\boldsymbol{\varepsilon}}^{\rm pl} = \dot{p}\mathbf{n} = \frac{3}{2}\alpha \sinh\beta f \frac{\mathbf{s} - \mathbf{x}}{J_2(\mathbf{\sigma} - \mathbf{\chi})} \tag{4}$$

The kinematic hardening evolution model consists of two hardening terms, one describing the initial portion of strain hardening and the second accounting for the later region of hardening (see Fig. 2), such that:

$$\boldsymbol{\chi} = \sum_{i=1}^{2} \boldsymbol{\chi}_{i}$$
⁽⁵⁾

The individual kinematic hardening evolution components are described using an Armstrong-Frederick kinematic hardening model [19], with temperature rate terms included to account for variations in the material parameters as a function of temperature:

$$\dot{\mathbf{\chi}}_{i} = \frac{2}{3} C_{i} \dot{p} \mathbf{n} - \gamma_{i} \mathbf{\chi}_{i} \dot{p} + \frac{1}{C_{i}} \frac{\partial C_{i}}{\partial T} \mathbf{\chi}_{i} \dot{T}$$
⁽⁶⁾

The isotropic hardening model, R, is also made up of two non-linear components, describing the initial softening region and the softening behaviour prior to the onset of failure:

$$R = \sum_{i=1}^{2} R_i \tag{7}$$

and the evolutions of the individual terms are described using the following equation [20]:

$$\dot{R}_{i} = b_{i} \left(Q_{i} - R_{i} \right) \dot{p} + \left(\frac{1}{b_{i}} \frac{\partial b_{i}}{\partial T} + \frac{1}{Q_{i}} \frac{\partial Q_{i}}{\partial T} \right) R_{i} \dot{T}$$
⁽⁸⁾

As described in [9], a negative value of Q enables the cyclic softening phenomena to be modelled at a continuum level.

FINITE ELEMENT IMPLEMENTATION

The above material model is implemented in a UMAT user material subroutine for use with the commercial FE code Abaqus. If viscoplastic behaviour has occurred, i.e. f > 0 and $(\partial f/\partial \sigma):(\partial \sigma/\partial t) > 0$, an implicit integration scheme is used to obtain a converged solution for the increment in accumulated effective plastic strain, such that:

$$\Delta p = \Delta p + \mathrm{d}\Delta p \tag{9}$$

and the iterative increment in effective plastic strain, $d\Delta p$, for the isothermal case is defined as:

$$d\Delta p = \frac{\varphi \cdot (\Delta p / \Delta t)}{\left[1 / \Delta t + 3GZ + ZC_i - Z\mathbf{n} : \mathbf{\chi}_i \gamma_i + Zb_i (Q_i - R_i)\right]}$$
(10)

where Z is $\partial \varphi / \partial \sigma$ and φ is defined in Annex A. The UMAT user subroutine requires the material Jacobian to be provided at every increment for which the UMAT is called. The material Jacobian relates the differential increment in stress, $\partial \delta \sigma$, to the differential increment in strain, $\partial \delta \varepsilon$, such that:

$$\mathbf{D} = \frac{\partial \delta \boldsymbol{\sigma}}{\partial \delta \boldsymbol{\varepsilon}} \tag{11}$$

To produce more efficient simulations with reduced runtime, the consistent tangent stiffness (CTS) matrix may be used to evaluate the material Jacobian. For the current model, the CTS is determined here as (see Annex A for derivation and definition of terms):

$$\delta \boldsymbol{\sigma} = Z_1 \delta \boldsymbol{\varepsilon} + \mathbf{I} Z_2 \mathbf{I} : \delta \boldsymbol{\varepsilon} + \mathbf{n} Z_3 \mathbf{n} : \delta \boldsymbol{\varepsilon} - \mathbf{n} Z_4 n_{\mathrm{tr}} \mathbf{I} : \delta \boldsymbol{\varepsilon} - \boldsymbol{\chi}_i Z_5 \mathbf{n} : \delta \boldsymbol{\varepsilon} + \boldsymbol{\chi}_i Z_5 n_{\mathrm{tr}} \mathbf{I} : \delta \boldsymbol{\varepsilon} + \mathbf{Z}_{\mathrm{E}} (Z_7 \mathbf{n} : \delta \boldsymbol{\varepsilon} - Z_8 n_{\mathrm{tr}} \mathbf{I} : \delta \boldsymbol{\varepsilon})$$
(12)

CALIBRATION OF THE MATERIAL PARAMETERS

The calibration and validation of the model for the present material is based on the program of HTLCF testing presented previously for this material, as part of the same research project, by Farragher et al. [17] and Hyde et al. [18]. The tests were carried out using the TMF test machine at the University of Nottingham and the HTLCF test rig at NUI Galway. These test programs include the tests employed here for calibration and validation, viz. strain-controlled isothermal fatigue and stress relaxation tests at 20 °C, 400 °C and 500 °C. The intended application of the calibrated material model is to the thermo-mechanical analysis of branched pipes in power plant header components, as studied by Farragher et al. [10], for example.

