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Community Topic Usage in Social Networks

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ABSTRACT
When studying large social media data sets, it is useful to reduce the dimensionality of both the network (e.g. by finding communities) and user-generated data such as text (e.g. using topic models). Algorithms exist for both these tasks, however their combination has received little attention and proposed models to date are not scalable (e.g.: [4]). One approach to such combined modelling is to perform community and topic modelling independently and later combine the results. In the case of overlapping communities, this combination requires a method for attributing each users topic usage to the communities in which she participates. This paper presents a Bayesian model for attributing individual documents to communities which balances the users proportional community membership with community topic coherence. Community topic usage is modelled with a Dirichlet distribution with fixed concentration parameter, leading to a well defined conjugate prior. Thought the prior is computationally expensive, the already reduced dimensionality in both topics and communities make a tractable algorithm feasible, even for large data sets. The model is applied to a corpus of tweets and twitter follower relations collected on hash tags used by people with eating disorders [14].

Keywords
topic models; community detection; Bayesian inference; conjugate prior; Dirichlet distribution; author community membership

1. INTRODUCTION
Several studies have found that communities in the twitter follower network can act as a kind of forum on particular topics of discussion [9, 8, 7]. In this scenario, tweets intended for such a forum would reflect those topics, whereas tweets by the same users that are intended for other audiences would show distinct topical content. It is the aim of the work presented here to distinguish the intended audience (in terms of follower network communities) of each tweet and in this way estimate the topics used by those communities. One would expect that many twitter users would be members of/contribute to multiple communities, thus one would expect such communities to be overlapping [9].

Approaches to linking social media texts with network communities have been studied previously. Java et.al. [9] performed overlapping community detection on the full Twitter network and identified coherent themes in key terms used by some inferred communities, though they were not clear on how the key terms were identified and did not provide numerical measures of such coherence. There were about 94,000 twitter users in April 2007 when they performed their study, thus scale was less of an issue than today (in 2015 there are over 300 million Twitter users).

Duan et.al. [4] developed a full Bayesian model incorporating both a stochastic block model for community detection and hierarchical Dirichlet process for topic detection. In this model, all of an authors documents are assigned to just one community (hence they do not overlap) and it’s scalability is questionable.

Li et.al. [10] present a different approach to combined community and topic detection by utilising extra thematic metadata — hash tags (twitter data) and publication venue (citation data). The twitter follower network was not utilised. In their model, communities (not documents) have topic mixtures and topics generate both words and hash tags/venues. The twitter data analysed was intended as a summary of hot topics over a 2 month period, in contrast to the data utilised here that intends to capture interactions within a restricted set of twitter communities over a longer period. In such a social data set, follower links are of great importance, as they represent the conduit over which interactions are possible.

Earlier, Li et.al. [11] applied a similar approach to what we propose here, combining the results of community detection and topic modelling and applying the resulting synthesis to social bookmarking data. The community model they ap-
plied, however, did not produce overlapping communities so a naive approach to inferring community topic proportions was effective.

In Section 2 I describe and develop the model, including the conjugate prior to the Dirichlet distribution. In Section 3 we present an algorithm based on Gibbs sampling for estimating the posterior. In Section 4 we develop two metrics for assessing model quality. In Section 5 we describe the data set and contributing topic and community detection models used as an example in this study. In Section 6 we present results showing that the model succeeds in its aims. In Section 7 we summarise the contribution and discuss future work and implications.

2. DOCUMENT ASSIGNMENT MODEL

Assigning documents to their authors communities is done according to two premises: the proportion of an authors documents in a community should reflect the authors proportional community membership and the topic proportions of documents assigned to a community should be somewhat coherent. This is operationalised by the following generative model. Authors community memberships and documents are modelled.

A authors, C communities and N documents are modelled.

![Figure 1: Generative Model](image)

Document community assignments C are generated by a fixed multinomial whose probabilities are the document authors community membership proportions α. For each document d assigned to community c, topic proportions θd are drawn from a Dirichlet distribution (with parameters ξc) for that community. A conjugate prior for the ξ is provided, parametrised by N0 and θ0 (see Section 2.1 for the construction of the prior). The model is summarised in Figure 1. Grey nodes indicate observed or pre-set values.

