The influence of creep on the settlement of foundations supported by stone columns

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THE INFLUENCE OF CREEP ON THE SETTLEMENT OF FOUNDATIONS SUPPORTED BY STONE COLUMNS

by

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Declaration

I, the undersigned, hereby declare that this thesis, entitled 'The influence of creep on foundations supported by stone columns' is entirely my own work. The thesis has not been submitted in whole or in part to any other University or Institution. All sources used have been acknowledged and referenced in the text.

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______________________________    Date: ________________________________

Brian Sexton
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Abstract

Vibro-replacement stone columns are widely used in geotechnical practice to improve the bearing capacity and settlement characteristics of weak natural soils and man-made fills, accelerate consolidation in fine soils and reduce soil liquefaction potential. The vibro-replacement technique is becoming increasingly popular for the treatment of soft cohesive (and often organic) soil deposits in which creep settlements can make up a significant proportion of the total settlement. The majority of field trials, numerical studies, and laboratory tests carried out to date have focused on estimating the improvement to bearing capacity or the reduction in settlement (almost exclusively primary settlement), with very little consideration given to the potential of stone columns to arrest long-term creep settlements. This research addresses this gap in knowledge by using two-dimensional finite element analysis techniques to assess the effectiveness of using vibro-replacement stone columns to treat creep-prone soils.

The majority of the numerical modelling work described in this thesis is based on axisymmetric unit cell models, implemented using PLAXIS 2D, using the Soft Soil Creep (SSC) model to represent the host soil. The SSC model is a three-dimensional isotropic elasto-viscoplastic model suitable for normally consolidated and lightly overconsolidated clays, silts, and peat. The granular column material has been modelled using the elasto-plastic Hardening Soil (HS) model. The soil profile adopted is that of the Bothkennar geotechnical test bed in Scotland, consisting of an overconsolidated crust overlying two layers of soft lightly overconsolidated Carse clay. Simplified single-layer profiles based on the Bothkennar parameters were used for preliminary numerical modelling purposes, before the full profile was considered.

An examination of the evolution of settlement improvement factor (defined as the ratio of the settlement of untreated to treated ground) with time, using PLAXIS 2D, indicates that the inclusion of creep leads to a lower ‘total’ settlement improvement factor than would be obtained for primary consolidation settlement alone. Parametric studies have indicated, as expected, that this effect is more pronounced in situations where creep settlements account for a greater proportion of the total settlement.
The numerical output has also been used to derive separate ‘primary’ and ‘creep’ settlement improvement factors. The ‘primary’ settlement improvement factors, which were found to be in relatively good agreement with a selection of pre-existing analytical formulations (which pertain to primary settlement only), tend to be much larger than the ‘creep’ settlement improvement factors. Nevertheless, the latter factors are greater than unity, suggesting that columns help reduce creep settlements.

Consideration of the variations of radial, vertical, and hoop stresses and strains with time and depth predicted by PLAXIS 2D has highlighted how creep influences the behaviour of the composite system, in particular the stress transfer process from soil to column. As the soil creeps, vertical stress is transferred from soil to column; the amount of stress transferred increases with depth. It is demonstrated in the thesis that the lower ‘total’ settlement improvement factors (owing to the presence of creep) occur because the columns, which have already yielded, are forced to carry additional vertical stress, inducing additional shear-plane formation. In addition, both the radial and hoop stresses in the soil are lower in treated ground than in untreated ground; these radial and hoop stresses are lower in a soil that creeps. The radial stress reduction means that the lateral support imparted onto the column by the soil is lower; this will also contribute to lower ‘total’ settlement improvement factors but is not as influential as the additional yielding caused by the vertical stress transfer process. The hoop stress reductions for the ‘with creep’ case are caused by the additional plastic deformation but do not contribute to the lower ‘total’ settlement improvement factors. The impact of creep on the stress transfer process for floating columns is similar to that for end-bearing columns; the soil is unloaded and the magnitude of this stress transfer (unloading) increases with depth.

Given that this geotechnical problem is being investigated numerically for the first time, it is appropriate that the emphasis of the work is on obtaining practical estimates of the likely behaviour of stone columns in creep-prone soils rather than on the subtleties of complex higher-order models. Nevertheless, selected analyses, repeated using the advanced Creep-SCLAY1S model (which incorporates anisotropy, bonding, and destructuration and is not yet commercially available with PLAXIS), have yielded very similar findings to those obtained using the SSC model. However, settlement improvement factors (‘primary’ and ‘total’) are lower when destructuration is considered because column presence triggers bond degradation.
The majority of existing analytical settlement design methods pertain to primary settlement only, and in the absence of further guidance, designers will tend to apply the same improvement factor to creep settlements as they have estimated for primary settlements. To overcome this, a simplified empirical design procedure that accounts for the influence of creep has been developed based on a parametric study carried out using the SSC model with a view to identifying appropriate variables which influence the aforementioned improvement factors. The parameters have been altered in a specific range so that they are representative of those typically encountered for soft (creep-prone) clays in practice. It is suggested that the method is used in conjunction with an existing primary settlement design method that captures all key features of primary settlement behaviour.

Finally, a novel procedure which uses Cylindrical Cavity Expansion (CCE) theory in conjunction with the conventional ‘wished-in-place’ installation technique to account for column installation has also been used for a selection of analyses. A two-step process was implemented: (i) CCE theory has been used to work out post-installation lateral earth pressure coefficients caused by the lateral expansion of the vibrating poker when columns are installed in soft clay, and (ii) the new earth pressure coefficients have been incorporated in a standard axisymmetric unit cell model to establish their influence on settlement improvement factors for an infinite column grid. This two-step approach can be used as an improvement upon the conventional ‘wished-in-place’ column installation technique, with larger settlement improvement factors predicted when installation (increased earth pressure coefficients) is taken into account. For stage (i), a comparison of the SSC model output with and without creep has been used to give an indication of the possible effect of column construction on lateral earth pressure coefficients surrounding columns in creep-prone soils. The stage (ii) output has indicated that the conclusions heretofore are unaffected if installation is or is not considered; incorporating creep leads to lower settlement improvement factors.
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<td>CCE</td>
<td>Cylindrical Cavity Expansion</td>
</tr>
<tr>
<td>CRS</td>
<td>Constant Rate of Strain</td>
</tr>
<tr>
<td>CSL</td>
<td>Critical State Line</td>
</tr>
<tr>
<td>CSS</td>
<td>Current Stress Surface</td>
</tr>
<tr>
<td>DMT</td>
<td>Dilatometer</td>
</tr>
<tr>
<td>EOP</td>
<td>End of primary</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>ICE</td>
<td>Institution of Civil Engineers</td>
</tr>
<tr>
<td>IL</td>
<td>Incremental Load</td>
</tr>
<tr>
<td>IRC</td>
<td>Irish Research Council</td>
</tr>
<tr>
<td>LIR</td>
<td>Load increment ratio</td>
</tr>
<tr>
<td>MRSS</td>
<td>Mobilised relative shear strength</td>
</tr>
<tr>
<td>MPM</td>
<td>Material Point Method</td>
</tr>
<tr>
<td>NC</td>
<td>Normally consolidated</td>
</tr>
<tr>
<td>NCS</td>
<td>Normal Consolidation Surface</td>
</tr>
<tr>
<td>OC</td>
<td>Overconsolidated</td>
</tr>
<tr>
<td>OCR</td>
<td>Overconsolidation ratio</td>
</tr>
<tr>
<td>POP</td>
<td>Pre-overburden pressure</td>
</tr>
<tr>
<td>PPT</td>
<td>Pore pressure transducer</td>
</tr>
<tr>
<td>RF</td>
<td>Restricted Flow</td>
</tr>
<tr>
<td>SBPM</td>
<td>Self-boring pressuremeter</td>
</tr>
<tr>
<td>SCF</td>
<td>Stress Concentration Factor</td>
</tr>
<tr>
<td>SERC</td>
<td>Science and Engineering Research Council</td>
</tr>
<tr>
<td>TCD</td>
<td>Trinity College Dublin</td>
</tr>
<tr>
<td>TST</td>
<td>Total stress transducer</td>
</tr>
<tr>
<td>VSC</td>
<td>Vibro Stone Column</td>
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### Soil model abbreviations

<table>
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<tr>
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<tr>
<td>ACM</td>
<td>Anisotropic Creep Model</td>
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<tr>
<td>ET</td>
<td>Equivalent Time</td>
</tr>
<tr>
<td>HS</td>
<td>Hardening Soil</td>
</tr>
<tr>
<td>LE</td>
<td>Linear Elastic</td>
</tr>
<tr>
<td>n-SAC</td>
<td>Non-associated structured anisotropic creep model</td>
</tr>
<tr>
<td>MC</td>
<td>Mohr Coulomb</td>
</tr>
<tr>
<td>MCC</td>
<td>Modified Cam Clay</td>
</tr>
<tr>
<td>SS</td>
<td>Soft Soil</td>
</tr>
<tr>
<td>SSC</td>
<td>Soft Soil Creep</td>
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Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$A$</td>
<td>Cross-sectional area of soil unit treated with granular material</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Cross-sectional area of granular column</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of soil in the unit cell ($A_s = A - A_c$)</td>
</tr>
<tr>
<td>$A/A_c$, $A/A_c$</td>
<td>Area-replacement ratio, reciprocal area-replacement ratio</td>
</tr>
<tr>
<td>$C$</td>
<td>Term used for to compare different soil models/scenarios in Chapter 8</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Swelling Index</td>
</tr>
<tr>
<td>$C_c$</td>
<td>Compression Index</td>
</tr>
<tr>
<td>$C_\alpha$</td>
<td>Coefficient of Secondary Compression / Creep Coefficient</td>
</tr>
<tr>
<td>$C_k$</td>
<td>Change of permeability index</td>
</tr>
<tr>
<td>$D_c$</td>
<td>Column Diameter</td>
</tr>
<tr>
<td>$E, E', E_u$</td>
<td>Young’s Modulus, Drained Young’s Modulus, Undrained Young’s Modulus</td>
</tr>
<tr>
<td>$E_i$</td>
<td>Initial/Tangent Modulus</td>
</tr>
<tr>
<td>$E_{50}$</td>
<td>Secant/Triaxial Modulus</td>
</tr>
<tr>
<td>$E_{oed}$</td>
<td>Oedometric Modulus</td>
</tr>
<tr>
<td>$E_{ur}$</td>
<td>Unload-reload Modulus</td>
</tr>
<tr>
<td>$E_c/E_s$</td>
<td>Modular Ratio</td>
</tr>
<tr>
<td>$EA$</td>
<td>Axial Stiffness</td>
</tr>
<tr>
<td>$EI$</td>
<td>Flexural Rigidity</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear Modulus</td>
</tr>
<tr>
<td>$G_s$</td>
<td>Specific Gravity</td>
</tr>
<tr>
<td>$H$</td>
<td>Element Height, Layer Thickness</td>
</tr>
<tr>
<td>$K$</td>
<td>Bulk Modulus</td>
</tr>
<tr>
<td>$K, K_0, K_a, K_p$</td>
<td>Coefficient of lateral earth pressure, at-rest, active, passive</td>
</tr>
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### List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$K_0^{nc}$</td>
<td>Coefficient of lateral earth pressure in the normally consolidated condition</td>
</tr>
<tr>
<td>$L, L_c$</td>
<td>Unit Cell Length, Column Length</td>
</tr>
<tr>
<td>$L_{crit}$</td>
<td>Critical depth at which end-bearing and bulging failure occur simultaneously</td>
</tr>
<tr>
<td>$M, M_c, M_e$</td>
<td>Slope of CSL, Slope of CSL in Compression, Slope of CSL in Extension</td>
</tr>
<tr>
<td>$R_f$</td>
<td>Failure Ratio (HS model)</td>
</tr>
<tr>
<td>$R_{inter}$</td>
<td>Strength reduction factor used to model the loss of strength at the interface</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Column Radius</td>
</tr>
<tr>
<td>$R$</td>
<td>Time resistance, $R = dt/d\varepsilon$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>Coefficient of determination</td>
</tr>
<tr>
<td>$U_y^e$</td>
<td>Yielding Factor: ‘Elastic degree of consolidation at the moment of column yielding’ (Castro &amp; Sagaseta, 2009)</td>
</tr>
</tbody>
</table>

- $a_0, a_f$: Initial cavity radius, final cavity radius (pertaining to CCE analyses)
- $c', c_u$: Effective Cohesion, undrained shear strength
- $c_v$: Coefficient of consolidation
- $e, e_0$: Void ratio, initial void ratio
- $k$: Constant dependent on column arrangement (square, triangular, or hexagonal)
- $k, k_x, k_y, k_0$: Permeability, horizontal permeability, vertical permeability, initial permeability
- $m$: Power dictating the stress dependency of soil stiffness (HS model)
- $n$: Settlement improvement factor, $n = \delta_{untreated}/\delta_{treated}$
- $n_{TOTAL}$: ‘Total’ settlement improvement factor (i.e. primary + creep)
- $n_{CREEP}$: ‘Creep’ settlement improvement factor
- $n_{PRIMARY}$: ‘Primary’ settlement improvement factor
- $n_0$: Priebe’s basic settlement improvement factor
- $n_1$: Priebe’s $n_0$ modified to account for the column compressibility
- $n_2$: Priebe’s $n_1$ modified to account for the bulk densities of the soil and column
- $p, p'$: Mean principal total stress, mean principal effective stress
- $p_a$: Applied load / load level
- $p_{lim}$: Limit pressure (CCE)
- $p_p$: Preconsolidation stress / pressure (3D)
- $p_{ref}^r$: Reference pressure
- $q$: Deviatoric stress
- $q_{as}, q_f$: Asymptotic value of the shear strength, ultimate deviatoric stress (HS model)
<table>
<thead>
<tr>
<th>Abbreviation</th>
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<tbody>
<tr>
<td>$r$</td>
<td>Radial distance from the axis of symmetry</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Time resistance number, $r_s = dR/dt$</td>
</tr>
<tr>
<td>$r, z, \theta$</td>
<td>Cylindrical coordinates (radial, vertical, circumferential)</td>
</tr>
<tr>
<td>$s$</td>
<td>Column Spacing</td>
</tr>
<tr>
<td>$s'$</td>
<td>$s' = (\sigma'<em>{yy} + \sigma'</em>{xx})/2$</td>
</tr>
<tr>
<td>$t, t_e, t_0, t'_e, t'$</td>
<td>Time, equivalent time, time origin defining the beginning of creep, EOP time, effective creep time</td>
</tr>
<tr>
<td>$t$</td>
<td>$t = (\sigma'<em>{yy} - \sigma'</em>{xx})/2$</td>
</tr>
<tr>
<td>$u, \Delta u$</td>
<td>Pore pressure, excess pore pressure</td>
</tr>
<tr>
<td>$u_v$</td>
<td>Vertical displacement</td>
</tr>
<tr>
<td>$w, w_l, w_p$</td>
<td>Moisture Content, Liquid Limit, Plastic Limit</td>
</tr>
<tr>
<td>$w_1, w_2$</td>
<td>Weighting Factors (Creep Settlement Design Method)</td>
</tr>
<tr>
<td>$z$</td>
<td>Depth, Yield Depth</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\Delta \sigma/\sigma$</td>
<td>LIR</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Bulk unit weight</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>Unit weight of pore fluid (water)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Settlement</td>
</tr>
<tr>
<td>$\varepsilon, \varepsilon' , \varepsilon'' , \varepsilon^c$</td>
<td>Total strain, elastic strain, plastic strain, viscoplastic/creep strain</td>
</tr>
<tr>
<td>$\varepsilon^H$</td>
<td>Natural/Logarithmic strain</td>
</tr>
<tr>
<td>$\varepsilon_{yy}, \varepsilon_{yye}$</td>
<td>Vertical (axial) strain, vertical strain at infinite time</td>
</tr>
<tr>
<td>$\varepsilon_{xx}, \varepsilon_{zz}$</td>
<td>Radial strain, hoop (tangential) strain</td>
</tr>
<tr>
<td>$\varepsilon_v, \varepsilon_d$</td>
<td>Volumetric strain, deviatoric strain</td>
</tr>
<tr>
<td>$\dot{\varepsilon}, \dot{\varepsilon}', \dot{\varepsilon}^c$</td>
<td>Total strain rate, elastic strain rate, viscoplastic/creep strain rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Viscosity constant (Newtonian dashpot), Stress ratio ($\eta = q/p'$)</td>
</tr>
<tr>
<td>$\kappa, \kappa^*$</td>
<td>Swelling Indices</td>
</tr>
<tr>
<td>$\lambda, \lambda^*$</td>
<td>Compression Indices</td>
</tr>
<tr>
<td>$\mu, \mu^*$</td>
<td>Creep Coefficients/Indices</td>
</tr>
<tr>
<td>$(\lambda^* - \kappa^<em>)/\mu^</em>$</td>
<td>Creep Ratio</td>
</tr>
<tr>
<td>$\nu, \nu', \nu_{ur}$</td>
<td>Poisson’s ratio, drained Poisson’s ratio, unloading-reloading Poisson’s ratio</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Specific volume, $\nu = 1 + e$</td>
</tr>
<tr>
<td>$\sigma_c, \sigma_s$</td>
<td>stress carried by the column, stress carried by the soil</td>
</tr>
<tr>
<td>$\sigma_c/\sigma_s$</td>
<td>SCF</td>
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### List of Abbreviations

<table>
<thead>
<tr>
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<th>Description</th>
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<tbody>
<tr>
<td>$\sigma, \sigma'$</td>
<td>Normal/vertical total stress, normal/vertical effective stress</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>Preconsolidation stress / pressure (1D)</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Yield stress</td>
</tr>
<tr>
<td>$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$</td>
<td>Total radial, vertical (axial), and hoop (tangential) stresses</td>
</tr>
<tr>
<td>$\sigma'<em>{xx}, \sigma'</em>{yy}, \sigma'_{zz}$</td>
<td>Effective radial, vertical (axial), and hoop (tangential) stresses</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Reference time</td>
</tr>
<tr>
<td>$\tau_{rel}$</td>
<td>Relative shear stress</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>Friction Angle</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Dilatancy Angle, Hyperbolic Law's Creep Parameter</td>
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### Creep-SCLAY1S model parameters

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<th>Description</th>
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<tr>
<td>$\alpha_0, \alpha$</td>
<td>Initial yield surface inclination, yield surface inclination</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Rate of yield surface rotation</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td>Effectiveness of shear and volumetric strains in rotating the yield surface</td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>Initial amount of bonding</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Rate of destructuration</td>
</tr>
<tr>
<td>$\xi_d$</td>
<td>Effectiveness of shear and volumetric strains in destroying the bonding</td>
</tr>
<tr>
<td>$M(\theta)$</td>
<td>Stress ratio at critical state</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Lode Angle</td>
</tr>
<tr>
<td>$\lambda_i, \lambda_i^*$</td>
<td>Intrinsic Compression Indices</td>
</tr>
<tr>
<td>$\dot{A}$</td>
<td>Rate of viscoplastic multiplier</td>
</tr>
<tr>
<td>$S_i$</td>
<td>Sensitivity</td>
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### Subscripts

<table>
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<th>Description</th>
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<td>$c$</td>
<td>Column</td>
</tr>
<tr>
<td>$s$</td>
<td>Soil</td>
</tr>
<tr>
<td>$0$</td>
<td>Initial</td>
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### Superscripts

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<td>$e$</td>
<td>Elastic</td>
</tr>
<tr>
<td>$c$</td>
<td>Creep</td>
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1. Introduction

1.1 Background

The vibro-replacement stone column technique is widely used in geotechnical practice to treat weak natural soils and man-made fills in order to form stable ground conditions for residential and light commercial and industrial structures, e.g. highway embankments, roads, gas plants, wastewater treatment plants (Sondermann & Wehr, 2004, 2012). Vibro-replacement stone columns (vertical inclusions of compact stone) are generally stronger and stiffer than the soil in which they are constructed and therefore increase bearing capacity and reduce settlement of the composite ground. The columns derive their load-carrying capacity from the lateral support imparted onto them by the surrounding soil. Additional benefits of vibro-replacement include the acceleration of consolidation in fine soils and the reduction of soil liquefaction potential, both of which arise due to the stone column drainage effect (Sondermann & Wehr, 2012).

The majority of research to date has focussed on estimating settlement reduction, almost exclusively to primary settlement, with very little consideration given to how stone columns arrest creep settlements (McCabe et al. 2009, Mitchell & Kelly 2013). Since stone columns are now being used extensively in soft cohesive (and often organic) deposits, careful consideration needs to be given to creep settlements. Analytical settlement design methods usually involve the direct prediction of a settlement improvement factor, \( n = \frac{\delta_{\text{untreated}}}{\delta_{\text{treated}}} \) (where \( \delta_{\text{untreated}} \) and \( \delta_{\text{treated}} \) are the settlements of the untreated ground (i.e. no columns) and the ground treated with stone columns respectively) before using this factor to predict settlements of treated ground according to \( \delta_{\text{treated}} = \frac{\delta_{\text{untreated}}}{n} \). The analytical formulations typically estimate the value of \( \delta_{\text{untreated}} \) (for a scenario in which the loading extends over a wide area) using Eq. 1.1, where \( p_a \) is the applied load, \( H \) is the thickness of the treated soil layer, and \( E_{\text{oed}} \) is the oedometric soil modulus. The Priebe (1995) method of estimating \( n \) for primary settlement is the most popular in European geotechnical practice, although other more rigorous methods have emerged in the last few years.

\[
\delta_{\text{untreated}} = \frac{p_a H}{E_{\text{oed}}} \quad (1.1)
\]
Given the increasing application of vibro-replacement to soft cohesive soils, McCabe et al. (2009) compiled a database of \( n \) values derived from measurements for both wide-area loading (e.g. embankments) and small-area loading (e.g. tanks, footings) on these soils. The database confirmed that \( n \) values tend to vary with the proportion of ground replaced with granular material in a similar fashion to that predicted by Priebe’s (1995) basic improvement factor (Eq. 2.8). However, the measured \( n \) values reported in the database compiled by McCabe et al. (2009) tend to be ‘lumped’ values, with no distinction between initial compression (which admittedly, tends to be a small proportion of the total settlement in soft soils), primary consolidation and creep settlements. The monitoring duration required to capture creep settlements reliably in the field serves as the main impediment in the latter case. This is unfortunate as creep settlements can be significant, if not dominant in normally consolidated cohesive and especially organic soils, in some cases contributing to in excess of 50% of the total settlement (e.g. Simons & Som 1970, Simons 1974). Additionally, stone columns (and vertical drains) accelerate consolidation so that creep may contribute to a significant proportion of the post-construction settlement.

While considerable laboratory-scale testing has also been carried out to investigate stone column behaviour (e.g. Muir Wood et al. 2000, McKelvey et al. 2004, Black et al. 2011), it tends to be limited by scale effects and a difficulty in replicating realistic boundary conditions. There are also difficulties associated with extrapolating long-term performance in the field from short-term laboratory tests (Mitchell & Kelly, 2013). Furthermore, the creep potential of reconstituted soil tends to be significantly less than that of an undisturbed soil sample in the field. It should be noted that the terms ‘creep’ and ‘secondary settlement’ are sometimes used interchangeably in the literature. However in this thesis, the term ‘creep’ is used to denote any compression under constant effective stress, whereas the term ‘secondary settlement’ is preferred for those ‘creep’ settlements occurring once primary consolidation has ceased, e.g. Le et al. (2011).

1.2 Aims

Other than an analytical formulation by Madhav et al. (2009, 2010), the development of suitable settlement improvements factors for creep settlements has not been considered in previous research, and in the absence of further guidance, designers sometimes apply the
same \( n \) value to creep settlements as they have estimated for primary settlements. The aim of this research is to carry out a comprehensive interrogation, using finite element (FE) analysis with appropriate constitutive models, of settlement improvement factors appropriate to stone columns in creep-prone clays. The following specific objectives have been identified:

(i) Confirm that 'primary' settlement improvement factors predicted by FE analyses are in keeping with predictions from the more advanced analytical design approaches for primary settlement.

(ii) Calculate separate ‘primary’ and ‘creep’ settlement improvement factors, in the process establishing how creep may influence the conventional ‘lumped’ or 'total' settlement improvement factor.

(iii) Identify the influence of appropriate soil and column variables on the aforementioned settlement improvement factors.

(iv) Examine the behaviour of the composite soil-column system to determine how creep affects the distributions of radial, vertical, and hoop stress and strain with depth.

(v) Assess the impact of soil model complexity on both the aforementioned settlement improvement factors and the distributions of stress and strain.

(vi) Develop a simplified design procedure that can be applied in conjunction with existing approaches to account for the influence of creep on settlement improvement factors.

1.3 Methodology

In this research, a total of 682 distinct two-dimensional axisymmetric analyses have been carried out using the PLAXIS 2D (Brinkgreve et al., 2011) FE program as the basis for establishing the influence of creep on settlement performance. The PLAXIS program is specific to geotechnical engineering and is capable of modelling a wide range of problems, e.g. groundwater flow analysis, tunnel construction, and ground excavation. The soil profile considered for the FE model is that of the Bothkennar geotechnical test bed in Scotland, consisting of an overconsolidated crust overlying two layers of soft lightly overconsolidated Carse clay. Simplified single-layer profiles based on Bothkennar parameters (with the stiff crust omitted in the interests of clarity) are used for preliminary numerical modelling purposes. The multi-layer profile has then been analysed to produce a more realistic model.
This multi-layer profile been used to examine the distributions of stress and strain with time and depth.

Heretofore, numerical studies with the primary goal of understanding stone column behaviour have tended to use either the Mohr Coulomb (MC) or Hardening Soil (HS) models (e.g. Ellouze & Bouassida 2009, Killeen & McCabe 2010), neither of which account for creep. In this thesis, the Soft Soil Creep (SSC) model (as commercially available with PLAXIS software) is used for the majority of the analyses. Given that this geotechnical problem is being investigated numerically for the first time, it is appropriate that the emphasis of the work is on obtaining practical estimates of the likely behaviour of stone columns in creep-prone soils rather than on the subtleties of complex higher-order models. Nevertheless, selected analyses have been repeated using the Creep-SCLAY1S model (Sivasithamparam et al., 2014), which incorporates anisotropy (Section 3.4.2), bonding (Section 3.4.3), and destructuration (Section 3.4.3); this model is not yet commercially available.

1.4 Thesis Outline

The literature review has been split into two chapters. In Chapter 2, the different methods of stone column construction and associated vibratory equipment are described and the stone column installation process is explained. Relevant field measurements, numerical studies and laboratory data pertaining to the behaviour of stone columns in soft soils are reviewed. A selection of popular analytical settlement design methods are then presented, most of which are based on the unit cell concept. These provide a frame of reference for the FE modelling later in the thesis.

In Chapter 3, different approaches that can be used to model time-dependent behaviour are discussed. This chapter also includes a description of a selection of three-dimensional elastoviscoplastic soil models, focusing on tracing how modelling the time-dependent behaviour of soft clays has evolved from the development of the isotropic SSC model up to the advanced Creep-SCLAY1S model.

A detailed description of the PLAXIS 2D material models that are used in the following chapters of the thesis is given in Chapter 4. The mesh sensitivity studies and other additional
checks carried out to ensure accurate and reliable FE results are also summarised in this chapter. Some recognised principles adopted when modelling stone column behaviour using the finite element method (FEM) have also been introduced (e.g. interface elements, installation effects); as have the additional considerations associated with modelling soft soil behaviour (e.g. drained/undrained behaviour). These checks and mesh sensitivity studies are described in detail in Appendix A.

The development of the Bothkennar FE model is described in Chapter 5. The profile was developed based on Killeen’s (2012) PLAXIS 3D Foundation HS model soil profile. The adopted soil profile has been comprehensively validated, e.g. by using the PLAXIS ‘Soil Test’ facility to calibrate the adopted soil parameters against the high quality test data reported in ICE (1992) and by verifying that the model predicts the long-term measured settlement performance of a pad footing at the Bothkennar site reported by Jardine et al. (1995). Equivalence of PLAXIS 2D and 3D Foundation output was also confirmed. The properties derived for the granular column material (modelled using the HS model) have also been described in this chapter, as has the development of simplified single-layer profiles (no crust) used for preliminary numerical modelling purposes.

The preliminary numerical modelling work based on the simplified single-layer profiles is described in Chapter 6. As the research progressed, different approaches were used to establish the influence of creep on the settlement behaviour of vibro-improved ground. These different approaches and their technical hierarchy are discussed. The results section of this chapter concentrates on establishing how creep influences the conventional ‘lumped’ or ‘total’ settlement improvement factor by examining how settlement improvement factors vary with time for models that do and do not account for creep. Attempts have also been made to establish separate ‘primary’ and ‘creep’ settlement improvement factors, and in turn the influence of modular ratio on each has been investigated. Further simulations have been carried out to establish how the proportion of primary to creep settlement influences predicted $n$ values.

In Chapter 7, the multi-layer Bothkennar profile has been analysed using the SSC model to investigate if the findings from Chapter 6 for the single-layer profiles also hold for the multi-layer profile. This chapter also concentrates on examining stress-strain behaviour, in
particular the variations of radial, vertical, and hoop stress and strain with time and depth, and how they are influenced by creep. Additional analyses have also been carried out to investigate if floating columns exhibit fundamentally different behaviour.

In Chapter 8, selected analyses on the multi-layer Bothkennar profile have been repeated using the Creep-SCLAY1S (a description of the theoretical background to the model has also been included). The results attained using this model have been compared to the SSC model output.

Attempts have been made in Chapter 9 to identify the parameters that have the largest influence on 'primary', 'total', and 'creep' settlement improvement factors and the corresponding stress transfer process from soil to column due to creep. To do so, a parametric study has been carried out by altering the soil parameters in a specific range representative of other soft clays, e.g. Swedish clays, which tend to be softer and more creep-prone than Bothkennar clay, Finnish clays, which tend to be firmer, and Norwegian clays, which tend to be comparable with Bothkennar. The majority of these profiles incorporate a crust. Rather than individually analyse each clay type, a ‘customised’ soil profile has been developed for investigation purposes (consisting of a 1m crust overlying 9m of soft clay). The FE output in this chapter has been used to develop an approach which incorporates creep into the design of granular columns. Emphasis was placed on developing an easy-to-use formulation that could be used by the practising engineer. The approach can be used in conjunction with any existing primary settlement design method that captures all key features of primary settlement behaviour; the reliability of different primary settlement design methods has been appraised in Sexton et al. (2013).

In Chapter 10, a novel procedure that has been used to model the stone column installation process is described. Cylindrical Cavity Expansion (CCE) theory has been used to work out post-installation lateral earth pressure coefficients (using the SSC model with and without creep) caused by the lateral expansion of the vibrating poker when columns are installed in soft clay. The new $K$ values have been incorporated in standard axisymmetric unit cell models to establish their influence on settlement improvement factors for an infinite column grid.
The conclusions and recommendations of the thesis are presented in Chapter 11.

**Appendix A** describes a number of preliminary checks have been carried out to guarantee the accuracy of the results obtained using the FEM. Additional results pertaining to the single-layer profiles are presented in **Appendix B**. Design equations pertaining to the Madhav *et al.* (2009, 2010) analytical formulation are presented in **Appendix C**.

**1.5 Publications**

The following papers have already been published in parallel with the development of this thesis:

   This paper is based on the material presented in Section 6.4. It involves a comparison of the commercially available HS, SS, and SSC models with a view to establishing separate ‘primary’ and ‘creep’ n values. The evolution of n with time (and how it is impacted upon by creep) was also examined therein.

   This study was carried out to establish which of the analytical design method(s), based on a comparison with numerical output, captured the factors affecting stone columns and their ability to arrest primary settlements most appropriately. This serves the purpose of establishing which method(s) may be suitable to extend with a view to incorporating creep in the design procedure. This paper does not appear in the body of the thesis but is referred to in places.

This preliminary numerical modelling, which was used to give a first estimation of separate 'primary' and 'creep' $n$ values, was published in ICGI 2012. The approach used in this paper is explained in Section 6.3.


This work, published in BCRI 2012, was carried out as a precursor to the paper submitted to Acta Geotechnica appraising the vibro settlement prediction methods.


This study, presented at ICIEGE 2013, involves modelling the stone column installation process using a procedure which involves CCE theory. It describes an approach that could be used as an improvement upon the conventional ‘wished-in-place’ column installation technique (Chapter 10). The HS model was used for the study published at ICIEGE 2013 but the SSC model has been used for the analyses described in Chapter 10.

The following paper, currently in preparation, is a development of the first publication in the previous list. It is based on the material presented in Section 6.5.

2. Literature Review I - Stone Column Behaviour

2.1 Introduction

In the first section of this chapter, the different components of soil settlement are described to provide context for subsequent references throughout the chapter. The vibro stone column construction process is then introduced and the different methods of column construction and vibratory equipment are described. A discussion follows on the technical aspects of the stone column installation process and some field, numerical, and laboratory studies that have assessed post installation soil properties are reviewed. Next, studies that have examined the settlement performance of stone columns in soft soils are presented. Finally, a selection of analytical settlement design methods are reviewed, and although the majority of these pertain to primary settlement only, they form an important frame of reference for modelling work presented later.

2.2 Settlement Components

Soil settlement typically consists of immediate (undrained) compression, primary consolidation, and secondary compression or creep (Figure 2.1). Tertiary compression may also be a feature of peats and soft clays that possess a significant organic content.

- Immediate compression occurs due to the distortion of soil particles in undrained conditions. No volume change occurs during this initial stage of settlement and no pore pressure dissipation takes place. The immediate settlement can be calculated using elastic theory, e.g. Christian & Carrier (1978), or estimated as a proportion of primary settlement dictated by soil type (Tomlinson, 1995).

- Primary consolidation settlement occurs due to the dissipation of excess pore pressures that develop during load application. When the excess pore pressures have fully dissipated, this marks the end of the primary (EOP) consolidation stage. The primary consolidation process is relatively well understood, initiated by the pioneering work of Terzaghi (1943).
Creep describes the volume change, unrelated to changes in effective stress, which results from the change in void ratio ($\Delta e$) upon readjustment of soil particles into more stable configurations. Considerable debate has raged about whether creep occurs concurrently with primary settlement or not, although nowadays the general consensus points towards the former (see Section 3.2). The subset of creep settlement occurring after primary settlement is complete is referred to by Le et al. (2011) as secondary compression, and it is this portion of creep that is more readily quantified; in general, using the creep coefficient, $C_\alpha$, see Eq. 2.1, where $t$ denotes time.

$$C_\alpha = \frac{\Delta e}{\Delta(\log t)}$$  \hspace{1cm} (2.1)

Tertiary compression occurs subsequent to secondary compression. It is characterised by a non-linear (increasing) slope in the settlement-log(time) plot (e.g. Augustesen et al., 2004), after a period of constant $C_\alpha$, i.e. $C_\alpha$ increases with time at constant effective stress.

![Diagram](image.png)

**Figure 2.1** Immediate compression, primary consolidation, secondary compression and tertiary compression

Greater detail on the creep process and associated approaches to modelling creep are provided in Chapter 3.
2.3 Vibro-Compaction and Vibro-Replacement

The vibro-compaction technique involves using a vibrating poker to compact soil particles into a denser arrangement. The technique was first used by the Keller company in 1936 to densify non-cohesive soils (Sondermann & Wehr, 2012), e.g. Figure 2.2. In cohesive or saturated soils, vibro-compaction would lead to too much liquefying of the soil so that it would either take too long or not occur at all, e.g. Sondermann & Wehr (2012).

![Figure 2.2 Vibro-Compaction and Vibro-Replacement ranges of applicability - adapted from Killeen (2012)](image)

The vibro-replacement / vibro stone column (VSC) technique, which involves filling a temporarily-stable cylindrical cavity with compact stone (filled and compacted in stages using the vibrating poker) overcomes the limitations of the vibro-compaction technique in cohesive soils, and is widely used in Europe to improve weak soils and man-made fills. It is now widely accepted that VSCs reduce settlement (Mitchell & Huber 1985, Watts et al. 2000), improve bearing capacity (Barksdale & Bachus, 1983), and accelerate consolidation (Munfakh et al. 1983, Han & Ye 2001, Castro & Sagaseta 2009). In addition, VSCs provide a suitable economic alternative to piled foundations in certain situations, e.g. for light residential or commercial structures supporting low/moderate loads, while the construction time associated with VSCs can also be significantly shorter than that associated with piling and alternative foundation solutions. Other attractions of the technique include the additional drainage provided by the columns (particularly useful in liquefaction-prone areas) and the ability to use columns in conjunction with ground-bearing slabs as opposed to a system of ground beams and suspended slabs.
2.4 Column Construction

2.4.1 Vibratory Equipment

The vibro-replacement process and equipment have been described in detail by Slocombe et al. (2000) and Sondermann & Wehr (2012). During operation, the vibrating poker is lowered to the required treatment depth using extension tubes suspended from a crane or Vibrocat; a purpose-built tracked base machine that ensures column verticality and allows for application of additional pull-down pressures during penetration and compaction, e.g. Sondermann & Wehr (2012). A typical poker, which weighs 15-40kN with a diameter of 300-500mm and a length of 2-5m, is depicted in Figure 2.3. The poker comprises an eccentric weight powered by a motor (providing the vibratory action), extension tubes to supply energy for the motor, an elastic coupling to prevent vibratory energy from being transferred to the extension tubes, and supply pipes for water or air, depending on the construction method employed (Section 2.4.2). Bottom feed (see Section 2.4.2) pokers also contain delivery tubes to supply the aggregate used in column construction to the poker tip.

![Diagram of a typical bottom feed poker](image)

**Figure 2.3** Typical bottom feed poker - Egan et al. (2008)
2.4.2 Construction Methods

Vibro stone columns can be constructed using either a top feed method or a bottom feed method using either wet or dry jetting processes. In the case of the top feed method, stone is tipped into a hole formed by the vibrating poker, whereas for the bottom feed method, stone is added through a delivery tube along the side of the poker and exits at the poker tip. In both cases, compaction is carried out in stages from the base of the hole upwards. Air is used to aid construction and maintain stability of the hole for the dry method whereas water is used for the same purpose for the wet method. Typical stages in the construction of both wet top feed and dry bottom feed columns are depicted in Figures 2.4 and 2.5.

![Figure 2.4 Stone column construction (Wet top feed) - Raju et al. (2004a)](image)

![Figure 2.5 Stone column construction (Dry bottom feed) - Raju et al. (2004a)](image)
Aggregate of size 40-75mm is used for the top feed method whereas for the bottom feed method, the size of the aggregate used ranges from 15-45mm (McCabe et al., 2007). The wet top feed method is suited to soft cohesive soil deposits where the ground water level is high while the dry top feed method is mainly used for firmer soil deposits with lower ground water levels. The dry bottom feed method is now the preferred construction technique in softer cohesive soil deposits, with its use now largely replacing the wet top feed method since its development in the 1970s, e.g. McCabe et al. (2009). The dry bottom feed method enables columns to be constructed in soils with low undrained shear strengths, \( c_u \ll 15-20 \text{kPa} \) (McCabe et al. 2009, Wehr 2013). Use of the wet method has waned in recent years with the disposal of ‘flush’ becoming an ever-increasing problem from an environmental standpoint.