The set of required parameters for the material model may be obtained using a standalone uniaxial implementation of the material model, partly following the work of Zhan [22]. As described in [12], the

 $\langle \mathbf{o} \rangle$

material parameters are divided into three distinct groups, the elastic, cyclic plastic and cyclic viscoplastic material parameters.

For the elastic material parameters, the Young's modulus, E, is easily identified from monotonic test data (the first quarter cycle of the initial loop). Poisson's ratio, v, is assumed to have a constant value of 0.3 throughout and the elastic material parameters are presented in Table 2.

Once the elastic constants have been determined, the isotropic hardening parameters, Q_i and b_i may be estimated. Firstly a value of Q (normally an asymptotic, saturated value of R) is obtained from the experimental data by plotting $R = \sigma_{0,\text{max}} \cdot \sigma_{\text{max}}$ against $p = \text{abs}(2N\varepsilon^p)$. As the experiments do not exhibit a clear softening saturation plateau, the value of Q is taken to be the cyclic softening value just prior to the onset of failure, as highlighted in Fig. 3. Splitting the experimental data into its primary, secondary and failure regions enables the values of Q_i to be determined, such that:

$$Q = Q_1 + Q_2 \tag{13}$$

where Q_1 is the value of isotropic hardening at the transition to the secondary softening regime. The values of b_i , which describe the rate of decay of R_i , are then fitted to the experimental data using a least squares approach. The least squares optimisation function to be minimised is:

$$\mathbf{F}(\boldsymbol{X}_{k}) = \sum_{j=1}^{m} \left(\sigma^{\text{th}}(\boldsymbol{X}_{k}) - \sigma^{\exp} \right)^{2}$$
⁽¹⁴⁾

where x_k represents the material parameters to be optimised. As Fig. 3 illustrates, the inclusion of an additional non-linear isotropic hardening parameter compared with a previous implementation of the model [12], has resulted in a greatly improved isotropic hardening model when compared with the experimental data.

The kinematic hardening material parameters are identified following the process outlined in [22, 23]. For uniaxial, isothermal loading, the stress may be split into its various components as:

$$\sigma = \chi + (R + k + \sigma_{v}) \operatorname{sgn}(\sigma - \chi)$$
⁽¹⁵⁾

Equations (6) and (8) may be integrated to give:

$$\chi_{i} = \frac{C_{i}}{\gamma_{i}} \left(1 - e^{-\gamma_{i} \varepsilon^{pi}} \right)$$
⁽¹⁶⁾

$$R_{i} = Q_{i} \left(1 - e^{-b_{i} p} \right) \tag{17}$$

Differentiating equation (15) with respect to the plastic strain (ε^{pl}) and rearranging gives:

$$\ln\left(\frac{\partial\sigma}{\partial\varepsilon^{pl}} - Q_i b_i e^{-b_i p}\right) = \ln(\mathcal{C}_i) - \gamma_i \varepsilon^{pl}$$
⁽¹⁸⁾

In equation (18), the term $\partial \sigma / \partial \varepsilon^{\text{pl}}$ may be evaluated using a process involving a Ramberg-Osgood smoothing function [22]. Assuming that the second kinematic hardening term dominates at the later stages of strain hardening, as illustrated in Fig. 2, plotting $\ln(\partial \sigma / \partial \varepsilon^{\text{pl}} - \partial R / \partial \varepsilon^{\text{pl}})$ against ε^{pl} enables the material parameters C_2 and γ_2 to be evaluated. Fig. 4 shows a typical correlation obtained using this procedure at a temperature of 500°C. The process is then repeated to obtain C_1 and γ_1 , by plotting $\ln(\partial \sigma / \partial \varepsilon^{\text{pl}} - \partial R / \partial \varepsilon^{\text{pl}} - \partial \chi_2 / \partial \varepsilon^{\text{pl}})$ against ε^{pl} . The cyclic plasticity material parameters for SA-P91 steel are shown in Table 3.