The probability of assignment of document d with author ad to community c, and the probability of a documents topic distribution θd are as follows:

$$P(d \in c) = \alpha_{ad c} \quad (1)$$

$$P(\theta_d | d \in c, \xi_c) = B(\xi_c)^{-1} \prod_t (\theta_{dt})^{\xi_{ct} - 1} \quad (2)$$

2.1 A Conjugate Prior For Dirichlet Distributions

The Dirichlet distribution is a member of the exponential family of distributions, and as such has a (conjugate) prior with a relatively simple, constant-dimensional Bayesian update. Given the equation for the T dimensional Dirichlet distribution with parameters ζ

$$P(\theta) = B(\zeta)^{-1} \prod_t \theta_t^{\zeta_t - 1} \quad (3)$$

$$B(\zeta) = \prod_t \Gamma(\zeta_t) / \Gamma(\sum_t \zeta_t) \quad (4)$$

where B is the beta function, it is easy to write down a candidate conjugate prior and corresponding posterior update after evidence \(\{\theta_1 \ldots \theta_N\}:

$$P_n(\zeta) \propto B(\zeta)^{-N_0} \prod_t (\theta_{0t})^{\zeta - 1} \quad (5)$$

$$P(\zeta | \theta_1 \ldots \theta_N) \propto B(\zeta)^{-(N_0 + N)} \prod_t \left( \theta_{0t} \prod \theta_{at} \right)^{\zeta - 1} \quad (6)$$

here, n ranges from 1 to N and t from 1 to T. The values for N0 and θ0 can be interpreted in terms of hypothetical prior observations: N0 being the number of prior observations and θ0 the element-wise product of those observations.

Note that due to the Γ(Σc ξ) term in B(ζ), this only defines a probability if Σc ξi is bounded. We could however multiply this candidate by an arbitrary function of ζ and it would remain a conjugate prior (ie: have convenient posterior form and update). For example we could choose to multiply by Γ(Σc ξ)−Σc ξi and the resulting function would have bounded integral (and could thus define a probability). For the purposes of this study, however, we chose instead to fix Σc ξi.

For convenience we will express ζ = Ξ with fixed concentration parameter Ξ = Σc ξc > 0, a scalar, and Σc ξc = 1. ξ ≥ 0. We can now write down the full probability of the model. Taking C_d to represent the allocated community for document d and N_c the number of documents allocated to community c, we have:

$$P(C, \theta, \xi | \alpha, \theta_0, N_0) = \prod_d P(d \in C_d)P(\theta_d | d \in C_d, \xi_d) \prod_c P(\xi_c | N_0, \theta_0)$$

$$\propto \prod_d \alpha_{ad c} \left( B(\Xi_{\xi_d})^{-1} \prod_t (\theta_{dt})^{\Xi_{\xi_d t} - 1} \right) \times \left( \prod_c B(\Xi_{\xi_c})^{-N_0} \prod_t (\theta_{0t})^{\Xi_{\xi_c t} - 1} \right)$$

$$= \prod_c B(\Xi_{\xi_c})^{-(N_0 + N_c)} \prod_{ad \in C_c} \prod_t (\theta_{dt} \prod_{ad \in C_c} \theta_{dt})^{\Xi_{\xi_c t} - 1} \quad (7)$$

3. ESTIMATION

To obtain a maximum a posteriori (MAP) estimate for document-community associations and community topic distributions, we use a modified Gibbs sampling algorithm not dissimilar to that used in [6]. The method iterates between sampling from the posterior distribution of document-community associations and MAP estimation of ξ with those associations fixed.