McCabe et al. (2009) compared the settlement performance of bottom feed and top feed columns measured in the field against design predictions obtained using Priebe's (1976) analytical formulation (see Section 2.7.1.2) adopting a friction angle for the stone (\( \phi'_{cc} \)) of 40°. In general, the bottom feed columns behaved better than predicted, whereas for top feed columns, the opposite was generally the case, e.g. Figure 2.6. This may indicate that \( \phi'_{cc} = 40° \) is a conservative assumption for bottom feed columns, with Herle et al. (2008) advocating the use of higher \( \phi'_{cc} \) values (in excess of 50°).

![Figure 2.6 Predicted versus measured settlement improvement factors for widespread loadings and footings - McCabe et al. (2009)](image-url)
2.5 Column Installation

2.5.1 Background

The installation of a stone column into any host soil invariably results in significant alterations to the stress regime in the ground (Egan et al., 2008). Column installation effects have been investigated by a number of authors, with the majority of studies reporting the benefits of the installation process arising due to the lateral soil displacement and remoulding caused by the vibrating poker as columns are installed, e.g. Debats et al. (2003), Kirsch (2006), Guetif et al. (2007) and Castro & Karstunen (2010). The benefits include:

- The lateral earth pressure coefficient of the in-situ soil increases from its at-rest value ($K_0$) to a post-installation $K$ value. Larger $K$ values and associated increased horizontal stresses in the soil lead to more resistance to lateral bulging of the granular material during load application.
- The installation process generates large excess pore pressures which dissipate during consolidation, thus resulting in increased mean effective stresses and consequently an increase in soil stiffness.

2.5.2 Field Measurements

Field measurements by Kirsch (2006) have indicated $K/K_0$ values in excess of unity post-construction, e.g. Figure 2.7, where the first set of field data (‘Field 1’) pertains to a group of 25 no. 9m long, 0.8m diameter stone columns installed in a silt layer ($K_0 = 0.91$) while the second set of data (‘Field 2’) was obtained for a group of 25 no. 6m long, 0.8m diameter columns installed in a sandy silt ($K_0 = 0.57$). The columns were installed in a square grid in both cases with measurements obtained using earth pressure cells and pore water pressure cells. The lower $K/K_0$ values close to the column were attributed to remoulding and dynamic effects. Kirsch (2006) has also reported increased pore water pressures (occurring immediately following column installation) and soil stiffnesses (up to almost 3 times the initial values, measured using Ménard pressuremeters, see Figure 2.8).
**Figure 2.7** $K/K_0 (= k_{measured}/k_{0,initial})$ versus $a/d_s$ (where $a/d_s$ represents the distance from the centre of the column grid divided by the diameter of one column) - adapted from Kirsch (2006) - also shown are upper and lower bound curves enveloping the $K/K_0$ values.

**Figure 2.8** $E/E_0 (= E_M/E_{M,initial})$ versus $a/d_s$ (where $a/d_s$ represents the distance from the centre of the column grid divided by the diameter of one column) - adapted from Kirsch (2006) - separate upper and lower bound curves enveloping the $E/E_0$ values for the two test fields are also shown.
Castro (2007) measured the excess pore pressure response (using piezometers, denoted ‘PZ’ in Figures 2.9 and 2.10) that occurred following the installation of a group of 7 no. 0.8m diameter, 9m long columns installed in a triangular pattern at a spacing of 2.8m in a soft clay (Figure 2.9). The pore pressures were found to increase during column construction (Figure 2.10), peaking when the vibratory probe reached the piezometer depth (the closer the piezometer to the column, the higher the recorded excess pore pressure). The pore pressures dissipated quickly following column construction, except for the last two columns to be constructed, where the degree of lateral restraint was greater.

![Figure 2.9 Column grid and instrumentation - Castro (2007)](image)

![Figure 2.10 Excess pore pressures during column construction - Castro (2007)](image)
A pore pressure response similar to Castro (2007) has been reported by Gäb et al. (2007) following the installation of a large grid of 37 no. columns (0.7m diameter, 14.5m long) in a triangular pattern at a spacing of ~ 1.70m. The columns were installed in 4 stages (see Figure 2.11), commencing with an outer ring, then a ring second from outside, the central column and finally the last 6 columns (columns outside the ‘test field’ were constructed thereafter). In this case, the maximum excess pore pressures occurred at a depth of 12m, near the base of the columns (the profile was multi-layered with sand above 11m depth and more than 50m of clay thereafter, and so the higher values were observed in the cohesive layer). The pore pressures dissipated quickly in the sand layer but slowly in the clay, with excess pore pressures remaining even one year after construction; the floating columns only penetrated the clay to a depth of 3.5m and so the drainage was not as quick below the base of the columns as it was above.

![Figure 2.11 Installation Sequence - Gäb et al. (2007)](image)

Stone column installation can also generate significant surface heave (Egan et al., 2008), the amount of which is dependent on the number of columns and the column spacing. Large column grids generate more surface heave than single columns or small groups (Egan et al., 2008). Heave measurements have only been recorded in a small number of studies, e.g. Watts et al. (2001), Castro (2007), Gäb et al. (2007) and McCabe et al. (2013). McCabe et al. (2013) have shown that the volume of heaved material displaced corresponds closely to the volume of the column and that the variation of heave magnitude with radial distance from the column agrees well with corresponding measurements for driven piles when normalised by the column radius.
2.5.3 Laboratory Experiments

While Castro et al. (2013) state that the reconstituted soils used in laboratory testing are not fully representative of natural clay behaviour, Lee et al. (2004), Weber et al. (2010), and Gautray et al. (2014) have made efforts to model column installation in the laboratory using centrifuge testing. Lee et al. (2004) measured the short-term radial stress and pore pressure changes that occurred subsequent to the installation of sand compaction piles (i.e. stone columns) in a remoulded and reconsolidated soft Singapore marine clay using total stress transducers (TSTs) and pore pressure transducers (PPTs). The installation process involved jacking a hollow casing into a soft clay bed and then withdrawing the casing while using an Archimedes screw to forcibly inject sand into the clay bed. The casing jack-in process resulted in a gradual stress and pore-pressure build-up, peaking at the point where the casing reached the transducer levels during the jack-in process (Line 1 in Figure 2.12). During the withdrawal and compaction process, the pore pressures and total stresses peak again at the transducer level (Line 3 in Figure 2.12). The magnitudes of the stresses and pore pressures decreased with distance from the sand compaction pile, consistent with the aforementioned field measurements of Castro (2007) and Gäb et al. (2007).

The study by Weber et al. (2010) concentrated on investigating the smear and disturbance effects that occur as a result of the column installation process, both of which negatively impact upon the consolidation performance (smear is when soil and column particles mix at the soil-column interface). Reconstituted samples of natural silty clay from Birmensdorf in Switzerland were used to form the clay bed, with the columns constructed from quartz sand. The columns were installed in the clay bed using a specially developed installation tool that replicates full-scale stone column installation using a dry, bottom-feed displacement method.

Environmental scanning electron microscopy and mercury intrusion porosimetry methods were used to identify three zones of disturbed soil post-installation (Figure 2.13): (i) a penetration zone where the sand particles are squeezed through the clay, (ii) a smear zone where the soil particles were significantly reoriented, and (iii) a densification zone (extending to 2.5 column radii (denoted $r_w$ in Figure 2.13) from the column axis) where the structure of the clay does not change but where compaction is measurable. Beyond this zone, no compaction or change of clay structure was observed.
Figure 2.12 (a) Horizontal stress and (b) pore pressure measurements - Lee et al. (2004)

Figure 2.13 Zones of disturbance due to stone column installation - Weber et al. (2010)
Gautray et al. (2014) used an electrical needle to measure the impedance (electrical resistivity) of the soil surrounding a column installed in a centrifuge using the installation tool developed by Weber et al. (2010). The needle can be used to measure the density increase in the soil subsequent to installation (impedance increases with increasing density). The results, which were compared with the observations made by Weber et al. (2010), indicated:

- In the densification zone identified by Weber et al. (2010), Gautray et al. (2014) found that the impedance was marginally less than that in the undisturbed zone (Figure 2.14b compared with Figure 2.14a). This is inconsistent with the findings in Weber et al. (2010). The impedance in the undisturbed zone was relatively uniform with depth.
- Within the smear zone (Figure 2.14c), an increased impedance was observed in the upper third of the profile, attributable to the compaction of the column during installation. This was also evident in the densification zone (Figure 2.14b).
- The significant drop of impedance below the top third of the profile in the smear zone was interpreted as being consistent with the findings of Weber et al. (2010), i.e. the clay particles have become reorientated, thus indicating that the smear zone will not be parallel to the axis of the column over the whole depth. This reorientation was also evident in the densification zone, although not to the same extent.
Literature Review I - Stone Column Behaviour

2.5.4 Numerical Studies

2.5.4.1 Global $K/K_0$ Increases

Several authors have analysed stone columns using 2D (e.g. Ambily and Gandhi, 2007) and 3D (e.g. Killeen and McCabe, 2010) FE analyses. Some of the analyses have involved ‘wished-in-place’ columns (no installation effects), in which $K$ is assumed to be unaffected by the vibratory action of the poker and subsequent compaction of the columns. Others have aimed to capture the installation effect by using ‘globally’ increased $K$ values; e.g. Gäb et al. (2008), based on approaches adopted by Priebe (1976, 1995) and Goughnour & Bayuk (1979b) have adopted $K = 1$, while Domingues et al. (2007a,b) have used $K = 0.7$ (between the conservative $K_0 = 1 - \sin \phi'$, for normally consolidated soils (Jaky, 1944) and $K = 1$, where $\phi'$ denotes the friction angle of the soil). Elshazly et al. (2006) have backfigured post-installation $K$ values by matching measured load-settlement behaviour in the field to FE analyses carried out using PLAXIS and found that $K$ values post-installation may range from 1.1 to 2.5, with best estimates of approximately 1.5. However, the assumption that $K$ values are the only sources of difference between measurements and FE output is questionable.

2.5.4.2 Cylindrical Cavity Expansion (CCE)

Although not directly representative of the vibratory action of the poker and the progressive compaction of the columns from the base upwards, CCE nevertheless provides a convenient means of simulating the lateral expansion of a granular column into the surrounding soil.
In practical situations, column installation effectively involves expanding a cavity from a zero initial radius to the final radius of the column \( (R_c) \). However, any expansion imposed in a numerical simulation must begin with a finite cavity radius to avoid the development of infinite circumferential strain. Consequently, it must be ensured that the internal cavity pressure reaches the limit pressure \( (p_{lim}) \), as defined by Gibson & Anderson (1961) in Eq. 2.2 \((p_0\) is the original in-situ horizontal total stress, \( E \) is the Young’s Modulus of the soil and \( \nu \) is its Poisson’s ratio).

\[
p_{lim} = p_0 + c_u \left(1 + \frac{E}{2c_u (1 + \nu)}\right)
\]  

(2.2)

Carter et al. (1979) have described the use of the cavity expansion technique in two different types of elasto-plastic soil, and report that doubling the cavity size is sufficient in most cases to reach an internal cavity pressure that will be within 6% of \( p_{lim} \); any further expansion beyond this will only cause further growth of the annular region of yielded soil. Since the cavity expansion process must start from a finite radius, the final cavity radius, \( a_f \), should be obtained by rearranging Eq. 2.3 (i.e. observing volume conservation), where \( a_0 \) is the initial cavity radius.

\[
a_f^2 - a_0^2 = R_c^2
\]  

(2.3)

Guetif et al. (2007) (and also Debats et al. (2003)) used CCE to evaluate the improvement to \( E \) as a result of column installation in a soft clay. The analysis was carried out using the PLAXIS 2D MC model by expanding a cylindrical hole of 'dummy material' (weak Young's modulus) from a radius of 0.25m (vibrating poker radius) to \( R_c = 0.55 \)m. The Updated Mesh option was used to account for the large strains generated during the installation phase, after which large excess pore pressures developed at the soil-column interface. Following the dissipation of excess pore pressure (~11 months after installation), the mean effective stresses increased by an average of 30% in an influence zone of \( \sim 6R_c \) around the column axis (Figure 2.15). The mean-effective stress increase (from \( p'_0 \) to \( p' \)) was used to evaluate the increased soil stiffness (from \( E_0 \) to \( E \)) using Eq. 2.4, where the power, \( m \), dictates the dependence of stiffness on stress level. The authors used \( m = 1 \) (logarithmic compression behaviour), which is typical for soft soils, e.g. Brinkgreve et al. (2011).
Literature Review I - Stone Column Behaviour

\[
\frac{E}{E_0} = \left( \frac{p'}{p'_0} \right)^m
\]  

(2.4)

**Figure 2.15** Normalised mean effective stress versus distance from column axis, \( r \), after consolidation - Guetif *et al.* (2007)

Castro & Karstunen (2010) have applied the cavity expansion technique to the undrained installation of a single 10m long 0.8m diameter stone column, confirming that column installation generates excess pore pressures (\( \Delta u \)), see Figures 2.16a and 2.16b (for depths of 3m and 7m respectively). The S-CLAY1 and S-CLAY1S models referred to in the legend incorporate anisotropy, and anisotropy and destructuration respectively. The excess pore pressures, which increase with depth, can be normalised by \( c_u \) to allow for a direct comparison between different depths (and soil models).

**Figure 2.16** Excess pore pressures after installation at depths of (a) 3m (b) 7m - Castro & Karstunen (2010)
The radius of influence for excess pore pressure (~13.5\(R_c\)) is constant with depth, and is dependent on the amount of soil surrounding the column that has reached a plastic state (the radius of influence is different for different parameters). These pore pressures dissipate following consolidation, leading to a mean effective stress increase, and in particular a horizontal effective stress increase (and hence \(K\)) in the soil, see Figure 2.17a.

In the same study, Castro & Karstunen (2010) reported an erasure of interparticle bonding at the column interface after the undrained expansion phase (limited to 4-5\(R_c\) for the numerical model in question) using the S-CLAY1S model. This led to a reduction in the undrained shear strength of the soil, the majority of which was recovered following consolidation, see Figure 2.17b. The soil models not accounting for destructuration (S-CLAY1 and Modified Cam Clay (MCC)) predicted an increased undrained shear strength following the consolidation phase. In general, the computed values of \(K/K_0\) were lower for the S-CLAY1S model than for the S-CLAY1 model (but \(K/K_0 > 1\) nevertheless) owing to the destructuration caused by column installation.

![Figure 2.17](image)  
**Figure 2.17** (a) \(K/K_0\) after consolidation, (b) \(c_u/c_{u0}\) after consolidation - Castro & Karstunen (2010)

Castro *et al.* (2013) have used also the S-CLAY1 and S-CLAY1S models to demonstrate changes in soil fabric caused by the installation process (see Figure 2.18), i.e. installation leads to particle reorientation (similar to the laboratory smear zone identified by Weber *et al.* (2010)).
2.6 Stone Column Settlement Behaviour

A significant amount of field trials, numerical studies, and laboratory experiments have been conducted to investigate stone column settlement behaviour. The majority of these concentrate on primary consolidation settlement. The dearth of data that exists regarding stone columns and their effect on long-term creep settlements in the field has been recognised by McCabe et al. (2009) and Mitchell & Kelly (2013). Creep effects can contribute to a significant proportion of the total settlement in soft cohesive soils (see Section 1.1).

2.6.1 Field Studies

The field performance of stone columns in soft clays and silts has been thoroughly reviewed by McCabe et al. (2009). The review paper focused on long-term settlements from full-scale load tests and construction projects (both published and unpublished data). The authors developed a comprehensive database of $n$ values (Figure 2.19) for wide area (i.e. embankments) and small area (i.e. footings) loading situations, in the process highlighting the effectiveness of stone columns at reducing total and differential settlements. However, the measured $n$ values in the McCabe et al. (2009) database tend to be ‘lumped’ values, with no distinction between initial compression (which admittedly tends to be low in soft soils), primary consolidation and creep settlements. The monitoring duration required to capture creep settlements reliably in the field is an obvious obstacle in this regard.
The potential of vibro stone columns to reduce long-term creep settlements in the field has received little attention, although Cooper & Rose (1999) and Watts et al. (2000) are notable exceptions.

Cooper and Rose (1999) noted a reduced creep rate following the construction of a stone column foundation (comprising wet bottom feed stone columns and vibro concrete columns) supporting a 7m high embankment to the River Avon bridge of the St. Philip’s Causeway in Bristol in the UK. The soil profile consists of thick alluvial deposits of soft clay and peat and thus creep behaviour was considered important. Excess pore pressures measured using piezometers indicated that primary consolidation was complete at an early stage and that the majority of post-construction settlement would be due to creep. Watts et al. (2000) performed a number of full-scale instrumented load tests comparing the effectiveness of vibro stone columns supporting a strip footing to a similar strip footing on untreated ground. The columns effectively reduced both the creep rate and the total settlement under sustained loading applied through kentledge blocks. In both studies, quantitative estimates of a ‘creep’ settlement improvement factor were not provided.
2.6.2 Numerical Studies

Numerous authors have investigated stone column settlement behaviour using numerical techniques with the aim of understanding how columns contribute to improving the settlement performance of soft/marginal soils; factors such as column length ($L_c$), column spacing ($s$), and column stiffness were investigated, e.g. Balaam et al. (1977), Poorooshab & Meyerhof (1997) and Domingues et al. (2007a,b). The majority of studies have used simplified 2D analysis techniques, e.g. plane strain (Gäb et al., 2008) or axisymmetry (Ambily & Gandhi 2007, Domingues et al. 2007a,b, Castro & Sagaseta 2011); 3D modelling has been used by Weber et al. (2008), Kamrat-Pietraszewska & Karstunen (2009), and Killeen (2012).

2.6.2.1 Numerical Approaches

There are a number of different approaches than can be used to model stone column behaviour:

- **Plane Strain**: Plane strain idealisations can be used to simplify complicated 3D problems for use in conjunction with the FEM. The plane strain concept can be used to model a situation where one dimension is large in comparison with the other two. Modelling stone column behaviour using the plane strain approach involves replacing the stone columns with stone walls (trenches) having an ‘equivalent’ overall plan area. This is useful for long foundations such as embankments and strip footings (Kok Shien, 2013).

- **Homogenization**: The homogenization technique involves modelling the stone column and treated soil as a single composite material with equivalent properties and is formulated assuming that the influence of the columns is uniformly and homogeneously distributed throughout the treated soil, e.g. Schweiger & Pande (1986). This method can be used for flexible footings and rigid rafts (‘equal vertical stress’ and ‘equal vertical strain’ assumptions, respectively) whereas the unit cell concept can only be used for rigid rafts.

- **Unit Cell (Axisymmetry)**: Axisymmetry is a 2D simplification applicable when an object/problem has rotational symmetry with respect to both the geometry and applied load (Mar, 2002). The geometry is usually defined using a cylindrical coordinate system.
\((r, z, \theta)\), where \(r\), \(z\), and \(\theta\) represent the radial, vertical, and circumferential directions respectively. The unit cell approach is based on the assumption of a large grid of regularly-spaced columns subjected to a uniform load, e.g. Figure 2.20. The area-replacement ratio, \(A_c/A\) (where \(A\) is the cross-sectional area of a unit cell treated with a single stone column of cross-sectional area, \(A_c\)), is used as a measure of the amount of in-situ soil replaced with stone and is dependent on the centre-to-centre column spacing, \(s\), and column diameter, \(D_c\) (Eq. 2.5), where \(k\) is a constant depending on the column arrangement. All of the columns are assumed to exhibit identical behaviour and so an analysis of one such column and its tributary soil area (hatched in Figure 2.20) is sufficient. Owing to the symmetry of the problem, the shear stresses along the perimeter of the unit cell can be assumed to be zero. The unit cell approach is valid except for columns near the edges of the loaded area (Balaam & Booker 1981, McKelvey et al. 2004), which are assumed to be in the minority for large groups, such as might be used to support embankments or large floor slabs, for example.

\[
\frac{A_c}{A} = \frac{1}{k} \left(\frac{D_c}{s}\right)^2
\]  

(Eq. 2.5)

- **3D Analyses:** The column-soil system can be modelled fully in three dimensions. Such analyses require significantly more computational effort.

![Figure 2.20 Typical column grids encountered in practice; (a) triangular (b) square (c) hexagonal](image)

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2.6.2.2 Numerical Results

The majority of numerical studies have tended to use the Linear Elastic (LE), MC, or HS models (e.g. Ellouze & Bouassida 2009, Killeen & McCabe 2010). Other studies have used more advanced constitutive models incorporating anisotropy and destructuration, e.g. the S-CLAY1 and S-CLAY1S models (Gäb et al. 2008, Kamrat-Pietraszewska & Karstunen 2009). However, none of these have accounted for creep deformation.

Findings from 2D studies

- Ambily & Gandhi (2007) demonstrated an improved settlement performance for closer column spacings and soils with higher undrained shear strengths using the MC model.
- Domingues et al. (2007a,b) carried out a series of parametric studies using a program incorporating Biot (1941) consolidation theory with the $p$-$q$-$\theta$ Model (extension of the MCC model, based on the Mohr-Coulomb failure criterion). Their study indicated that the settlement and consolidation behaviour of stone columns supporting an embankment on soft soil improved (reduced horizontal and vertical displacements and accelerated consolidation) as column stiffness and $A_c/A$ increased.
- Castro & Sagaseta (2011) demonstrated an accelerated rate of consolidation settlement for closely-spaced columns using the MC model. The authors also investigated the variation of stress concentration factor (SCF = $\sigma_c/\sigma_s$, where $\sigma_c$ is the stress in the column and $\sigma_s$ is the stress in the soil) with time and found that it remained relatively constant after yielding, although it was overestimated by analytical solutions assuming full lateral confinement, see Section 2.7.1.1.

Findings from 3D studies

Killeen (2012) thoroughly analysed the majority of variables affecting stone column settlement behaviour, e.g. $A_c/A$, $L_e$, column stiffness, column strength, and column installation effects, using the HS model in conjunction with PLAXIS 3D Foundation (Brinkgreve et al., 2007). The findings can be summarised as follows:
• The settlement performance improves with increasing $A_c/A$, $L_c$, column stiffness, and column strength. The interaction between these variables was also found to be important, e.g. longer columns are more effective at closer spacings.

• The influences of the different variables have been linked to modes of deformation, e.g. punching failure, block failure, and bulging failure. These deformation modes were examined by analysing the distribution of shear strain within the column and surrounding soil. The variation of SCF with depth was also linked to the mode of deformation.

• Punching failure was observed for short columns at close spacings whereas bulging failure occurred in long columns at wider spacings. The bulging failure mode is dependent upon the lateral support provided by the soil and occurs near the surface where overburden stresses are lower. Block failure, where all the columns act together and punch into the underlying soil, is an extension of the punching failure mode and occurs in large groups of closely-spaced columns.

• Accounting for column installation by using an increased $K$ value contributes to an improved settlement performance; $K$ has a higher influence when columns are spaced further apart.

• The presence of a stiff crust, typical of soft clay profiles, contributes to increased column confinement in the upper layers, thus transferring column bulging to deeper layers, which in turn enhances the column’s bearing capacity and settlement performance. This highlights the significance of careful installation practices and the consequences of damaging the stiff crust material. This was notable in the field trials carried out by Serridge & Sarsby (2008).

• The influence of column position beneath a footing is important; improved settlement performance is achieved when columns are spaced closer to the footing edge, attributed to the fact that higher stress concentrations occur beneath the edge of rigid footings.

Three-dimensional modelling work carried out by Hanna et al. (2013) and Kok Shien (2013) has been used to identify how the mode of failure of a single column differs to a column group. The findings are consistent with both Killeen (2012) and with the laboratory findings reported in the next section; single columns fail by bulging whereas column groups exhibit multiple deformation modes, e.g. the outer columns bend outward (more pronounced in longer columns) and the centre columns bulge (but at greater depths compared to single columns).
2.6.3 Laboratory Studies

2.6.3.1 Analysing stone column performance in the laboratory

A significant amount of laboratory-scale testing has been carried out to investigate stone column behaviour (e.g. Charles & Watts 1983, Muir Wood et al. 2000, McKelvey et al. 2004, Black et al. 2011). Accurately modelling stone column behaviour in the laboratory can be hindered by scale effects, sample disturbance, and difficulties in replicating field boundary conditions. To date, the majority of laboratory work, with the exception of Moorhead (2013), has focused on either bearing capacity or short-term settlement response. Modelling long-term creep behaviour in the laboratory would require additional considerations, including:

(i) Remoulding effects: Significantly more creep occurs in undisturbed soils than in remoulded soils. Additionally, remoulding decreases the rate of secondary/creep compression, e.g. Mesri (1973).

(ii) Sample disturbance: Sample disturbance is very difficult to avoid in laboratory testing but is important when analysing primary consolidation and creep effects. It leads to difficulties in evaluating the preconsolidation pressure reliably, one of the main factors dictating soil response with respect to both time and stress compressibility (Degago, 2011); in general, lower quality samples are characterised by lower preconsolidation pressures and hence lower apparent overconsolidation ratios (OCRs). This effect has been studied in detail by Degago (2011), e.g. if the OCR is underestimated, an elasto-plastic model may fortuitously yield good predictions because the lower OCR would compensate for the missing creep effects. However, if OCR is estimated correctly, an elasto-viscoplastic model is necessary to reliably predict the settlement response.

(iii) Temperature effects: According to Fox & Edil (1996), laboratory and field tests conducted on samples of Middleton peat from the US have shown an increased rate of creep compression with increased temperature. Early work by Buisman (1936) also showed there to be more creep at higher temperatures and so laboratory testing should generally be carried out at average in-situ soil temperatures in order to correspond with field conditions.

(iv) Layer thickness: Consolidation takes significantly longer in thick soil layers than thin layers, i.e. field versus laboratory (see Section 3.2), and so monitoring creep behaviour
in the laboratory has its advantages. Yin (1999) suggested a minimum duration of one week for oedometer tests examining creep behaviour in the laboratory. The prediction of creep settlements longer than the duration of the test involves extrapolating the test data and any proposed fitting functions (e.g. logarithmic, hyperbolic).

(v) **Strain rate effect:** Laboratory strain rates tend to be larger than field strain rates (e.g. Figure 2.21). Clay soils exhibit significant rate-dependency (Graham et al., 1983); higher strain rates yield higher undrained shear strengths and higher preconsolidation pressures, e.g. Segikuchi & Ohta (1977), Qu et al. (2010), see Figure 2.22 (although advocates of Hypothesis A (see Section 3.2) assume that the $e - \log \sigma$ relationship is the same in both the laboratory and in the field, where $\sigma$ denotes stress).

![Figure 2.21](image)

**Figure 2.21** Field strain rates versus laboratory strain rates - adapted from Leroueil et al. (2006)

![Figure 2.22](image)

**Figure 2.22** Influence of strain-rate on preconsolidation pressure - Yin et al. (2010)
2.6.3.2 Investigation techniques

The majority of laboratory studies have used reconstituted soil samples, although the techniques employed in each study varied considerably:

- Hughes & Withers (1974) used radiographic techniques to examine the deformational behaviour of a single column installed in normally consolidated kaolin clay.
- Muir Wood et al. (2000) used an exhumation technique to examine the deformed shapes of model columns constructed in reconstituted kaolin clay. Subsequent to the loading phase, the quartz sand used to form the columns was sucked out and a wire framework for building a sculpture was inserted in the hole left by the column. Plaster of Paris was poured into the hole and allowed to set before the clay was excavated and the columns exhumed. The exhumation technique enabled the load transfer mechanisms that occurred between the columns and surrounding soil to be evaluated.
- McKelvey et al. (2004) used a transparent clay-like material (specially developed at Trinity College Dublin) to enable visual examination of the failure mechanisms and deformed shapes of model floating columns supporting strip footings, pad footings and circular footings. A digital camera was used to monitor column deformation as the tests proceeded.

A number of different methods have been used to construct model columns, the majority of which involved using helical augers, e.g. McKelvey et al. (2004), Sivakumar et al. (2004), Wehr (2004).

2.6.3.3 Findings (critical depth and mode of deformation)

The main findings can be summarised as follows:

- Hughes & Withers (1974), looking at the behaviour of single floating columns, identified a critical depth \( L_{\text{crit}} \) at which end-bearing and bulging failure occurred simultaneously as \( 4D_c \); columns with \( L_c/D_c < 4 \) would fail in end-bearing (punching) before bulging. Additionally, negligible strains were observed below depths of \( 4D_c \). This led the authors to postulate that columns longer than \( 4D_c \) will not provide additional load-carrying
capacity increases since the ultimate capacity of a column is dependent on the lateral resistance provided by the soil in the bulging zone.

- Muir Wood et al. (2000) concentrated on identifying the mode of deformation (and also the manner in which columns transfer load to the surrounding clay) in groups of floating columns using the aforementioned exhumation technique. Their findings indicated that columns which are not prevented from expanding radially by closely adjacent columns will bulge whereas closely-spaced columns will form diagonal shear planes (points A and B respectively in Figure 2.23). Short columns will penetrate the underlying clay (load transmitted to base, Figure 2.23a) whereas long columns will not because less load is transmitted to the base (see Figure 2.23b).

![Figure 2.23 Photographs of deformed sand columns (a) short columns, (b) long columns - Muir Wood et al. (2000)](image)

- McKelvey et al. (2004) demonstrated that bulging was significant in the upper zones of long columns with little or no load transferred to the base whereas shorter columns tended to bulge over their entire length and thus exhibited a punching mechanism as significant load was transferred to the base. This is consistent with the findings of Hughes & Withers (1974). McKelvey et al. (2004) also noted that columns increased the load-carrying of the treated clay (although columns longer than $6D_c$ did not contribute any additional capacity, thus suggesting an optimum column length for bearing capacity). However, it was suggested that columns longer than $6D_c$ would improve settlement performance because they increase the stiffness of the composite soil-column sample.
• Sivakumar *et al.* (2004) measured the load-carrying capacity increase obtained using sand columns of different lengths to reinforce specimens of kaolin clay under both unit cell and foundation loading (supported by a single column) conditions. The results showed that full-depth columns substantially increased the load-carrying capacity of the clay; floating columns also contributed to increased load-carrying capacities in the majority of cases. Their findings indicated an optimum column length of $5D_c$ under foundation loading, beyond which no additional capacity was achieved, consistent with earlier findings by Narasimha Rao *et al.* (1992).

• Black *et al.* (2011) investigated the influence of $L_c/D_c$ and $A_c/A$ on the performance of single isolated columns and small and large column groups. Their findings point to an optimum $A_c/A$ of between 30% and 40% for settlement control purposes, noting that settlement control can be equally effective using either short columns ($L_c/D_c < 6$) at close spacings or longer columns ($L_c/D_c > 6$) at wider spacings. Additionally, they established a critical length of $8D_c$-$10D_c$ beyond which columns did not provide any additional settlement control.

### 2.6.3.4 Moorhead (2013)

Moorhead (2013) carried out a series of laboratory tests to investigate the effectiveness of granular columns at reducing each component of settlement; initial settlement, primary consolidation settlement and creep. The tests were carried out on reconstituted samples of kaolin and Belfast sleech, soils considered to have very little creep potential and significant creep potential respectively. The testing programme was separated into two components:

• The first set of tests were carried out in 1D loading chambers with a diameter of 254mm and a height of 100mm. They focused on examining the settlement performance of a rigid raft (entire surface area loaded) supported by either a single end-bearing column at $A_c/A = 15\%$ ($L_c/D_c = 1$) or a group of seven end-bearing columns ($L_c/D_c = 2.7$) at an equivalent $A_c/A$. The $L_c/D_c$ ratios are lower than would typically be encountered in practice.

• The second part examined the behaviour of an isolated footing supported by a single end-bearing granular column at $A_c/A = 33\%$. These tests were carried out in 3D test chambers, incorporating flexible boundary conditions, large enough to test samples 300mm in
diameter and 400mm high. Load was applied through a circular footing 70mm in diameter supported by a 40mm diameter column \((L_c/D_c = 10)\).

Casagrande's (1936) method was used to identify the EOP consolidation times and also the initial and primary consolidation settlements. The creep rates were quantified based on the slopes of the settlement-log(time) curves after the EOP time.

The findings for the raft loading scenario were as follows:

- The initial settlements were negligible at each stage of loading in all tests.
- In the kaolin, both the single column and the seven-column group accelerate consolidation (fourfold and sevenfold increases with respect to the untreated case respectively). The \(n\) values for primary settlement ranged from 2.4-6.0 at low bearing pressures when the clay was overconsolidated (OC). For higher pressures, the corresponding \(n\) values for the normally consolidated (NC) condition ranged from 1.3-1.8. The corresponding \(n\) values for creep ranged from 1.6-2.4 and 1.0-1.3 in the OC and NC states respectively.
- For the slease, the \(n\) values for primary settlement were in the range 3.7-5.7 and 1.2-3.8 for the OC and NC states respectively and the corresponding creep \(n\) values were in the range 1.8-3.0 and 1.2-2.7 respectively. The consolidation time increases were fivefold and tenfold for the single and group configurations with respect to the untreated case respectively.
- In general, the granular columns are more effective at low pressures when the samples are in an OC state, with \(n\) values for primary settlement larger than those for creep in all cases. The \(n\) values for creep become negligible at very high pressures.

For the isolated foundation loading:

- The \(n\) values for initial settlement were approximately 1.0 in all cases.
- In the kaolin, the \(n\) values for primary settlement were lower than the raft loading case, ranging from 1.3-1.4. The \(n\) values for creep ranged from 1.0-1.4.
• In the sleech, the $n$ values for primary settlement ranged from 2.4-2.5 and the $n$ values for creep from 2.5-2.8.

• This suggests a material dependent response. However, the authors believed that the higher $n$ values in the sleech may be misleading; the initial conditions in the sleech bed for the unreinforced and reinforced cases were different; for the reinforced case, the undrained shear strength of the clay bed was 20% larger. This would explain the larger $n$ values for primary settlement. In addition the organic content in the reinforced clay bed was lower, which would intuitively produce lower $C_a$ values, even in the absence of granular columns.

The main conclusion from this study is that columns have no effect on initial settlement, reduce consolidation settlement at low pressures but are not as effective at higher pressures, and only have a minor influence on reducing creep. The findings need to be substantiated for different values of $L_c$ and $A_c/A$.

2.7 Stone Column Design Methods

2.7.1 Settlement

Numerous semi-empirical and analytical approaches exist for the prediction of the settlement improvement offered by the vibro-replacement technique in weak or marginal soil deposits, the first of which was suggested by Greenwood (1970) using a set of empirical curves. The majority of the settlement prediction methods (which tend to assume end-bearing columns) are based on the unit cell assumption, with a small number based on plane strain (e.g. Van Impe & De Beer, 1983) or homogenization techniques (e.g. Schweiger & Pande 1986, Lee & Pande 1998). The settlement prediction approaches are reviewed in detail in Sexton et al. (2013) with a view to establishing which approach(es) is/are comparable with a series of PLAXIS 2D axisymmetric unit cell analyses on an end-bearing column.

Analytical design methods typically relate $n$ to $A_c/A$, and/or modular ratio, $E_c/E_s$ (where $E_c$ is the modulus of the column and $E_s$ is the modulus of the soil). A number of other influential variables have been considered in analytical formulations; these include the effect of installation, load level ($p_u$), $\phi'$, and the dilatancy angle of the column material ($\psi_c$). The
published solutions account for these variables in different ways, although few capture all of them. Additionally, the majority of design methods tend to pertain to primary settlement only, a notable exception being Madhav et al. (2009, 2010), which accounts for creep. This method is discussed in Section 2.7.1.3.

2.7.1.1 Underlying Theoretical Considerations in Vibro-Replacement Design

Introduction

Elastic vs. Elastic-Plastic
The elastic-plastic methods are typically based on the Mohr-Coulomb failure criterion, with some assuming that the granular material deforms at constant volume as it yields ($\psi_c = 0^\circ$), and others accounting for dilation of the granular column material at yield using a constant $\psi_c$. Balaam & Booker (1985) and Pulko & Majes (2005) have highlighted that elastic-plastic methods are preferable to purely elastic methods because the elastic methods tend to overpredict the settlement improvement offered by column installation, especially for high modular ratios. This overprediction is as a result of the fact that elastic methods overpredict the SCF.

Vertical and/or Radial Deformation
Approaches to modelling the behaviour of the column-soil system vary; some, such as Han & Ye (2001) account only for vertical deformation, while others account for both radial and vertical deformation. For elastic methods that consider vertical deformation only, the SCF is equal to the ratio of the oedometric moduli of the column and soil materials ($E_c/E_s$). Elastic solutions that consider both radial and vertical deformation result in slightly lower SCFs
(lateral deformation reduces SCFs, e.g. Balaam & Booker (1981)). However, these SCFs will still be overpredicted because yielding of the column material is not considered (column yielding and plastic strains will reduce SCFs). Barksdale & Bachus (1983) have suggested that SCFs in practice range from 3-10 depending on the column spacing adopted in the field.

**Installation**

The densification effect resulting from column installation and subsequent bulging has been accounted for in different ways. Priebe (1976) has assumed an increase in $K$ following column installation to the liquid earth pressure of the soil ($K = 1$). Other methods allow for the input of different values depending on the designer’s discretion: Baumann & Bauer (1974) have limited allowable $K$ values to the range $K_0 < K < 1/K_0$; Goughnour & Bayuk (1979a) have limited allowable $K$ values to the range $K_0 < K < K_p$, where $K_p$ denotes the passive earth pressure coefficient of the soil; Borges *et al.* (2009) have formulated their closed-form expression based on fitting curves to the results of numerical analyses assuming $K = 0.7$ (between $K_0 = 1 - \sin \phi'$ and $K = 1$); Van Impe and Madhav (1992) have suggested the use of an increased oedometric soil modulus depending on the method of installation and the column spacing.

**Drained vs. Undrained + Consolidation**

Solutions have been developed for drained conditions and for undrained conditions with a follow-up consolidation period to allow for the dissipation of excess pore pressure. The undrained plus consolidation solutions (e.g. Han & Ye 2001, Castro & Sagaseta 2009) have been based on Barron’s (1948) solution for vertical drains (Barron's (1948) solution assumes that the vertical stress on the soil is constant during the consolidation process), but with modified coefficients of consolidation used to account for the fact that the columns carry a considerable proportion of the applied load (vertical drains have a much reduced stiffness and diameter when compared to stone columns). The Castro & Sagaseta (2009) solution has been derived for the case of an elastic-plastic column (radial deformation has been considered) while Han & Ye (2001) have based their solution on an elastic column subjected to full lateral confinement.
2.7.1.2 Unit Cell Approaches

A flow chart (from Sexton et al., 2013) detailing the development and origin of the majority of the design methods based on the unit cell approach is presented in Figure 2.24. These unit cell methods are reviewed in detail in Sexton et al. (2013).