The value of the cyclic yield stress, k, is obtained by plotting stress versus plastic strain and identifying k from the linear (vertical) portion of the reversal loading, as highlighted in Fig. 5.

The cyclic viscoplastic material parameters are determined using the stress relaxation material response. During a stress relaxation (dwell) test, the strain is held constant and thus:

$$\dot{\varepsilon} = \dot{\varepsilon}^{\rm el} + \dot{\varepsilon}^{\rm pl} = 0 \tag{19}$$

Rearranging and integrating equation (19), the stress as a function of time may be defined as:

$$\sigma = \frac{2}{\beta} \tanh^{-1} \left(\tanh\left(\frac{\beta(\sigma_0 - \chi - k)}{2}\right) e^{-\alpha\beta E(t-t_0)} \right) + \chi + k$$
⁽²⁰⁾

where cyclic softening (*R*) is assumed to have a negligible effect during a stress relaxation test. The parameters α and β are then identified using equation (20) and the least squares fitting procedure (see equation (14)). Fig. 6 illustrates the stress relaxation results predicted by the model during the calibration of the cyclic viscoplastic material parameters and the cyclic viscoplastic material parameters are shown in Table 2.

Figs. 7 and 8 illustrate the final calibrated accuracy of agreement achieved with the experimental data, for the initial and 100^{th} cycles at a strain-rate of 0.1 %/s and a strain range of ±0.5 %.

RESULTS: VALIDATION OF THE MATERIAL MODEL

The material model was validated via comparison with results of isothermal fatigue tests conducted at alternative strain-rates of 0.033 %/s and 0.025 %/s. Figures 9 and 10 show the typical correlation obtained with the experimental data for various cycles at the stated temperatures and strain-rates. In all cases, the predicted results correlate closely with the experimental data, illustrating the capability of the hyperbolic sine material model to accurately capture the constitutive behaviour of the SA-P91 steel, including the cyclic stress-strain, cyclic softening and strain-rate effects. The quality of agreement achieved at 20 °C is similar to that presented in Figs. 9 and 10.

DISCUSSION

The results presented for the FE implementation of the material model illustrate the capability to capture complex high temperature cyclic viscoplastic material behaviour, in this case for a service-aged 9Cr steel. Fig. 1 demonstrates the potential associated with using a hyperbolic sine flow rule, with respect to extrapolation to conditions typically observed in modern plant. This capability leads to the determination of strain-rate independent material parameters, which is of vital importance for life prediction at the component level.

This benefit of the hyperbolic sine cyclic viscoplasticity model is also demonstrated in the current set of results, during both the calibration and validation stages. As stress relaxation behaviour covers a range of strain-rates, Fig. 6 illustrates that the material model has successfully captured the strain-rate effect across a range of temperatures. Figs. 7 and 8 show the cyclic stress-strain response achieved during the calibration process and the subsequent correlation achieved with the experimental data in Figs. 9 and 10 illustrates that the model captures the strain-rate effect outside the calibrated regime.

The inclusion of a material Jacobian based on the consistent tangent stiffness matrix for the material model gives a 10 % reduction in runtime for the uniaxial simulations performed here. This result is highly beneficial as the generation of an efficient material model to capture the highly non-linear material behaviour of 9Cr steels will enable efficient simulation of complex component geometries, e.g. see [10]. This will in turn facilitate efficient prediction of component failure.

This paper also illustrates some key characteristics which are required for material parameter identification. In particular, the calibration of the cyclic viscoplastic parameters, α and β , is dependent on the full set of material parameters and any inaccuracies in the other material parameters hinders the ability of the model to capture the viscous behaviour of the material. This is most notable for the kinematic

hardening parameters, where a poor choice of constants which may correlate well with the cyclic data for a given strain-rate, will produce deviations between the predicted and measured relaxation data.

Future work is focused on achieving cyclic viscoplastic material parameters from longer-term stress relaxation tests to ensure that more accurate relaxation behaviour is captured by the model, particularly at higher temperatures, where strain-rate effects become more important. The possibility of generating global optimisation techniques, such as implemented in [24, 25], will also be investigated. However, to ensure that the strain-rate effect is captured accurately, it is imperative that any optimisation technique does not remove the physical basis of the current set of material parameters.