To sample document community allocations, we need the conditional probability of a documents community membership given the current value of ξ. Omitting inconsequent conditional dependencies and terms independent of d and c, we obtain:
\[ P(d \in c|\xi, \theta) \propto P(d \in c|\xi)P(\theta_d|\xi) \]
\[ \propto \alpha_{dc}B(\Xi_{\xi_d})^{-1}\prod_t(\theta_{dt})^{\Xi_{dt}-1} \tag{8} \]

For the MAP estimation of \( \xi \) we need its conditional probability given current document allocations. Again omitting inconsequent dependencies and terms independent of \( \xi \) and \( c \), and writing \( \theta_c \) for the set of topic proportions for documents in \( c \), we obtain:

\[ P(\xi|C, \theta) \propto P(\theta_c|C, \xi_c)P(\xi_c) \]
\[ \propto \left( \prod_{d\in c}B(\Xi_{\xi_d})^{-1}\prod_t(\theta_{dt})^{\Xi_{dt}-1} \right) \]
\[ \times \left( B(\Xi_{\xi_c})^{-N_0}\prod_t(\theta_{dt})^{\Xi_{dt}-1} \right) \]
\[ = B(\Xi_{\xi})^{-(N_0+N_c)}\prod_t(\theta_{dt})^{\Xi_{dt}-1} \tag{9} \]

Estimates for \( \xi_c \) were obtained from Equation (9) via numerical optimisation. With fixed \( \Xi \) and due to the logarithmic convexity of the Gamma function for positive real numbers, this expression can be seen to be logarithmically concave, thus numerical optimisation of its log can be expected to behave reasonably, as was found to be the case.

Neither Equation (8) nor (9) scale well, however due to the already reduced dimensionality of the input data through topic modelling and community detection algorithms, it has proved tractable on large data sets.

4. METRICS OF MODEL QUALITY

In this unsupervised setting, comparison to a ground truth is impossible. The numerical metrics below attempt therefore to assess the efficacy of the model in terms of the models goals. The metrics were applied both to estimated models and to naive document allocation via community membership proportions (\( \alpha \)) alone. Results are presented in Table 1.

**Community Topic Coherence.**

To capture how effective the models had been at resolving coherent community topic proportions, the conditional entropy of community allocations \( C \) given community topic proportions \( T \) was employed.

\[ H(C|T) = -\sum_c P(c)\sum_t P(t|c)\log_2 P(t|c) \tag{10} \]

This quantity captures the amount of extra information (measured in binary bits) needed to obtain the community document allocations given knowledge of community topic proportions. If the documents associated with a community are faithful to the topic proportions of that community, you would expect this to be low. On the other hand, if they have a diversity in their topic mixes, much extra information would be needed to identify them.

Taking \( N_a \) to be the number of documents from author \( a \) and recalling \( N \) represents the total number of documents, \( \alpha_{ac} \) the affinity of author \( a \) for community \( c \), and \( \theta_{dt} \) the topic proportions of document \( d \), probabilities for naive allocation can be written as follows:

\[ P_{\text{naive}}(c) = \sum_a P(a)P(c|a) = \sum_a \frac{N_a}{N} \alpha_{ac} \]
\[ = \frac{1}{N} \sum_a N_a \alpha_{ac} \]

\[ P_{\text{naive}}(t|c) = \frac{P(t, c)}{P(c)} = \frac{\sum_a P(a)P(t|a)P(c|a)}{\sum_a P(a)P(c|a)} = \frac{\sum_a P(a)\sum_{\xi_{dt} \in a} \theta_{dt}}{\sum_a \sum_{\xi_{dt} \in a} \theta_{dt}} \tag{11} \]

For the estimated models, we use MAP estimates of document allocations for community probability and expected values of the posterior community Dirichlet distributions, which are just their parameters \( \xi_c \), for conditional topic probabilities.

\[ P_{\text{estimated}}(c) = \frac{N_c}{N} \]
\[ P_{\text{estimated}}(t|c) = \xi_{ct} \tag{12} \]

To assess individual communities, we can also calculate the entropy \( H(c|T) \) for some community \( c \):

\[ H(c|T) = -\left[ P(c)\sum_t P(t|c)\log_2 P(t|c) + P(\neg c)\sum_t P(t|\neg c)\log_2 P(t|\neg c) \right] \tag{13} \]