![Flow Chart of Unit Cell Approaches](image)

**Figure 2.24 Development of Settlement Prediction Methods (Unit Cell)**

The ‘Equilibrium Method’

The simplest analytical approach to stone column design is known as the ‘equilibrium method’. The approach is based on elastic theory and has been described by Aboshi et al. (1979). It is based on vertical equilibrium between the soil and the columns (Eq. 2.6) with oedometric (i.e. elastic behaviour with full lateral confinement) conditions in the soil. This approach necessitates prior knowledge of the SCF, see Eq. 2.7 (e.g. experience/field measurements) whereas other methods such as Priebe (1976, 1995) have used CCE theory to establish the SCF.

\[
p_a \cdot A = \sigma_c \cdot A_c + \sigma_s \cdot (1 - A_c)
\]

\[
n = 1 + \frac{SCF - 1}{A/A_c}
\]
Priebe (1976, 1995)

Despite its semi-empirical basis, Priebe’s (1995) method has become one of the most popular design methods (European practice) for evaluating $n$ for vibro-improved ground. Priebe’s (1995) method is an extension of Priebe’s (1976) method in which CCE theory has been used to evaluate the radial strain assuming zero vertical strain (and hence the SCF). Priebe (1976) makes a number of simplifying assumptions to calculate a basic improvement factor, $n_0$: bulging is constant over the length of the column, the column material is incompressible, and the bulk densities of the soil and column are neglected, see Eq. 2.8 (Eq. 2.8 is based upon a Poisson’s ratio for the soil, $\nu_s$, of 0.33, although the method allows for different Poisson’s ratios). Priebe’s (1995) method represents a further development which accounts for the column compressibility ($n_1$) and the bulk densities of the soil and column materials ($n_2$).

$$n_0 = 1 + \frac{A_c}{A} \left[ \frac{5 - \frac{A_c}{A}}{4 \left(1 - \frac{A_c}{A}\right) \tan^2 \left(\frac{45 - \phi'_c}{2}\right)} - 1 \right]$$

(2.8)

Analytical Methods

Balaam & Booker’s (1981) analytical elastic approach accounts for both vertical and radial displacement. It forms the basis for later solutions developed by Balaam & Booker (1985), Han & Ye (2001), Pulko & Majes (2005), Castro & Sagaseta (2009), and Pulko et al. (2011), all of which are closed-form with the exception of Balaam & Booker (1985). This method is iterative, requiring numerical implementation to obtain a solution.

The elastic-plastic methods derived by Pulko & Majes (2005), Castro & Sagaseta (2009), and Pulko et al. (2011) account for dilation of the granular column material (constant $\psi_c$) at yield whereas Priebe’s (1976, 1995) method assumes the granular column material to deform at constant volume ($\psi_c = 0^\circ$). Pulko & Majes (2005) and Castro & Sagaseta (2009) are elastic-plastic extensions of the earlier elastic solution developed by Balaam & Booker (1981) for drained conditions. Castro & Sagaseta (2009) have considered an undrained loading situation followed by a consolidation process to allow for the dissipation of excess pore pressures whereas Pulko & Majes (2005) and Pulko et al. (2011) have studied the unit cell problem under drained conditions. As noted by Castro & Sagaseta (2009), both approaches are
Literature Review I - Stone Column Behaviour

considered to be limiting cases of the real situation because load application is not rapid enough to be considered as undrained nor slow enough to be considered as a drained process.

The method developed by Pulko et al. (2011), which deals with encased stone columns, is an extension of the previous solution derived by Pulko & Majes (2005). The new method by Pulko et al. (2011) can also be applied to non-encased stone columns by setting the encasement stiffness to zero. The solutions derived by Castro & Sagaseta (2009) and Pulko & Majes (2005) ignored the elastic strains in the column during its plastic deformation whereas the newer solution by Pulko et al. (2011) has taken them into account.

Figure 2.25 (from Castro & Sagaseta (2009)) shows the different stress paths followed depending on whether the problem is studied under drained or undrained (plus consolidation) conditions.

![Stress paths in the column (subscript c); (a) elastic case (b) at yielding (c) elastic-plastic case - Castro & Sagaseta (2011)](image)

**Figure 2.25** Stress paths in the column (subscript c); (a) elastic case (b) at yielding (c) elastic-plastic case - Castro & Sagaseta (2011)
For an elastic column, both approaches produce the same result. For a yielding column (elastic-plastic case), although the stress paths are different, the final settlements are very similar (provided that the drained solutions account for elastic strains of the column during its plastic deformation), as shown by Castro & Sagaseta (2009) using FE analyses. For drained analyses that neglect the elastic strains of the column during its plastic deformation (e.g. Pulko & Majes, 2005), the final settlement will be underpredicted. For undrained plus consolidation solutions (e.g. Castro & Sagaseta, 2009), neglecting the elastic strains of the column during its plastic deformation leads to negligible error in the solution. The differences between the drained and undrained plus consolidation analyses will effectively vanish provided the drained solutions account for the elastic strains of the column during its plastic deformation (i.e. Castro & Sagaseta (2009) and Pulko et al. (2011) will produce almost identical solutions for non-encased columns).

Summary/Appraisal
An appraisal of a number of empirical and theoretical solutions for evaluating $n$ values in the context of comparable PLAXIS 2D axisymmetric unit cell analyses on end-bearing columns by Sexton et al. (2013) has indicated the following:

- The majority of elastic-plastic methods yield similar predictions as the modular ratio increases (more realistic for soft soils), highlighting the fact that regardless of the basis or corresponding assumptions made in the derivation of each method, predicted $n$ values are in reasonable agreement with one another.
- The predicted $n$ values and SCFs obtained using Castro & Sagaseta (2009) and Pulko et al. (2011) offer the best agreement with the FE analyses for a wide range of input parameters, e.g. $p_a$, $\phi'$, $\psi$, and $K$. This was the main conclusion arising from this work.
- Small differences between the numerical and analytical results were attributed to the stress dependency of soil stiffness present in the numerical model (a more realistic assumption); the analytical solutions assume that the soil behaves in a linear elastic manner.
- Sexton et al. (2013) have suggested that the Castro & Sagaseta (2009) and Pulko et al. (2011) design methods should be used more often in geotechnical practice because they give more realistic results and allow for the consideration of significantly more input data.
2.7.1.3 Madhav et al. (2009, 2010)

The method developed by Madhav et al. (2009, 2010) is based on an extension to an earlier method by Shahu et al. (2000), also based on the unit-cell assumption (end-bearing columns). However, in contrast to the methods in the Sexton et al. (2013) flowchart (Figure 2.27), the method developed by Shahu et al. (2000) incorporates the effect of a granular mat in order to combat/reduce the high stress concentrations that occur near the top of the granular columns. Shahu et al. (2000) have used elastic theory to calculate the settlement of the granular column. The elastic assumption is reasonable provided adequate thickness of mat is provided but $n$ values will be overpredicted in the majority of other cases (especially in soft soils with large modular ratios), see Section 2.7.1.1.

Madhav et al. (2009, 2010) have extended the Shahu et al. (2000) method to account for creep based on the principle that the stress on the soil will decrease as it creeps while the column (which does not creep) will carry the surplus load. A similar load transfer mechanism has also been suggested by Mitchell & Kelly (2013), who have noted that the increment of load transferred to the column as the soil creeps has yet to be quantified. As a result of the stress unloading, the soil becomes overconsolidated; however, this overconsolidation effect is different to Bjerrum’s (1967) overconsolidation effect due to ageing. The design equations for both the Shahu et al. (2000) and the Madhav et al. (2009, 2010) methods are presented in Appendix C.

2.7.2 Bearing Capacity

Thorburn & McVicar (1968) were the first to suggest an empirical design approach for evaluating the bearing capacity of stone column-improved ground. Analytical solutions have since been developed to predict the increase in bearing capacity of column-improved ground, e.g. Hughes & Withers (1974), Madhav & Vitkar (1978), Bouassida & Hadari (1995), Etezad et al. (2006). The approach developed by Hughes & Withers (1974) is the most popular bearing capacity prediction approach. It is formulated using pioneering CCE theory (Gibson & Anderson, 1961) to calculate the ultimate capacity of a single stone column based on the undrained shear strength of the soil, the in-situ radial stress at the bulging depth, and $\phi'$. Whereas the Gibson & Anderson (1961) approach is based on a frictionless material, Vesic
(1972) has developed a solution to include soils with both friction and cohesion (both solutions coincide for frictionless soils). The aforementioned approaches have been developed to predict the ultimate bearing capacity of single stone columns. Etezad et al. (2006) have developed a theoretical model to predict the bearing capacity of a group of columns. The theoretical model has been prompted by prior numerical modelling and experimental data.

2.8 Summary and Implications of Literature Review I

The stone column technique is becoming increasingly popular for treating soft creep-prone soils. The majority of field studies have measured 'lumped' $n$ values, with no division between 'primary' and 'creep' $n$ values. This research will aim to address this gap in the literature by calculating separate 'primary' and 'creep' $n$ values and establishing how the conventional 'lumped' $n$ value (which includes primary and creep contributions) varies with time. Numerical studies (the majority of which use simplified procedures to account for column installation effects) investigating stone column behaviour have used models that do not consider viscous behaviour, and laboratory studies, although informative, tend to be limited by scale effects and sample disturbance. Moorhead (2013) is the first to investigate stone column creep effects in the laboratory.

A thorough review of the vibro-replacement design methods also highlights the gap that exists with regard to the effectiveness of the stone column technique in creep-prone soils, a notable exception being Madhav et al. (2009, 2010). However, this method does not account for yielding of the granular material and so significantly overpredicts $n$. In this research, a simplified design procedure that accounts for the influence of creep will also be developed (Chapter 9). This method can be used in conjunction with an existing primary settlement design method that captures all key features of primary settlement behaviour (investigated in Sexton et al. (2013)). A novel procedure that can be used to incorporate installation effects into numerical models has also been explored in Chapter 10. The majority of numerical studies to date have tended to used either 'wished-in-place' columns and/or global $K/K_0$ increases. The procedure described in Chapter 10 involves a two-step process which uses CCE theory (which has been used by numerous authors including Castro & Karstunen (2010)) in conjunction with the 'wished-in-place' installation technique.
3. Literature Review II - Modelling Time-Dependent Behaviour

3.1 Introduction

The purpose of this chapter is to introduce the important technical aspects that need to be considered with regard to modelling creep behaviour using the FEM. The FEM is an approximate technique which can be used to solve complicated engineering problems for which exact analytical solutions do not exist. It involves discretising/dividing the continuum of interest into a number of smaller subdivisions (finite elements), interconnected through nodes located on the element boundaries. An approximate form for the solution is assumed in each finite element (usually in terms of nodal values) and then techniques such as the principle of virtual work, the weighted residual method, or the variational method can be used to evaluate the strains and stresses in the continuum.

The first section explains the difference between Hypotheses A and B and summarises the recent work that has been carried out which supports the latter hypothesis. The next section describes different classes of constitutive models for describing the time-dependent behaviour of soft soils and the concepts that have been used in their development. The final section summarises a selection of relevant 3D elasto-viscoplastic constitutive models.

3.2 Hypothesis A versus Hypothesis B

The question of whether creep acts as a separate phenomenon in parallel with primary consolidation was first posed by Ladd et al. (1977). Two different theories, Hypothesis A (e.g. Mesri & Choi 1985, Feng 1991) and Hypothesis B or isotache models (e.g. Šuklje 1957, Nash & Ryde 2001, see Section 3.3.1.2) have emerged to explain the influence of creep during the primary consolidation phase (Figure 3.1). Hypothesis A postulates that the strain at EOP is independent of sample thickness, whereas Hypothesis B assumes that creep is significant during primary consolidation, meaning that thicker soil layers (in field conditions) show larger EOP strains than thinner (laboratory) layers. Isotache models (Hypothesis B) are based on the principle that the prevailing creep rate at any time is defined by the current state of the soil only, namely the current void ratio and effective stress, e.g. Degago et al. (2011).
Degago et al. (2011) have noted that time-dependent settlements of thick soft clay layers in the field tend to be analysed based on the behaviour of thin laboratory samples (despite the significantly longer primary consolidation times associated with the thicker field layers). Accordingly, the authors thoroughly reviewed and evaluated a selection of previously published laboratory and field experiments that were specifically conducted with the aim of examining the effect of layer thickness on the time-dependent compressibility of soft soil layers, e.g. Feng (1991), Aboshi (1973), Imai & Tang (1992), and Konovalov & Bezvolev (2005) and concluded that the time-dependent behaviour of clay is best described by Hypothesis B. Experimental observations that were previously used to support Hypothesis A have been explained consistently using a numerical model based on the isotache concept (SSC model) while it has been highlighted that other results used to support Hypothesis A have been presented in a misleading fashion, i.e. the data was plotted in terms of change in nominal strain rather than total nominal strain (Degago et al., 2011). In an earlier study, Kabbaj et al. (1988) also validated Hypothesis B for four specific test embankments: Berthierville, St. Alban, Gloucester, and Väsby by comparing effective stress-strain curves obtained from laboratory oedometer tests with in-situ curves.

![Figure 3.1 Hypotheses A and B](image-url)
3.3 Model Classification

Constitutive models for describing the time-dependent behaviour of soft soils can be classified as either empirical models, rheological models, or general stress-strain-time models (Liingaard et al., 2004). A comprehensive review of each class of model has been carried out by Liingaard et al. (2004); most of which are discussed herein.

3.3.1 Empirical Models

Empirical models are generally obtained by fitting mathematical/constitutive expressions to experimental data, e.g. the results of constant rate of strain (CRS) tests. The expressions tend to be closed-form and limited to the boundary conditions or loading situations for which they have been formulated (Liingaard et al. 2004, Bodas Freitas 2008). However, the models/expressions can be used as a basis for developing 3D constitutive relations in generalised stress space.

3.3.1.1 Semilogarithmic Creep Law

The Semilogarithmic Creep Law uses $C_\alpha$ to describe the linear relationship between settlement (or void ratio) and the logarithm of time (e.g. as observed in conventional oedometer tests). This was first proposed by Buisman (1936) to match experimental data obtained from a series of short and long duration step-loading oedometer tests on samples of clay and peat. The model requires a time origin ($t_0$) defining when creep begins. The vertical strain ($\varepsilon_{yy}$) during a creep period from $t_0$ to a time, $t$, is calculated as in Eq. 3.1, where $e_0$ is the initial void ratio (note that $C_\alpha/[1+e_0]$ is sometimes denoted $C_{\alpha,0}$, i.e. the creep coefficient in terms of strain).

$$\varepsilon_{yy} = \frac{C_\alpha}{1+e_0} \log \left( \frac{t_0 + t}{t_0} \right) \quad (3.1)$$

$C_\alpha/C_c$ Concept

Mesri & Godlewski (1977) developed the $C_\alpha/C_c$ concept by examining the relationship between the compression index, $C_c$, and $C_\alpha$ for a variety of natural soils. For the purposes of
their investigation, the symbols $C_c (\Delta e / \Delta [\log \sigma])$ and $C_\alpha (\Delta e / \Delta [\log t])$ were used to represent the compressibility in both the compression and recompression ranges. The $C_\alpha/C_c$ concept is based on the theory that the magnitude and behaviour of $C_\alpha$ with time mirrors the magnitude and behaviour of $C_c$ with stress. $C_\alpha$ and $C_c$ increase as effective stresses approach the preconsolidation pressure. The values peak at or just beyond the preconsolidation pressure, then decrease slightly and remain constant thereafter. For most natural soils, $C_\alpha/C_c$ values lie between 0.025 and 0.10. $C_\alpha/C_c$ values for inorganic soils generally lie in the range 0.025-0.06. High $C_\alpha/C_c$ values relate to highly organic soils such as peats and muskegs.

The validity of the $C_\alpha/C_c$ concept has been questioned by Watabe et al. (2012), who have highlighted that the $C_\alpha/C_c$ concept, while appearing valid for the strain rate range encountered in typical laboratory testing, is not applicable for the lower strain rates encountered in the field. Leroueil (2006) has also suggested that the $C_\alpha/C_c$ ratio will decrease with decreasing strain rate.

**Limit Creep / Constant $C_\alpha$**

The assumption of whether or not a constant $C_\alpha$ can be used to describe secondary/creep compression has received much debate. Investigations by Berre and Iversen (1972) and Leroueil et al. (1985) have shown that the slope of the settlement-log(time) curve decreases with time (i.e. indicating that secondary/creep compression may eventually cease, although Mesri & Vardhanabhuti (2005) have suggested that creep will not come to a complete stop, but will be ‘imperceptible’ at large times).

Yin (1999) developed a hyperbolic creep function to predict non-linear creep behaviour (see Eq. 3.2) to overcome the fact that logarithmic functions yield infinite settlements at infinite times. If the ratio $\psi/\nu$ (where $\nu = 1 + e$ is the specific volume) is constant, then the hyperbolic law simplifies to the logarithmic law, with $\psi = C_\alpha / \ln(10)$. In general, $\psi/\nu$ decreases with time ($\varepsilon_{yy\infty}$ is the vertical strain when time is infinite and $\psi_0' = \psi_0/\nu$ at time $t = 0$), where $\varepsilon_{yy\infty}$, $\psi_0$, and $t_0$ are model parameters.

$$
\varepsilon_{yy} = \frac{\psi}{\nu} \ln \left( \frac{t_0 + t}{t_0} \right)
$$

where

$$
\frac{\psi}{\nu} = \frac{\psi_0'}{1 + (\psi_0'/\psi_{yy\infty}) \ln[(t_0 + t)/t_0]}
$$

(3.2)
Oedometer tests carried out on samples of soft Hong Kong marine clay were used to examine its validity. Robinson (2003) has further evaluated the use of a hyperbolic creep function, concluding that it is suitable for predicting creep behaviour occurring both before and subsequent to the point at which excess pore pressures have fully dissipated.

In contrast to the majority of numerical models discussed in Section 3.4 (which tend to use logarithmic laws), the 3D elasto-viscoplastic constitutive ‘ET’ (Equivalent Time) model developed by Bodas Freitas et al. (2011) incorporates the hyperbolic law proposed by Yin (1999). For a hyperbolic law, the vertical spacing between any two isotaches or equivalent time lines (concepts explained in the Section 3.3.1.2) decreases at increasing times whereas models incorporating a linear logarithmic law have evenly spaced isotaches at all times (Bodas Freitas et al., 2011).

3.3.1.2 Isotache Models

The isotache concept (strain rate approach), which was first proposed by Šuklje (1957), is based on a unique relationship between the creep rate, the current stress state, and the current strain or void ratio ($\dot{\varepsilon}$-$\sigma'$-$\varepsilon$), irrespective of the previous loading history (Bodas Freitas, 2008). The model is based on a set of ‘isotaches’ or ‘time lines’ which are a system of parallel $e$- $\log \sigma$ curves representing void ratio or strain after a constant time of delayed compression. The model acknowledges the existence of creep during primary consolidation (a theory first proposed by Taylor & Merchant (1940)). The model also accounts for the dependence of time-dependent strains on layer thickness, permeability, and drainage conditions (Olsson, 2010).

Bjerrum (1967) developed a model similar to Šuklje's (1957) isotache model, also assuming that primary consolidation and creep are not separate processes. The Bjerrum (1967) model also explains how ageing affects the apparent preconsolidation pressure and OCR of normally consolidated clays. In Bjerrum's model (Figure 3.2), each line (or isotache) represents a specific time of sustained loading and corresponds to the equilibrium void ratio at different values of effective stress.
Bjerrum (1967) has separated the compression into two components: (i) instant compression and (ii) delayed compression (Figure 3.3).

- Instant compression is the settlement that would occur if the excess pore pressures generated due to an increase in effective stress dissipated instantaneously, i.e. if the soil were to behave as a drained material. The effective stress increase causes a reduction in void ratio until an equilibrium value is reached, at which point the structure effectively supports the overburden pressure.
- Delayed compression represents the void ratio / volume reduction at unchanged effective stress.

Instant and delayed compression are distinct from primary and secondary compression. Primary and secondary compression are separated based on the dissipation of excess pore pressures following load application whereas the terms instant and delayed compression describe the reaction of the clay to an increase in effective stress. The dotted curve in Figure 3.3 (defining instant and delayed compression) shows what would happen if the pore water in
the soil could not sustain the effective stress increase, i.e. the applied pressure would be instantaneously transferred to the soil structure as an effective pressure. However, due to the viscosity of the water, the pore pressure will dissipate gradually (corresponding to a gradual effective stress increase). This response is portrayed by the solid curve in Figure 3.3. The time required for this dissipation of pore pressure is dependent on layer thickness, permeability, and drainage conditions and so this division is unsuitable for describing soil behaviour with respect to effective stress (Bjerrum, 1967).

![Figure 3.3 Instant and delayed compression versus primary and secondary compression - Bjerrum (1967)](image_url)

The additional curve in Figure 3.2 shows that the undrained shear strength increases as a result of the void ratio reduction that occurs during delayed compression. The decreased void ratio (and hence water content) means that the number of contact points between clay particles increases and hence the cohesive component of the shear strength in plastic clays will increase. This increased strength means that cohesive clays develop a reserve resistance against further compression (the clay behaves as if overconsolidated). This is commonly referred to as an apparent preconsolidation pressure, i.e. the clay has ‘aged’ (this ‘ageing’ effect in natural clays is different to overconsolidation caused by groundwater level fluctuations, erosion, weathering, cementation, etc. (Liingaard et al., 2004). Bjerrum’s ideas
were expressed mathematically by Garlanger (1972). Garlanger used void ratio \((e)\) as opposed to engineering strain \((\varepsilon)\), which was used by Buisman (1936). Butterfield (1979) then used natural strain (or logarithmic strain), \(\varepsilon^H\) (see Eq. 3.3), noting that it supersedes engineering strain for cases involving large strain.

\[
\varepsilon^H = \ln \left( \frac{1 + e}{1 + e_0} \right)
\]  

(3.3)

Yin & Graham (1989a,b, 1994) introduced the concept of ‘equivalent time’ to describe the 1D time-dependent behaviour of clay soils because Bjerrum’s (1967) time lines for a constant duration of loading are not unique in all cases (they do not include the influence of the rate of straining) and so cannot describe the full range of laboratory procedures, e.g. multistage loading, relaxation tests, and CRS tests. The equivalent time model is also capable of modelling unload-reload behaviour and overconsolidated stress states. The concept of equivalent time used by Yin & Graham (1989a,b, 1994) corresponds to an equivalent loading time rather than an absolute time or duration of loading (Bodas Freitas, 2008). The equivalent time relates to a unique creep strain rate; larger equivalent times are associated with smaller creep strain rates.

Similar to the work of Bjerrum (1967), Yin & Graham (1989a,b, 1994) have divided the settlement into instant and delayed compression. However, in the equivalent time approach, the instant deformation is considered to be elastic whereas Bjerrum (1967) considers it as compression that occurs simultaneously with effective stress application (i.e. elastic-plastic), assuming no hydro-dynamic lag. Therefore, in the equivalent time model, the position of the instant time line occurs at higher void ratios than in Bjerrum's (1967) model. The model is formulated using a combination of an equivalent time, a reference time line, an instant time line and a limit time line (Figure 3.4). These have been explained by Liingaard et al. (2004) and Bodas Freitas (2008):

- The equivalent time \((t_e)\) is the time needed to creep from the reference time line \((t_e = 0)\) to the current value of vertical strain and vertical effective stress under constant effective stress.
• The reference time line is the reference state for calculating equivalent time. Equivalent times below the reference time line are positive whereas equivalent times above the reference time line are negative. The reference time line corresponds to the elastic-plastic line in soils that do not creep.

• The instant time line defines instantaneous purely elastic strains (in contrast to Bjerrum’s (1967) elastic-plastic instant time line), i.e. it describes the instant elastic response of the soil skeleton that occurs due to effective stress changes.

• The limit time line \( (t_e = \infty) \) defines the time beyond which the creep rate is zero.

![Figure 3.4 Equivalent times, instant time line, reference time line and limit time line - Liingaard et al. (2004)](image)

### 3.3.2 Rheological Models

According to Liingaard et al. (2004), rheological models tend to be used to gain a conceptual understanding of time effects in soil. The models, which have typically been developed for metals, steel, and fluids, describe uniaxial conditions and tend to be presented in either closed-form or differential form. There are three main categories of rheological model: (i) the differential approach, (ii) engineering theories of creep, and (iii) the hereditary approach. All three categories have been reviewed in detail in Liingaard et al. (2004), with only those based
on the differential approach described herein; the engineering theories of creep are more suitable for materials where the stress states are below the yield limit (e.g. metals and concrete) whereas the hereditary approach (based on the principle that the current strain can be obtained by integration over the entire loading history) is generally considered too complex for describing the behaviour of geomaterials (Liingaard et al., 2004).

### 3.3.2.1 Differential Approach

In the differential approach, material behaviour is represented by a combination of elementary models, e.g. the elastic Hookean spring, the viscous Newtonian dashpot, and the Saint Vernant plastic slider (Bodas Freitas, 2008), see Figure 3.5. The superscripts $e$ and $v$ in Figure 3.5 denote elastic and viscous, $E$ is the spring constant, $\eta$ is the viscosity constant, and $\sigma_y$ is the yield stress above which plastic deformations are allowed, with the stress difference $\sigma - \sigma_y$ often termed the overstress (Liingaard et al., 2004).

![Diagram of linear spring, viscous dashpot, and plastic slider](image)

**Figure 3.5** Hookean elastic spring, Newtonian viscous dashpot, Saint Venant plastic slider - Liingaard et al. (2004)

The most popular models used in the field of geomechanics are the Maxwell model, the Kelvin-Voigt model, and the Bingham model. The Maxwell model consists of a spring and a dashpot connected in series, the Kelvin-Voigt model consists of a spring and a dashpot in
parallel, and the Bingham model consist of a dashpot and slider in parallel which are connected in series with a linear spring (see Figure 3.6, where $\varepsilon^e$ and $\varepsilon^{vp}$ denote elastic and viscoplastic strains). The Bingham model concept has close parallels with overstress theory (see Section 3.3.3.1). The response is purely elastic and time-independent when the applied stress is below $\sigma_y$. The time-dependent part of the model, consisting of the parallel unit with the slider and dashpot, becomes active when the applied stress is above $\sigma_y$. The total strain rate is composed of the elastic and viscoplastic strain rates.

![Bingham Model](image)

**Figure 3.6 Bingham model - Liingaard et al. (2004)**

The linear constitutive relationships applied in these rheological models are too simple to accurately capture nonlinear soil behaviour. Additionally, extension of these models to 3D space is complicated (Liingaard et al., 2004).

### 3.3.3 General stress-strain-time models

General stress-strain-time models are capable of describing the rate-dependent behaviour of soils under a variety of different loading conditions. These include elasto-viscoplastic models and viscoelastic-viscoplastic models (not discussed). The majority of elasto-viscoplastic models are based on overstress theory or non-stationary flow surface (NSFS) theory. There are also some alternative approaches which are not mentioned here.

#### 3.3.3.1 Overstress Theory

The most popular general stress-strain-time models for describing time-dependent soil behaviour have been based on Perzyna’s overstress theory (1963, 1966). Overstress models
assume a static yield surface exists, inside which only purely elastic strains develop. This idea is similar to a traditional time-independent yield surface dividing elastic and plastic strains (Liingaard et al., 2004). In overstress models, elastic strains are time-independent while inelastic strains are time-dependent. The total strain rate is composed of an elastic strain rate and a viscoplastic strain rate.

The elastic strain rate is calculated using Hooke’s law while the viscoplastic strain rate is calculated using a flow rule. The size of the viscoplastic strain increment depends on the distance between the current loading surface and the static yield surface (Bodas Freitas et al., 2011). This is termed the overstress, and is the 3D equivalent of the stress difference, $\sigma-\sigma_y$, in the Bingham model (Bodas Freitas, 2008).

In overstress models, inelastic strains are not related to the stress history but rather to the current state of stress only (Liingaard et al., 2004). This differs to traditional elastoplastic theory which implies a consistency rule so that stresses outside the yield surface are not possible (in overstress models, the stress state can be on, within, or outside the yield surface). Overstress models cannot model tertiary creep (increasing creep strain rate, see Augustesen et al. (2004)) and hence undrained creep rupture (Adachi et al., 1987) because they predict an increased undrained shear strength with increasing viscoplastic (creep) strain rate (Bodas Freitas et al., 2011).

The models based on the overstress theory described here have been further classified by Yin et al. (2010) as ‘conventional overstress models’. Parameter determination for these models requires very low loading rate laboratory tests, which tend not to be feasible, and thus overstress models tend not to be suitable for practical use (Yin et al., 2010). Additionally, Yin et al. (2010) have noted that the fundamental theory behind conventional overstress models is flawed because experimental data indicate that viscoplastic strains always occur, thus implying that the static yield surface never actually exists. ‘Extended overstress models’, which assume viscoplastic strains inside the static yield surface, overcome this limitation.
3.3.3.2 Non-stationary Flow Surface (NSFS) Theory

The NSFS theory is an extension to the theory of classical elastoplasticity, the major difference concerning the definition of the yield condition. In classical elastoplasticity, the yield condition does not change with time when plastic strains remain constant, whereas in NSFS, the yield condition depends on time, changing every moment, even when viscoplastic strains remain constant (Liingaard et al., 2004), i.e. ‘stationary’ versus ‘non-stationary’ yield surface. As is the case with overstress theory, the total strain rate in NSFS theory is composed of an elastic strain rate and a viscoplastic strain rate.

NSFS models cannot capture time effects in the overconsolidated region, i.e. they cannot satisfactorily reproduce creep/relaxation phenomena when these processes are initiated from stress states within the yield surface (Liingaard et al. 2004, Bodas Freitas 2008). Liingaard et al. (2004) have summarised the differences between overstress theory and NSFS theory in tabular form. The Sekiguchi-Ohta (1977) model, developed for normally consolidated clays, is probably the most well-known model in this category. The model can capture creep rupture in undrained conditions (Liingaard et al., 2004) whereas overstress models cannot. Additionally, NSFS models yield time-independent elasto-plastic behaviour when loading at a very fast strain rate (thus mimicking the soil response that would be found using an equivalent time-independent elasto-plastic model) whereas overstress models would only yield an elastic response (Bodas Freitas, 2008).

3.4 Three-dimensional Elasto-Viscoplastic Models

This section describes a selection of 3D isotropic and anisotropic elasto-viscoplastic soil models, some of which additionally account for bonding and destructuration (the inviscid counterparts to these models are not discussed). The models described in this section, all of which are based on Hypothesis B, are classified as general stress-strain-time models (see Section 3.3.3).

The focus of this section is to highlight how modelling the time-dependent behaviour of soft clays has evolved since the isotropic SSC model (Vermeer et al. 1998, Vermeer & Neher 1999) was implemented in the PLAXIS FE code. The elasto-viscoplastic models used in this
thesis (SSC model and Creep-SCLAY1S model) are described in detail in Chapters 4 and 8 respectively. The purpose of this section is not to review every elasto-viscoplastic soil model but rather to trace how the Creep-SCLAY1S model evolved from earlier models to provide a frame of reference for the analyses in Chapter 8.

### 3.4.1 Isotropic Models

The **SSC model** (Vermeer *et al.* 1998, Vermeer & Neher 1999) is classified as an ‘*extended overstress model*’ because creep strains are permitted inside the yield surface so that there is no purely elastic region. The SSC model can also be referred to as a ‘*Creep model*’ (Yin *et al.*, 2010). ‘*Creep models*’ use $C_a$ as the soil viscosity input parameter. The SSC model is a 3D isotropic model suitable for normally consolidated clays, silts, and peat (Brinkgreve *et al.*, 2011). A 1D form of the model was first developed based on pioneering work by Buisman (1936), Bjerrum (1967), and Garlanger (1972) and has since been extended to its 3D form. The development of the model has been described in detail by Vermeer *et al.* (1998) and Vermeer & Neher (1999). The model is based on the isotache concept (*Hypothesis B*) proposed by Šuklje (1957), see Section 3.3.1.2.

As noted by Vermeer & Neher (1999), the majority of oedometer tests carried out to investigate secondary compression behaviour tend to be based on step loading rather than natural loading processes (which tend to be continuous or transient in nature). As a result, the 1D equations proposed by Buisman (1936) and others first need to be formulated differentially to enable a straight-forward extension to a 3D form. The procedure is described by Vermeer & Neher (1999) and Brinkgreve *et al.* (2011).

The **isotropic EVP model** developed by Yin *et al.* (2002) is also denoted a ‘*Creep model*’ based on the classification by Yin *et al.* (2010). It has been developed based on the ‘*equivalent time*’ concept of Yin & Graham (1989a,b, 1994). It incorporates a non-linear creep function and can be used to model the behaviour of normally consolidated and lightly overconsolidated clays.
3.4.2 Anisotropic Models

Anisotropic elasto-viscoplastic soil models have been developed as extensions to the EVP and SSC models, e.g. Zhou et al. (2005), Leoni et al. (2008). Clay soils exhibit anisotropy in both strength and stiffness and to disregard it could mean inaccurate predictions of soil response under loading (Zdravkovic et al. 2002, Grimstad et al. 2010). The Anisotropic Creep model (ACM, Leoni et al. 2008) uses rotated MCC ellipses as contours of volumetric creep strain rates and a rotational hardening law (Wheeler et al., 2003) to account for changes in anisotropy with viscous strains (the rotational hardening law is explained in Section 8.3). In contrast, the anisotropic EVP model developed by Zhou et al. (2005) is based on fixed anisotropy (i.e. the model accounts for initial anisotropy but does not incorporate a rotational hardening law for describing changes in anisotropy).

Yin et al. (2010), noting that the assumption made regarding the flow direction of viscoplastic strain in the aforementioned models has some limitations, have proposed a new anisotropic elasto-viscoplastic soil model based on extended overstress theory. The ‘Creep models’ incorrectly predict strain-softening behaviour for undrained triaxial tests on isotropically consolidated samples (and the stress path cannot cross the critical state line (CSL) for normally consolidated clay) because they assume that the viscoplastic volumetric strain rate is independent of the stress state. As a result, unrealistically large viscoplastic volumetric strain rates are predicted as the stress state approaches critical state, whereas in reality (i.e. experimental results), the viscoplastic volumetric strain rate should be nearly zero (Yin et al., 2010). This leads to instability in the model due to the excessively large volume contraction. The new model overcomes this deficiency by making the volumetric strain rate dependent on the stress ratio, \( \eta = q/p' \) (as \( \eta \) approaches the CSL, the volumetric strain rate diminishes accordingly to zero rather than suddenly ‘jumping’ to zero to comply with a zero volumetric strain condition at critical state).

The majority of models discussed in this section assume isotropy of elastic behaviour as incorporation of elastic anisotropy adds enormous complexity to soil models in which the anisotropy is not fixed, i.e. evolving anisotropy (Wheeler et al., 2003).
3.4.3 Anisotropic Models with Bonding and Destructuration

Soft soils and most other geological materials exhibit bonding/structure between their particles. According to Burland (1990) and Leroueil & Vaughan (1990), the effects of structure should be considered equally important to basic soil mechanics concepts such as initial void ratio and stress history in determining soil behaviour. ‘Destructuration’ refers to the progressive breakdown/degradation of bonds during straining (Leroueil et al., 1979). The soil models described in this section incorporate bonding and destructuration using the concept of an intrinsic yield surface, first used by Gens & Nova (1993). An intrinsic yield surface is an additional yield surface introduced to represent the behaviour of an equivalent unbonded soil (which would yield at lower stresses than its bonded equivalent). The difference in the size of the yield and intrinsic yield surfaces is a measure of the amount of bonding (Karstunen et al., 2006).

Yin et al. (2011) extended the Yin et al. (2010) model to account for bonding and destructuration. The model, termed the AniCreep model, takes account of both initial anisotropy and also the development/erasure of anisotropy due to viscoplastic strains using a rotational hardening law (same basis as the Yin et al. (2010) model). The EVP-SCLAY1S model is also capable of accounting for anisotropy, bonding, and destructuration. The model, described by Yin & Karstunen (2008), is based on the overstress theory of Perzyna (1963, 1966) and the elasto-plastic S-CLAY1S model (Koskinen et al. 2002, Karstunen et al. 2005). The S-CLAY1S model accounts for anisotropy, bonding, and destructuration but does not include viscous effects (see Section 8.2). The EVP-SCLAY1S model is categorised as a ‘conventional overstress model’ and so does not predict viscoplastic strains inside the yield surface.

The majority of soil models discussed here assume an associated flow rule, which tends to be a reasonable approximation when used in conjunction with an inclined yield surface (i.e. inclined reference axis), e.g. Wheeler et al. (2003). However, Grimstad et al. (2010), arguing that this assumption may not prove valid for all natural clays (based on experimental observations made by Feng (1991)), have developed the non-associated structured anisotropic creep (n-SAC) model. The n-SAC model uses Janbu’s (1969) time-resistance concept to incorporate creep effects.
According to Janbu (1969), the resistance of any medium is defined as the incremental cause divided by the incremental effect (the resistance is the tangent to the cause-effect curve when the behaviour is non-linear). The time resistance, $R$, is defined as $R = dt/dε$. $R$ varies with time and stress. Janbu (1969) has highlighted that the long-term time resistance for predicting the amount of secondary/creep compression in clays varies linearly with time, i.e. $r_s = dR/dt = \text{constant}$ (see Figure 3.7). This long-term dimensionless time resistance number, $r_s$, is the input creep parameter used for the n-SAC model and can be determined from a standard incremental load (IL) oedometer test. This parameter is not as familiar as the widely used $C_a$ or its isotropic equivalent, $μ*$. The n-SAC model also enables a creep limit, beyond which creep will not occur, to be introduced by either defining a maximum time or maximum OCR value.

![Figure 3.7](image.png)

**Figure 3.7** Graphical determination of the time resistance number from an IL oedometer test - adapted from Grimstad *et al.* (2010)

The key feature of this model is that the time resistance concept is introduced on the viscoplastic multiplier rather than on the viscoplastic volumetric strain so that the viscoplastic volumetric strain can be either positive or negative depending on whether the stress state is on the ‘dry’ or ‘wet’ side of critical state. This is advantageous because it allows soil behaviour on the ‘dry’ side of critical state to be properly captured (e.g. highly overconsolidated soils). In addition, post-peak deviatoric stress jumps observed for rate variations in undrained shear tests can be properly captured (Grimstad *et al.*, 2010). The models by Yin *et al.* (2010, 2011) have accomplished this using an extended version of the overstress concept (Grimstad *et al.*, 2010).
Models which assume constant contours of volumetric creep strain (e.g. SSC or ACM) yield unrealistic creep strains for almost all stress paths (Ollson, 2013). Additionally, anisotropic elasto-viscoplastic soil models incorporating the evolution of anisotropy using a rotational hardening law will predict a continually rotating yield surface because viscous strains will always exist whereas their inviscid counterparts will not (Ollson, 2013).