CONCLUSIONS

A hyperbolic sine, unified cyclic viscoplastic material model is implemented in a UMAT user subroutine and applied to the high temperature, low cycle fatigue behaviour of a service-aged P91 steel. Some key findings are:

- The identification and implementation of a consistent tangent stiffness matrix for the material Jacobian gives significant runtime savings for modelling uniaxial test cases, leading to more efficient computational modelling of complex geometries and loading conditions.
- For temperatures above 400 °C, the choice of the 13 independent material parameters must be determined with care to capture the strain-rate effect accurately.
- In a unified formulation, the identification of the optimum viscous parameters is influenced by the cyclic plasticity parameters. This requires careful correlation with the measured stress relaxation behaviour to ensure more reliable extrapolation to strain-rates outside the regime of calibration.
- The addition of a second non-linear isotropic hardening term results in significantly better correlation with experimental softening data.

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ANNEX A: CONSISTENT TANGENT STIFFNESS

For axisymmetric loading conditions, the material Jacobian is given as:

$$\mathbf{D} = \frac{\partial \delta \boldsymbol{\sigma}}{\partial \delta \boldsymbol{\epsilon}} = \begin{pmatrix} \frac{\partial \delta \sigma_{11}}{\partial \delta \varepsilon_{11}} & \frac{\partial \delta \sigma_{11}}{\partial \delta \varepsilon_{22}} & \frac{\partial \delta \sigma_{11}}{\partial \delta \varepsilon_{33}} & \frac{\partial \delta \sigma_{11}}{\partial \delta \varepsilon_{12}} \\ \frac{\partial \delta \sigma_{22}}{\partial \delta \varepsilon_{11}} & \frac{\partial \delta \sigma_{22}}{\partial \delta \varepsilon_{22}} & \frac{\partial \delta \sigma_{22}}{\partial \delta \varepsilon_{33}} & \frac{\partial \delta \sigma_{22}}{\partial \delta \varepsilon_{12}} \\ \frac{\partial \delta \sigma_{33}}{\partial \delta \varepsilon_{11}} & \frac{\partial \delta \sigma_{33}}{\partial \delta \varepsilon_{22}} & \frac{\partial \delta \sigma_{33}}{\partial \delta \varepsilon_{33}} & \frac{\partial \delta \sigma_{33}}{\partial \delta \varepsilon_{12}} \\ \frac{\partial \delta 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The material Jacobian may be obtained by defining the consistent tangent stiffness (CTS) matrix, i.e. by relating the differential stress increment, $\delta\Delta\sigma$, to the differential strain increment, $\delta\Delta\epsilon$. For the current model, the tensor normal may be defined as [26]:

$$\mathbf{n} = \frac{3}{2} \frac{\mathbf{s}^{\text{tr}} - \mathbf{x}_{\text{t}}}{\sigma_{\text{e}}^{\text{tr}}} = \frac{3}{2} \frac{\mathbf{s} - \mathbf{x}}{\sigma_{\text{e}}}$$
(A2)

where the superscript tr denotes a trial value and the subscript t denotes the previous increment. Rearranging equation (A2) and applying the differential operator gives:

$$\delta \mathbf{s} = \frac{\sigma_{e}}{\sigma_{e}^{tr}} \delta \mathbf{s}^{tr} + \left(\mathbf{s}^{tr} - \mathbf{x}_{t} \right) \left(\frac{\sigma_{e}^{tr} \delta \sigma_{e} - \sigma_{e} \delta \sigma_{e}^{tr}}{\left(\sigma_{e}^{tr} \right)^{2}} \right) + \delta \mathbf{x}$$
(A3)

where $\delta \mathbf{x}_t = 0$ as \mathbf{x}_t is a constant. As $\mathbf{n}:\mathbf{n}=3/2$ and using equation set (A9) below, equation (A3) becomes:

$$\delta \mathbf{s} = \frac{\sigma_{e}}{\sigma_{e}^{tr}} \delta \mathbf{s}^{tr} + \frac{2}{3} C_{i} \delta \Delta p \mathbf{n} - \gamma_{i} \boldsymbol{\chi}_{i} \delta \Delta p + \left(\frac{\mathbf{s}^{tr} - \mathbf{x}_{t}}{\sigma_{e}^{tr}}\right) \left(\mathbf{n} : \delta \mathbf{s}^{tr} - 3G \delta \Delta p - (C_{i} - \mathbf{n} : \gamma_{i} \boldsymbol{\chi}_{i}) \delta \Delta p - \frac{\sigma_{e}}{\sigma_{e}^{tr}} \mathbf{n} : \delta \mathbf{s}^{tr}\right)$$
(A4)