Again it is useful to compare entropies from a naive model and an estimated model. We already have formulae for \( P(c) \) and \( P(t|c) \) (Equations 8 and 9). For a naive model, we have:

\[ P_{\text{naive}}(\neg c) = \frac{1}{N} \sum_a N_a (1 - \alpha_{ac}) \]
\[ P_{\text{naive}}(t|\neg c) = \frac{\sum_a (1 - \alpha_{ac}) \sum_{\xi_{dt} \in a} \theta_{dt}}{\sum_a (1 - \alpha_{ac}) N_a} \tag{14} \]

and for the estimated models, we have:

\[ P_{\text{estimated}}(\neg c) = \frac{N - N_c}{N} \]
\[ P_{\text{estimated}}(t|\neg c) = 1 - \xi_c \tag{15} \]

**Faithfulness to Author Community Membership.**

There can be a tension between respecting author community affinities and creating coherent community topic distributions. A model that produces excellent community topic distributions may require documents to be allocated in different proportions to their authors community affinities.
To assess this disparity, we use the Hellinger distance between estimated author community affinities calculated from document assignments and the actual affinities used as inputs to the model. Kullback-Leibler divergence was also considered, however this leads to uninformative infinite divergences if the estimate for a community is zero and the actual affinity non-zero.

\[
H(P_\alpha(c), P_{\text{estimated}}(c)) = \frac{1}{\sqrt{2}} \sum_c \left( \sqrt{P_\alpha(c)} - \sqrt{P_{\text{estimated}}(c)} \right)^2
\]

\[
= \frac{1}{\sqrt{2}} \sum_c \left( \frac{\sum_a \alpha_{ac}N_a}{N} - \sqrt{\frac{N_c}{N}} \right)^2
\]

Community membership of authors in the Twitter follower network is an indication of who they listen to. The model presented here makes the assumption that documents are divided between those communities in similar proportions to the number of links to those communities, but this may not be the case. The links represent the mix of sources of tweets that a user sees, whereas the documents assigned to a community represent tweets intended to be seen by that community. Proportions of active and passive communication may not always coincide. For example, other users followed for interest as sources of information are unlikely to be considered as targets for published tweets.

As such, we may not necessarily expect complete symmetry between listening (represented here by follower links and α) and speaking (represented by tweets and their allocation to communities), and low similarity may be acceptable. Note that many community affinities are zero, meaning no links exist to members of that community and no communication is possible. In these cases, the affinity is always respected (see Equation 8).

5. DATA SET

Hash tags have been identified as potential symbols of community membership [12, 3]. Drawing on this observation, tweets were collected on a selection of Twitter tags such as #proana, #edproblems and #thinspiration found to be used by the Twitter “pro-anorexia” and eating disorder community between December 2012 and December 2014[14]. During data collection, the lists of friends and followers of the author of each tweet were also collected.

Text Data and Topic Model.
Retweets were removed and Tweets were tokenised by standardising numerous text emoticon forms, isolating punctuation as individual word tokens and converting mixed case words to lower case (all caps words were retained). Url’s, #tags, @mentions and apostrophised words (eg: “didn’t”) were left unchanged. Further pre-processing included removal of word tokens appearing less than 5 times and removal of tweets with less than 3 word tokens. This resulted in a corpus of 262,736 documents and a vocabulary of 18,713 words. A standard latent Dirichlet allocation [2] topic model with 20 topics was inferred for the resulting corpus. The LDA Dirichlet prior on topic/word probabilities was set to \( \beta = 0.01 \). Writing \( N \) for the number of words, \( D \) the number of documents in corpus, \( T \) the number of topics, the parameter for the LDA Dirichlet prior on document/topic probabilities was set \( \alpha = 0.05N/DT \). This value allocates 5% of the probability mass for smoothing. A previous study found evidence that a topic model such as this can have some ability to resolve social psychological constructs such as identity salience [15].

Network Data and Community Model.