The **Creep-SCLAY1S model**, described by Sivasithamparam et al. (2014), also uses the concept of a constant rate of viscoplastic multiplier to calculate creep strain rate (a similar idea to what has been adopted by Grimstad et al. (2010) in the n-SAC model). This is in contrast with the ACM, which uses the concept of contours of constant volumetric creep strain rate which means it is limited in some aspects as listed below, see Karstunen et al. (2013), Sivasithamparam et al. (2013, 2014):

- The ACM is limited to the ‘wet’ side of critical state (a zero volumetric strain rate condition is imposed at critical state, and hence there exists no ‘dry’ side of critical state).
- Unrealistic strain softening behaviour is predicted in undrained triaxial tests.
- The ACM uses an associated flow rule, leading to a ‘jump’ in the volumetric creep rate when approaching critical state (i.e. volumetric creep rate suddenly reduces to zero). The Creep-SCLAY1S model overcomes this problem because creep is formulated using the concept of a constant rate of viscoplastic multiplier.
- The ACM also experiences problems reproducing step-changes in strain rate and hence cannot appropriately simulate the isotache behaviour observed in natural clays.

The formulation of the Creep-SCLAY1S model is described in detail in Section 8.3. The viscosity input parameter required for the Creep-SCLAY1S model \((C_\alpha \text{ or } \mu^*)\) is instantly recognizable, making the model attractive from an international modelling perspective (same input parameter as the SSC model and the ACM). The viscosity parameters used in the AniCreep, EVP-SCLAY1S and n-SAC models are not as straight-forward to establish. As noted by Jostad & Degago (2010), it would be convenient if all creep models had a common set of input parameters so that any output differences would be easier to comprehend and also a common database of parameters could be assembled to study the creep behaviour of all clays.
3.5 Summary and Implications of Literature Review II

Ladd et al. (1977) were the first to pose the question of whether creep occurs concurrently with primary settlement or whether creep only begins once primary consolidation is finished, theories now referred to as Hypothesis B and Hypothesis A respectively. A thorough review carried out by Degago et al. (2011) has suggested that that the time-dependent behaviour of clay is best described by Creep Hypothesis B. Kabbaj et al. (1988) also validated Hypothesis B for four specific test embankments in an earlier study.

Three different classes of model exist for describing time-dependent soil behaviour; empirical models, rheological models, and general stress-strain-time models. Empirical models are limited to the specific situations from which they have been derived and tend to be closed-form, although they can be used as the basis for developing 3D constitutive models. Rheological models, which also tend to be closed-form, are useful to gain a conceptual understanding of time-dependent soil behaviour. General stress-strain-time models are 3D and are often formulated incrementally and so are readily adaptable for numerical implementation into FE code. The majority of general stress-strain-time models described in Section 3.4 are based on either conventional or extended overstress theory. These models have evolved and now incorporate anisotropy, bonding, and destructuration.

The majority of the FE modelling work described in this thesis has been carried out using the commercially available elasto-viscoplastic SSC model, based on constant contours of volumetric strain rate. This assumption can result in unrealistic creep strains. The results obtained using the SSC model (Chapters 6 and 7) have been corroborated by comparison with the Creep-SCLAY1S model (Chapter 8), which uses the concept of a constant rate of viscoplastic multiplier. This allows soil behaviour on the ‘dry’ side of critical state to be properly captured and also means the model is capable of reproducing the post-peak deviatoric stress jumps observed for rate variations in undrained shear tests.
4. Material Models and Preliminary Finite Element Considerations

4.1 Introduction

The PLAXIS 2D program can be used to analyse and model a wide range of geotechnical problems, e.g. groundwater flow, excavation, embankment construction, tunnel construction, etc. This chapter introduces the principles of material modelling and gives detailed descriptions of the different material models that have been used in the subsequent chapters of the thesis. Also included is a brief description of some of the preliminary checks that have been carried out to guarantee the accuracy of the FE results, including some aspects particular to the modelling of stone column behaviour with the FEM. These checks and mesh sensitivity studies are described in more detail in Appendix A. The axisymmetric unit cell FE model used for the modelling work in the subsequent chapters of the thesis is described to provide context for the preliminary checks.

4.2 Material Modelling

The main difference between geotechnical analysis and other types of structural analysis is the complexity associated with real soil behaviour and the presence of pore fluid (Mar, 2002). Soil is a multi-phase material which consists of solid particles and pore fluid, e.g. water and air (Mar, 2002). The stress-strain response tends to be highly non-linear with strength and stiffness strongly dependent on the particular stress or strain level (Potts and Zdravkovic, 1999) and on the stress path followed (Mar, 2002).

Soil behaviour is further complicated by the fact that it exhibits anisotropy, permeability, dilatancy, and time-dependency or creep (Mar, 2002). In contrast to steel and other structural materials, modelling real soil behaviour requires advanced material/constitutive models to account for its multi-faceted nature. The accuracy depends on both the appropriateness of the selected material model for the problem in question and the ability to choose suitable parameters for the chosen model.
Material models (differing in rigour and assumptions made) approximate real soil behaviour using mathematical equations to formulate stress-strain relationships. The material models that have been used in the subsequent chapters of the thesis (SSC, SS, and HS) are described in this section. The most fundamental soil models commercially available with PLAXIS 2D are also introduced (LE, MC) and a brief description of the MCC model is included as a precursor to the SS and SSC models.

### 4.2.1 Linear Elastic (LE) model

The LE model is based on Hooke's Law of isotropic elasticity. Material behaviour is dictated by $E$ and $v$. The model is unsuitable for capturing soil behaviour accurately. Alternative stiffness moduli (which are related as shown in Eq. 4.1) such as the shear modulus ($G$), bulk modulus ($K$), or $E_{oed}$ can be used as auxiliary input parameters.

$$
G = \frac{E}{2(1+\nu)} \quad ; \quad K = \frac{E}{3(1-2\nu)} \quad ; \quad E_{oed} = \frac{(1-\nu)E}{(1-2\nu)(1+\nu)}
$$

### 4.2.2 Mohr Coulomb (MC) model

The linear elastic perfectly plastic MC model is universally recognisable in geotechnical engineering. Hooke's Law is used to describe elastic behaviour before the onset of yielding. Failure is dictated by the Mohr-Coulomb yield criterion. In addition to $E$ and $v$, the model requires three additional input parameters, $\phi'$, $c'$ (cohesion), and $\psi$. The yield surface assumes the shape of a cone with a hexagonal cross-section in principal stress space, see Figure 4.1 (the corners of the yield surface sometimes lead to computational problems). The model involves six yield functions and six plastic potential functions in the $p'-q$ plane. The yield functions are defined by $\phi'$ and $c'$ with the plastic potential functions dictated by $\psi$. The MC model assumes (unrealistically) a constant rate of dilation during shearing (leading to negative pore pressures in undrained conditions).
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Figure 4.1 Mohr Coulomb yield surface in principal stress space ($c' = 0$ kPa) - Brinkgreve et al. (2011)

4.2.3 Modified Cam Clay (MCC) model

The MCC model (Roscoe and Burland, 1968) is suitable for modelling the behaviour of normally consolidated clays (Brinkgreve et al., 2011). It is an extension of the original Cam Clay model proposed by Roscoe et al. (1963). The yield surface of the MCC model is elliptical in $p'$-$q$ space which makes it more suitable for constitutive FE modelling than the original model (in which the tear shaped yield surface incorporates a discontinuity). Stress states within the yield surface result in purely elastic strains. The size of the yield surface is dictated by the magnitude of the preconsolidation pressure (Mar, 2002). The model is based on a logarithmic relationship between mean effective stress ($p'$) and $e$. The behaviour in primary loading (Eq. 4.2) is dictated by the compression index ($\lambda$) while the unload-reload behaviour (Eq. 4.3) is dictated by the swelling index ($\kappa$). The CSL describes the relationship between $p'$ and $q$ (deviator stress) at failure, see Eq. 4.4. The parameter $M$ (Eq. 4.4) determines the height of the ellipse and hence the shape of the yield surface. The model is based on the Drucker-Prager failure criterion (conical yield surface in principal stress space).

$$e - e_0 = \lambda \ln \left( \frac{p'}{p'_0} \right)$$  \hspace{1cm} (4.2)
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\[ e - e_0 = \kappa \ln \left( \frac{p'}{p'_0} \right) \]  \hspace{1cm} (4.3)

\[ q = Mp'; \ M = \frac{6\sin\phi'}{3 - \sin\phi'} \]  \hspace{1cm} (4.4)

### 4.2.4 Soft Soil (SS) model

The SS model is suitable for modelling the primary compression behaviour of near normally-consolidated clay-type soils (Brinkgreve et al., 2011). Similar to the MCC model, the SS model is based on a logarithmic stress-strain relationship, e.g. Eqs. 4.5 and 4.6 for primary loading and unloading-reloading respectively (Figure 4.2). The unload-reload response is assumed to be elastic, as denoted by the ‘e’ superscripts in Eqs. 4.5 and 4.6. The model is formulated in terms of volumetric strain (\(e_v\)) instead of void ratio with \(\lambda^*\) and \(\kappa^*\) related to \(\lambda\) and \(\kappa\) as defined in Eq. 4.7. The relationship between the isotropic material parameters (\(\lambda^*\) and \(\kappa^*\)) and the conventional 1D compression and swelling indices (\(C_c\) and \(C_s\)) is described by Eq. 4.8. There is no unique relationship between \(\kappa^*\) and \(C_s\), because the ratio of horizontal to vertical stress changes during 1D unloading (Brinkgreve et al., 2011). An approximation is obtained assuming an isotropic stress state during unloading so that the horizontal and vertical stresses are equal.

\[ e_v - e_v^0 = \lambda^* \ln \left( \frac{p'}{p'_0} \right) \]  \hspace{1cm} (4.5)

\[ e_v^e - e_v^{e0} = \kappa^* \ln \left( \frac{p'}{p'_0} \right) \]  \hspace{1cm} (4.6)

**Figure 4.2** Logarithmic stress-strain relationship - Brinkgreve et al. (2011)
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\[
\kappa^* = \frac{\kappa}{(1 + e_0)}; \quad \lambda^* = \frac{\lambda}{(1 + e_0)} \quad (4.7)
\]

\[
\kappa^* \approx \frac{2C_s}{(1 + e_0) \ln10}; \quad \lambda^* = \frac{C_s}{(1 + e_0) \ln10} \quad (4.8)
\]

The yield surface of the SS model is based on the MCC model but for the SS model the yield surface is decoupled from the failure line so that failure is not necessarily related to critical state. This gives better \(K_0\) predictions than the MCC model which predicts unrealistically high \(K_0\) values in the NC range. The SS model is based on the Mohr-Coulomb failure criterion but the slope of the failure line is smaller than the slope of the \(M\)-line (Figure 4.3).

The parameter \(M\) (Eq. 4.9) in the SS model is used to determine the height of the ellipse in the \(p'\)-\(q\) plane, where \(v_{ur}\) denotes Poisson’s ratio for unloading-reloading. The height of the ellipse is responsible for the ratio of horizontal to vertical stresses in primary 1D compression and so the value of \(M\) should be compatible with a known value of \(K_0^{nc}\) (coefficient of lateral earth pressure in the normally consolidated condition) in primary 1D compression.

\[
M = \sqrt{\frac{(1 - K_0^{nc})^2}{(1 + 2K_0^{nc})^2} + \frac{(1 - K_0^{nc})(1 - 2v_{ur})(\lambda^*/\kappa^* - 1)}{(1 + 2K_0^{nc})(1 - 2v_{ur})\lambda^*/\kappa^* - (1 - K_0^{nc})(1 + v_{ur})}} \quad (4.9)
\]

**Figure 4.3** Yield surface of the SS model in the \(p'\)-\(q\) plane - Brinkgreve et al. (2011)

The total strain (\(\varepsilon\)) is composed of an elastic component, \(\varepsilon^e\), and a plastic component, \(\varepsilon^p\) (see Eq. 4.10, rate effects are not included). Stress changes below the yield cap induce elastic strains. Once the yield stress is exceeded, plastic strains are developed. The mean yield stress is updated based on the amount of accumulated plastic strain (Degago et al., 2011). As a
result, an infinite number of ellipses exist during loading (Figure 4.3), each corresponding to a different preconsolidation stress \( (p_p) \). In order to ensure the cap remains in the compression zone (e.g. when an effective cohesion \( (c') \) is assigned to the failure surface or when \( c' = 0 \)), a ‘threshold’ ellipse (Figure 4.3) with a minimum \( p_p = c' \cot \phi' \) or one stress unit is defined.

\[
\varepsilon = \varepsilon^e + \varepsilon^p
\] (4.10)

### 4.2.5 Soft Soil Creep (SSC) model

The SSC model is the only elasto-viscoplastic soil model commercially available with PLAXIS software and therefore it has been used as the basis for the majority of the numerical modelling work described in this thesis. The total strain \( (\varepsilon) \) in the SSC model is composed of an elastic component \( (\varepsilon^e) \) and a viscoplastic (creep) component \( (\varepsilon^c) \), see Eq. 4.11. The viscoplastic component can be further divided into strains during and after consolidation respectively (see Figure 4.4), where \( \sigma'_0 \) is the initial effective stress, \( \sigma' \) is the final effective stress, \( \sigma_{p0} \) is the initial preconsolidation stress before loading, \( \sigma_{pc} \) is the preconsolidation stress at EOP, and \( t' = t - t_c \) is the effective creep time (with \( t \) and \( t_c \) denoting the times from the beginning of loading and to EOP respectively). The parameter \( \tau_c \) can be determined from a standard oedometer test using the construction developed by Janbu (1969), e.g. Figure 4.5. The physical meaning of \( \tau_c \) is still unresolved (Brinkgreve et al., 2011).

\[
\varepsilon = \varepsilon^e + \varepsilon^c = A \ln \left( \frac{\sigma'}{\sigma'_0} \right) + B \ln \left( \frac{\sigma_{pc}}{\sigma_{p0}} \right) + C \ln \left( \frac{\tau_c + t'}{\tau_c} \right)
\] (4.11)

![Figure 4.4 Idealised stress-strain curve from an oedometer test - Brinkgreve et al. (2011)](image-url)
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Figure 4.5 Consolidation and creep behaviour in a standard oedometer test - Brinkgreve et al. (2011)

The parameters $A$, $B$, and $C$ are related to the conventional 1D parameters as shown in Eq. 4.12.

$$A = \frac{C_s}{(1+e_0) \ln 10} \quad ; \quad B = \frac{C_e - C_s}{(1+e_0) \ln 10} \quad ; \quad C = \frac{C_\alpha}{(1+e_0) \ln 10}$$ (4.12)

The yield surface of the SSC model is similar to the MCC model, but, based on Bjerrum’s (1967) concept, the preconsolidation stress ($\sigma_p$) is assumed to depend on the amount of viscoplastic strain that is accumulated with time (Degago et al., 2011), i.e. the yield surface of the model expands due to creep, e.g. Eq. 4.13.

$$\sigma_p = \sigma_{p0} \exp \left( \frac{\varepsilon^c}{B} \right)$$ (4.13)

The elastic strain rate depends on the rate of increase of mean effective stress whereas the viscoplastic strain rate depends on the current state of effective stress and strain relative to a reference creep rate (defined at a reference intrinsic time corresponding to a reference preconsolidation pressure on the reference isotache). For an IL oedometer test with a loading duration of 1 day, the reference time ($\tau$) is usually 1 day for load steps on the normal consolidation line. However, the value of the assumed preconsolidation pressure is related to the duration of the test and thus a different test time would lead to a different preconsolidation pressure (Leoni et al., 2008).
Eqs. 4.11 and 4.13 describe the relationship between accumulated creep and time under a constant effective stress. They have been converted into a differential form (Eq. 4.15) by deriving an expression for $\tau_c$ (Eq. 4.14); for solving transient or continuous loading problems, the constitutive law needs to be in differential form, e.g. Vermeer & Neher (1999). Additionally, all inelastic strains are assumed to be time-dependent so that the total strain rate is composed of an elastic strain rate and a time-dependent creep part ($\dot{\varepsilon}^e + \dot{\varepsilon}^c$).

\[
\tau_c = \tau \left( \frac{\sigma_{pc}}{\sigma'} \right)^{\frac{B}{C}} \tag{4.14}
\]

\[
\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^c = A \ln \left( \frac{\dot{\sigma}'}{\sigma'} \right) + \frac{C}{\tau} \left( \frac{\dot{\sigma}'}{\sigma_p} \right)^{\frac{B}{C}} \tag{4.15}
\]

The stress invariants, $p'$ and $q$ are then used for the extension of the 1D model to the 3D model. A new stress measure, $p^{eq}$ (Eq. 4.16), is defined, which is constant on ellipses in the $p'-q$ plane (Figure 4.6). $p^{eq}$ is equivalent to $\sigma'$ in the 1D model and $p_p^{eq}$ (a generalised preconsolidation pressure) is equivalent to $\sigma_p$.

\[
p^{eq} = p' - \frac{q^2}{\frac{2}{M^2}(p' + c' \cot \phi')} \tag{4.16}
\]

Figure 4.6 $p^{eq}$ ellipses in the $p'-q$ plane - Brinkgreve et al. (2011)
For the 3D model, the isotropic material parameters \((\kappa^*, \lambda^*, \text{ and } \mu^*)\) are used in place of the 1D parameters \((A, B, \text{ and } C)\), e.g. Eq. 4.17. The creep coefficient, \(\mu^*\), can be determined from a standard oedometer test by plotting \(\varepsilon_v\) against \(\ln t\), e.g. Figure 4.7. The time period must be sufficiently long to ensure that the slope of the settlement-time curve is a straight line after consolidation. If the time period is not long enough, the consolidation rate will not be low enough and the creep rate will be overestimated because the consolidation rate will be mistakenly added to the creep rate, e.g. Waterman & Broere (2005).

\[
\kappa^* \approx 2A \; ; \; B = \lambda^* - \kappa^* \; ; \; \mu^* = C = \frac{C}{1 + e_0} \ln 10
\]  (4.17)

**Figure 4.7** \(\mu^*\) determination \((\varepsilon_v \text{ versus } \ln t)\) - Waterman & Broere (2005)

The 3D form of Eq. 4.15 is given by Eq. 4.18, with \(p^c_p\) related to \(p^eq\) as shown in Eq. 4.19 (3D form of Eq. 4.13).

\[
\dot{\varepsilon} = \dot{\varepsilon}^c + \dot{\varepsilon}^p = \kappa^* \ln \left( \frac{p^eq}{p^p} \right) + \frac{\mu^*}{\tau} \left( \frac{p^eq}{p^p} \right)^{\lambda^* - \kappa^*} \mu^*
\]  (4.18)

\[
p^c_p = p^eq_p \exp \left( \frac{\varepsilon^c}{\lambda^* - \kappa^*} \right)
\]  (4.19)

In addition to volumetric creep strains, soft soils will also exhibit deviatoric creep strains. The formulation of the 3D model was completed (to include deviatoric creep strains) by adopting a flow rule for the rate of creep strain based on the assumption that creep strain is simply a time-dependent plastic strain (Vermeer & Neher, 1999).
The volume change during a creep period, $\Delta t$, is given by Eq. 4.20. The creep ratio, $$(\lambda^* - \kappa^*)/\mu^*$$, governs the change of creep rate with time, where $\mu^*$ specifies the creep rate at the reference time, $\tau$ ($\tau = 1$ day for the SSC model).

$$\Delta \varepsilon^c_v = \mu^* \ln \left( 1 + \frac{\Delta t}{\tau} \left( \frac{p_{eq}^c}{p_{eq}^p} \right)^{\frac{\lambda^* - \kappa^*}{\mu^*}} \right)$$

(4.20)

The OCR defines the creep rate at which the PLAXIS calculation starts:

- The amount of creep is heavily dependent on the OCR. The correct value of the OCR should take into account the time elapsed since the soil was formed and started creeping (Waterman & Broere, 2004).
- Even for ‘normally consolidated soils’, Brinkgreve (2001) recommends using an initial OCR value greater than 1.0 so that the predicted initial settlement velocity is not excessive (using OCR = 1.0 gives an unrealistic initial creep rate). In such situations, it is recommended to reset $K_0 = K_{0,nc}$ as opposed to the value ($K_0 > K_{0,nc}$) suggested by PLAXIS (Brinkgreve, 2001).
- The initial OCR can be estimated based on the age of the soil in question, e.g. Degago (2011). It is unrealistic to have a ‘normally consolidated soil’ with an OCR of 1.0, as in general, soft clays will display a small overconsolidation effect due to ageing (e.g. Degago, 2011) and water table fluctuations. Using an OCR of 1.0 to represent a ‘normally consolidated clay’ would imply the clay to be only 1 day old (Degago, 2011).

The SSC model is based on the Mohr-Coulomb failure criterion. The parameter $M$ (Eq. 4.9) governs the height of the ellipse (similar idea to the SS model), where $M$ is again determined based on $K_{0,nc}$, i.e. failure does not correspond to critical state. A time step or interval should be defined for the SSC model, otherwise the yield cap (which is responsible for plastic deformations) will not be able to move and as a result the model will predict an unrealistically stiff soil response.
4.2.6 Hardening Soil (HS) model

The HS model is a hyperbolic elasto-plastic model which models the dependence of stiffness moduli on stress level but does not account for viscous effects such as creep and stress relaxation. The model is based on the Mohr-Coulomb failure criterion and its yield surface can expand due to plastic strains. The formulation of the model has been described in detail by Schanz et al. (1999). The MCC and SS models are based on volumetric (compression) hardening. These models work well on the ‘wet’ side when volumetric strains dominate but are not representative on the ‘dry’ side. In the HS model, the volumetric hardening has been complemented by deviatoric (shear) hardening to overcome this.

The HS model is an extension of the hyperbolic model developed by Duncan & Chang (1970). The formulation of the shear hardening yield surface is based on a hyperbolic relationship between $\varepsilon_{yy}$ and $q$ in primary triaxial loading, e.g. Figure 4.8, where $E_i$ is the initial modulus, $E_{50}$ is the secant/triaxial modulus, $q_a$ is the asymptotic value of the shear strength, $q_f$ is the ultimate deviatoric stress (based on the Mohr-Coulomb failure criterion) and $R_f$ is the failure ratio ($R_f = q_f/q_a$).

$$q$$

$q_a$

$q_f$

asymptotic

failure line

$E_i$

$E_{50}$

$E_{tr}$

$E_{xx}$

$\varepsilon_{yy}$

Figure 4.8 Hyperbolic stress-strain relationship in primary loading for a standard drained triaxial test - Brinkgreve et al. (2011)

In addition to the plastic shear strains that develop in primary deviatoric loading, the model accounts for plastic volumetric strains occurring in triaxial stress states based on an adaptation of Rowe’s (1962) stress-dilatancy theory. A second yield surface (cap yield surface) is introduced to account for the plastic volumetric strains that occur in isotropic
compression, without which it would not be possible to formulate a model with independent input of $E_{50}$ and $E_{oed}$. The size of the cap yield surface is governed by the preconsolidation stress. The stress dependency of soil stiffness is dictated by the power, $m$. The model requires the input of three stiffness parameters at a chosen reference pressure ($p^{ref}$), $E_{50}^{ref}$, $E_{oed}^{ref}$ and $E_{ur}^{ref}$ (unload-reload modulus). The position of the shear hardening yield surface is largely controlled by $E_{50}$ while the position of the yield cap (compression hardening) is largely controlled by $E_{oed}$. The yield surfaces are plotted in principal stress space in Figure 4.9. Both yield surfaces assume the shape of the Mohr-Coulomb failure criterion in the $p'\cdot q$ plane.

![Figure 4.9 HS model yield surfaces in principal stress space for a cohesionless soil - Schanz et al. (1999)](image)

For soft soils ($m = 1$), the stiffness parameters can alternatively be entered in terms of $C_c$ and $C_s$, which are related to the stiffness moduli as in Eq. 4.21.

\[
E_{oed}^{ref} = \frac{2.3(1 + e_0) p^{ref}}{C_c} ; \quad E_{ur}^{ref} = \frac{2.3(1 + e_0)(1 + \nu)(1 - 2\nu) p^{ref}}{(1 - \nu)C_s} ; \quad E_{50}^{ref} = 1.25E_{oed}^{ref} \quad (4.21)
\]

### 4.2.7 Soil model comparisons

The SSC, SS, and HS models are compared in this section. Comparisons between these three models are the basis for 'Approach B' in Section 6.4.
4.2.7.1 Constitutive Model Yield Surfaces

For the SS and SSC models, $M$ should be compatible with a known value of $K_0^{nc}$ in 1D compression. In contrast, the yield surface of the HS model (which incorporates a more complex and realistic hardening mechanism) is shaped differently in order to model a more realistic $K_0^{nc}$ value in 1D compression. Brinkgreve et al. (2011) have suggested that the modelling capabilities of the SS model are superseded by the HS model. The HS and SS/SSC models will only be in agreement for oedometer-type loading paths, e.g. Vermeer (1999).

4.2.7.2 ‘Soil Test’ Facility

In this thesis, different approaches (introduced in Chapter 6) have been used to assess the effectiveness of using stone columns to arrest creep settlements. These approaches are based on comparing PLAXIS output for different soil models (HS, SS, and SSC) and for different scenarios using the SSC model (i.e. with creep and without creep). As a precursor to this, the PLAXIS ‘Soil Test’ facility has been used to compare the undrained behaviour of a soft clay using the three different soil models for five different scenarios:

1. SS model (standard).
2. HS model (standard).
3. HS model with the mobilised relative shear strength to one (MRSS = 1), in effect turning off the shear hardening yield surface so that only the cap hardening remains. This cannot be effected in a full FE calculation.
4. SSC model (standard).
5. SSC model with $\mu^*$ set to the minimum allowable value.

The soil properties used for these comparisons correspond to those quoted in Table 5.3 for the single-layer profiles described in Sections 4.3.1 and 5.4. The soil stiffnesses for the single-layer profiles have been altered to achieve a desired $E_r/E_s$, and as such the simulated soil response will not be directly comparable with the test results reported by Atkinson et al. (1992), Hight et al. (1992a), and Smith et al. (1992). The purpose of this section is solely to assess how the models behave relative to one another. Section 5.3.4 will focus on quantitative comparisons with the test data reported in ICE (1992).
Isotropically consolidated undrained (ICU) triaxial compression tests

- Firstly, isotropically consolidated undrained (ICU) triaxial compression tests have been simulated; the stress-strain behaviour is shown in Figure 4.10a. The SS and SSC model stiffness responses are similar; the HS model predicts a softer stiffness response. Similar deviator stresses at failure are predicted for all three models.
- The response of the SSC model is strain-rate dependent; a higher strain rate leads to a higher deviator stress and thus a higher shear strength.
- The corresponding stress paths in \( p' - q \) space are compared in Figure 4.10b. In this case, the inviscid model stress paths are almost coincident while the SSC model stress paths show strain-rate dependence; higher strain rates result in less of a reduction to the mean effective stress which in turn contributes to a larger deviator stress at failure.
- When the MRSS is set to one for the HS model, its response is very similar to the SS model (note that the HS and HS (MRSS = 1) lines are coincident in the \( p' - q \) plot in Figure 4.10b).
- The simulations using the SSC model with \( \mu^* \) set to the minimum allowable value highlight that creep is not only difference between the SS and SSC models; the SSC (\( \mu^* \approx 0 \)) and SS model lines on Figures 4.10a and 4.10b should collapse on one another if it were. The behaviour for SSC (\( \mu^* \approx 0 \)) is no longer strain-rate dependent but the stiffness response in triaxial compression is different nonetheless.
- In Figure 4.10a, the SSC model with \( \mu^* \approx 0 \) predicts softening post-peak whereas the SS model does not. The post-peak softening behaviour predicted by the SSC model occurs because the viscoplastic volumetric strain rate is independent of the stress state (see Section 3.4.2). The SS and SSC models also deal with plasticity in different ways. Note that the SS and SSC (\( \mu^* \approx 0 \)) model curves are coincident in Figure 4.10b until the stress path reaches the CSL, at which point the SSC (\( \mu^* \approx 0 \)) viscoplastic strain suddenly jumps to zero.

Incrementally-loaded (IL) oedometer tests

An incrementally-loaded (IL) oedometer test with a load-step duration of 1 day has also been simulated in order to compare the behaviour of the models in 1D compression (similar to the unit cell scenario modelled in the following chapters). In this case (Figure 4.10c, \( \sigma'_{yy} \) denotes
the vertical/axial effective stress), the models are in better agreement. As expected the SSC model predicts a larger preconsolidation pressure than either of the inviscid models (although the difference is minor); this is due to the influence of creep, i.e. Bjerrum (1967). In the normally consolidated range, the differences between the predicted $C_c$ or $\lambda^*$ values in all cases do not exceed 10%.
Figure 4.10 ‘Soil Test’ Output (a) ICU Triaxial: $q$ vs. $\varepsilon_{yy}$ (b) ICU Triaxial: $q$ vs. $p'$ (c) Oedometer: $\varepsilon_{yy}$ vs. $\log \sigma'_{yy}$
4.3 Preliminary Finite Element Considerations

A number of preliminary checks have been carried out to guarantee the accuracy of the results obtained using the FEM. These checks, which are described in detail in Appendix A, are summarised here. Additional considerations relevant to modelling stone column behaviour with the FEM are also discussed. A description of the axisymmetric unit cell FE model is included to provide a frame of reference for the subsequent checks.

4.3.1 Unit Cell Model

In the same vein as the work of Debats et al. (2003), Ambily & Gandhi (2007) and others, axisymmetric unit cell analyses have been used for the majority of the FE modelling carried out in this thesis. A small number of 3D analyses (described in Section 5.3) have been carried out using PLAXIS 3D Foundation to validate the adopted soil profile. A column diameter of $D_c = 0.6\text{m}$ (column radius, $R_c = 0.3\text{m}$) was adopted as typical of columns in soft cohesive soil deposits constructed with a standard 430mm diameter poker, e.g. Watts et al. (2000), Bell (2004). The initial analyses have been carried out for a simplified single-layer profile (5m deep unit cell), see Figure 4.11. Further analyses have been carried out for longer single-layer unit cells measuring 10m and 15m in length (Figure 4.12). These column lengths correspond to $L_c/D_c$ ratios of 8.33, 16.67, and 25.00 respectively.

![Figure 4.11 5m Unit Cell Model](image)
The soil parameters for the simplified profiles have been based upon the heavily-researched Bothkennar Carse clay test site in Scotland. For these initial studies, the stiff crust has been omitted in the interests of clarity and the soil stiffnesses have been altered to achieve desired $E_c/E_s$ ratios. Additional unit cell analyses have also been carried out for the multi-layer Bothkennar profile consisting of a 1.5m crust overlying two layers of Carse clay. The soil profiles and soil parameters are described in detail in Chapter 5.

Boundary conditions applied to the unit cell models are reflective of oedometric conditions; the vertical boundaries are restricted laterally (i.e. roller boundaries) while the base is fixed in all directions. The vertical boundaries are modelled as closed consolidation boundaries while the top and bottom layers permit groundwater flow. The columns have been ‘wished-in-place’ (as used in previous studies by Killeen & McCabe (2010) and Gäb et al. (2008)) and are modelled as end-bearing on a hard stratum. Initial stress generation has been carried out using the $K_0$ procedure; the $K_0$ procedure can be used in situations in which the top surface in the soil model is horizontal and all remaining soil layers, including the water table, are parallel to the surface, e.g. Brinkgreve et al. (2011).
The majority of the unit cell analyses have been carried out for a range of commonly-encountered reciprocal area-replacement ratios, $3 < A/A_c < 10$, as identified in the McCabe et al. (2009) database. Different $A/A_c$ values have been obtained by varying the unit cell diameter (i.e. the spacing, $s$, in Eq. 2.5) while maintaining $D_c = 0.6m$.

### 4.3.2 Mesh Sensitivity and Iterative Control Parameters

The following points have been considered in this section:

- The sensitivity of the FE output has been examined to assess both the impact of element type (6-node or 15-node, see Section A2.1) and mesh coarseness (see Section A2.3). The mesh sensitivity studies described in Appendix A have been carried out for the 5m unit cell with the soil modelled using the SSC model. These studies indicate that meshes consisting of ~2000 or more nodes produce sufficiently accurate predictions for the maximum displacement ($u_y$) at the surface of the unit cell and for the average mean effective stresses ($p'$) in the column and soil at the end of the final analysis phase. Based on the results, fine meshes consisting of 6-noded elements were deemed sufficient for this unit cell. Similar mesh sensitivity studies for the multi-layer Bothkennar soil profile also confirmed the appropriateness of a fine mesh consisting of 6-noded elements.

- The choice of element type influences the time steps adopted by the PLAXIS program when carrying out consolidation analyses. The program selects appropriate time steps using the automatic time-stepping procedure (see Section A2.2.1), and although it is possible to overwrite the default setting to use time-steps smaller than the suggested minimum value, this can lead to oscillating pore pressures (e.g. Vermeer & Verruijt, 1981). The mesh sensitivity studies have been carried out using the default setting to isolate the influence of element type and size.

- The influences of altering some other default iterative control parameters used by the FE program are also discussed in Appendix A (see Sections A2.2.2 and A2.2.3). These include the desired maximum and minimum settings (which encourage the program to use larger or smaller steps), the tolerated error (which ensures that local and global equilibrium errors remain within acceptable limits), and the arc-length control procedure (used in load-controlled calculations to accurately determine failure loads).
4.3.3 Modelling Stone Column Behaviour using the Finite Element Method (FEM)

Additional aspects that need to be considered when modelling stone column behaviour with the FEM include:

- **Interface elements (see Section A3.1):** The majority of numerical studies investigating stone column behaviour, including those in this thesis, have declined to use interface elements at the boundary between the granular column material and the in-situ soil, e.g. Ambily and Gandhi (2007), Domingues et al. (2007a,b), Gäb et al. (2008). One series of analyses carried out as part of this research using interface elements has indicated that the time-settlement behaviour and hence the resulting \( n \) values are relatively unaffected.

- **Choice of model for stone column (see Section A3.2):** The HS model has been used in this thesis to model the granular column material. The influence of using the LE and MC models for the column material has been examined. The results indicate that the MC and HS models yield similar \( n \) values but the LE model significantly overpredicts \( n \) values (i.e. column yielding is not considered, see Section 2.7.1.1).

- **Column installation (see Section A3.3):** The ‘wished-in-place’ column installation technique, which has been used for the majority of analyses in this thesis, may result in out-of-equilibrium stresses which need to be restored prior to load application. This has been achieved using a plastic nil-step, which needs to have a time interval when applied in conjunction with the SSC model. A parametric study has been carried out to establish an appropriate nil-step duration that does not impact upon the final settlement.

4.3.4 Other Considerations

The following additional aspects have been considered:

- **Permeability (k, see Section A4.1):** Parametric studies have been carried out to investigate how the permeability of the clay may affect the FE output. The results indicate that although increasing the permeability accelerates the rate of consolidation, the corresponding \( n \) values are unaffected.

  **Change of Permeability Index (\( C_k \), see Section A4.2):** \( C_k \) defines how the permeability changes during a consolidation analysis. The permeability changes according to Eq. 4.22,
where $k$ is the updated permeability and $k_0$ is the initial permeability. Parametric studies have been carried out to assess whether using the default setting of $C_k = 1 \times 10^{15}$ (no change of permeability with void ratio during consolidation) or $C_k = 0.5e_0$ (Leroueil et al., 1992) has any influence on the resultant $n$ values. The outcome of this study was as expected; using $C_k = 0.5e_0$ results in a marginally slower rate of excess pore pressure dissipation but the corresponding $n$ values are unaffected. The default setting has been used for the single-layer profiles (Chapter 6) while $C_k$ has been defined $= 0.5e_0$ for the multi-layer profile (Chapters 7 and 8).

$$\log \left( \frac{k}{k_0} \right) = \frac{\Delta e}{C_k}$$ (4.22)

- **Modelling Soft Soil Behaviour (see Section A4.3):** Soil behaviour can be modelled as drained or undrained. Drained behaviour is suitable for highly permeable materials for which no excess pore pressures are generated during loading whereas undrained behaviour should be used for low permeability soils which develop considerable pore pressures when loaded. PLAXIS includes three options for modelling undrained behaviour. The Undrained A approach (undrained effective stress analysis with effective stiffness and effective strength parameters) has been used in this thesis. Analyses were carried out which have verified that $n$ values are similar whether modelling the clay as a drained material or as an undrained material with a follow-up consolidation period.

- **Updated Mesh Analyses (see Section A4.4):** The influence of using an Updated Mesh on predicted $n$ values has been examined for both the single-layer and multi-layer profiles. The results indicate that the final settlements are less when using the Updated Mesh option but the corresponding $n$ values are unaffected. Hereafter, the single-layer profiles have been analysed assuming no mesh geometry change as settlement proceeds but the Updated Mesh option has been used for the multi-layer profile to account for the larger displacements.

- **Oedometer simulations (see Section A4.5):** Oedometer simulations were carried out using the SSC model prior to the axisymmetric unit cell modelling work described in the subsequent chapters. These served the purpose of: (i) verifying that the output values of $C_c$ and $C_o$ corresponded to their respective inputs, (ii) establishing the effect of using an Updated Mesh on these parameters, (iii) identifying how the creep parameter is affected
by the preconsolidation pressure, and (iv) confirming that the $C_c$ and $C_a$ outputs are unaffected if modelling on a larger scale.

### 4.3.5 One-dimensional compression behaviour validation

The settlements without columns for the 5m, 10m, and 15m unit cells for the different soil models and soil stiffnesses are validated using 1D compression formulae. The percentage differences between the PLAXIS 2D output and the 1D compression formulae predictions are less than 4% in all cases, affirming the reliability of both the modelling procedure and the PLAXIS output.