Thus, the only unknown is the differential increment in effective plastic strain, which is defined as:

$$\delta \Delta p = \varphi(\mathbf{\sigma}, \mathbf{\chi}, R) \Delta t = (\delta \varphi(\mathbf{\sigma}) : \delta \mathbf{\sigma} + \delta \varphi(\mathbf{\chi}) : \delta \mathbf{\chi} + \delta \varphi(R) \delta R) \Delta t$$
(A5)

where

$$\varphi = \dot{p} = \alpha \sinh\beta \left(\sigma_{\rm e}^{\rm tr} - 3G\Delta p - R - k\right) \tag{A6}$$

The derivatives required in equation (A5) are:

$$\delta \varphi(\mathbf{\sigma}) = \alpha \beta \cosh \beta (\sigma_{e}^{tr} - 3G\Delta p - R - k)\mathbf{n} = Y\mathbf{n}$$

$$\delta \varphi(\mathbf{\chi}) = -\alpha \beta \cosh \beta (\sigma_{e}^{tr} - 3G\Delta p - R - k)\mathbf{n} = -Y\mathbf{n}$$

$$\delta \varphi(R) = -\alpha \beta \cosh \beta (\sigma_{e}^{tr} - 3G\Delta p - R - k) = -Y$$

Thus, equation (A5) becomes:

$$\delta\Delta p = \frac{Y\mathbf{n} : \delta\sigma^{\mathrm{tr}}}{\left(\frac{1}{\Delta t} + 3GY + YC_{\mathrm{i}} - Y\mathbf{n} : \gamma_{\mathrm{i}}\boldsymbol{\chi}_{\mathrm{i}} + Yb_{\mathrm{i}}(Q_{\mathrm{i}} - R_{\mathrm{i}})\right)} = \frac{Y\mathbf{n} : \delta\sigma^{\mathrm{tr}}}{D}$$
(A7)

Substituting back into equation (A4) and using the equations listed below in (A9) and (A10), the CTS is given by:

$$\delta \boldsymbol{\sigma} = Z_1 \delta \boldsymbol{\varepsilon} + \mathbf{I} Z_2 \mathbf{I} : \delta \boldsymbol{\varepsilon} + \mathbf{n} Z_3 \mathbf{n} : \delta \boldsymbol{\varepsilon} - \mathbf{n} Z_4 n_{\rm tr} \mathbf{I} : \delta \boldsymbol{\varepsilon}$$
(A8)
$$- \boldsymbol{\chi}_i Z_5 \mathbf{n} : \delta \boldsymbol{\varepsilon} + \boldsymbol{\chi}_i Z_5 n_{\rm tr} \mathbf{I} : \delta \boldsymbol{\varepsilon} + \mathbf{Z}_{\rm E} (Z_7 \mathbf{n} : \delta \boldsymbol{\varepsilon} - Z_8 n_{\rm tr} \mathbf{I} : \delta \boldsymbol{\varepsilon})$$

where $\mathbf{Z}_{E}=(\mathbf{s}^{tr}-\mathbf{x}_{t})$ and Z_{i} are material parameters related to the constants obtained in Tables 2 and 3, as well as the von Mises stress, σ_{e} , and trial stress, σ_{e}^{tr} and described in Table A1. Equation (A8) may be differentiated with respect to the strain tensor to populate the matrix in equation (A1). The following two sets of equations are used extensively throughout the development of the CTS:

$$\delta \sigma_{e} = \mathbf{n} : \delta(\mathbf{s} - \mathbf{x})$$

$$\delta \sigma_{e}^{tr} = \mathbf{n} : \delta \mathbf{s}^{tr}$$

$$\delta \sigma = \delta \sigma^{tr} - 2G \delta \Delta p \mathbf{n}$$

$$\delta \mathbf{s} = \delta \mathbf{s}^{tr} - 2G \delta \Delta p \mathbf{n}$$

$$\delta \mathbf{x} = \frac{2}{3} C_{i} \delta \Delta p \mathbf{n} - \gamma_{i} \mathbf{\chi}_{i} \delta \Delta p + \frac{1}{C_{i}} \frac{\partial C_{i}}{\partial T} \mathbf{\chi}_{i} \Delta T$$

$$\delta R = b_{i} (Q_{i} - R_{i}) \delta \Delta p + \left(\frac{1}{b_{i}} \frac{\partial b_{i}}{\partial T} + \frac{1}{Q_{i}} \frac{\partial Q_{i}}{\partial T}\right) R_{i} \Delta T$$
(A9)