We consider only mutual follower links as they indicate some possibility of mutual interaction, a feature we expect of social communities. Initial analysis of the collected network data indicates that many follower links had not been removed since near the beginning of data collection, and in fact the distribution of “last polled times” is roughly linear in time. A rudimentary survival analysis (the median link age for links where both creation and removal events were observed\(^1\)) indicated links on average lasted approximately 96 days. For the community analysis, links that had not been polled within 96 days of the last observation (December 2014) were discarded. Degree one nodes were removed as they are highly likely to be connected to other, unobserved, nodes and communities in the larger Twitter network. The resulting network has 66,744 nodes and 927,594 edges. Overlapping communities were inferred using a mixed membership stochastic block model [5, 1]. Visualisation of the network with community observations (Figure 2) reveals a relatively small number of fairly distinct, well separated communities with the remainder highly interconnected. For this reason we chose to make three models, one with the inferred 183 communities, and two with a smaller number of communities (50 and 20).

Combination.
When combining the two forms of data for this analysis, only users who appeared in both were retained. That is, users who had at least one tweet retained for the topic model as well as a (recently observed) link retained in the network data. This resulted in 15,515 users and 133,851 of their tweets.

6. RESULTS

Models were inferred for several values of Ξ and compared to naive document allocation via community membership proportions \( \alpha \) alone using the metrics presented in Section 4. Initial experiments suggested a value of Ξ ≃ 600 would perform well and models were also estimated with Ξ = 100 and Ξ = 30 for comparison. Results are summarised in Table 1.

Overall Assessment.
As expected, the larger value of Ξ produced more resolved community topic proportions (lower entropy scores) and were less faithful to the community membership information inferred from the network data (Table 1). The increased distance to community membership information was however small compared to the improved resolution of topics for communities, thus higher values of Ξ should be preferred.

The implementation used for experiments presented here assigned batches of 100 documents between estimations of ξ, for communities whose membership had changed. Estimations of ξ were done with scipy.optimize.minimize using the “Nelder-Mead” method [13], a hill climbing simplex method.

\(^1\) weighted by the opportunity to observe links of that duration given the observation window
Figure 2: Network Visualisation with 183 Communities. Colours represent different communities, node size indicates bridgedness. Note few distinct, separated communities and many highly interconnected communities.
The clear associations between topics and communities testifies to the efficacy of the community detection and topic modelling algorithms and supports the hypothesis that communities in the mutual follower network often define fora for discussion or sharing along a particular theme. It also begs the question as to possible biases in the model — is it too good to be true? Further investigation of the model, perhaps using synthetic data sets with known properties may be appropriate to allay such suspicions.

7. CONCLUSIONS

This paper presents an approach to identifying Twitter communities and their topics of discussion. Existing efficient methods for community detection and topic modelling are leveraged and a novel Bayesian model and inference algorithm are developed to associate tweets with communities in which their authors participate.

A Dirichlet distribution is used to model community topic usage, and a conjugate prior for the Dirichlet distribution is developed. A modified Gibbs sampling procedure incorporating alternate sampling of document/community allocations and MAP estimation of community topic Dirichlet distributions is used to estimate the posterior. The MAP estimation step requires costly numerical optimisation, however due to the already reduced dimensionality of the problem (from text topic modelling and network community detection), this remains tractable for reasonably large data sets.

The model is applied to a collection of 262,736 tweets and 441,655 user follow relations collected from public tweets related to “pro-anorexia” and eating disorders. A substantial improvement of community topic coherence is demonstrated relative to a naïve approach that utilises author community membership alone. Results show very distinct community topic usage for more than half the communities. This is a strong result, supporting the hypothesis that communities in the mutual follower network often serve as fora for particular themes, however it is also suggestive of possible inherent bias in the model which should be further investigated.

The principles used and design of the algorithms presented here are a step to understanding the relations between community structures and topic usage with the future aim of developing a joint model of community detection in author networks and topic modelling of document content.

Possible extensions of the model include estimation of the scaling parameter for community topic proportions and introducing a parameter to moderate the relative strength of author community membership and community topic coherence during inference.

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9. REFERENCES

Figure 3: Community topic allocations with 50 communities, $\Xi = 100$. Data point area proportional to expected sum of document topic proportions for a community. Communities ordered by Equation 13.