### 4.4 Summary

In this chapter, the commercially available PLAXIS 2D soil models that have been used in this research have been described in detail. The inviscid SS model is suitable for normally consolidated clay-type soils and is based on a similar logarithmic stress-strain relationship to the MCC model. The SS model is based on the Mohr-Coulomb failure criterion and yields better $K_0$ predictions than the MCC model in the NC range. The elasto-viscoplastic SSC model is also based on the MCC model, but, based on Bjerrum’s (1967) concept, the yield surface expands due to ageing or creep. The inviscid HS model includes deviatoric hardening to complement volumetric hardening which makes it suitable for overconsolidated soils. The second half of this chapter briefly introduces some important considerations in FE modelling of stone columns and some of the checks that have been carried out to guarantee the accuracy of the FE results. These considerations and checks are described in more detail in Appendix A.
5. Finite Element (FE) Model Parameters

5.1 Introduction

The primary focus of this research is to ascertain how creep influences stone column behaviour in soft soils. The Bothkennar soft clay test site, located on the southern bank of the Forth estuary, near Grangemouth in Scotland, was purchased by the UK Science and Engineering Research Council (SERC) in 1987 as a national soft clay test bed. Bothkennar provides a suitable soil profile for the purpose of the FE study; the creep ratio, \((\lambda^*-\kappa^*)/\mu^*\), of Bothkennar clay may be considered as a ‘mid-range’ value of the selection of soft clay sites worldwide shown in Figure 5.1.

![Figure 5.1 (\(\lambda^*-\kappa^*\))/\(\mu^*\) versus \(\mu^*\) for worldwide clays](image)

The first section of this chapter describes the Bothkennar test site and the associated parameters adopted for the PLAXIS 2D FE model. An adapted version of the HS model soil profile used by Killeen (2012), based on ICE (1992), is used. Killeen’s (2012) research focused on examining a wide range of behavioural aspects relevant to small groups of stone columns supporting small area footings. However, creep was not considered, so additional material parameters have been derived for the SSC model in this thesis.

The PLAXIS 2D profile is then validated against Killeen's (2012) PLAXIS 3D Foundation profile (which has in turn been validated against a field load test described by Jardine et al.)
(1995) for an unreinforced rigid pad footing). Additionally, simulations carried out using the PLAXIS 'Soil Test' facility in this thesis (see Section 5.3.4) have been used to calibrate the adopted soil parameters with high quality test data reported by Atkinson et al. (1992), Hight et al. (1992a), and Smith et al. (1992).

The Bothkennar profile used by Killeen (2012) is a multi-layer profile consisting of a crust overlying two layers of Carse clay. The initial FE analyses described in Chapter 6 are based on simplified single-layer profiles (with the stiff crust omitted in the interests of clarity, see Sections 4.3.1 and 5.4). The parameters are based on the Bothkennar parameters but the complexities associated with interpreting the results for a multi-layer profile are avoided (the multi-layer Bothkennar profile is used in Chapters 7 and 8). The behaviour of stone columns in creep-prone soils has received very little attention and so it was deemed prudent to start this research by studying single-layer profiles initially, described in the penultimate section of this chapter (Section 5.4). The final section summarises the material parameters used for the stone backfill, also based on Killeen (2012).

5.2 Bothkennar Soil Profile

5.2.1 Site Description, Soil Classification and Clay Mineralogy

The Bothkennar soft clay test site is located close to the Kincardine Bridge midway between Edinburgh and Glasgow (Figure 5.2a). The Bothkennar soft clays were deposited under stable estuarine conditions and tend to be relatively uniform across the site (ideal for geotechnical research) apart from a laminated zone in the south-east corner (Nash et al., 1992a), Figure 5.2b.

The Bothkennar soil can be classified as a silty clay using the A-line of the BS5930 (1999) plasticity chart. This classification is based on clay fractions ranging from 35-50% measured by Paul et al. (1992) using ultrasonic methods to disperse the clay. The clay can be subdivided into four main facies types (bedded, laminated, mottled and weathered), e.g. Paul et al. (1992). It is highly structured and possesses a significant organic content of between 3% and 5% (measured by loss on ignition) with notably lower and higher values evident in the laminated and mottled facies respectively (Hight et al., 1992b). As reported by Hight et
al. (1992b), the clay mineralogy and composition is relatively uniform but it does exhibit natural variability in structure, fabric and state depending on the facies type.

![Figure 5.2](image)

**Figure 5.2** (a) Bothkennar Site Location (b) Bothkennar Site Plan - Nash et al. (1992a)

The majority of the in-situ testing was carried out in the south-west corner of the site (as marked in Figure 5.2b). The geotechnical profile for a borehole at this location is shown in Figure 5.3. The natural moisture content ($w$) ranges from 30-75% while the liquid limit ($w_L$) ranges from 50-80%. The plastic limit ($w_P$) is relatively constant at 25%. The profile of bulk unit weight (derived from samples recovered using Delft continuous and piston sampling) is relatively constant below the weathered crust layer at approximately 1600-1650kg/m$^3$. 
Finite Element (FE) Model Parameters

Figure 5.3 Geotechnical profile at borehole D1 in the south-west corner of the site - Nash et al. (1992a)

The Bothkennar soil profile adopted by Killeen (2012) consists of a 1.5m silty clay crust overlying two lightly overconsolidated upper and lower Carse clay layers (Figure 5.4). The clay is in turn underlain by a gravel/bedrock layer. The groundwater conditions at the site are hydrostatic. The water table is located at a depth of 0.5-1.0m below the surface, e.g. Nash et al. (1992a), Leroueil et al. (1992), Jacobs and Coutts (1992).

Figure 5.4 Bothkennar Soil Profile
5.2.2 Initial conditions

The geological history of the Bothkennar clay reported by Hight et al. (1992b) indicates a maximum unloading of 15kPa due to erosion, which would have given rise to a profile of OCR reducing with depth from about 1.25 at 5m below ground level to 1.15 at 15m. However, incremental load oedometer tests carried out by Nash et al. (1992a) indicate a yield stress ratio (equivalent to OCR) of between 1.5 and 1.6 for the lower Carse clay, e.g. Figure 5.5. Nash et al. (1992a) have suggested that these higher OCR values (than would have occurred due to the unloading process alone) are due to ageing of the Carse clay. The structured nature of the natural clay at Bothkennar could also contribute to the higher values, e.g. Hight et al. (1992b). Based on these findings, the initial stress state for the FE model has been generated using a pre-overburden pressure (POP) of 15kPa for the crust and upper Carse clay layers and an OCR of 1.5 for the lower Carse clay. These two different methods of defining the initial stress state are described by Brinkgreve et al. (2011), e.g. Figure 5.6.

![Figure 5.5 Yield stress profile - Nash et al. (1992a)](image-url)
Nash et al. (1992a) have carried out a series of spade cell, self-boring pressuremeter (SBPM), and Marchetti dilatometer tests to evaluate the initial horizontal stress profile, which can be used to obtain suitable $K_0$ values for the FE model. The variation of $K_0$ with depth calculated from the different in-situ tests is scattered, e.g. Figure 5.7. The $K_0$ values, which tend to be high in the crust and less than 1.0 below it, decrease slightly with depth but generally lie between 0.6 and 0.9 for most of the profile (with a reduction locally at a depth of about 7m). These in-situ test results have been used by Killeen (2012) to deduce $K_0$ values of 1.5, 1.0, and 0.75 for the crust, upper Carse clay, and lower Carse clay layers respectively.

5.2.3 Strength characteristics

Undrained shear strengths reported by Nash et al. (1992a) using both laboratory (unconsolidated undrained (UU) triaxial tests on specimens prepared from piston and Laval samples) and in-situ techniques (field vane tests, SBPM tests, dilatometer (DMT) piezocone...
tests) indicate that the shear strength of the Bothkennar clay increases linearly with depth, Figure 5.8. The uncorrected field vane strengths tend to be larger than the unconsolidated undrained triaxial strengths while the SBPM values tend to overlap the two. The measured undrained shear strengths are considerably higher than would be expected for a normally consolidated clay with a plasticity index in the region of 40%. These high values may be as a result of bonding of the clay due to ageing (Nash et al., 1992a), the angularity of the silt particles (Paul et al., 1992) or the fact that the clay has a significant organic content (Nash et al., 1992a).

![Figure 5.8](image)

Figure 5.8 Variation of undrained shear strength with depth (a) unconsolidated undrained triaxial tests (b) field vane tests (c) SBPM tests, DMT piezocone tests - Nash et al. (1992a)

Allman & Atkinson (1992) conducted a series of triaxial stress path tests on samples of one-dimensionally normally consolidated and lightly overconsolidated reconstituted Bothkennar soil. The soil samples were obtained between depths of 3.5m and 6.5m below ground level and were reconstituted at their natural water content. Critical state friction angles of $\phi'_s = 34^\circ$ in compression and $\phi'_s = 37^\circ$ in extension have been computed based on CSLs given by $M_c = 1.38$ and $M_e = -1.00$ ($c' = 0\text{kPa}$) respectively. The authors have attributed the relatively high friction angles to the significant proportion of silt-sized grain particles and the high organic content.

However, Hight et al. (1992b) have noted that these selections ($34^\circ$ and $37^\circ$) form a lower bound to the data at large strains for the intact soil. Friction angles for the Bothkennar clay
generally varied between 36° and 45° ($c' = 0$ kPa) at strains of 15-20% for intact soil samples, with particularly high values evident in specimens from the bedded and laminated facies. The authors concluded that a critical state is not reached in triaxial tests on Bothkennar clay samples possessing a well-ordered fabric.

The PLAXIS Undrained A (Section A4.3) option has been used to model the Bothkennar Carse clay for the purposes of the FE model. A critical state friction angle of 34° has been used in conjunction with nominal cohesion values of 3kPa and 1kPa (e.g. Killeen, 2012) for the crust and clay layers respectively for numerical stability purposes. Killeen (2012) has attributed the higher adopted nominal cohesion in the crust to the lower overburden stress. In addition, a dilatancy angle of $\psi_s = 0^\circ$ has been chosen on the basis that lightly overconsolidated clays tend to exhibit very little dilatancy. In other FE studies, friction angles ranging from 34°-37° have been used (see Table 5.1). These fall within the range quoted by Hight et al. (1992b). However, as explained in Section 5.3.4.6, $\phi' = 34^\circ$ is more comparable with other soft clay $\phi'$ values, e.g. Table 9.1.

### 5.2.4 Stiffness parameters

The stiffness parameters adopted by Killeen (2012) are based on a comprehensive testing programme of IL oedometer tests, CRS tests, and restricted flow (RF) tests carried out by Nash et al. (1992b) on both intact and reconstituted clay samples obtained from various depths using a 200mm thin-walled Laval sampler. The variation of void ratio with depth was determined assuming a constant specific gravity, $G_s = 2.68$ for the whole profile. The IL tests involved 20-30 daily load increments (four equal load increments up to the in-situ vertical effective stress, 10kPa load increments to determine the preconsolidation/yield stress and larger load increments with a load increment ratio (LIR = $\Delta \sigma/\sigma$) of 1.0 thereafter up to 2000kPa). The CRS tests were carried out at different strain rates ranging from 0.015mm/min to 0.00015mm/min in order to determine its effect. The compressibility of the Bothkennar clay at stresses well above the yield stress was found to be relatively constant at $0.3 < C_r/(1+e_0) < 0.4$, e.g. Figure 5.9a. The behaviour of the reconstituted specimens were notably different, e.g. Figure 5.9b (samples from a depth of about 5m). The test data presented in Figure 5.9b also reflect strain rate effects (faster strain rates result in higher yield stresses) and sample variability.
Results from the IL testing programme indicate that the maximum rate of creep occurs just after yield, thus highlighting the link between creep and the structural breakdown at yield (Nash et al., 1992b). $C_\alpha/C_c$ was found to remain relatively constant in the range 0.03-0.05 ($C_\alpha$ was determined from the slope of void ratio versus log(time) curves 16-20 hours after load increment application). Triaxial stress path testing on reconstituted soil samples by Allman & Atkinson (1992) yielded $\lambda = 0.181$. Allman & Atkinson (1992) also investigated the swelling behaviour and concluded that $\kappa$ was relatively constant at about 0.025. Killeen (2012) used the ratio of $\lambda/\kappa$ measured by Allman & Atkinson (1992) to work out $C_s$ from the $C_c$ values presented by Nash et al. (1992b) for the natural intact soil samples assuming $\lambda/\kappa = C_c/C_s$.

The stiffness behaviour will be appreciably different depending on whether the soil profile for the FE model is built using the test results on intact soil samples or reconstituted soil samples, e.g. the compression index for natural soil ($\lambda_*$) is almost 2.5 times larger than that for reconstituted soil ($\lambda_i*$). Killeen (2012) has used the intact soil samples as the basis for the HS model stiffness parameters. The same stiffness parameters are adopted in this thesis for the SSC model. The ICE (1992) publications have also been used by a range of other authors (Table 5.1) to develop a suitable FE soil profile for Bothkennar. These parameters exhibit significant variability that is dependent on both the interpretation of the test data and the adopted soil model.
• Nash (2001), Castro & Karstunen (2010) and Killeen (2012) have used the 1D compression data on intact soil reported by Nash et al. (1992) for the selection of the compression index.

• Killeen (2012) has then used the ratio of $\lambda/\kappa$ derived by Allman & Atkinson (1992) for reconstituted soil to work out $\kappa$ whereas Castro & Karstunen (2010) have used the actual $\kappa$ value for reconstituted soil derived by Allman & Atkinson (1992). Nash (2001) appears to have used an intermediate value.

• The remaining FE studies have used lower compression indices closer to those for reconstituted soil. The choice is dependent on model type, e.g. for models that include bonding and destructuration, the value entered should correspond to that for reconstituted soil, e.g. Karstunen et al. (2013).

Killeen (2012) derived a value of $m = 1$ (power dictating the stress dependency of soil stiffness) for the lower Carse clay at Bothkennar based on the 1D test data presented by Nash et al. (1992b). Insufficient data were available to derive a value of $m$ for the upper Carse clay and crust layers and so $m = 1$ was also used. The same values have been adopted in this research.

5.2.5 Permeability characteristics

The variation of the hydraulic conductivity (permeability) of the clay with depth has been studied by Leroueil et al. (1992) using both in-situ (e.g. pushed-in-place piezometers, self-boring permeameters, BAT system) and laboratory (e.g. oedometer cells, triaxial cells, radial flow cells) methods. The authors have noted that self-boring permeameters tend to produce the most representative profile (the pushed-in-place piezometers and BAT system tend to underestimate the permeabilities). The laboratory measurements generally fall between the outer limits of these measurements. The results show a hydraulic conductivity anisotropy ratio ($k_x/k_y$, where $k_x$ and $k_y$ denote the horizontal and vertical permeabilities respectively) falling between 1.5 and 2.0 with $C_k = 0.5e_0$. The hydraulic conductivity profile was found to be similar to the water content profile, increasing up to 5m, remaining constant to 8m, and then decreasing with depth. Permeabilities measured by Nash et al. (1992b) using CRS tests fall in a similar range to those measured by Leroueil et al. (1992) with $C_k$ values generally in the region of 0.40-0.49$e_0$. 
<table>
<thead>
<tr>
<th>Authors</th>
<th>Soil model</th>
<th>$\phi'$ ($^\circ$)</th>
<th>$\lambda^*$</th>
<th>$\lambda_i^*$</th>
<th>$\kappa^*$</th>
<th>$\mu^*$</th>
<th>$\lambda*/\kappa*$</th>
<th>$\lambda_i*/\kappa*$</th>
<th>$\lambda*/\mu*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Killeen (2012)</td>
<td>HS model</td>
<td>34.0</td>
<td>0.162</td>
<td>-</td>
<td>0.023</td>
<td>-</td>
<td>7.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Castro &amp; Karstunen (2010), Castro et al. (2014)</td>
<td>S-CLAY1 &amp; S-CLAY1S models</td>
<td>34.6</td>
<td>0.160</td>
<td>0.06</td>
<td>0.0067</td>
<td>-</td>
<td>24.0</td>
<td>9.0</td>
<td>-</td>
</tr>
<tr>
<td>Nash (2001)</td>
<td>EVP BRISCON model</td>
<td>-</td>
<td>0.131-0.189</td>
<td>-</td>
<td>0.0089</td>
<td>0.0053</td>
<td>15.0</td>
<td>-</td>
<td>25.0</td>
</tr>
<tr>
<td>Bodas Freitas et al. (2011)</td>
<td>3D EVP model based on overstress theory</td>
<td>36.0</td>
<td>0.084</td>
<td>-</td>
<td>0.0105</td>
<td>0.0038</td>
<td>8.0</td>
<td>-</td>
<td>22.3</td>
</tr>
<tr>
<td>Liu et al. (2013)</td>
<td>MCC model with anisotropy &amp; destructurati on</td>
<td>34.0</td>
<td>-</td>
<td>0.085</td>
<td>0.0088</td>
<td>-</td>
<td>-</td>
<td>9.66</td>
<td>-</td>
</tr>
<tr>
<td>Karstunen et al. (2013), Sivasithamparan et al. (2013)</td>
<td>Creep-SCLAY1 model &amp; ACM</td>
<td>36.9</td>
<td>0.100</td>
<td>0.06</td>
<td>0.0067</td>
<td>0.0051</td>
<td>15.0</td>
<td>9.0</td>
<td>19.6</td>
</tr>
<tr>
<td>Kamrat-Pietraszewska et al. (2008)</td>
<td>S-CLAY1 &amp; S-CLAY1S models</td>
<td>37.10</td>
<td>0.100</td>
<td>0.05</td>
<td>0.0067</td>
<td>-</td>
<td>15.0</td>
<td>7.5</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 5.1** Variation in parameters chosen for Bothkennar

### 5.2.6 Adopted soil parameters

The adopted soil parameters for the multi-layer soil profile are summarised in Table 5.2. The parameters correspond to those used by Killeen (2012). The additional $C_a = 0.04 C_c$ values (i.e. $\mu^* = 0.04 \lambda^*$) for the SSC model soil profile used in thesis were calculated based on the
results of the IL testing programme carried out by Nash et al. (1992b). The parameters for the simplified single-layer profiles will be described in Section 5.4.

<table>
<thead>
<tr>
<th></th>
<th>Crust</th>
<th>Upper Carse Clay</th>
<th>Lower Carse Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Depth (m)</strong></td>
<td>0.0 - 1.5</td>
<td>1.5 - 2.5</td>
<td>2.5 - 14.5</td>
</tr>
<tr>
<td><strong>γ (kN/m³)</strong></td>
<td>18.0</td>
<td>16.5</td>
<td>16.5</td>
</tr>
<tr>
<td><strong>OCR</strong></td>
<td>-</td>
<td>-</td>
<td>1.5</td>
</tr>
<tr>
<td><strong>POP</strong></td>
<td>15.0</td>
<td>15.0</td>
<td>-</td>
</tr>
<tr>
<td><strong>K₀</strong></td>
<td>1.5</td>
<td>1.0</td>
<td>0.75</td>
</tr>
<tr>
<td><strong>φ'(°)</strong></td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td><strong>∅'(kPa)</strong></td>
<td>3.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>ψ'(°)</strong></td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>ε₀</strong></td>
<td>1.0</td>
<td>1.2</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>λ</strong></td>
<td>0.015</td>
<td>0.049</td>
<td>0.162</td>
</tr>
<tr>
<td><strong>κ</strong></td>
<td>0.002</td>
<td>0.006</td>
<td>0.023</td>
</tr>
<tr>
<td><strong>μ</strong></td>
<td>0.0006</td>
<td>0.0020</td>
<td>0.0065</td>
</tr>
<tr>
<td><strong>m (power)</strong></td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>νₑₑ</strong></td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
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<tr>
<td><strong>Kₑₑ (m/day)</strong></td>
<td>1 x 10⁻⁴</td>
<td>1 x 10⁻⁴</td>
<td>1 x 10⁻⁴</td>
</tr>
<tr>
<td><strong>kₚₚ (m/day)</strong></td>
<td>6.9 x 10⁻⁵</td>
<td>6.9 x 10⁻⁵</td>
<td>6.9 x 10⁻⁵</td>
</tr>
</tbody>
</table>

**Table 5.2 Bothkennar Material Parameters**

### 5.3 Validation of Soil Profile

The adopted soil profile is validated as follows:

- **Jardine et al.** (1995) described two field tests on an unreinforced rigid pad footing at the Bothkennar site. The first test, simulated by Killeen (2012) to validate the adopted HS model profile in PLAXIS 3D Foundation, has also been simulated using the SSC model in this thesis to investigate potential soil model differences. The second test, which incorporated significant creep settlements, was also simulated in this thesis to investigate the suitability of the SSC model parameters.

- Simulations have been carried out to compare PLAXIS 2D and PLAXIS 3D Foundation in order to confirm the validity of using Killeen’s (2012) 3D parameters for the 2D
program. Time-settlement and stress-strain behaviour has been compared using both the HS and SSC models.

- The PLAXIS 2D undrained shear strength profile has been validated against the high quality test data reported by Nash et al. (1992a). Using the PLAXIS Undrained A (Section A4.3) option means that the undrained shear strength is an output rather than an input parameter and thus needs to be validated against a known shear strength profile.
- PLAXIS 'Soil Test' facility simulations have been compared with the triaxial test data published in ICE (1992).

5.3.1 Field Tests

Killeen (2012) has validated the adopted HS model PLAXIS 3D Foundation soil profile by simulating an instrumented field load test on an unreinforced rigid pad footing at the Bothkennar site. The simulated field load test (Pad A) was documented by Jardine et al. (1995). An additional field load test (Pad B) was also documented by Jardine et al. (1995) but its behaviour was not modelled by Killeen (2012) because the test incorporated significant creep settlements over a two-year period. The loading rates for Pads A and B are presented in Figure 5.10. Pad A was loaded to failure over 3 days while Pad B was loaded to 67% of the ultimate bearing capacity of Pad A (the load on Pad B was maintained and monitored for more than 2 years).

![Figure 5.10 Loading rates for Pads A and B - Killeen (2012)](image-url)
5.3.1.1 Pad A

Pad A was 2.2m square and 0.8m thick. It was loaded using kentledge blocks. Loading was paused overnight and whenever the settlement rate exceeded 8mm/hour. The footing was modelled in PLAXIS 3D Foundation as a linear elastic material with $E = 30$GPa, $\nu = 0.15$ and $\gamma = 24$kN/m$^3$. The Carse clay layers were modelled as undrained materials (owing to the short duration of the load test) while the crust layer was modelled as a drained material. The simulated load-settlement behaviour for both the HS and SSC models is compared to the field measurements in Figure 5.11.

![Pad A load-settlement behaviour](image)

**Figure 5.11** Pad A load-settlement behaviour (PLAXIS 3D Foundation versus Field Measurements)

The HS model load-settlement curve in Figure 5.11 corresponds to that in Killeen (2012). The load-settlement behaviour is in relatively good agreement with the field data, highlighting the validity of the HS model soil parameters. However, the stiffness response is underestimated at low load levels. Killeen (2012) has attributed the underestimated HS model stiffness response to the following:

- The model parameters were determined from large-strain laboratory tests.
- The HS model does not account for increased stiffness at small strains (e.g. the HSsmall model).
The field test was carried out in an area with a 0.3m shelly layer (Nash et al. 1992a, Jardine et al. 1995) with the Carse clay occurring at a shallower depth (this would lead to a stiffer response in undrained loading).

The SSC model load-settlement behaviour is in better agreement with the field data at small strains, with deviation after an applied pressure of ~80kPa.

5.3.1.2 Pad B

The field test on Pad B (2.4m square and 0.8m thick) included a 25kPa load-unload cycle (to evaluate the ground response at small strains) prior to the application of a 90kPa load, corresponding to about 67% of the ultimate bearing capacity of Pad A. Load-settlement behaviour is documented in Figure 5.12. The SSC model stiffness response at small strains is much closer to the field measurements than the HS model (the HSsmall model would be more apt in this case). The simulated time-settlement behaviour is compared to the field data in Figure 5.13. As noted by Jardine et al. (1995), Pad B included significant creep settlements under the sustained load. The SSC model response is in good agreement with the field data, while (as expected), the inviscid HS model will under-predict the long-term settlement.

![Figure 5.12 Pad B load-settlement behaviour (PLAXIS 3D Foundation versus Field Measurements)](image-url)
In this section, unit cell settlement behaviour for PLAXIS 2D and PLAXIS 3D Foundation is compared in order to confirm the validity of using the 3D parameters for the 2D program. Firstly, load-settlement and time-settlement behaviour have been examined for both the HS and SSC models for the untreated case. The load-settlement behaviour under a 50kPa load, applied through a 0.6m thick concrete footing \((E = 30\text{GPa}, \nu = 0.15, \gamma = 24\text{kN/m}^3)\) over the entire surface area of the unit cell is plotted in Figure 5.14 and the time-settlement behaviour is plotted in Figure 5.15. The PLAXIS 2D and PLAXIS 3D Foundation predictions are in excellent agreement.
Additionally, the behaviour of the unit cell has been compared with Killeen’s (2012) HS model results at $A/A_c = 8$. Killeen (2012) has examined vertical strain ($\varepsilon_{yy}$), horizontal strain ($\varepsilon_{xx}$), and SCF with depth for an infinite grid of columns using PLAXIS 3D Foundation. The PLAXIS 2D and PLAXIS 3D Foundation predictions are presented in Figure 5.16; $\varepsilon_{xx}$ and $\varepsilon_{yy}$ are in excellent agreement (Figures 5.16a and 5.16b). The maximum values occur at a depth of approximately 3m below ground level (the results in Killeen (2012) are presented as depth below footing level whereas the results in Figure 5.16 are presented as depth below ground level).

The variation of SCF with depth predicted by PLAXIS 2D and PLAXIS 3D Foundation is compared in Figure 5.16c. For convergence reasons, Killeen (2012) has used $K_0^{nc} = 0.50$ for the granular column material. Two sets of PLAXIS 2D predictions are included in this figure, the first using $K_0^{nc} = 0.50$ and the second using a more realistic value of $K_0^{nc} = 1 - \sin\phi'_{c} = 0.296$. The following conclusions can be drawn:

- When using the same values for $K_0^{nc}$, the SCF with depth predicted by PLAXIS 2D is in good agreement with PLAXIS 3D Foundation.
- When using $K_0^{nc} = 0.296$, PLAXIS 2D yields a more or less constant SCF with depth. However, using $K_0^{nc} = 0.50$ results in a reduction to the stress carried by the column and a corresponding increase (although minor) to the stress carried by the soil, hence yielding a reducing SCF with depth. This results in more settlement and thus lower $n$ values, i.e. the columns are not as effective.

**Figure 5.15** Time-settlement behaviour (PLAXIS 2D versus PLAXIS 3D Foundation)
For the HS model, $K_0^{nc}$ controls the position of the cap yield surface ($q = M\rho'$); using a lower $K_0^{nc}$ results in a larger value of $M$ and hence a larger elastic region (thus there will be less settlement and the column will be able to take more load thus giving a higher SCF).

The more realistic value of $K_0^{nc} = 1 - \sin \phi' = 0.296$ has been used for the PLAXIS 2D work in this thesis.

![Figure 5.16](image_url) Distribution of (a) $\varepsilon_{yy}$, (b) $\varepsilon_{xx}$, and (c) SCF with depth (PLAXIS 2D versus PLAXIS 3D Foundation, HS model, $A/A_c = 8$)

### 5.3.3 Undrained Shear Strength Profile

The PLAXIS 2D undrained shear strength profile is compared with average field vane strengths and average UU triaxial strengths reported by Nash et al. (1992a) in Figure 5.17. These measurements show a linear increase of shear strength with depth. SBPM measurements overlapping the field and UU strengths are also included. The match between the measured and predicted undrained shear strengths is quite good, confirming the suitability of the Undrained A approach. The predicted undrained shear strengths are independent of whether the soil profile is modelled using the SSC model or the inviscid HS or SS models.
Extensive laboratory testing on the Bothkennar clay has been carried out by a number of different Universities in the UK (e.g. Bristol University, City University London, Surrey University, Imperial College) and by Laval University in Canada. The results of both unconsolidated undrained (UU) triaxial compression tests and undrained triaxial compression tests on samples reconsolidated to estimated in-situ stresses (CK\textsubscript{0}U) on Bothkennar clay have been reported by Atkinson \textit{et al.} (1992), Hight \textit{et al.} (1992a), and Smith \textit{et al.} (1992). Only the SSC model has been used in conjunction with the 'Soil Test' Facility for comparison with the test data. The majority of the FE simulations in this thesis are carried out with the SSC model. The SS and HS models are only used as the basis for 'Approach B' in Chapter 6 and as such the previous soil model comparisons in Section 4.2.7.2 are sufficient.

\textbf{Figure 5.17} Undrained Shear Strength profile - FE Validation

5.3.4 ‘Soil Tests’ (Lower Carse Clay)
5.3.4.1 Sampling Procedures

Soil samples for the different testing programmes have been obtained using Laval Samplers, Sherbrooke Samplers and Piston Samplers. Sherbrooke (least disturbance) and Laval samples are preferable to Piston samples (significant disturbance) - when compared with the latter, the former were found to have retained much higher effective stresses after trimming. As a result, the Sherbrooke samples exhibit higher peak strengths in both triaxial compression and extension, e.g. Hight et al. (1992a). The piston samples are unsuitable for reliable peak strength and/or yield stress predictions. The majority of test specimens were prepared using either wire saw trimming (using a wire and soil lathe to shave a sample to the required diameter) or tubing (pushing a tube into a larger tube sample). Atkinson et al. (1992) have noted that wire saw trimming causes the least sample disturbance.

5.3.4.2 Summary of Testing Programmes

- Smith et al. (1992) tested intact samples of Carse clay obtained from depths of 5.3-6.2m. The samples were anisotropically consolidated to the in-situ effective stresses following a swelling loop to mimic the recent stress history (thus retracing the soil’s light overconsolidation), and were then sheared at an axial strain rate of 4.5% per day.
- Atkinson et al. (1992) carried out a series of undrained triaxial compression tests on samples reconsolidated to the in-situ anisotropic stress state (no swelling loop). The soil samples were retained from depths of 12.6m and 15.4m using a Laval sampler. Four different methods were used to trim the sample for testing in the triaxial apparatus (wire saw trimming, tubing, and two methods described by Landva (1964)). The samples were sheared at an axial strain rate of 5% per day up to a maximum axial strain of 15%.
- The CK0U tests reported by Hight et al. (1992a) were used to assess the effect of sample disturbance and sample quality on the Bothkennar clay. These results were obtained from tests run at Imperial College (reconsolidation paths involved a swelling loop) and tests run at Surrey University (no swelling loop). The Laval and Sherbrooke samples were obtained from depths of 5-8m. The specimens were prepared for testing using wire saw trimming. Undrained shearing was carried out at an axial strain rate of 4.5% per day at Imperial and 5% per day at Surrey.
### 5.3.4.3 Comparison with Smith et al. (1992)

Figure 5.18a compares the SSC model stress-strain response for the lower Carse clay with the test data presented by Smith et al. (1992), where \( p'_0 \) is the mean effective stress at the beginning of shearing. The SSC model underpredicts the peak undrained shear strength when adopting a critical state friction angle of \( \phi'_s = 34^\circ \). The significant angular silt content contributes to the very high peak strengths.

As noted in Section 5.2.3, the Bothkennar clay samples possessing a well-ordered fabric tend not to reach a critical state; using a value of 40\(^\circ\) in conjunction with the SSC model (Figure 5.18a) gives a better match to the measured data. The stress-paths in \( p'\)-\( q \) space also illustrate this approach (Figure 5.18b). Additionally, the SSC model cannot fully reproduce the post-peak strength loss due to destructuration evident in Figure 5.18a. This would require a more advanced constitutive model, e.g. the Creep-SCLAY1S model. The reason for the post-peak softening behaviour predicted by the SSC model is discussed in Sections 3.4.2 and 4.2.7.2.

### 5.3.4.4 Comparison with Atkinson et al. (1992)

The SSC model stress-strain response is compared to the test data reported by Atkinson et al. (1992) in Figures 5.19a and 5.20a at depths of 12.6m and 15.4m respectively. In general, the predicted peak strength is in relatively good agreement with the test data but the model is again incapable of accounting for the post-peak strength loss due to destructuration. The corresponding stress paths in \( p'\)-\( q \) space also show good agreement with the test results.
(Figures 5.19b and 5.20b). Atkinson et al. (1992) noted that the peak strength predictions indicate a mobilised friction angle of 39-40° owing to the fact that none of the samples reach a critical state. SSC model simulations using $\phi'_s = 40^\circ$ yield peak strength predictions that are in excellent agreement with the test data (Figures 5.19-5.20).

![Figure 5.19](image1)

**Figure 5.19** Comparison with Atkinson et al. (1992) - 12.6m depth samples (a) CK0U Triaxial: $q$ vs. $\varepsilon_{yy}$ (b) CK0U Triaxial: $q$ vs. $p'$

![Figure 5.20](image2)

**Figure 5.20** Comparison with Atkinson et al. (1992) - 15.4m depth samples (a) CK0U Triaxial: $q$ vs. $\varepsilon_{yy}$ (b) CK0U Triaxial: $q$ vs. $p'$

### 5.3.4.5 Comparison with Hight et al. (1992a)

The SSC model response is again in good agreement with the test data from Hight et al. (1992a) in Figures 5.21-5.22 (plotted in terms of $t = (\sigma'_{yy} - \sigma'_{xx})/2$ and $s' = (\sigma'_{yy} + \sigma'_{xx})/2$, where $\sigma'_{xx}$ denotes the radial effective stress) using $\phi'_s = 34^\circ$ but the agreement is much improved adopting $\phi'_s = 40^\circ$. 

![Figure 5.21](image3)

![Figure 5.22](image4)
Finite Element (FE) Model Parameters

Figure 5.21 Comparison with Hight et al. (1992a) - Imperial College Tests (a) CK0U Triaxial: $t$ vs. $\varepsilon_{yy}$ (b) CK0U Triaxial: $t$ vs. $s'$

Hight et al. (1992a) have also reported on CK0U tests carried out at City University on specimens trimmed from Laval, Sherbrooke, and Piston samples. These are summarised in Figure 5.23. Simulating these tests with the SSC model yields the same conclusions as all previous cases, e.g. the intact samples do not reach critical state and so a higher friction angle is necessary to provide a good match with the measured data. This is clearly evident in Figure 5.23a which has been taken from Hight et al. (1992a).
5.3.4.6 Summary

- The SSC model produces a relatively good match with the measured data. In all cases, the strain at which the peak axial stress is reached is comparable although the peak undrained shear strength tends to be lower.

- This can be explained by the fact that the intact samples do not reach a critical state. However, PLAXIS requires the input of the critical state friction angle and as such, the selection of 34° based on the work of Allman & Atkinson (1992) was deemed appropriate (as noted by Bodas Freitas (2008), Allman & Atkinson's (1992) selection forms a lower bound to the data at large strains for intact soil samples). There is a predisposition by some authors to use a higher critical state angle for the Carse clay, e.g. Table 5.1, but using $\phi' = 34^\circ$ is more comparable with other soft clay $\phi'_s$ values, e.g. Table 9.1.

- Advanced constitutive models (not yet commercially available) would be needed to account for the anisotropic nature of the Carse clay and also its loss of bonding due to destructuration. Further 'Soil Test' facility simulations using the Creep-SCLAY1S model will be described in Section 8.5.
5.4 Simplified Single-Layer Profiles

Simplified single-layer profiles measuring 5m, 10m, and 15m in length (with the stiff crust omitted in the interests of clarity, see Section 4.3.1) are used for the initial FE analyses described in Chapter 6:

- The soil parameters are largely based upon the Bothkennar lower Carse clay ($\lambda^* = 0.162$, $\kappa^* = 0.023$, $\mu^* = 0.0065$) but the complexities associated with analysing a multi-layer profile are avoided. The water table is located at the soil surface in all cases.
- The soil stiffnesses have been varied (as executed by Six et al. (2012)) in order to investigate the influence of $E_c/E_s$ on the ability of columns to arrest primary and creep settlements. Four different nominal $E_c/E_s$ values (defined as the ratio of the constrained/oedometric moduli at a reference pressure of 100kPa), of 5, 10, 20, and 40, values spanning the same range as adopted by Balaam & Booker (1981), Poorooshasb & Meyerhof (1997), Castro & Sagaseta (2009), and Six et al. (2012) have been used. The clay properties for each modular ratio are presented in Table 5.3.
- Apart from $K$ (which has been set equal to the at-rest value of $K_0 = 1 - \sin \phi' = 0.44$), the parameters in Table 5.3 are the only parameters that have been changed (solely to isolate the effect of $E_c/E_s$) with respect to the values quoted by Killeen (2012) for the lower Carse clay at Bothkennar (with the $\lambda^*/\kappa^*$ and $\lambda^*/\mu^*$ ratios remaining fixed).
- The soil stiffnesses in each case are denoted $4E_s$, $2E_s$, $E_s$, and $0.5E_s$ representing modular ratios of 5, 10, 20, and 40 respectively.
- The creep ratio, ($\lambda^* - \kappa^*)/\mu^*$, is constant at ~21 for all SSC model simulations. Waterman & Broere (2005) have suggested this value should normally be between 5 and 25, with low values representing soft soils with considerable creep and high values representing stiff soils with little creep. In most practical cases, $10 < (\lambda^* - \kappa^*)/\mu^* < 20$. 
Finite Element (FE) Model Parameters

Table 5.3 Soil parameters for modular ratios of 5, 10, 20, and 40

<table>
<thead>
<tr>
<th>Modular Ratio ($E_c/E_s$)</th>
<th>Soil Stiffness Notation</th>
<th>$\lambda^*$</th>
<th>$\kappa^*$</th>
<th>$\mu^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$4E_s$</td>
<td>0.0072</td>
<td>0.0010</td>
<td>0.00029</td>
</tr>
<tr>
<td>10</td>
<td>$2E_s$</td>
<td>0.0144</td>
<td>0.0021</td>
<td>0.00058</td>
</tr>
<tr>
<td>20</td>
<td>$E_s$</td>
<td>0.0288</td>
<td>0.0041</td>
<td>0.00115</td>
</tr>
<tr>
<td>40</td>
<td>$0.5E_s$</td>
<td>0.0575</td>
<td>0.0082</td>
<td>0.00230</td>
</tr>
</tbody>
</table>

It should be noted that because the soil models used are not linear elastic, the soil stiffness depends on depth, stress level and OCR; so therefore the values of $E_c/E_s$ quoted above are only an estimate of what is actually modelled. The values of $E_c/E_s$ would only be exact for a normally consolidated soil for which the reference pressures in the soil and column materials are equal.

As noted by Killeen (2012), the lower Carse clay is highly compressible. The average modular ratio is approximately 100. This is much higher than quoted in the aforementioned studies but it is not unrealistic, e.g. Barksdale & Bachus (1983), Domingues et al. (2007a,b) and Borges et al. (2009) have all investigated stone column behaviour for modular ratios as high as 100 while Hanna et al. (2013) have analysed stone column behaviour for a modular ratio of 116.5.