where the effects of the temperature rate terms are independent of strain and go to zero. The second set of equations are [26]:

$$\delta \mathbf{s}^{\text{tr}} = 2G\delta \boldsymbol{\zeta} = 2G\delta \boldsymbol{\varepsilon} - \frac{2}{3}G(\delta \boldsymbol{\varepsilon} : \mathbf{I})\mathbf{I}$$

$$\delta \mathbf{s} = \delta \boldsymbol{\sigma} - K\mathbf{I}\mathbf{I} : \delta \boldsymbol{\varepsilon}$$
(A10)

Table 1: Chemical composition of the SA-P91 steel prior to industrial application [17].

Element	С	Mn	Si	Ν	Cr	Mo	Nb/C b	Cu	V	Al	Р	Ni
wt%	0.1	0.42	0.26	0.058	8.48	0.94	0.073	-	0.204	0.007	0.013	0.19

 Table 2: Identified elastic and cyclic viscoplastic material constants.

<i>T</i> (°C)	E (GPa)	α (/s)	β (/MPa)		
20	211	1.0×10 ⁻⁸	0.12		
400	185	2.1×10 ⁻⁷	0.07		
500	170	5.4×10 ⁻⁶	0.04		

 Table 3: Cyclic plasticity material parameters.

<i>T</i> (°C)	k (MPa)	C ₁ (MPa)	? 1	C ₂ (MPa)	¥2	<i>Q</i> ₁ (MPa)	b_1	Q ₂ (MPa)	b_2
20	160	390400.8	1612.8	68696.1	420.1	-47.8	148.0	-33.7	2.9
400	135	352921.2	2306.6	48825.1	416.9	-41.9	70.0	-41.2	1.6
500	130	245241.8	2700.4	47954.1	319.7	-52.8	38.2	-51.9	1.4

Table A1: Definition of the terms in the CTS.

 Z_{7}

$$Z_{1} = 2G \frac{\sigma_{e}}{\sigma_{e}^{\text{tr}}}$$

$$Z_{2} = K - \frac{1}{3}Z_{1}$$

$$K = \text{bulk modulus}$$

$$Z_{3} = 2G \frac{2}{3}C_{i}\frac{Y}{D}$$

$$Z_{4} = \frac{1}{3}Z_{3}$$

$$Z_{5} = 2G\gamma_{i}\frac{Y}{D}$$

$$Z_{6} = \frac{1}{3}Z_{5}$$

$$= 2G \left(\frac{1}{\sigma_{e}^{\text{tr}}} - \frac{1}{\sigma_{e}^{\text{tr}}}3G\frac{Y}{D} + \mathbf{n} : \mathbf{\chi}_{i}\frac{1}{\sigma_{e}^{\text{tr}}}\gamma_{i}\frac{Y}{D} - \frac{\sigma_{e}}{\sigma_{e}^{\text{tr}^{2}}}\frac{Y}{D}\right)$$

$$Z_{8} = \frac{1}{3}Z_{7}$$



Figure 1: Comparison of the hyperbolic sine material model with experimental data for a 9Cr steel [21], illustrating the extrapolation capability of the model.



Figure 2: Kinematic hardening regions for material parameter identification.



Figure 3: Comparison of the isotropic hardening model with the experimentally measured softening data for a strain-rate of 0.1 %/s and strain range of ± 0.5 % at 500 °C.



Figure 4: Kinematic hardening parameter identification for the later stages of strain hardening at a temperature of 500 °C.



Figure 5: Identification of the cyclic yield stress value at a temperature of 400 °C.



Figure 6: Identification of the cyclic viscoplastic material parameters at temperatures of 400 $^{\circ}$ C and 500 $^{\circ}$ C.



Figure 7: Comparison of the model predicted results with experimental data, for the initial and 100^{th} cycles at 400 °C and a strain-rate of 0.1 %/s.



Figure 8: Comparison of the model and experimental stress-strain response at a temperature of 500 $^{\circ}$ C and a strain-rate of 0.1 %/s for the initial and 100th cycles.



Figure 9: Comparison of FE predicted results with experimental data for the initial and 100th cycles at a temperature of 400 °C and a strain-rate of 0.033 %/s.



Figure 10: Comparison of FE predicted results with experimental data for the initial and 100^{th} cycles at a temperature of 500 °C and a strain-rate of 0.025 %/s.

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