5.5 Properties of Granular Column Material

Suitable material parameters for the stone backfill have also been derived from Killeen (2012), e.g. Table 5.4. The bulk unit weight of 1900kg/m$^3$ used by Killeen (2012) was selected based on the work of Mitchell & Huber (1985), Domingues et al. (2007a,b) and Gäb et al. (2008). A friction angle of 45° has been selected for the stone, representative of bottom feed columns (e.g. McCabe et al., 2009), while $\psi_c$ was calculated as $\psi_c = \phi'c - 30^\circ$ (e.g. Bolton, 1986). The nominal cohesion ($c' = 1$kPa) has been used for numerical stability. $E_{oed}^{ref}$ was assumed equal to $E_{50}^{ref}$ and $E_{ur}^{ref}$ was taken as $3E_{50}^{ref}$, as recommended by Brinkgreve et al. (2011). The values of $E_{oed}^{ref}$, $E_{50}^{ref}$, and $E_{ur}^{ref}$ for the stone quoted in Table 5.4 are based on Gäb et al. (2008), as is the value of the power, $m = 0.3$. The horizontal and vertical permeabilities of 1.7m/day were chosen based on Elshazly et al. (2008). These $k$ values are
lower than would typically be associated with parent column material because they allow for
the infiltration of silt and clay particles into the column during installation.

<table>
<thead>
<tr>
<th>$\gamma$ (kN/m$^3$)</th>
<th>$\phi'_c$ (%)</th>
<th>$\psi_c$ (%)</th>
<th>$c'$ (kPa)</th>
<th>$E_{\text{oed}}^{\text{ref}}$ (MPa)</th>
<th>$E_{50}^{\text{ref}}$ (MPa)</th>
<th>$p_{\text{ref}}$ (kPa)</th>
<th>$m$ (-)</th>
<th>$k_x$ (m/day)</th>
<th>$k_y$ (m/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.0</td>
<td>45</td>
<td>15</td>
<td>1</td>
<td>70</td>
<td>70</td>
<td>100</td>
<td>0.3</td>
<td>1.7</td>
<td>1.7</td>
</tr>
</tbody>
</table>

**Table 5.4** Properties of the granular column material

Killeen (2012) has noted that the Young's moduli quoted in Table 5.4, at a reference pressure
of 100kPa, are higher than those back-calculated by Barksdale & Bachus (1983) from
measured field settlements (30-58MPa). Killeen (2012) has attributed the differences to the
method of column construction. The values quoted by Barksdale & Bachus (1983) pertain to
top feed columns. Bottom feed columns are likely to yield better settlement performance and
therefore higher moduli. However, this author has noticed that the confining pressure has a
significant influence on the working value in the FE model, e.g. at a confining pressure of
50kPa (closer to the confining pressure in the subsequent numerical simulations), the $E_{50}$
value for the granular column material is approximately 57MPa (calculated using Eq. 5.1).
This value falls in the range quoted by Barksdale & Bachus (1983).

$$E = E^{\text{ref}} \left(\frac{P}{P^{\text{ref}}}\right)^m$$

(5.1)

**5.6 Summary**

The Bothkennar PLAXIS 3D Foundation HS model soil profile used by Killeen (2012)
consists of two layers of lightly overconsolidated Carse clay overlain by a weathered crust
layer. The additional parameters necessary for the PLAXIS 2D SSC model profile used in
this research were obtained from Nash et al. (1992b). The profile for the 2D model has been
validated against field tests carried out by Jardine et al. (1995) and also using the PLAXIS
'Soil Test' facility. Simplified single-layer profiles based on the Bothkennar parameters have
been used for the initial FE modelling work described in Chapter 6.
6. Preliminary Numerical Modelling (Single-Layer Profiles)

6.1 Introduction

This chapter describes the axisymmetric FE analyses carried out for the single-layer profiles, described in Sections 4.3.1 and 5.4, measuring 5m, 10m, and 15m in length. The evolution of settlement improvement factor with time is examined and separate ‘primary’ and ‘creep’ settlement improvement factors are evaluated (\(n_{\text{PRIMARY}}\) and \(n_{\text{CREEP}}\)). To the author’s knowledge, this is the first study in which the evolution of \(n\) with time has been examined. This chapter mainly focuses on deformational behaviour of granular columns, whereas the distribution of stress and strain is examined in detail later in Chapters 7, 8, and 9.

The first section of this chapter describes the PLAXIS 2D analysis stages. As the research project evolved, different approaches (A, B, and C) were used to analyse the behaviour of columns in creep-prone soils; each approach is described in turn and the results for each are presented. The influence of modular ratio, column length, and column spacing on settlement performance is thoroughly examined.

6.2 General Analysis Stages

The stages in each analysis are as follows:

- Initial stresses are generated using the \(K_0\) procedure (see Section 4.3.1).
- The columns are ‘wished-in-place’ (see Section 2.5.4.1).
- A plastic nil-step with a small time interval (see Section 4.3.3 and Appendix A) is applied to restore any out-of-equilibrium stresses generated by the ‘wished-in-place’ installation.
- A plate element is placed over the surface of the unit cell. The plate properties are arbitrary; its purpose is merely to provide a sufficiently stiff (axial stiffness, \(EA = 5 \times 10^6\) kN/m, flexural rigidity, \(EI = 8.5 \times 10^3\) kNm²/m, \(v = 0\)) loading platform and to prevent any substantial differential settlements between the surfaces of the column and the soil, e.g. Brinkgreve et al. (2011).
• A 100kPa load is applied in undrained conditions (the unreinforced rigid pad footing at the Bothkennar site reached an ultimate bearing capacity of 138kPa, see Section 5.3.1.1).
• A consolidation phase is then allowed; after full pore pressure dissipation, creep is observed for the SSC model. Settlements cease completely at this point for the SS and HS models, i.e. there are no viscous effects. The duration of the consolidation phase has been set at 1000 years. While this exceeds the design life of structures typically treated with granular columns (40 to 100 years, e.g. Mitchell & Kelly (2013)), the actual duration selected in PLAXIS 2D is of little importance; \( n \) can be established at different times.

6.3 Approach A: Extrapolation Method

6.3.1 Description

The SSC model, based on the isotache concept (see Section 3.3.1.2), predicts creep occurring concurrently with primary consolidation. This renders Casagrande's (1936) method unsuitable for separating primary and creep settlement components (and hence separate \( n \) values) under a given load increment. For an initial estimation of the amount of primary and creep settlement present, a method of extrapolation assuming a constant rate of creep from 1 day after loading onwards has been used (see Figure 6.1). This approach (Approach A) has been used for the isolated case of \( L = 5\text{m} \) and \( E_c/E_s = 20 \) (i.e. soil stiffness, \( E_s \), see Table 5.3).

![Figure 6.1 Separating primary and creep settlement using extrapolation](image_url)

By extending the straight-line portion of the settlement-log(time) plot back to the 1 day line, assuming the immediate (undrained) compression to be small for soft soils and that creep
occurring before 1 day is insignificant (reasonable for overconsolidated soils), the primary and creep settlements (and hence $n_{\text{primary}}$ and $n_{\text{creep}}$) can then be evaluated.

### 6.3.2 Validation

The validity of the method was verified for $5 < (\lambda^* - \kappa^*)/\mu^* < 25$ (typical range, see Section 5.4) by varying only $\mu^*$, see Figure 6.2. The intersection point of all five time-settlement curves in Figure 6.2 occurs at a time of 1 day, indicative of little or no prior creep. The creep settlements obtained using this method are dependent on the length of the consolidation period; although the $n_{\text{creep}}$ values, calculated based on the slopes of the settlement-log(time) plots after EOP, are not.

**Figure 6.2** Validation of separation method for $5 < (\lambda^* - \kappa^*)/\mu^* < 25$

### 6.3.3 Results

Settlement-log(time) plots for the untreated case and at different values of $A/A_c$ are presented in Figure 6.3. For the untreated case, primary consolidation takes approximately 500 days, with creep alone observed thereafter. The columns significantly accelerate consolidation, consistent with Munfakh *et al.* (1983), Han & Ye (2001), and Castro & Sagaseta (2009). With the inclusion of highly permeable granular columns having $A/A_c = 10$, the EOP consolidation time is in the region of 50 days, and decreases thereafter for lower $A/A_c$ values.
The data in Figure 6.3 were interpreted as described in Section 6.3.1 and the corresponding $n$ values are presented in Figure 6.4. Also included in Figure 6.4 are predictions obtained using a selection of analytical settlement design methods (for primary settlement, see Section 2.7.1.2). For this case, the method yielded $n_{\text{primary}}$ values that were in good agreement with the recent Castro & Sagaseta (2009) and Pulko et al. (2011) analytical predictions while the $n_{\text{creep}}$ values, although positive, were significantly lower. ‘Total’ settlement improvement factors ($n_{\text{total}}$, i.e. primary plus creep) are effectively a weighted average of the primary and creep components (dependent on the percentages of primary and creep settlement in the untreated profile).

**Figure 6.3** Settlement vs. log(time); $L = 5\text{m}, E_c/E_s = 20$

**Figure 6.4** Approach A Results; $L = 5\text{m}, E_c/E_s = 20$
6.4 Approach B: Comparison of commercially available SSC, SS, and HS models

6.4.1 Description

Approach A will not hold in every case and so a new approach (Approach B) was devised in which settlements and corresponding $n$ values were derived from analyses carried out with the clay modelled using the SSC, SS, and HS models. Alternative versions of $n$ are considered in (i)-(iii) below. The symbol $\delta$ is used for settlements, with TOTAL or PRIMARY denoted in the subscript, as well as the model used and whether the settlements relate to treated or untreated ground.

(i) The SSC model can be used on its own to work how $n$ values for total settlement (i.e. primary plus creep, see Eq. 6.1) vary with time.

\[
n_{TOTAL(SSC)} = \frac{\delta_{TOTAL(SSC) - UNTREATED}}{\delta_{TOTAL(SSC) - TREATED}}
\]  

(ii) Analyses using the SS model (using equivalent soil parameters to those inputted for the SSC model) allow for a direct estimate of $n_{PRIMARY}$, see Eq. 6.2.

\[
n_{PRIMARY(SS)} = \frac{\delta_{PRIMARY(SS) - UNTREATED}}{\delta_{PRIMARY(SS) - TREATED}}
\]

(iii) An equivalent $n_{PRIMARY}$ can also be calculated using the HS model (see Eq. 6.3). Soil parameters equivalent to those used for the SSC and SS models are used for these analyses.

\[
n_{PRIMARY(HS)} = \frac{\delta_{PRIMARY(HS) - UNTREATED}}{\delta_{PRIMARY(HS) - TREATED}}
\]

Appropriate comparisons between these $n$ values are identified below:

(i) Both $n_{PRIMARY(SS)}$ and $n_{PRIMARY(HS)}$ can be compared with analytical methods to provide context for the FE predictions.
(ii) A comparison between \( n_{\text{TOTAL(SSC)}} \) and \( n_{\text{PRIMARY(SS)}} \) can be used as a gauge of the influence of creep; somewhat justified as the SSC and SS models share a common yield surface.

(iii) A comparison between \( n_{\text{TOTAL(SSC)}} \) and \( n_{\text{PRIMARY(HS)}} \), although less theoretically justifiable, has value as the modelling capabilities of the HS model supersede those of the SS model (Section 4.2.7.1).

6.4.2 Results

6.4.2.1 End of primary (EOP) consolidation

Settlement-log(time) plots for untreated soil predicted by the SSC, SS, and HS models are presented in Figure 6.5 for the 5m long unit cells. The soil stiffnesses are annotated on the figures (\( E_s \) is defined in Table 5.3). These figures illustrate:

- EOP settlements for the inviscid SS and HS models are approximately equal in all cases.
- The EOP consolidation times determined using Casagrande’s (1936) method (identified approximately in Figure 6.5) associated with the SSC model are slightly larger than those for the HS or SS models; this is consistent with an increase in consolidation time due to creep. Pore pressure dissipation plots generated in PLAXIS indicate slightly larger EOP times.
- Primary consolidation occurs more quickly in the stiffer soil (\( 4E_s \), EOP \( \approx 125 \) days) than in the softer soil (\( 0.5E_s \), EOP \( \approx 1000 \) days), corresponding to backfigured average coefficients of consolidation (using Casagrande's (1936) method) for the clay of \( c_v \approx 20 \) m\(^2\)/year and 2.5 m\(^2\)/year respectively.
- The EOP consolidation times for the intermediate cases (\( 2E_s \) and \( E_s \)) are in the region of 250 days and 500 days respectively.

The settlement-log(time) plots for the 10m and 15m long unit cells exhibit similar trends and are in keeping with Terzaghi’s (1943) theory of consolidation; primary consolidation takes approximately \( (10/5)^2 = 4 \) times longer for the 10m long unit cells and \( (15/5)^2 = 9 \) times longer for the 15m long unit cells. The relevant 10m and 15m unit cell plots for the SSC model are presented in Appendix B for illustration purposes.
Settlement-log(time) plots (SSC model) for different $A/A_c$ values for the 5m unit cell are presented in Figures 6.6a-c for $E_s/E_s = 5, 10, \text{ and } 40$ respectively. The corresponding plot for $E_s/E_s = 20$ has been presented in Figure 6.3. The granular columns significantly accelerate the consolidation process for all modular ratios, and the duration reduces further for lower $A/A_c$ values. The same trends are observed for the longer unit cells and for the analyses carried out with the clay modelled using the inviscid models; the 10m and 15m SSC model output is...
presented in Appendix B. The consolidation time reductions are comparable with those reported by Kok Shien (2013), who also modelled stone column behaviour using axisymmetry under a 100kPa load instantaneously applied through a rigid plate element.

\[ \frac{E_c}{E_s} = 5 \]  
\[ \frac{E_c}{E_s} = 10 \]  
\[ \frac{E_c}{E_s} = 40 \]

Figure 6.6 Settlement vs. log(time); SSC model, \( L = 5 \text{m} \) (a) \( E_c/E_s = 5 \) (b) \( E_c/E_s = 10 \) (c) \( E_c/E_s = 40 \)
6.4.2.2 Evolution of settlement improvement factor with time

The evolution of $n$ with time for the 5m unit cell is plotted in Figures 6.7-6.10 ($E_c/E_s = 5, 10, 20$ and $40$) for the SSC, SS, and HS models for $A/A_c = 10, 6,$ and $3$. These plots are derived from the relevant data in Figures 6.3 and 6.6. The results highlight the importance of considering the time since the start of consolidation because $n$ is time-dependent.

![Figure 6.7](image-url) Evolution of $n$ with time - $L = 5m$, $E_c/E_s = 5$

![Figure 6.8](image-url) Evolution of $n$ with time - $L = 5m$, $E_c/E_s = 10$
Upon instantaneous loading of the unit cell, the excess pore water pressure in the (undrained) soil carries the entire applied load so that the stress on the column starts from zero. As consolidation proceeds, the stresses are gradually transferred from the soil to the free-draining column material. Examination of Figures 6.7-6.10 indicates the following:

- At any early stage (up to ~10 days), the differences in $n$ values for all three models are relatively minor. The apparently unfavourable values of $n \leq 1$ (which have no practical
significance) reflect the fact that the settlement of treated ground occurs more rapidly than that of untreated ground, and as such, a true appraisal of the long-term magnitude of \( n \) can only be made once full pore pressure dissipation is satisfied in both cases.

- The \( n \) values thereafter increase, until the time at which EOP for the untreated case occurs, at which point a constant \( n \) (‘steady-state’) is reached.
- The \( n_{TOTAL(SSC)} \) values reduce slightly at large times beyond EOP (although at impractically large times); this is more noticeable at higher modular ratios.
- The \( n_{TOTAL(SSC)} \) and \( n_{PRIMARY(SS)} \) values are consistently lower than the \( n_{PRIMARY(HS)} \) values. The ICU triaxial tests simulated in Section 4.2.7.2 indicate that the HS model predicts a softer stiffness response than the SS model. As a result, the stresses after column yielding are higher in the column and lower in the soil than the corresponding stresses when using the SS model. This results in a higher SCF for the HS model and thus \( n_{PRIMARY(HS)} > n_{PRIMARY(SS)} \).
- The ‘steady-state’ \( n_{TOTAL(SSC)} \) and \( n_{PRIMARY(SS)} \) values are similar, especially at lower \( A/A_c \) values.
- The similarity between the \( n_{TOTAL(SSC)} \) and \( n_{PRIMARY(SS)} \) values would suggest that creep does not influence \( n \) for the scenario modelled and thus a common \( n \) may suffice in stone column design.
- However, a comparison of the \( n_{TOTAL(SSC)} \) and \( n_{PRIMARY(HS)} \) values indicates that the presence of creep reduces the overall \( n \) values.

The evolution of \( n \) with time for the 10m and 15m unit cells, presented in Appendix B, is similar to that for the 5m unit cell; although the ‘steady-state’ \( n \) values are reached at larger times owing to the greater drainage path lengths.

### 6.4.2.3 Comparison of 'primary' and 'creep' settlement improvement factors with analytical solutions

The \( n_{PRIMARY} \) values (after EOP) calculated using the inviscid soil models have been compared to predictions obtained using a selection of analytical settlement design methods (Section 2.7.1.2) in Figure 6.11 (the corresponding figures for the 10m and 15m unit cells are presented in Appendix B). For the purposes of clarity, the \( n_{TOTAL(SSC)} \) values have not been
plotted (because as is evident in Figures 6.7-6.10, the ‘steady-state’ $n_{TOTAL(SSC)}$ and $n_{PRIMARY(SS)}$ values are similar).

Also included in Figure 6.11 are ‘creep’ settlement improvement factors ($n_{CREEP}$) calculated using the SSC model. These $n_{CREEP}$ values (see Eq. 6.4) are calculated using the slopes of the settlement-log(time) plots to deduce $\mu^*$ after the complete dissipation of excess pore pressures. The relationship between $\mu^*$ and the slope of this straight-line creep portion (see Figure 6.12) is given in Eq. 6.5. Essentially, $n_{CREEP}$ is only applicable after EOP, i.e. for pure creep. This approach gives the same value for $n_{CREEP}$ as calculated using Approach A.

\[
n_{CREEP} = \frac{\mu^*(SSC) - UNTREATED}{\mu^*(SSC) - TREATED}
\]

\[
Slope = \frac{\Delta H}{\log(t_2/t_1)} = 2.30\mu^*H = 2.30\frac{C_0}{(1+e_0)\ln 10}H
\]

**Figure 6.11** Analytical comparisons - $L = 5m$ (a) $E_i/E_s = 5$ (b) $E_i/E_s = 10$
Preliminary Numerical Modelling (Single-Layer Profiles)

Figure 6.11 Analytical comparisons - $L = 5$ m (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

Figure 6.12 Slope of settlement-log(time) plot used to work out ‘creep’ settlement improvement factors
**‘Primary’ settlement improvement factors**

- For the lowest modular ratio (i.e. $E_c/E_s = 5$), Balaam & Booker’s (1981) elastic method is in reasonable agreement with the $n_{\text{PRIMARY}}$ values, but for modular ratios of 10, 20 and 40, the method significantly overpredicts $n$ (yielding of the granular material is ignored and the SCF is too large).

- For all unit cell lengths, $n_{\text{PRIMARY(SS)}}$ and $n_{\text{PRIMARY(HS)}}$ are in relatively good agreement with the majority of the analytical methods and especially the recent Castro & Sagaseta (2009) and Pulko *et al.* (2011) approaches. This gives general confidence in the FE modelling procedure.

- For the lower modular ratios, the $n_{\text{PRIMARY(SS)}}$ values are in better agreement with the analytical elastic-plastic methods (Castro & Sagaseta 2009, Pulko *et al.* 2011) while for the higher modular ratios, the $n_{\text{PRIMARY(HS)}}$ predictions are superior. The methods developed by Castro & Sagaseta (2009) and Pulko *et al.* (2011) are in almost perfect agreement with the numerical predictions of $n_{\text{PRIMARY(HS)}}$ for $E_c/E_s = 20$ and 40 for all unit cell lengths. Coincidentally, the $n_{\text{PRIMARY(HS)}}$ values for the $E_c/E_s = 20$ scenario are almost identical to the corresponding $n_{\text{PRIMARY}}$ values for Approach A in Section 6.3.3.

- The PLAXIS 2D $n$ values increase marginally with column length for all $E_c/E_s$ values. The analytical methods also predict marginally bigger $n$ values for longer columns.

Possible reasons for the better agreement between the $n_{\text{PRIMARY}}$ values calculated using the PLAXIS soil models and those predicted by the elastic-plastic methods (Priebe 1995, Castro & Sagaseta 2009, Pulko *et al.* 2011) at higher modular ratios include:

- In the case of Priebe (1995), the predictions are better for the more ‘realistic’ higher modular ratios due to the semi-empirical nature of the method. This is due to the assumption of a significant bulging mechanism which is more prevalent in softer soils, (e.g. CCE theory has been used by Hughes and Withers (1974) to model the lateral bulging failure of a single column and hence predict its ultimate bearing capacity while Priebe (1995) has also used CCE theory as the basis for calculating $n_0$, Eq. 2.8).

- For Castro & Sagaseta (2009) and Pulko *et al.* (2011), the reason for the better predictions at higher modular ratios is likely to be due to the variability of soil stiffness with stress level. The analytical formulations assume a constant stiffness modulus for the soil and
column. However, the PLAXIS models account for the stress dependency of stiffness. As a result, the modular ratio used in the analytical solutions will not be exactly the same as that in the FE calculations. For low modular ratios, the confining pressure in the column will be lower and so it will not take as much of the load as it would take for higher modular ratios. Accordingly, the confining pressure in the soil will be higher at lower modular ratios. In general, the differences between the analytical and FE predictions will be more evident in situations where elastic strains are more prominent (e.g. low $A/A_c$ values).

‘Creep’ settlement improvement factors

- The $n_{CREEP}$ values suggest that stone columns are helpful in reducing creep settlements, with greater improvement at lower $A/A_c$ values.
- However, the $n_{CREEP}$ values tend to be lower than $n_{PRIMARY}$ and less than approximately 1.5 for $A/A_c > 6$ in all cases.
- The $n_{CREEP}$ values are also lower than the $n_{TOTAL(SSC)}$ values.

6.4.2.4 Influence of modular ratio on 'primary' and 'creep' settlement improvement factors

The influence of $E_c/E_s$ on $n_{PRIMARY(SS)}$, $n_{PRIMARY(HS)}$, and $n_{CREEP}$ is studied in this section. The influence of $E_c/E_s$ on $n_{PRIMARY}$ in Figures 6.13 and 6.14 for the 5m, 10m, and 15m unit cells indicates the following:

- $E_c/E_s$ has a significant influence on $n_{PRIMARY}$, regardless of $A/A_c$ or $L_c$. This is the premise on which some analytical settlement design methods (especially elastic methods) are based; as $E_c/E_s$ increases, there is a corresponding increase in $n$, e.g. Balaam & Booker (1981), Borges et al. (2009).
- The $n_{PRIMARY}$ values begin to converge as $E_c/E_s$ increases as its influence becomes less dominant; this is more evident for the 5m unit cell than it is for the 15m unit cell. Kirsch (2004) also noticed that the influence of $E_c/E_s$ on $n$ diminished above $E_c/E_s \approx 50$, e.g. Figure 6.15; the study pertained to a foundation supported by four columns.
Figure 6.13 'Primary' Settlement Improvement Factors - SS model (a) $L = 5$ m (b) $L = 10$ m (c) $L = 15$ m
Figure 6.14 ‘Primary’ Settlement Improvement Factors - HS model (a) $L = 5\text{m}$ (b) $L = 10\text{m}$ (c) $L = 15\text{m}$
The influence of $E_c/E_s$ on $n_{\text{CREEP}}$ is plotted in Figure 6.16. As $E_c/E_s$ increases, the $n_{\text{CREEP}}$ values remain almost unchanged; slight differences are evident for the longer columns (15m unit cells) at close spacings.

The reason for the slight differences for the longer columns at closer spacings is due to the absence of yielding at depth, confirmed in the PLAXIS output by looking at plots of ‘plastic points’ (Figures 6.17, 6.18, and 6.19). The ‘plastic points’ option in the PLAXIS output shows the stress points that are in a plastic state, e.g. stresses lying on the Mohr-Coulomb failure surface (red points) or on the shear hardening envelope (green points). A plastic point is indicated in the PLAXIS output only when the stress state is exactly on the relevant yield/hardening surface. In some cases, when the stress state is reversed by a small amount, the plastic point is no longer shown. However, the stress point is still effectively in a plastic state. Therefore the lower ‘horizon’ of the plastic point PLAXIS output has been used to confirm the presence of yielding.
Figure 6.16 'Creep' Settlement Improvement Factors - SSC model (a) $L = 5m$ (b) $L = 10m$ (c) $L = 15m$
Preliminary Numerical Modelling (Single-Layer Profiles)

Figure 6.17 Column Yielding, $L = 5$ m ($A/A_c = 3$) (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$

Figure 6.18 Column Yielding, $L = 10$ m ($A/A_c = 3$) (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$
6.4.2.5 Effect of \((\lambda^{*}-\kappa^{*})/\mu^{*}\)

The clay properties used in the previous simulations are representative of Bothkennar clay with the creep ratio, \((\lambda^{*}-\kappa^{*})/\mu^{*} \approx 21\). For the untreated 5m long unit cell, the relative percentages of primary/creep settlement after 1, 10, 30, 100, and 1000 years are 89/11, 85/15, 84/16, 82/18, and 79/21 respectively (and so the \(n_{\text{TOTAL(SSC)}}\) values are dominated by primary settlement). These percentages have been worked out using 1D compression theory (e.g. Section A4.6) and verified using PLAXIS 2D.

The \(n_{\text{CREEP}}\) values calculated in Section 6.4.2.3 are likely to have a greater influence in reducing \(n_{\text{TOTAL(SSC)}}\) for soils with lower \((\lambda^{*}-\kappa^{*})/\mu^{*}\) ratios. This theory has been tested by carrying out a series of analyses with a higher \(\mu^{*}\) so that the ratio of \((\lambda^{*}-\kappa^{*})/\mu^{*} = 5\), i.e. at the lower end of the range quoted in Figure 5.1. The \(\lambda^{*}\) and \(\kappa^{*}\) values were left unchanged. The relative percentages of primary/creep settlement after 1, 10, 30, 100, and 1000 years in this case are 66/34, 58/42, 55/45, 52/48, and 53/47 respectively.

The evolution of \(n\) with time for the 5m unit cell \((E_{c}/E_s = 20)\) is plotted in Figure 6.20 for \(A/A_{c} = 10, 6, \) and 3. The \(n_{\lambda^{*}-\kappa^{*}}/\mu^{*} = 5\) values are lower than the \(n_{\lambda^{*}-\kappa^{*}}/\mu^{*} = 21\) values, supporting the theory that the lower \(n_{\text{CREEP}}\) values will have a greater influence on the \(n_{\text{TOTAL(SSC)}}\) values.
when creep makes up a larger proportion of the total settlement. The \( n_{TOTAL(SSC)} \) values for this new case are lower than both \( n_{PRIMARY(HS)} \) and \( n_{PRIMARY(SS)} \). This would suggest that accounting for creep gives lower \( n \) values than would be obtained if primary consolidation alone was considered (based on a comparison of the SSC model output with the SS and HS model outputs). However, it also highlights that the conclusions obtained using Approach B are parameter dependent; Approach C, described in Section 6.5 overcomes this limitation.

\[ \frac{(\lambda^* - \kappa^*)}{\mu^*} = 21 \]
\[ \frac{(\lambda^* - \kappa^*)}{\mu^*} = 5 \]

**Figure 6.20** Influence of \( \frac{(\lambda^* - \kappa^*)}{\mu^*} \) - SSC model; \( L = 5 \text{m}, E_c/E_s = 20 \) (a) \( A/A_c = 10 \) (b) \( A/A_c = 6 \) (c) \( A/A_c = 3 \)
It is also interesting to note that there are larger differences between the \((\lambda^* - \kappa^*)/\mu^* = 5\) and \((\lambda^* - \kappa^*)/\mu^* = 21\) \(n\) values at the higher end of modular ratios considered in this study than at the lower modular ratios, e.g. Figure 6.21 \((E_c/E_s = 5)\). This occurs because the \(n_{\text{primary}}\) values are much higher than the \(n_{\text{creep}}\) values at higher modular ratios and so a larger effect is observed in the weighted average. The corresponding figures for \(E_c/E_s = 10\) and 40 are presented in Appendix B.

**Figure 6.21** Influence of \((\lambda^* - \kappa^*)/\mu^*\) - SSC model; \(L = 5\text{m}, E_c/E_s = 5\) (a) \(A/A_c = 10\) (b) \(A/A_c = 6\) (c) \(A/A_c = 3\)
6.5 Approach C: SSC model with and without creep

6.5.1 Description

Approach C is based on the use of the SSC model alone. The detail is as follows:

- Two sets of analyses are performed; one set for the 'normal' case using a standard creep coefficient for the soil and the other set using a very low creep coefficient to eliminate creep behaviour. In the latter case, the soil state will closely follow the reference compression line independently of the applied strain rate so that for practical purposes, the response may be considered time-independent, e.g. Bodas Freitas et al. (2011). Direct comparison between the two sets is used to isolate the impact of creep.
- This approach is based on a fair like-for-like comparison and is technically superior to approaches A and B.
- PLAXIS does not permit the use of $\mu^* = 0$. This would be unrealistic because only purely elastic behaviour would be predicted; plastic strains are incorporated in the creep/viscoplastic strain component, e.g. Bodas Freitas et al. (2011).
- Additionally, the use of very low creep coefficients leads to numerical difficulties so the selection of the most appropriate value tends to be a trial and error process. In general, it was found that using $\mu^*$ values in the range $1\times10^{-5}$ to $1\times10^{-4}$ tends to yield suitable predictions (the settlements without columns are in good agreement with both 1D compression theory (Section A4.6) and the inviscid SS and HS model predictions, e.g. Figure 6.22). These low values of $\mu^*$ correspond to between 1% and 10% of the $\mu^*$ values for the simplified profile with soil stiffness, $E_s$ (Table 5.3), and to between 0.2% and 2% of the creep coefficient for the Bothkennar lower Carse clay (Table 5.2).
6.5.2 Results

6.5.2.1 Dependence of ‘primary’ settlement improvement factors on soil model

The settlement improvement factors values calculated using the SSC model without creep are denoted \( n_{\text{PRIMARY(SSC, } \mu^* = 0)} \). The following points are relevant:

- There are slight numerical differences between the \( n_{\text{PRIMARY}} \) values calculated using the SSC model without creep, the HS model, and the SS model. The model dependence of \( n_{\text{PRIMARY}} \) is highlighted in Figure 6.23 for the 5m unit cell. The corresponding plots for the 10m and 15m unit cells are presented in Appendix B.
- For the 5m long profiles, \( n_{\text{PRIMARY(HS)}} > n_{\text{PRIMARY(SSC, } \mu^* = 0)} > n_{\text{PRIMARY(SS)}} \) in all cases. The same trend holds for the 10m and 15 long unit cells.
- For higher modular ratios \( (E_c/E_s = 20 \text{ and } E_c/E_s = 40) \), the \( n_{\text{PRIMARY(SSC, } \mu^* = 0)} \) values begin to approach the \( n_{\text{PRIMARY(HS)}} \) values.
- Settlement improvement factors obtained using Castro & Sagaseta (2009) and Pulko et al. (2011) are also included in Figure 6.23 to provide context for the PLAXIS 2D \( n_{\text{PRIMARY}} \) values.
Figure 6.23 ‘Primary’ settlement improvement factors - L = 5m (a) $E_s/E_s = 5$ (b) $E_s/E_s = 10$
(c) $E_s/E_s = 20$ (d) $E_s/E_s = 40$
6.5.2.2 Evolution of settlement improvement factor with time

The evolution of $n$ with time for the 5m unit cell is plotted in Figure 6.24 ($E_c/E_s = 20$) for $A/A_c = 10, 6$, and $3$ respectively. The standard creep coefficient is used for the case with creep so that the creep ratio, $(\lambda^* - \kappa^*)/\mu^* \approx 21$. Definitive conclusions can be drawn from these comparisons:

- The $n_{TOTAL(SSC)}$ values are less than the $n_{PRIMARY(SSC, \mu^* \approx 0)}$ values at all $A/A_c$ values. This indicates that the incorporation of creep leads to a lower settlement improvement factor than would be obtained had primary consolidation settlement been considered alone.
- Larger differences between $n_{PRIMARY(SSC, \mu^* \approx 0)}$ and $n_{TOTAL(SSC)}$ would be observed if a larger creep coefficient were used for the case incorporating creep because the lower $n_{CREEP}$ values would hold greater weighting.
- After EOP, the $n$ values for the case with creep continue to reduce marginally, i.e. the effect of the lower $n_{CREEP}$ values on $n_{TOTAL(SSC)}$ is perceptible after EOP.

The same conclusions can be drawn from the 10m and 15m unit cell profiles. These results are presented in Appendix B, as are the remaining 5m unit cell results for $E_c/E_s = 5, 10$, and $40$. Larger differences between $n_{TOTAL(SSC)}$ and $n_{PRIMARY(SSC, \mu^* \approx 0)}$ are observed in situations where $n_{PRIMARY(SSC, \mu^* \approx 0)}$ is much larger than $n_{CREEP}$, e.g. at close spacings (e.g. $A/A_c = 3$) or at higher modular ratios (e.g. $E_c/E_s = 40$), supporting the inference made in Section 6.4.2.5.
Figure 6.24 Approach C - $L = 5m$, $E_c/E_s = 20$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$


6.6 Summary

Three different approaches (A, B, and C) have been described in this chapter with a view to establishing the effect of using vibro stone columns to treat creep-prone soils. These approaches have been deployed to analyse the settlement behaviour of simplified single-layer profiles. The results indicate that incorporating creep yields lower settlement improvement factors than would be achieved if only primary consolidation settlement was considered.

The analyses have identified that the relative proportions of primary and creep settlement are relevant, as is the length of time after construction over which the ground performance is of interest. The influences of modular ratio, column length, and column spacing have also been investigated. For practical purposes, the design of stone columns ignoring creep appears unconservative. If considerable creep is present, lower $n$ values should be applied by practitioners.

Approach C will be used for the remainder of the analyses in this thesis as it is technically superior to approaches A and B (for which the conclusions obtained are parameter/scenario dependent).
7. Analysing the multi-layer Bothkennar profile using the SSC model

7.1 Introduction

This chapter describes the simulations carried out on the multi-layer Bothkennar profile using the SSC model. In the first half of the chapter, the focus is placed on investigating whether the findings in Chapter 6 for the single-layer profiles also hold for the more realistic multi-layer profile. The variations of the relevant stresses and strains with time and depth are then examined in detail with a view to establishing how creep manifests itself in the combined soil-column system. Additional analyses have also been carried out to establish behavioural differences for floating columns. The analyses in this chapter have been carried out using the Updated Mesh option to account for large displacements.

This chapter is subdivided as follows:

(i) The time-settlement behaviour for the multi-layer profile predicted by the SSC model with and without (denoted $\mu^* \approx 0$ hereafter) creep is presented in Section 7.2; differences with the single-layer profile results are pointed out.
(ii) The evolutions of $n$ with time with and without creep are compared in Section 7.3. The calculated $n$ values are compared with those from analytical solutions in Section 7.4 to provide context.
(iii) In Section 7.5, the variations of stress with time at discrete positions in both the soil and column are examined to establish the influence (if any) of creep. The stress variations with time for the untreated cases with and without creep are presented as a frame of reference.
(iv) Full profiles of stress and strain with depth in the soil and column at a fixed point in time (after EOP) are then examined in Section 7.6. The corresponding profiles for floating columns are examined in Section 7.7.
7.2 Time-settlement behaviour

The analysis stages are analogous to those described in Section 6.2 for the single-layer profiles; a 100kPa load is applied in undrained conditions with a follow-up consolidation period to allow pore pressure dissipation. Approach C is used to establish the influence of creep. Settlement-log(time) plots generated using the SSC model for the untreated case with and without creep are presented in Figure 7.1. The corresponding dissipation of excess pore pressure at mid-depth of the lower Carse clay with and without creep is plotted in Figure 7.2.

![Figure 7.1 Settlement vs. log(time) for untreated soil](image1)

![Figure 7.2 Excess pore pressure at mid-depth of lower Carse clay vs. log(time) for untreated soil](image2)
The following points are of interest:

- The EOP consolidation times with and without creep are approximately 40,000 days (~100 years) and 15,000 days (~40 years) respectively. The long consolidation times arise due to (i) the long drainage path, (ii) the low clay permeability, and (iii) the high clay compressibility, e.g. Nash (2001), and are consistent with a backfigured coefficient of consolidation of $c_v \approx 1.6 \text{ m}^2/\text{year}$ using the Casagrande (1936) method.

- The pore pressure dissipation plots indicate longer EOP consolidation times than would be obtained based on the settlement-log(time) plots using Casagrande’s (1936) method. Similar observations have been made by Moorhead (2013) from smaller-scale laboratory tests.

- The slight kink in the pore pressure curve after approximately 900 days (Figure 7.2) for the case without creep occurs due to the change in stiffness of the clay as it passes through yield, e.g. Nash (2001).

Settlement-log(time) plots for different $A/A_e$ values are plotted in Figures 7.3 and 7.4 for the cases without and with creep respectively. In both cases, the columns significantly accelerate consolidation; the consolidation time reductions are approximately 50-fold, 100-fold, and 200-fold for $A/A_e = 15, 10, \text{ and } 6$ respectively. These reductions are consistent with the results for the 15m long single-layer profile (which has a comparable drainage path length) in Chapter 6.

![Figure 7.3 Settlement vs. log(time) - very low creep coefficient ($\mu^* \approx 0$)](image-url)
7.3 Evolution of settlement improvement factor with time

The evolution of $n$ with time for the cases with and without creep is presented in Figure 7.5 at $A/A_c = 15, 10, 6,$ and $3.$ This figure clearly illustrates that incorporating creep results in lower $n$ values than would be obtained if only primary consolidation settlement was considered; this is in keeping with the findings in Chapter 6. The percentage differences between the $n_{\text{PRIMARY}(\text{SSC}, \mu^* \approx 0)}$ and $n_{\text{TOTAL}(\text{SSC})}$ values are larger than those for the single-layer profiles in Chapter 6 for two reasons:

(i) For the multi-layer profile, the relative percentages of primary/creep settlement after 1, 10, 30, 100, and 1000 years are 73/27, 66/34, 63/37, 60/40, and 55/45 respectively and so the weighted effect of creep is more prominent than for the single-layer profile.

(ii) The $n_{\text{PRIMARY}(\text{SSC}, \mu^* \approx 0)}$ values for the multi-layer profile are larger than the corresponding $n_{\text{PRIMARY}(\text{SSC}, \mu^* \approx 0)}$ values for the single-layer profiles; this is mainly due to the larger initial $K_0$ values. As will be shown in Section 9.5.1, $K_0$ affects $n_{\text{PRIMARY}(\text{SSC}, \mu^* \approx 0)}$ more than either $n_{\text{TOTAL}(\text{SSC})}$ or $n_{\text{CREEP}}$ and so the percentage differences between $n_{\text{PRIMARY}(\text{SSC}, \mu^* \approx 0)}$ and $n_{\text{TOTAL}(\text{SSC})}$ will be larger. In general, percentage differences between ‘primary’ and ‘total’ settlement improvement factors increase as $n$ increases.
Figure 7.5 Evolution of \( n \) with time (a) \( A/A_c = 15 \) (b) \( A/A_c = 10 \) (c) \( A/A_c = 6 \) (d) \( A/A_c = 3 \)
7.4 Comparison of settlement improvement factors with analytical solutions

The \( n_{\text{PRIMARY}(\text{SSC}, \mu^* \approx 0)} \) and \( n_{\text{TOTAL}(\text{SSC})} \) values (after EOP) have been compared to the analytical predictions for primary settlement in Figure 7.6. The analytical methods, which are formulated in general for single-layer profiles, have been modified to account for the additional stresses from the overlying layers for the multi-layer profile. For Pulko et al. (2011), the final yield depth to which plastic strains appear in the column (yielding starts at the surface and progresses downward as the applied load increases) needs to be modified and the settlements should be integrated from the top to the bottom of each layer rather than from the surface. The Castro and Sagaseta (2009) solution uses a factor \( U_y^e \) (elastic degree of consolidation at the moment of column yielding) to work out whether or not the column is in a plastic state (if \( U_y^e > 1 \), no yielding takes place, otherwise yielding of the granular material occurs). This factor also needs to be modified.

![Comparison of settlement improvement factors with analytical solutions](image)

**Figure 7.6** Comparison of settlement improvement factors with analytical solutions

From an examination of Figure 7.6, the following can be deduced:

- As was the case with the single-layer profiles, the analytical predictions by Castro & Sagaseta (2009) and Pulko et al. (2011) are in excellent agreement with the PLAXIS 2D \( n_{\text{PRIMARY}(\text{SSC}, \mu^* \approx 0)} \) values; slight differences are only evident at \( A/A_c = 3 \).
• Additionally, Priebe’s $n_2$ (1995) predictions are also in very good agreement with the PLAXIS 2D $n_{\text{primary(SSC, } \mu^* = 0)}$ values, with slight deviation for closely-spaced columns (e.g. $A/A_c < 5$).
• The $n_{\text{TOTAL(SSC)}}$ values are lower than the $n$ values predicted by the analytical methods at all $A/A_c$ values, confirming that incorporating creep results in lower $n$ values.

7.5 Variation of stress with time at fixed points

The variations of vertical, radial, and hoop stress with time (with and without creep) are examined in this section. These variations of stress with time have been examined at four stress points (shown in Figure 7.7), two at the surface of the column and soil (C1 and S1) and two at a depth of 8.5m (mid-depth of the lower Carse clay layer) in the column and soil (C2 and S2). Although rarely measured in practice, SCFs would most likely be measured at the surface due to ease of installing the instrumentation, e.g. Killeen (2012).

The stress variations with time are a useful starting point to establish the pattern of behaviour at a given stress-point. In Section 7.6, the distributions of stress and strain with depth will be examined to highlight the effect of creep throughout the entire depth of the profile at a fixed point in time.

![Figure 7.7 Stress points at which the variations of stress with time are examined](image)
7.5.1 Variation of stress with time (untreated case)

The variations of vertical, radial, and hoop stress with time for the untreated case with and without creep are presented in Figure 7.8 to provide context for the subsequent comparisons.

![Figure 7.8 Variation of stresses with time for untreated soil: (a) $\sigma'_{yy}$ (b) $\sigma'_{xx}$ (c) $\sigma'_{zz}$]
Examination of Figure 7.8 indicates the following:

- At mid-depth, the long-term stresses are slower to develop for the case with creep, corresponding to larger EOP times. At the surface, the drainage path length is shorter so the EOP time is relatively unaffected by creep.
- The vertical \( (\sigma'_{yy}) \) stresses at the surface and at mid-depth are comparable after EOP for the cases with and without creep; the soil carries the entire applied load of 100kPa.
- After EOP, the radial \( (\sigma'_{xx}) \) and hoop \( (\sigma'_{zz}) \) stresses for the case with creep are marginally larger than those without creep. This occurs due to an increase of \( K_0 \) during creep, which can be predicted according to Eq. 7.1, e.g. Mesri & Castro (1987).

\[
K_0 = (1 - \sin \phi') \left( \frac{t}{t_p} \right)^{-\frac{C_a/C_c}{1-C_{c_t}/C_c}} \sin \phi' \quad (7.1)
\]

- The radial and hoop stress reductions up to \(~3000\) days for the case with creep occur because there is a slight pore pressure build-up due to creep (as evident in Figure 7.2) before net dissipation begins to take effect; the kink is explained in Section 7.2.

**7.5.2 Variation of vertical stress with time for the treated case**

The variations of vertical stress with time at \( A/A_c = 3 \) are compared to the untreated case in Figures 7.9 (surface, C1 and S1) and 7.10 (mid-depth, C2 and S2). \( A/A_c = 3 \) was chosen for presentation purposes because the mechanisms of interest are most obvious at this spacing. It is apparent that:

- The stone columns reduce the vertical stress carried by the soil (seen by comparing the untreated soil stresses with the soil stresses at \( A/A_c = 3 \)).
- For the case without creep, the stresses on the soil and column are constant after EOP, which is approximately 3 days for \( A/A_c = 3 \), based on Figures 7.3 and 7.4.
- For the case with creep, stress is transferred from the soil to the column as the soil creeps. The additional increment of stress transferred to the already yielded column reduces its efficacy. Optimum column design should induce plasticity but the additional yielding
caused by the additional stress transferred to the column due to creep results in a suboptimal performance (i.e. lower $n$ values).

The same trend holds at all $A/A_c$ values considered in this study ranging from $3 < A/A_c < 15$, not all of which are presented for clarity purposes. However, the stress transfer mechanism is more prevalent at closer spacings.

**Figure 7.9** Variation of vertical stress ($\sigma_{yy}'$) with time at the surface

**Figure 7.10** Variation of vertical stress ($\sigma_{yy}'$) with time at mid-depth

The corresponding variations of SCF (see Section 2.6.2.2) with time for $A/A_c = 3$ are plotted in Figure 7.11. Without creep, the SCF at the surface (based on points C1 and S1) is 6.50. With creep, the SCF at the surface increases from 7.85 after 100 days to 8.75 after 30 years. The corresponding SCF without creep based on points C2 and S2 alone is 3.81 after 30 years;
with creep, the SCF based on these points is 5.4 after 30 years. In general, the SCFs for the ‘with creep’ case are much higher than the corresponding SCFs for the ‘without creep’ case.

For context, average SCFs in the crust and lower Carse clay layers for the ‘without creep’ case at $A/A_c = 15, 10, 6,$ and 3 are compared to SCFs predicted by Castro & Sagaseta (2009) in Table 7.1 (the Castro & Sagaseta (2009) SCFs are denoted ‘C&S’). The predictions are in good agreement for $A/A_c = 15$ and 10 but the PLAXIS values in the lower Carse clay are lower than those of Castro & Sagaseta (2009) at closer spacings, particularly at $A/A_c = 3$. As noted in Section 2.7.1.2, the Castro & Sagaseta (2009) solution fails to account for elastic strains in the column during its plastic deformation; at closer spacings, these elastic strains are more important and could account for the differences between the FE output and the analytical predictions.

<table>
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<th>$A/A_c$</th>
<th>C&amp;S</th>
<th>PLAXIS</th>
<th>C&amp;S</th>
<th>PLAXIS</th>
<th>C&amp;S</th>
<th>PLAXIS</th>
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<td>6.22</td>
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<td>5.54</td>
<td>5.45</td>
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<td>5.29</td>
</tr>
<tr>
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<td>4.98</td>
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<td>4.96</td>
<td>4.88</td>
<td>4.91</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Table 7.1 Comparison of PLAXIS SCFs without creep with Castro & Sagaseta (2009)
7.5.3 Variations of radial and hoop stresses with time for the treated case

The variations of radial and hoop stress with time in the soil at mid-depth at $A/A_c = 3$ are compared to those for the untreated case in Figures 7.12 and 7.13. These figures illustrate:

- The radial and hoop stresses in the soil at $A/A_c = 3$ for the case without creep are constant after EOP. The radial stress is approximately 90% of that for the untreated case and the hoop stress is approximately 80% of that for the untreated case.
- The corresponding radial and hoop stresses for the case with creep reduce after EOP. Because of the radial stress reduction, the lateral support imparted onto the column by the soil diminishes. This leads to additional bulging of the granular column material (giving rise to more settlement) and a lower load-carrying capacity.
- The percentage reductions in hoop stress due to creep are larger than the corresponding radial stress reductions.

In reality, particle rearrangement and the decay of organic material will lead to a reduction in soil volume. This may also contribute to additional bulging of the granular column in soils with significant creep. The corresponding radial and hoop stresses in the stone columns show a degree of scatter in some cases (caused by shear-plane formation, see Section 7.6.1.2); examining their variations with time at a particular stress-point is thus problematic.

Figure 7.12 Variation of radial stress ($\sigma'_{xx}$) in the soil with time at mid-depth
Analyzing the multi-layer Bothkennar profile using the SSC model

7.6 Profiles of stress and strain with depth

In this section, the distributions of stress and strain with depth in the soil and column are compared after a period of 100 years (which is after EOP for the untreated case). The stress and strain profiles at each depth in the soil have been obtained at the same radius as the plane through points S1 and S2 in Figure 7.8. However, the stress and strain profiles in the column require averaging over the column cross-section and therefore the Microsoft Excel ‘Pivot Table’ function has been used for this purpose; as used by Killeen (2012). The steps in obtaining the data using the ‘Pivot Table’ function are as follows:

(i) Select all of the relevant data (stress, strain, or depth) in PLAXIS 2D.
(ii) Copy the data into Microsoft Excel.
(iii) Create a ‘Pivot Table’ using the data.
(iv) Use the ‘Pivot Table’ to calculate the average stress or strain value at each depth.
(v) Plot the relevant averaged data with depth.

7.6.1 Stress profiles in the soil

7.6.1.1 Vertical stress profiles

The distributions of vertical stress in the soil with depth for \( A/A_c = 3 \) and 15 (closest and widest spacings considered in this study) are plotted in Figures 7.14 and 7.15 respectively for
the cases without and with creep. Also included in these figures are the stresses carried by the untreated soil in each case to provide a frame of reference. The profiles are plotted from the ground surface. Since the Updated Mesh option was used, the final ground surface will be lower when there is more settlement (i.e. for the analyses incorporating creep). The following points are noteworthy:

- The stone columns reduce the stress carried by the soil at both $A/A_c = 3$ and 15. This trend also holds for $3 < A/A_c < 15$.
- For the case without creep (Figures 7.14a and 7.15a), the stress reduction is uniform with depth. This is reflected by the relatively constant ‘distance/gap’ between the untreated and treated ‘lines’ in Figures 7.14a and 7.15a, illustrated using green arrows and markers for optical purposes.
- For the case with creep (Figures 7.14b and 7.15b), the stress reduction is larger and increases with depth as a result of the stress transfer process from soil to column due to creep (as discussed in Section 7.5.2). This indicates that the magnitude of stress transferred from the soil to the granular column increases with depth.

**Figure 7.14** Profiles of vertical stress in the soil for $A/A_c = 3$; (a) SSC ($\mu^* = 0$), (b) SSC
Analysing the multi-layer Bothkennar profile using the SSC model

**Figure 7.15** Profiles of vertical stress in the soil for $A/A_c = 15$; (a) SSC ($\mu^* \approx 0$), (b) SSC

### 7.6.1.2 Radial stress profiles

The corresponding distributions of radial stress in the soil with depth for $A/A_c = 3$ and 15 are presented in Figures 7.16 and 7.17 respectively. The following can be deduced from these figures:

- Without creep (Figures 7.16a and 7.17a), the radial stresses for the untreated and treated cases are similar; the stone columns mobilise the majority of this when bulging.
- With creep (Figures 7.16b and 7.17b), the radial stress is less than the corresponding radial stress for the untreated case at close spacings ($A/A_c = 3$). However, little or no change is observed at wide spacings ($A/A_c = 15$).
Analysing the multi-layer Bothkennar profile using the SSC model

Figure 7.16 Profiles of radial stress in the soil for $A/A_c = 3$; (a) SSC ($\mu^* \approx 0$), (b) SSC

Figure 7.17 Profiles of radial stress in the soil for $A/A_c = 15$; (a) SSC ($\mu^* \approx 0$), (b) SSC
Stress oscillations are visible in the profiles of radial stress with depth in Figures 7.16 and 7.17, especially for the analyses incorporating creep at $A/A_c = 3$ (Figure 7.16b). These oscillations are predominantly caused by additional column yielding which manifests itself in the form of (additional) shear-plane formation in the column; mesh refinement studies have been carried out to verify that the oscillations are not mesh dependent.

The shear planes can be seen by examining the plots of total shear strain in Figure 7.18. In general, shear planes tend to form in closely-spaced columns (e.g. $A/A_c = 3$, Figure 7.18a). For the case with creep, significantly more shear planes form (e.g. Figure 7.18b). Columns at wider spacings (e.g. $A/A_c = 15$), which are not radially inhibited by closely adjacent columns, will tend to bulge more. This is consistent with the findings reported by Muir Wood et al. (2000), see Section 2.6.3.3. The oscillations disappear below the level of shear plane-formation (yielding begins at the surface and progresses downward with time).

**Figure 7.18** Total shear strains at $A/A_c = 3$; (a) SSC ($\mu^* \approx 0$), (b) SSC
Other factors which may contribute to the oscillations are:

- **Radial stress reductions**: For the analyses with creep, the radial stresses are lower so there is less lateral restraint to resist column bulging.
- **Pore pressure oscillations**: Pore pressure oscillations that occur due to viscoplastic creep strains; essentially ‘constitutive model oscillations’.

The base of the unit cell is fixed in the horizontal and vertical directions. For the analyses without creep, the radial strain ($\varepsilon_{xx}$) in the soil ‘tapers’ to zero at the base of the unit cell, e.g. Figure 7.19a. However, there is a sharp decrease of radial strain at the base for the case with creep, e.g. Figure 7.19b. This explains the sudden drops or ‘jumps’ observed in the profiles of radial stress with depth at the base in Figures 7.16b and 7.17b. Analyses have been carried out in Section 7.7 which show that these ‘jumps’ also occur at the base of floating columns; albeit for a different reason (see Section 7.7.2).

**Figure 7.19** Profiles of radial strain in the soil for $A/A_c = 3$; (a) SSC ($\mu^* \approx 0$), (b) SSC
7.6.1.3 Hoop stress profiles

The distributions of hoop stress in the soil with depth for $A/A_c = 3$ and 15 are presented in Figures 7.20 and 7.21 respectively. The plots are presented in a similar format to those for radial stress. The following points are noteworthy:

- With and without creep, the hoop stresses in the soil are lower than the corresponding hoop stresses for the untreated case; the hoop stress reduction is larger at closer spacings.
- The observed hoop stress reductions are larger than the corresponding radial stress reductions (seen by comparing Figures 7.20 and 7.21 with Figures 7.16 and 7.17).
- Without creep (Figures 7.20a and 7.21a), the hoop stress reduction is more prominent towards the surface of the soil profile in the vicinity of the bulging depth (which can be approximated from Figure 7.19 as being between 3m and 4m). In general, plastic deformation leads to the dissipation of energy; more deformation at the bulging depth leads to more of a stress reduction than at the base where the corresponding strains are less.
- With creep (Figures 7.20b and 7.21b), the hoop stress reductions are larger than those without creep because there is more plastic deformation in existence throughout the full depth of the profile (see Figure 7.19).
Analyzing the multi-layer Bothkennar profile using the SSC model

**Figure 7.20** Profiles of hoop stress in the soil for $A/A_c = 3$; (a) SSC ($\mu^* \approx 0$), (b) SSC

**Figure 7.21** Profiles of hoop stress in the soil for $A/A_c = 15$; (a) SSC ($\mu^* \approx 0$), (b) SSC
7.6.2 Stress and strain profiles in the column

7.6.2.1 Column stress profiles

The distributions of vertical, radial, and hoop stress in the column for $A/A_c = 3, 6,$ and $15$ are presented in Figures 7.22, 7.23, and 7.24 respectively. The key findings are:

- The total load (vertical stress) taken per column is larger at wider spacings.
- The output shows a significant degree of scatter in regions where column bulging occurs or shear planes form. The scatter for the ‘with creep’ case progresses much deeper due to the additional yielding and subsequent shear-plane formation.
- The vertical stress carried by the column increases due to creep (comparison of Figures 7.22a and 7.22b). This is more evident at depth than at the surface in these figures due to the scatter.
- The radial stresses for the case without creep (Figure 7.23a) are similar in magnitude to those for the case with creep (Figure 7.23b), although significantly more scatter is evident in the latter case. There is a slight radial stress reduction for the case with creep for $A/A_c = 3$ (this corresponds with the radial stress reduction in the soil, i.e. radial equilibrium is maintained).
- The hoop stress changes (Figures 7.24a and 7.24b) observed in the columns mirror the radial stress changes.
Analysing the multi-layer Bothkennar profile using the SSC model

Figure 7.22 Profiles of vertical stress in the column; (a) SSC ($\mu^* \approx 0$), (b) SSC

Figure 7.23 Profiles of radial stress in the column; (a) SSC ($\mu^* \approx 0$), (b) SSC
Figure 7.24 Profiles of hoop stress in the column; (a) SSC ($\mu^* \approx 0$), (b) SSC

7.6.2.2 Column strain profiles

The distributions of vertical, radial, and hoop strain ($\varepsilon_{yy}$) in the column for $A/A_c = 3, 6, \text{ and } 15$ are presented in Figures 7.25, 7.26, and 7.27 respectively. Key observations include:

- Maximum column bulging, which occurs at the weakest point in the soil profile (top of the lower Carse clay layer), is larger for the analyses incorporating creep.
- The vertical, radial, and hoop strains are larger at wider spacings (both with and without creep); column bulging is more pronounced because columns are freer to expand radially, e.g. Muir Wood et al. (2000).
- For the case with creep, there are significant oscillations in the distributions with depth (shear-plane formation, see Section 7.6.1.2).
- The distribution of vertical strain (Figure 7.25) with depth appears to be related to that of the radial and hoop strains (Figures 7.26 and 7.27); this is consistent with the results obtained by Killeen (2012) based on the use of the HS model in PLAXIS 3D Foundation.
Analysing the multi-layer Bothkennar profile using the SSC model

**Figure 7.25** Profiles of vertical strain in the column; (a) SSC ($\mu^* \approx 0$), (b) SSC

**Figure 7.26** Profiles of radial strain in the column; (a) SSC ($\mu^* \approx 0$), (b) SSC
Analysing the multi-layer Bothkennar profile using the SSC model

Figure 7.27 Profiles of hoop strain in the column; (a) SSC ($\mu^* \approx 0$), (b) SSC

7.7 Floating column analyses

The impact of creep on the stress transfer process for floating stone columns is examined in this section; 10m long columns terminated in the lower Carse clay layer have been adopted for these analyses (Figure 7.28). Axisymmetric analysis techniques based on Approach C have again been used to establish the influence of creep; the analysis stages are identical to those described in Section 6.2 for end-bearing columns ($p_a = 100kPa$).
Analysing the multi-layer Bothkennar profile using the SSC model

Figure 7.28 Floating stone column terminated at a depth of 10m in the lower Carse clay

Plots showing the settlement-log(time) behaviour and evolution of $n$ with time for floating columns have not been presented for space purposes. In summary, the floating columns accelerate consolidation, although obviously not to the same extent as end-bearing columns. The $n$ values for floating columns are lower than the corresponding $n$ values for end-bearing columns; differences are most pronounced at close spacings. Closely-spaced columns transfer a significant proportion of load to the base; this load is supported by a rigid stratum for end-bearing columns and a compressible stratum for floating columns (the rigid stratum provides much more support), e.g. Killeen (2012). The analyses incorporating creep give lower $n$ values than those without creep, consistent with the findings heretofore.

7.7.1 Vertical stress profiles

The distributions of vertical stress in the soil with depth for the cases without and with creep for the floating columns at $A/A_c = 3$ are plotted in Figure 7.29 (profiles of vertical stress for the untreated soil are also included to provide a frame of reference). The stress profile for the end-bearing columns at $A/A_c = 3$ (previously presented in Figure 7.14) is also superimposed on Figure 7.29. The findings are as follows:

- As was the case for end-bearing columns, the floating columns reduce the vertical stress carried by the soil (above the column base).
The stress reduction is larger and increases with depth for the case with creep.

The floating columns punch into the underlying soil resulting in the development of vertical stress and strain below the base; this is why the stresses for the treated case are larger than those for the untreated case below the base; these stresses are approximately equal for the ‘creep’ and ‘no creep’ cases. The stress reduction above the base outweighs the stress increase below the base; this is reflected in the $n$ values.

**Figure 7.29** Profiles of vertical stress in the soil for $A/A_c = 3$: (a) SSC ($\mu^* = 0$), (b) SSC

### 7.7.2 Radial stress profiles

The distributions of radial stress and strain in the soil with depth for the cases without and with creep for the floating columns at $A/A_c = 3$ are plotted in Figures 7.30 and 7.31 respectively. These figures indicate that the behaviour is very similar to that of end-bearing columns (superimposed on Figure 7.30, previously presented in Figure 7.16). Without creep (Figure 7.30a), the radial stress for the treated case is similar to that for the untreated case. With creep (Figure 7.30b), the radial stress in the soil is lower; the stress oscillations (shear
planes) are also evident for these floating columns. As with the end-bearing columns, the ‘jump’ in radial stress at the base of the column is also present. In this case, the base of the stone column is not fixed and so the ‘jump’ is not caused by a zero lateral strain boundary condition imposed at the base. For the floating columns, these ‘jumps’ occur at the base due to a change in the failure mechanism; at the base, the radial strain reduces (Figure 7.31) because the floating columns punch into the underlying soil resulting in a sudden lateral strain reduction and therefore a stress ‘jump’.

![Graph](image-url)

**Figure 7.30** Profiles of radial stress in the soil for $A/A_c = 3$; (a) SSC ($\mu^* \approx 0$), (b) SSC

### 7.7.3 Hoop stress profiles

The distributions of hoop stress in the soil with depth for the floating columns at $A/A_c = 3$ without and with creep are plotted in Figure 7.32. The pattern is similar to that for end-bearing columns (superimposed on Figure 7.32, previously plotted in Figure 7.20). Both with and without creep, the hoop stresses in the soil are reduced in comparison with the untreated case. The hoop stress reduction is larger for the case with creep (more plastic deformation).
Analysing the multi-layer Bothkennar profile using the SSC model

**Figure 7.31** Profiles of radial strain in the soil for $A/A_c = 3$; (a) SSC ($\mu^* \approx 0$), (b) SSC

**Figure 7.32** Profiles of hoop stress in the soil for $A/A_c = 3$; (a) SSC ($\mu^* \approx 0$), (b) SSC
7.8 Summary

The multi-layer Bothkennar profile has been analysed using Approach C in conjunction with the SSC model. The findings can be summarised as follows:

- Incorporating creep leads to lower settlement improvement factors than would be obtained had primary consolidation been considered alone. This is consistent with the findings in Chapter 6 for the single-layer profiles.
- The percentage differences between the $n_{\text{primary(SSC, } \mu^*=0)}$ and $n_{\text{total(SSC)}}$ values for the multi-layer profile are larger than those for the single-layer profiles in Chapter 6 because creep contributes to a larger proportion of the total settlement and so its weighted effect is more prominent.
- Creep results in a stress transfer process; as the soil creeps, vertical stress is transferred from the soil to the stone column. The additional load carried by the column induces additional yielding and (additional) shear-plane formation in closely-spaced columns. The magnitude of the vertical stress transferred from soil to column increases with depth and is more prevalent in closely-spaced columns. The additional increment of stress transferred to the already yielded column reduces its efficacy, resulting in a suboptimal performance, leading to lower $n$ values for the ‘with creep’ case.
- The radial and hoop stresses in the soil for the treated case are lower than the corresponding radial and hoop stresses in the soil for the untreated case. The hoop stress reductions are larger than the corresponding radial stress reductions. With creep, both the radial and hoop stresses for the treated case continue to reduce after EOP whereas the corresponding stresses for the ‘without creep’ case are constant after EOP.
- The radial stress reduction for the ‘with creep’ case means that the lateral support imparted onto the column by the soil is lower; this will also contribute to lower $n$ values but is not as influential as the additional yielding caused by the vertical stress transfer.
- The additional plastic deformation for the ‘with creep’ case is the main reason for the hoop stress reduction. However, the hoop stress reduction appears to be an effect rather than a cause of the larger settlements (and hence lower $n$ values) for the ‘with creep’ case.
- The influence of creep on the behaviour of floating columns is consistent with its influence on end-bearing columns.
8. Analysing the multi-layer Bothkennar profile using the Creep-SCLAY1S model

8.1 Introduction

This chapter describes further analyses on the multi-layer Bothkennar profile that have been performed using the Creep-SCLAY1S model, which requires implementation into the PLAXIS FE code as a user-defined model. The Creep-SCLAY1S model incorporates anisotropy, bonding, and destructuration, which can be ‘switched off’ individually or in various combinations by adjusting the input parameters (e.g. Section 8.5). This work was performed during an academic visit to Chalmers University of Technology in Gothenburg, Sweden, in October/November 2013. The purpose of this study is to assess whether using this more advanced model instead of the SSC model impacts upon the general conclusions made heretofore, rather than understanding the reasons for any quantitative differences in the model outputs. This chapter is subdivided as follows:

(i) The background and formulation of the Creep-SCLAY1S model are described in Sections 8.2 and 8.3.
(ii) The development of additional model parameters for the Bothkennar clay, over and above those already presented in Chapter 5, is described in Section 8.4.
(iii) 'Soil Test' facility simulations carried out comparing the Creep-SCLAY1S model response with the SSC model response and with the test data reported in ICE (1992) are described in Section 8.5.
(iv) The approach used to analyse the multi-layer profile using this model is described in Section 8.6, the results of which are presented and discussed in Section 8.7.

8.2 Background to the Creep-SCLAY1S model

The development of the Creep-SCLAY1S model is described in detail by Sivasithamparam et al. (2014). The model uses the concept of a constant rate of viscoplastic multiplier to calculate creep strain rate, and so overcomes some of the limitations associated with the
ACM (Anisotropic Creep model), which uses the concept of contours of constant volumetric creep strain rate. This is discussed in Section 3.4.3.

The anisotropy and destructuration components of the model are formulated using the constitutive equations from the rate independent S-CLAY1 (Wheeler et al., 1997, 2003) and S-CLAY1S (Koskinen et al., 2002) models:

- The S-CLAY1 model is an extension of the MCC model which uses an inclined yield surface (angle of inclination, $\alpha$) and a rotational hardening law to describe the development/erasure of anisotropy due to plastic straining.
- The yield curve inclination of the S-CLAY1 model changes due to both plastic volumetric strains and plastic shear strains (thus predicting a unique yield curve inclination at critical state) whereas an earlier model by Dafalias (1986) assumes that any changes to the inclination of the yield curve are caused by plastic volumetric strains only (Wheeler et al. 2003, Grimstand & Degago 2010).
- The S-CLAY1 model also incorporates the basic volumetric hardening law that describes the change to the size of the yield curve due to plastic volumetric strain (same as MCC model).
- The S-CLAY1S model is an extension to the S-CLAY1 model that accounts for bonding and destructuration using an intrinsic yield surface and a third hardening law to describe the breakdown of bonding caused by plastic straining.
- Both models assume an associated flow rule and ignore the anisotropy of elastic behaviour as this could potentially result in 21 additional elastic parameters (Wheeler et al., 2003).

8.3 Formulation of the Creep-SCLAY1S model

In this section, the formulation of the model is presented in triaxial stress space (based on the stress invariants, $p'$ and $q$, see Figure 8.1), for which the anisotropy can then be defined by a single scalar parameter (Wheeler et al., 2003). However, in order to model principal stress rotation effects, stress and strain tensors are used for the generalised form of the model (stress
invariants cannot be used for an anisotropic model), with anisotropy defined by a deviatoric fabric tensor, e.g. Wheeler et al. (2003), Ollson (2013).

\[ \epsilon = \epsilon^e + \epsilon^c \]  

\( \hat{\epsilon} = \hat{\epsilon}^e + \hat{\epsilon}^c \)  

**Figure 8.1** Yield surfaces of the Creep-SCLAY1S model in triaxial stress space

The total strain rate (\( \hat{\epsilon} \)) is composed of an elastic component (\( \hat{\epsilon}^e \)) and a creep component (\( \hat{\epsilon}^c \)), see Eq. 8.1. Similar to extended overstress models, the Creep-SCLAY1S model has no purely elastic domain.

In the Creep-SCLAY1S model, the hardening laws depend on viscoplastic/creep shear and volumetric strains. The rotational hardening law describing the changing inclination of the yield surface due to creep strains takes the form shown in Eq. 8.2, where \( \omega \) and \( \omega_d \) are two additional soil constants, \( d\epsilon_v^c \) and \( d\epsilon_d^c \) are the increments of creep volumetric and deviatoric strains respectively, and \( (\cdot) \) denote Macaulay brackets. \( \omega_d \) controls the relative effectiveness of the shear and volumetric strains in determining the overall target value for \( \alpha \) and \( \omega \) controls the absolute rate at which \( \alpha \) approaches the target value.
Analysing the multi-layer Bothkennar profile using the Creep-SCLAY1S model

\[
d\alpha = \omega \left( \left( \frac{3\eta}{4} - \alpha \right) \langle d\varepsilon_v^c \rangle + \omega_d \left( \frac{\eta}{3} - \alpha \right) |d\varepsilon_d^c| \right)
\]

(8.2)

The destructuration hardening law takes the form shown in Eq. 8.3, where \( \zeta \) and \( \zeta_d \) are two additional soil constants controlling the absolute rate of destructuration and the relative effectiveness of viscoplastic volumetric and deviatoric strains respectively in destroying the bonding (i.e. the degree of bonding, \( \chi \), will reduce to zero).

\[
d\chi = -\zeta \chi \left( |d\varepsilon_v^c| + \zeta_d |d\varepsilon_d^c| \right)
\]

(8.3)

The initial amount of bonding (\( \chi_0 \)) relates the size of the natural yield surface (\( p_{p0} \)) to the size of the intrinsic yield surface (\( p_{p0i} \)), see Eq. 8.4.

\[
p_{p0} = \left( 1 + \chi_0 \right) p_{p0i}
\]

(8.4)

Similar to the SSC model and the ACM, the yield surface (normal consolidation surface, NCS) evolves with creep volumetric strains according to Eq. 4.19. The equivalent mean stress measure (\( p^{eq} \)) in this case (i.e. the intersection of the current stress surface (CSS) with the \( p' \) axis, see Figure 8.1) is given by Eq. 8.5, where \( M(\theta) \) is the stress ratio at critical state, which in this model has been made a function of Lode Angle (\( \theta \)) to incorporate a smooth failure surface, see Sivasithamparam (2014).

\[
p^{eq} = p' - \frac{(q - \alpha p')^2}{(M^2(\theta) - \alpha^2)p'}
\]

(8.5)

In contrast to the SSC model and the ACM, which calculate the volumetric creep strain rate according to Eq. 8.6, the Creep-SCLAY1S model uses the rate of viscoplastic multiplier (\( \dot{\lambda} \)), see Eq. 8.7. The additional term \( (M^2(\theta) - \alpha^2)K_{0nc}/(M^2(\theta) - \eta^2K_{0nc}) \) is added to ensure that the creep strain under oedometer conditions corresponds to the concept of constant contours of volumetric creep strain rate, where the subscript \( K_{0nc} \) denotes the normally consolidated stress state.
Analysing the multi-layer Bothkennar profile using the Creep-SCLAY1S model

\[ \dot{e}_v^C = \frac{\mu^*}{\tau} \left( \frac{p^{eq}}{p_p^{eq}} \right)^{\frac{\lambda^*}{\mu^*}} \]  

\[ \dot{\lambda} = \frac{\mu^*}{\tau} \left( \frac{p^{eq}}{p_p^{eq}} \right)^{\frac{\lambda^*}{\mu^*}} \left( \frac{M^2(\theta) - \alpha^2 K_0^{nc}}{M^2(\theta) - \alpha^2 K_0^{nc}} \right) \]  

8.4 Additional Model Parameters

The determination of model parameters for the Creep-SCLAY1S model is straightforward. The anisotropy parameters, \( \alpha_0 \) (initial yield surface inclination), \( \omega_d \), and \( \omega \) can be calculated from the critical state friction angle using Eqs. 8.8, 8.9, and 8.10 where \( \eta_0 = q_0/p_0' = 3(1-K_0^{nc})(1+2K_0^{nc}) \).

\[ \alpha_0 = \frac{\eta_0^2 + 3\eta_0 - M^2}{3} \]  

\[ \omega_d = \frac{3}{8} \frac{4M^2 - 4\eta_0^2 - 3\eta_0}{\eta_0^2 - M^2 + 2\eta_0} \]  

\[ \omega = \frac{1}{\lambda^* \ln} \frac{10M^2 - 2\alpha_0 \omega_d}{M^2 - 2\alpha_0 \omega_d} \]  

In some cases, Eq. 8.10 can result in an unrealistic value of \( \omega \), in which case the empirical correlation suggested by Zentar et al. (2002) should be used instead (Eq. 8.11).

\[ \frac{10}{\lambda} \leq \omega \leq \frac{20}{\lambda} \]  

The initial amount of bonding, \( \chi_0 \), can be calculated based on the sensitivity, \( S_0 \), see Eq. 8.12.

\[ \chi_0 = S_0 - 1 \]  

The other destructuration parameters (\( \xi_d \) and \( \xi \)) can be calibrated using the optimisation procedure described in Koskinen et al. (2002). For this model, the intrinsic compression and creep indices, \( \lambda_i^* \) and \( \mu_i^* \) (measured from oedometer tests on reconstituted samples), should
be used as opposed to the $\lambda^*$ and $\mu^*$ (measured from oedometer tests on natural samples) values used for the SSC model. The intrinsic compression indices used for the crust, upper Carse clay, and lower Carse clay layers are shown in Table 8.1, taken from Karstunen et al. (2013). Also shown are the compression and creep indices used for natural clay for comparison purposes.

<table>
<thead>
<tr>
<th></th>
<th>Crust</th>
<th>Upper Carse Clay</th>
<th>Lower Carse Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^*$</td>
<td>0.0152</td>
<td>0.0494</td>
<td>0.1621</td>
</tr>
<tr>
<td>$\lambda_i^*$</td>
<td>0.0056</td>
<td>0.0183</td>
<td>0.0600</td>
</tr>
<tr>
<td>$\mu^*$</td>
<td>0.0006</td>
<td>0.0020</td>
<td>0.0065</td>
</tr>
<tr>
<td>$\mu_i^*$</td>
<td>0.0002</td>
<td>0.0007</td>
<td>0.0024</td>
</tr>
</tbody>
</table>

Table 8.1 Compression indices for the multi-layer Bothkennar profile

The additional soil parameters required for the FE model are summarised in Table 8.2. The $\omega$, $\zeta_d$, and $\zeta$ parameters have been calibrated by Karstunen et al. (2013) for the Bothkennar Carse clay. The slopes of the CSLs in compression ($M_c$) and extension ($M_e$) have been reported by Allman & Atkinson (1992), see Section 5.2.3.

<table>
<thead>
<tr>
<th>$M_c$</th>
<th>$M_e$</th>
<th>$a_0$</th>
<th>$\omega_d$</th>
<th>$\omega$</th>
<th>$\chi_0$</th>
<th>$\zeta_d$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.375</td>
<td>-1.00</td>
<td>0.5267</td>
<td>0.9281</td>
<td>50</td>
<td>8</td>
<td>0.2</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 8.2 Additional Material Parameters

8.5 ‘Soil Tests’ (Lower Carse Clay)

The PLAXIS 'Soil Test' facility has been used in conjunction with the Creep-SCLAY1S model to simulate the CK$_0$U triaxial tests reported by Atkinson et al. (1992), described in Section 5.3.4. The stress-strain response is plotted in Figures 8.2a and 8.3a at depths of 12.6m and 15.4m respectively and the corresponding stress paths are plotted in Figures 8.2b and 8.3b. These simulations have been carried out using a critical state friction angle of $\phi'_s = 34^\circ$. Three separate scenarios have been considered for the Creep-SCLAY1S model:
(i) Anisotropy and destructuration are ‘switched off’ by setting the relevant parameters to zero. These isotropic analyses enable a straight-forward comparison with the isotropic SSC model output.

(ii) The anisotropy parameters are ‘switched on’. Direct comparison with the isotropic Creep-SCLAY1S model results enables the influence of anisotropy to be established.

(iii) Both the anisotropy and bonding/destructuration parameters are ‘switched on’. The influence of soil destructuration can then be examined.

ISO is used hereafter to denote the isotropic response of the Creep-SCLAY1S model with anisotropy and destructuration ‘switched off’. ANIS is used to denote the anisotropic response with destructuration ‘switched off’, whereas A&D denotes the response with both anisotropy and bonding/destructuration ‘switched on’. The SSC model response using $\phi_s' = 34^\circ$ has been included on Figures 8.2 and 8.3 for comparison purposes.

It should be noted that for the isotropic analyses, $M_e$ has been set equal to $M_c = 1.375$. Additionally, for the isotropic analyses, the anisotropy parameters, $\alpha$, $\omega_d$, and $\omega$ have been set to zero; the rotational hardening law (Eq. 8.2) is ‘switched off’ and $K_0$ will be overpredicted, analogous to the MCC model. The intrinsic compression and creep indices quoted in Table 8.1 are used for the analyses incorporating destructuration.
Analysing the multi-layer Bothkennar profile using the Creep-SCLAY1S model

Figure 8.2 Comparison with Atkinson et al. (1992) - 12.6m depth samples (a) CK₀U Triaxial: q vs. εᵧᵧ (b) CK₀U Triaxial: q vs. p’

Figure 8.3 Comparison with Atkinson et al. (1992) - 15.4m depth samples (a) CK₀U Triaxial: q vs. εᵧᵧ (b) CK₀U Triaxial: q vs. p’
Figures 8.2 and 8.3 indicate the following:

- The predicted undrained shear strength for the scenario incorporating bonding and destructuration is in relatively good agreement with that measured by Atkinson et al. (2013).
- The peak undrained shear strength for this bonding and destructuration scenario is higher than that predicted by the other scenarios considered (isotropic, anisotropic, and SSC model) due to the bonded nature of the soil (i.e. since $\lambda_i^* \ll \lambda^*$). The peak undrained shear strengths for the isotropic and anisotropic cases are in relatively good agreement with those predicted by the SSC model.
- In all cases, a larger friction angle would provide a better match with the measured data because a critical state tends not to be reached in triaxial tests on Bothkennar clay samples possessing a well-ordered fabric (see Section 5.3.4).
- The Creep-SCLAY1S model will only predict post-peak softening for the case that incorporates destructuration. As noted in Sections 3.4.2, 4.2.7.2, and 5.3.4.3, the post-peak softening behaviour predicted by the SSC model occurs because the viscoplastic volumetric strain rate is independent of the stress state.

### 8.6 Analysis Approach

The approach used to establish the influence of creep on stone column behaviour is equivalent to Approach C for the SSC model, described in Section 6.5. Two sets of analyses are performed with the Creep-SCLAY1S model; the first set using the standard creep coefficient for the soil and the second set using a very low creep coefficient as before. The analysis stages are the same as those in the previous chapters, described in Section 6.2 ($p_a = 100\text{kPa}$). For the Creep-SCLAY1S model, these two sets of analyses (with and without creep) will be implemented for the three separate scenarios introduced in Section 8.5:

(i) The $n$ values with and without creep for the isotropic case will be denoted $n_{\text{TOTAL(ISO)}}$ and $n_{\text{PRIMARY(ISO)}}$ respectively. Direct comparison with the SSC model output will enable the influence of the chosen constitutive model, if any, to be identified.
(ii) The $n$ values with and without creep for the anisotropic case will be denoted $n_{\text{TOTAL(ANIS)}}$ and $n_{\text{PRIMARY(ANIS)}}$ respectively.
(iii) The \( n \) values with and without creep for the analyses incorporating anisotropy, bonding, and destructuration will be denoted \( n_{TOTAL(A&D)} \) and \( n_{PRIMARY(A&D)} \) respectively.

### 8.7 Results

The results section will be subdivided as follows:

- The time-settlement behaviour predicted by the Creep-SCLAY1S model for the three different scenarios for the untreated soil profile will be compared with the SSC model output to provide context for the subsequent comparisons.
- The \( n_{PRIMARY} \) and \( n_{TOTAL} \) values predicted by the Creep-SCLAY1S model for the different scenarios will be compared to those predicted by the SSC model. Settlement-log(time) plots comparing the untreated and treated cases for each scenario are not presented.
- Comparisons will be made with a selection of analytical settlement design methods.
- The evolution of \( n \) with time for each individual scenario will be examined to establish (i) if this model also predicts that the incorporation of creep leads to lower \( n \) values, and (ii) if incorporating anisotropy and/or bonding/destructuration affects the conclusions.
- The distribution of stress with depth will be examined to establish if/why the behaviour of the Creep-SCLAY1S model differs to that of the SSC model.

#### 8.7.1 Time-settlement behaviour - untreated case

Settlement-log(time) plots for the untreated case without and with creep for the Creep-SCLAY1S and SSC models are presented in Figures 8.4 and 8.5 respectively. These figures indicate:

- The time-settlement behaviour predicted by the Creep-SCLAY1S model for the isotropic and anisotropic cases is almost identical (both with and without creep). This will be the case for 1D loading provided the anisotropy parameters are derived based on the \( K_0 \) state.
- The behaviour predicted by the SSC model is comparable, although the final settlements are marginally lower.
• The settlements predicted by the Creep-SCLAY1S analyses incorporating bonding and destructuration are less than those for the other cases (since $\lambda_i^* \ll \lambda^*$ and $\mu_i^* \ll \mu^*$).

**Figure 8.4** Settlement vs. log(time) plots for untreated soil without creep

**Figure 8.5** Settlement vs. log(time) plots for untreated soil with creep
8.7.2 Comparison of settlement improvement factors

The $n_{\text{PRIMARY}}$ and $n_{\text{TOTAL}}$ values for the different models/scenarios are presented in Figures 8.6 and 8.7. At all values of $A/A_c$, the highest $n$ values arise for the isotropic Creep-SCLAY1S model analyses and the lowest for the analyses incorporating destructuration.

The overpredicted $K_0$ values for the isotropic case (see Section 8.6) result in larger horizontal soil stresses (presented in Section 8.7.5.1) which provide more resistance against bulging of the granular columns. This results in lower settlements for the treated case at all values of $A/A_c$, and since the settlement of untreated soil for the isotropic and anisotropic cases is similar (the lateral strains for the untreated case (1D) are negligible), the $n_{\text{PRIMARY(ISO)}}$ values are greater than the $n_{\text{PRIMARY(ANIS)}}$ values.

The values of $\lambda_i^*$ and $\mu_i^*$ for the case incorporating bonding and destructuration are lower than the values of $\lambda^*$ and $\mu^*$ for the other analyses; the soil profile is thus stiffer and lower $n$ values would be expected (lower $E_c/E_s$). However, the $n$ values are still lower than would be expected because $n$ is not very sensitive to soil stiffness above a threshold value, as illustrated in Section 6.4.2.4. There is a further reason for these lower $n$ values: the presence of columns leads to additional bond degradation (in comparison with the untreated case). The extent to which destructuration should be accounted for in design will depend on the initial amount of bonding, $\chi_0$, which is dictated by the soil sensitivity, $S_t$ (see Eq. 8.12). In highly sensitive soils (e.g. Swedish clays, see Table 9.1), destructuration will be more problematic than in soils with a lower sensitivity, e.g. Batiscan clay (Table 9.1).
Figure 8.6 Comparison of \( n_{\text{PRIMARY}} \) values

Figure 8.7 Comparison of \( n_{\text{TOTAL}} \) values

Analytical predictions obtained using Priebe (1995), Castro & Sagaseta (2009), and Pulko et al. (2011) are superimposed with the \( n_{\text{PRIMARY}} \) values obtained using the different models/scenarios in Figure 8.8. The \( n \) values predicted by Castro & Sagaseta (2009) and Pulko et al. (2011) fall between the \( n_{\text{PRIMARY(ANIS)}} \) and \( n_{\text{PRIMARY(A&D)}} \) for \( 4 < A/A_c < 15 \). The \( n_{\text{PRIMARY(ISO)}} \) and \( n_{\text{PRIMARY(A&D)}} \) values are larger and smaller than the analytical predictions respectively. However, it should be noted that the analytical predictions in Figure 8.8 are not applicable to the case incorporating bonding and destructuration; a lower soil stiffness (corresponding to \( \lambda_i^{*} \)) should be used to obtain the relevant lower \( n \) values.
Figure 8.8 Comparison of $n_{\text{PRIMARY}}$ values with analytical solutions

### 8.7.3 Evolution of settlement improvement factor with time

The evolutions of $n_{\text{PRIMARY}}$ and $n_{\text{TOTAL}}$ with time for the three different scenarios with the Creep-SCLAY1S model are compared in Figure 8.9 at $A/A_c = 6$. Also included in Figure 8.9 are the corresponding $n$ values obtained using the SSC model (presented previously in Figure 7.5c). In all cases, the $n$ values are less than unity initially because the settlement of treated ground occurs more rapidly than that of untreated ground; however, these $n$ values are of no practical significance. Regardless of the model/scenario, the ‘steady-state’ $n_{\text{TOTAL}}$ values after EOP are less than the corresponding $n_{\text{PRIMARY}}$ values; this holds at all values of $A/A_c$; $A/A_c = 6$ is chosen for illustrative purposes.
Analysing the multi-layer Bothkennar profile using the Creep-SCLAY1S model

**Figure 8.9** Evolution of $n$ with time at $A/A_c = 6$: (a) SSC Model (b) Creep-SCLAY1S model (ISO) (c) Creep-SCLAY1S model (ANIS) (d) Creep-SCLAY1S model (A&D)
As was apparent in Figures 8.5 and 8.6, $n$ values are different for each model/scenario. The relative differences between the $n_{TOTAL}$ and $n_{PRIMARY}$ values for each case are investigated in Figure 8.10 by plotting $(n_{TOTAL} - 1)$ against $(n_{PRIMARY} - 1)$ at different values of $A/A_c$; for an untreated soil, both $n_{TOTAL}$ and $n_{PRIMARY}$ will be equal to 1. Each datapoint in Figure 8.10 corresponds to a $n_{PRIMARY}$ value and a $n_{TOTAL}$ value at a single value of $A/A_c$. Best-fit lines have been added to each figure, along with their corresponding coefficients of determination ($R^2$). The relationship takes the form shown in Eq. 8.13, where $C$ is the slope of the line; smaller $C$ values indicate larger differences between $n_{PRIMARY}$ and $n_{TOTAL}$. For the SSC model, and for the isotropic and anisotropic cases with the Creep-SCLAY1S model, $0.61 < C < 0.64$, suggesting that the relative values of $n_{TOTAL}$ and $n_{PRIMARY}$ are independent of model type and of whether anisotropy is taken into account. For the case incorporating anisotropy and bonding/destructuration, the value of $C$ is higher because (i) the $n_{PRIMARY(A&D)}$ values are lower to begin with and (ii) $\mu_i^* << \mu^*$ so the weighted effect of creep is less visible. Note that $C$ would be smaller if the ratio of creep to primary settlement was larger and vice versa.

![Figure 8.10](image-url)
(n_{TOTAL} - 1) = C \cdot (n_{PRIMARY} - 1) \quad (8.13)

8.7.4 Profiles of stress in the soil with depth

The distributions of stress in the soil with depth without and with creep for the different models/scenarios at $A/A_c = 3$ and 15 are compared in this section. The corresponding stress profiles for the untreated case are presented in Section 8.7.4.1 to provide a frame of reference for the subsequent comparisons.

8.7.4.1 Stress profiles for the untreated case

Firstly, it should be noted that the initial stresses for all models/scenarios are equal (based on the same input parameters). The stresses for the untreated case without and with creep (both after 100 years) are presented in Figures 8.11 and 8.12. The main points to note are:

- Without creep, the profiles of vertical stress with depth (Figure 8.11a) are almost identical in all cases apart from the case incorporating bonding and destructuration; the difference occurs because the final settlements are almost 50% lower for this case (e.g. Figure 8.3) and the stresses are calculated based on updated nodal coordinates. A similar comment applies to the vertical stress profiles for the 'with creep' case (Figure 8.12a).
- The radial stresses with and without creep (Figures 8.11b and 8.12b) are almost equivalent in all cases except for the Creep-SCLAY1S model isotropic case (overpredicted $K_0$, see Section 8.6). Note that the hoop stresses are equal to the radial stresses for the untreated case.
Analysing the multi-layer Bothkennar profile using the Creep-SCLAY1S model

**Figure 8.11** Stress profiles for the untreated case without creep (a) $\sigma'_{yy}$ (b) $\sigma'_{xx}$

**Figure 8.12** Stress profiles for the untreated case with creep (a) $\sigma'_{yy}$ (b) $\sigma'_{xx}$
8.7.4.2 Vertical stress profiles

The vertical stress profiles in the soil at $A/A_e = 3$ are plotted in Figure 8.13 for the different models and scenarios without and with creep respectively. Lines are included on the figure, for optical purposes only, to help highlight differences between the ‘creep’ and ‘no creep’ profiles. Examination of this figure indicates that for each model/scenario (both with and without creep), the stone columns reduce the vertical stress carried by the soil in comparison with the untreated case (comparing Figure 8.13 with Figures 8.11a and 8.12a). The stress reduction is larger for the analyses incorporating creep, in keeping with discussions in Sections 7.6.2 and 7.7.1.1. The stress transfer due to creep (i.e. the stress reduction in the soil) is smallest for the analyses incorporating destructuration (Figure 8.13b) because of the lower creep coefficient (i.e. $\mu_{i*} << \mu^*$).

8.7.4.3 Radial stress profiles

The corresponding radial stress profiles in the soil at $A/A_e = 3$ are presented in Figure 8.14; lines have also been included for ease of comparison. The radial stresses in the soil are lower for the analyses incorporating creep (comparison of Figures 8.14a and 8.14b). The magnitudes of the radial stress reductions are independent of model type. Shear-planes (i.e. stress oscillations) extend to a greater depth for the anisotropic case with the Creep-SCLAY1S than for the isotropic case (overpredicted horizontal stresses, see Section 8.7.5.1, provide more resistance against lateral column bulging, and result in the formation of less shear-planes in the column). The magnitudes of the shear-planes are similar for the ‘anisotropy and destructuration’ case, despite the lower creep coefficient ($\mu_{i*} << \mu^*$); for this scenario, column presence triggers creep strains which result in further bond degradation (especially if close to the preconsolidation pressure).
Analysing the multi-layer Bothkennar profile using the Creep-SCLAY1S model

Figure 8.13 Profiles of vertical stress in the soil for $A/A_c = 3$ (a) No Creep (b) Creep

Figure 8.14 Profiles of radial stress in the soil for $A/A_c = 3$ (a) No Creep (b) Creep
8.7.4.4 Hoop stress profiles

The corresponding hoop stress profiles in the soil at $A/A_c = 3$ are presented in Figure 8.15. The hoop stresses without creep (Figure 8.15a) are almost identical in all cases. With creep (Figure 8.15b), the corresponding hoop stresses are lower. For the Creep-SCLAY1S model, a larger hoop stress reduction occurs for the anisotropic case than for the isotropic case (additional plastic deformation and shear plane formation leads to a larger hoop stress reduction). The hoop stress reduction predicted by the SSC model is larger than that predicted by the isotropic Creep-SCLAY1S model analyses (less lateral deformation due to the overpredicted $K_0$); however, the profiles with depth are approximately parallel. The profiles for the analyses incorporating anisotropy and bonding/destructuration are approximately parallel to those for the anisotropic case, although the hoop stress reduction is lower because $\mu_{i^*} \ll \mu^*$. 

![Figure 8.15 Profiles of hoop stress in the soil for $A/A_c = 3$ (a) No Creep (b) Creep](image_url)


8.8 Summary

In this chapter, the multi-layer Bothkennar profile has been analysed using the Creep-CLAY1S model. Three different scenarios have been considered; (i) isotropy, (ii) anisotropy, and (iii) anisotropy and bonding/destructuration. The output has been compared to the SSC model output developed in Chapter 7. The main findings are as follows:

- For all three scenarios with the Creep-CLAY1S model, incorporating creep leads to lower settlement improvement factors than would be obtained had primary consolidation been considered alone. This is consistent with the findings obtained using the SSC model.
- The ratios of ‘total’ to ‘primary’ settlement improvement factors are almost identical for the ‘isotropic’ and ‘anisotropic’ cases, suggesting that the findings regarding the effectiveness of stone columns with regards arresting creep settlements are independent of model type and of whether anisotropy is taken into account. A smaller ratio is observed for the case incorporating ‘anisotropy and bonding/destructuration’ because (i) the ‘primary’ settlement improvement factors for this scenario are lower to begin with and (ii) the 'intrinsic' creep index is less than the creep index for natural clay so the effect of creep on the weighted average is less visible.
- For the analyses incorporating bonding and destructuration, the 'primary' settlement improvement factors are lower because the columns will destroy some of the bonding and the ‘total’ settlement improvement factors are lower because creep strains trigger additional bond degradation.
- The extent to which destructuration should be accounted for in design will depend on the initial sensitivity of the clay; in highly sensitive clays, destructuration will be more of a factor.
9. The influence of creep on stone columns: applicability to other clays

9.1 Introduction

In the first half of this chapter, a parametric study has been carried out using the SSC model with a view to establishing the soil parameters that have the largest influence on 'primary', 'total', and 'creep' settlement improvement factors and the corresponding stress transfer process from soil to column due to creep. The soil parameters have been altered in a specific range so that they represent the range of parameters typically encountered for soft (creep-prone) clays in practice. In the second half of the chapter, a design approach has been developed based on the FE output to account for creep in the design of granular columns. The approach can be used in conjunction with any primary settlement design method, although it is suggested to use one that captures all key features of primary settlement behaviour; see Sexton et al. (2013).

9.2 Soft clay soil parameters

Soil parameters for a selection of different soft clays at well-researched sites around the world are summarised in Table 9.1. Creep ratios, \((\lambda^* - \kappa^*)/\mu^*\), have already been plotted against \(\mu^*\) in Figure 5.1 to put the Bothkennar values in context of this range. Creep ratios for Norwegian clays tend to be comparable to those at Bothkennar. Swedish clays tend to be very soft with \(\lambda^*\) values double those of Bothkennar clay; additionally, they tend to have a lower creep ratio, \((\lambda^* - \kappa^*)/\mu^*\), and so creep contributes more to the total settlement. Finnish clays tend to be firmer with lower \(\lambda^*\) values and higher creep ratios. Creep ratios as low as \((\lambda^* - \kappa^*)/\mu^* = 5\) have been reported by Lopes (2011) for the soft soils of the Tagus Basin in Portugal. However, this appears to be an outlier because the high \(\mu^*/\lambda^*\) ratio of 0.146 appears out of kilter with the correlation proposed by Mesri & Godlewski (1977) for a variety of natural soils, see Section 3.3.1.1. Nash & Ryde (2001) have attributed the high \(\mu^*/\lambda^*\) ratio of the estuarine alluvium bordering the estuary of the River Severn near Avonmouth to the presence of minor peat bands.
<table>
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<th>Soil Type</th>
<th>$\phi'$ (°)</th>
<th>$S_r$</th>
<th>$\lambda^*$</th>
<th>$\kappa^*$</th>
<th>$\mu^*$</th>
<th>$(\lambda^<em>-\kappa^</em>)/\mu^*$</th>
<th>References</th>
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<td>Bothkennar Clay</td>
<td>34.0</td>
<td>5-13</td>
<td>0.162</td>
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<td>0.0065</td>
<td>21.43</td>
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<td>Väsby Clay</td>
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<td>0.038</td>
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<td>Skå-Edeby Clay</td>
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<td>0.050</td>
<td>0.0185</td>
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<td>Bäckebo Clay</td>
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<td>Haney Clay</td>
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<td>0.0020  -0.0040</td>
<td>22.25 -44.50</td>
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<td>38.57</td>
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<td>Drammen Clay</td>
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<td>10-12</td>
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<td>22.00</td>
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<td>Ellingsrud Clay</td>
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</tr>
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<td>Batiscan Clay, Canada</td>
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<td>0.013</td>
<td>0.0055</td>
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<td>Yin &amp; Karstunen (2011)</td>
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<td>Saint-Herblain Clay, France</td>
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<td>1.0</td>
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<td>0.007</td>
<td>0.0074</td>
<td>16.12</td>
<td>Yin &amp; Karstunen (2011)</td>
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<tr>
<td>Hong Kong Marine Deposits (HKMD)</td>
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<td>0.0027</td>
<td>23.03</td>
<td>Yin &amp; Karstunen (2011)</td>
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<td>0.0177</td>
<td>5.20</td>
<td>Lopes (2011)</td>
</tr>
</tbody>
</table>

Table 9.1 Soil parameters for worldwide clays
9.3 Soil profile

The parameter variability shown in Table 9.1 demonstrates that clay properties vary significantly at different locations around the world. A standardised stratigraphy as shown in Figure 9.1 was preferred for the parametric study, rather than modelling each soil profile individually; this enables the effect of changing a single parameter to be isolated. This adopted profile (crust + clay) was preferred to a single-layer profile because the majority of clay sites reported in the literature tend to possess a crust (formed by weathering and groundwater level fluctuations).

The majority of parameters for the ‘base case’ are based on the crust and lower Carse clay layers of the multi-layer Bothkennar profile documented in Table 5.2. The water table is located at a depth of 1.0m. The analysis stages are analogous to those described in Section 6.2. Approach C is used to evaluate the influence of creep; for the ‘base case’, the relative percentages of primary/creep settlement to the total settlement of untreated ground under $p_a = 100$ kPa after 1, 10, 30, 100, and 1000 years are 79/21, 74/26, 71/29, 68/32, and 64/36 respectively, although a wider range was captured by the parametric study. The Updated Mesh option has not been used for these analyses. The ‘base case’ simulations are carried out with $K_0 = 0.54$ for the clay, calculated using Eq. 9.1 (Brinkgreve et al., 2011). A higher value of $K_0 = 1.0$ is used for the crust.

$$K_0 = K_0^{uc} OCR - \frac{v_{sr}}{1 - v_{sr}} (OCR - 1)$$  \hfill (9.1)
9.4 ‘Base Case’ Results

The ‘base case’ results are presented in this section to provide a frame of reference for the subsequent parametric study as the adopted profile is different to those in Chapters 6-8.

9.4.1 Settlement improvement factors

The \( n_{\text{PRIMARY}(\text{SSC}, \mu^* = 0)} \), \( n_{\text{TOTAL}(\text{SSC})} \) (both after EOP), and \( n_{\text{CREEP}(\text{SSC})} \) values for the base case are plotted in Figure 9.2. ‘Total’ settlement improvement factors are effectively a weighted average of ‘primary’ and ‘creep’ settlement improvement factors; the percentage differences between \( n_{\text{PRIMARY}(\text{SSC}, \mu^* = 0)} \) and \( n_{\text{TOTAL}(\text{SSC})} \) (relative to \( n_{\text{PRIMARY}(\text{SSC}, \mu^* = 0)} \)) are larger at closer spacings (36% difference at \( A/A_c = 3 \) versus 15% difference at \( A/A_c = 10 \)), consistent with the findings heretofore.

![Figure 9.2] n_{\text{PRIMARY}(\text{SSC}, \mu^* = 0)}, n_{\text{TOTAL}(\text{SSC})}, and n_{\text{CREEP}(\text{SSC})}

9.4.2 Soil and column stresses

The average vertical effective stresses in the soil and stone column after 100 years (without and with creep) for \( A/A_c = 3, 6, \) and 10 are plotted in Figure 9.3. The average stress in the soil (Figure 9.3a) for the ‘with creep’ case is lower than the ‘without creep’ case; the stress unloaded due to creep is transferred to the column (Figure 9.3b). Accordingly, SCFs are higher for the ‘with creep’ case (Figure 9.4), as discussed in Section 7.5.2. The stress transfer
process from soil to column due to creep is more prevalent at closer spacings (i.e. $A/A_c = 3$), as demonstrated in Sections 7.5.2 and 7.6.1.1.

Figure 9.3 Average $\sigma'_{yy}$ after 100 years; (a) soil, (b) stone column

Figure 9.4 SCFs after 100 years

9.4.3 Stress distributions with depth

The distributions of radial, vertical, and hoop stress in the soil with depth after 100 years are presented in Figure 9.5 at $A/A_c = 3$ with and without creep. These graphs will be discussed in the context of the subsequent parametric study in Section 9.5.


Figure 9.5 Profiles of stress in the soil for $A/A_c = 3$ after 100 years; (a) $\sigma''_{xx}$ (b) $\sigma''_{yy}$ (c) $\sigma''_{zz}$

9.5 Parametric Study

The parametric study will assess the effect of a range of different soil parameters ($C_c$ or $\lambda^*$, $C_s$ or $\kappa^*$, $C_o$ or $\mu^*$, $K_0$, $\phi'$). The parameters of the crust layer are fixed throughout, with only the relevant parameter in the clay layer altered. The influence of load level ($p_a$) will also be investigated. In all cases, the effects of the different parameters on $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$, $n_{\text{TOTAL}(SSC)}$, and $n_{\text{CREEP}(SSC)}$ will be presented, as will their effect on the average vertical effective stresses in the soil and column. Profiles with depth will only be presented in cases of interest.

9.5.1 Effect of $K_0$

The effect of $K_0$ on $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$, $n_{\text{TOTAL}(SSC)}$, and $n_{\text{CREEP}(SSC)}$ is shown in Figure 9.6. The initial horizontal stresses generated in PLAXIS increase as $K_0$ increases; the $n$ values should increase accordingly because the larger horizontal stresses provide more lateral support to resist column bulging. Figure 9.6 indicates that $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$ increases as $K_0$ increases but $n_{\text{CREEP}(SSC)}$ is relatively unchanged. Accordingly, percentage differences between $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$ and $n_{\text{TOTAL}(SSC)}$ increase as $K_0$ increases. The average vertical effective stresses in the soil and column after 100 years for the different $K_0$ values are compared in
Figure 9.7. As expected, the vertical stresses (with and without creep) are similar for all three $K_0$ values, with a maximum difference of ~5% for the ‘without creep’ case.

Figure 9.6 Effect of $K_0$ on $n_{\text{PRIMARY}}(\text{SSC}, \mu^* = 0)$, $n_{\text{TOTAL}}(\text{SSC})$, and $n_{\text{CREEP}}(\text{SSC})$

Figure 9.7 Effect of $K_0$ on average $\sigma'_{yy}$ after 100 years; (a) soil, (b) stone column
The influence of creep on stone columns: applicability to other clays

The distributions of radial, vertical, and hoop stress in the soil with depth after 100 years for \( K_0 = 1.0 \) are presented in Figure 9.8 for \( A/A_c = 3 \) with and without creep. The magnitude of the vertical stress reduction due to creep is similar to the base case (comparing Figures 9.8b and 9.5b). However, the radial stress reductions in the soil due to creep are dependent on the initial \( K_0 \) value of the in-situ soil:

- For the lower \( K_0 \) value considered (\( K_0 = 0.54 \)), the radial stresses in the soil for the ‘without creep’ and ‘with creep’ cases are similar in magnitude, see Figure 9.5a (although the shear planes appear to progress deeper for the ‘with creep’ case because the additional vertical stress carried by the stone column due to creep induces additional yielding). Accordingly, radial stress reductions in the soil do not seem to contribute significantly to shear plane formation in the stone columns.
- For the higher \( K_0 \) value considered (\( K_0 = 1.0 \)), the radial stresses in the soil for the case with creep are lower than those for the case without creep (Figure 9.8a); the granular columns appear to trigger viscoplastic creep strains that reduce the larger horizontal stresses that arise due to the \( K_0 \) increase.

The latter is comparable with the conclusions in Chapter 7 for the multi-layer Bothkennar profile (which has similarly high initial \( K_0 \) values). Accordingly, the findings in Chapter 7 regarding the radial stress reductions in the soil due to creep are parameter dependent.

The hoop stress differences between the ‘without creep’ and ‘with creep’ cases are also larger for the \( K_0 = 1.0 \) scenario (Figure 9.8c) than for the \( K_0 = 0.54 \) scenario (Figure 9.5c); as with the radial stresses, column presence triggers viscoplastic strains which result in stress reductions.
The influence of creep on stone columns: applicability to other clays

Figure 9.8 Profiles of stress in the soil for $A/A_c = 3$ after 100 years ($K_0 = 1.0$); (a) $\sigma'_{xx}$ (b) $\sigma'_{yy}$ (c) $\sigma'_{zz}$

9.5.2 Effect of $\mu^*$

As established in Section 6.4.2.5, higher $\mu^*$ values (lower creep ratios) result in lower $n_{TOTAL(SSC)}$ values because the weighted effect of creep has more influence. In this parametric study, $\mu^*$ has been both halved and doubled from the base case value, resulting in creep ratios of 42.8 and 10.7 respectively, spanning the range in Figure 5.1. The findings in Figure 9.9 are consistent with those in Section 6.4.2.5; $n_{TOTAL(SSC)}$ reduces as $\mu^*$ increases (the $n_{PRIMARY(SSC, \mu^* \approx 0)}$ values are unaffected). It is interesting to note that at close spacings, the $n_{CREEP(SSC)}$ values increase marginally for lower $\mu^*$ values because full yielding of the granular material does not take place. The average vertical effective stresses in the soil and column after 100 years for the different $\mu^*$ values are compared to the ‘without creep’ case in Figure 9.10. For higher $\mu^*$ values, additional stress is unloaded from the soil (Figure 9.10a) and transferred to the column (Figure 9.10b). Consequently, the SCFs increase as $\mu^*$ increases (Figure 9.11).
The influence of creep on stone columns: applicability to other clays

**Figure 9.9** Effect of $\mu^*$ on $n_{TOTAL(SSC)}$ and $n_{CREEP(SSC)}$

**Figure 9.10** Effect of $\mu^*$ on average $\sigma'_{yy}$ after 100 years; (a) soil, (b) stone column
The influence of creep on stone columns: applicability to other clays

Figure 9.11 Effect of $\mu^*$ on the SCFs after 100 years; (a) soil, (b) stone column

The corresponding profiles of vertical, radial, and hoop stress in the soil with depth for $A/A_c = 3$ are compared to the ‘without creep’ case in Figure 9.12. The vertical stress reduction (Figure 9.12b) is most significant at the base. The additional stress transfer for higher $\mu^*$ values induces additional yielding, which in turn leads to the formation of more shear planes, i.e. the radial stress oscillations (Figure 9.12a) are more pronounced at greater depths for higher $\mu^*$ values. The hoop stress reductions (Figure 9.12c) are also greater for higher $\mu^*$ values; more plastic deformation results in more energy dissipation and hence more stress reduction.

Figure 9.12 Profiles of stress in the soil for $A/A_c = 3$ after 100 years (effect of $\mu^*$); (a) $\sigma'_{xx}$ (b) $\sigma'_{yy}$ (c) $\sigma'_{zz}$
9.5.3 Effect of $\kappa^*$

The effect of both halving and doubling $\kappa^*$ (to 0.012 and 0.046) on $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$, $n_{\text{TOTAL}(SSC)}$, and $n_{\text{CREEP}(SSC)}$ is shown in Figure 9.13. Again, these $\kappa^*$ values are towards the lower and higher end respectively of those quoted in Table 9.1. Lower $\kappa^*$ values result in higher $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$ and $n_{\text{TOTAL}(SSC)}$ values, although the latter do not increase to the same extent as the former because the $n_{\text{CREEP}(SSC)}$ values are relatively unaffected by $\kappa^*$. The higher $n$ values at lower $\kappa^*$ values can be explained as follows: for lower $\kappa^*$ values, the settlements of untreated and treated ground reduce. However, the settlement of untreated ground will only reduce marginally (lightly overconsolidated soil), whereas the settlement of treated ground will experience more of a reduction as the columns reduce the stress carried by the soil (resulting in an overconsolidation effect) so that $\kappa^*$ has more of an influence. Accordingly, the denominator ($n = \delta_{\text{untreated}}/\delta_{\text{treated}}$) reduces more than the numerator and so $n$ increases.

![Figure 9.13 Effect of $\kappa^*$ on $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$, $n_{\text{TOTAL}(SSC)}$, and $n_{\text{CREEP}(SSC)}$](image)

The average vertical effective stresses in the soil and column for the three different $\kappa^*$ values without and with creep are plotted in Figure 9.14. The amount of stress transferred from the soil to the column due to creep is relatively unaffected by $\kappa^*$; this indicates that the the load transfer process from soil to column due to creep is not influenced by the unload-reload index.
The influence of creep on stone columns: applicability to other clays

Figure 9.14 Effect of $\kappa^*$ on average $\sigma'_{yy}$ after 100 years; (a) soil, (b) stone column

9.5.4 Effect of $\lambda^*$

The effect of $\lambda^*$ on $n_{\text{PRIMARY(SSC}}, \mu^* = 0)$, $n_{\text{TOTAL(SSC)}}$, and $n_{\text{CREEP(SSC)}}$ is shown in Figure 9.15. The influence of using a value of $\lambda^* > 0.162$ has not been studied as the soil profile would be too soft to provide sufficient lateral support for granular columns without some form of geotextile encasement. Lower $\lambda^*$ values correspond to higher oedometric soil moduli; accordingly $E_c/E_s$ reduces, resulting in lower $n$ values. The average vertical effective stresses in the soil and column after 100 years for the different $\lambda^*$ values, compared in Figure 9.16, indicate that the stresses are relatively independent of $\lambda^*$ for the range of values considered in this study (softer creep-prone soils).
The influence of creep on stone columns: applicability to other clays

Figure 9.15 Effect of $\lambda^*$ on $n_{\text{PRIMARY(SSID, } \mu^* = 0)}$, $n_{\text{TOTAL(SSID)}}$, and $n_{\text{CREEP(SSID)}}$

Figure 9.16 Effect of $\lambda^*$ on average $\sigma'_{yy}$ after 100 years; (a) soil, (b) stone column
9.5.5 Effect of $\phi'$

The friction angle of the Bothkennar Carse clay ($\phi' = 34^\circ$) is higher than that of other soft clays reported in the literature (see Table 9.1), attributable to the significant amount of angular silt particles and the relatively high organic content. The effect of $\phi'$ on $n$ (in the range $26^\circ$ to $34^\circ$) is shown in Figure 9.17; both $n_{\text{PRIMARY}}(\text{SSC, } \mu^* = 0)$ and $n_{\text{TOTAL}}(\text{SSC})$ increase marginally as $\phi'$ increases ($n_{\text{CREEP}}(\text{SSC})$ is unaffected). The increases are attributable to an increased $K_0$, which is automatically updated when $\phi'$ is changed, e.g. Eq. 9.1 with $K_0^{nc} = 1 - \sin \phi'$. The average vertical effective stresses in the soil and column after 100 years are unaffected, e.g. Figure 9.18 (as was the case for the different $K_0$ values, see Figure 9.7).

![Figure 9.17 Effect of $\phi'$ on $n_{\text{PRIMARY}}(\text{SSC, } \mu^* = 0)$, $n_{\text{TOTAL}}(\text{SSC})$, and $n_{\text{CREEP}}(\text{SSC})$](image-url)
Figure 9.18 Effect of $\phi'$ on average $\sigma'_{yy}$ after 100 years; (a) soil, (b) stone column

9.5.6 Effect of $p_a$

The effect of load level on $n_{\text{PRIMARY(SSC, $\mu^* = 0$), n_{\text{TOTAL(SSC)}}}$, and $n_{\text{CREEP(SSC)}}$ is presented in Figure 9.19. At lower load levels, $n$ values are larger; stone columns are more effective because there is less yielding. As a result, $n_{\text{CREEP(SSC)}}$ values at $p_a = 50\text{kPa}$ are larger than those at either $p_a = 100\text{kPa}$ or $p_a = 200\text{kPa}$, which are almost coincident. The average vertical effective stresses in both the soil and column increase as $p_a$ increases (e.g. Figure 9.20, after 100 years). The stress transfer process from soil to column due to creep is more significant at $p_a = 50\text{kPa}$ than at $p_a = 100\text{kPa}$ or $p_a = 150\text{kPa}$. At $p_a = 50\text{kPa}$ without creep, the column has not fully yielded and so there is more scope for stress transfer from soil to column due to
creep. However, \( p_a = 50\text{kPa} \) is lower than would be typically used in design, e.g. Mitchell & Huber (1985), Castro & Sagaseta (2009), Ellouze & Bouassida (2009).

**Figure 9.19** Effect of \( p_a \) on \( n_{\text{PRIMARY}}(\text{SSC}, \mu^* = 0) \), \( n_{\text{TOTAL}}(\text{SSC}) \), and \( n_{\text{CREEP}}(\text{SSC}) \)

**Figure 9.20** Effect of \( p_a \) on average \( \sigma'_{xy} \) after 100 years; (a) soil, (b) stone column
Profiles of radial, vertical, and hoop stress in the soil after 100 years for $A/A_c = 3$, presented in Figure 9.21 at $p_a = 50\text{kPa}$, indicate:

- The stresses are lower than those at $p_a = 100\text{kPa}$ (see Figure 9.5).
- The radial stress reduction due to creep is relatively comparable with the $p_a = 100\text{kPa}$ case (comparing Figures 9.21a and 9.5a).
- The vertical stress transfer process is more significant closer to the surface for $p_a = 50\text{kPa}$ than for $p_a = 100\text{kPa}$ (seen by comparing Figures 9.21b and 9.5b).
- The hoop stress reduction due to creep is also more pronounced closer to the surface for $p_a = 50\text{kPa}$ (Figure 9.21c) than for $p_a = 100\text{kPa}$ (Figure 9.5c).

![Profiles of stress in the soil for $A/A_c = 3$ ($p_a = 50\text{kPa}$) after 100 years; (a) $\sigma_{xx}'$ (b) $\sigma_{yy}'$ (c) $\sigma_{zz}'$.](image)

**Figure 9.21** Profiles of stress in the soil for $A/A_c = 3$ ($p_a = 50\text{kPa}$) after 100 years; (a) $\sigma_{xx}'$ (b) $\sigma_{yy}'$ (c) $\sigma_{zz}'$.

The relative impacts of increasing the different parameters on $n_{\text{PRIMARY(SSC)}}$, $\mu^* \approx 0$, $n_{\text{TOTAL(SSC)}}$, $n_{\text{CREEP(SSC)}}$, and the stress transfer process from soil to column due to creep are summarised in Table 9.2 (↑ indicates an increase, ↓ indicates a decrease, and ↔ indicates no effect).
The influence of creep on stone columns: applicability to other clays

<table>
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<tr>
<th></th>
<th>( n_{\text{PRIMARY(SSL, } \mu^* = 0)} )</th>
<th>( n_{\text{TOTAL(SSL)}} )</th>
<th>( n_{\text{CREEP(SSL)}} )</th>
<th>Stress transfer from soil to column due to creep</th>
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<td>( \phi' )</td>
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<td>( p_a )</td>
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Table 9.2 Relative impacts of increasing \( K_0, \mu^*, \kappa^*, \lambda^*, \phi'_i \), and \( p_a \) on \( n_{\text{PRIMARY(SSL, } \mu^* = 0)} \), \( n_{\text{TOTAL(SSL)}} \), \( n_{\text{CREEP(SSL)}} \), and the stress transfer process from soil to column due to creep

9.6 Incorporating creep into the vibro-replacement design process

Granular columns accelerate consolidation, and so creep can contribute to a significant proportion of the post-construction settlement. The FE output in Chapters 6-9 has indicated that \( n \) values in creep-prone soils will be overpredicted unless creep is accounted for in design. The Madhav et al. (2009, 2010) analytical formulation, as of yet the only attempt to incorporate creep into the design of granular columns, has been introduced in Section 2.7.1.3 (the formulation is presented in Appendix C). The mechanism of stress transfer from soil to column due to creep suggested by Madhav et al. (2009, 2010) is in qualitative agreement with the FE output, as is the decay of \( n \) with time after EOP. As the creep coefficient increases, the method predicts that the corresponding \( n \) values will reduce further and the amount of stress transferred from soil to column will increase. This is also consistent with the PLAXIS 2D FE output. However, the method is based on elastic theory, and so \( n \) values will be overpredicted unless adequate thickness of granular mat is provided. Additionally:

(i) The method is based on the assumption that primary consolidation is finished before creep begins (Hypothesis A).

(ii) The method assumes the soil is normally consolidated so that only \( C_c \) influences primary settlement, e.g. Eq. C3 (\( C_s \) is only taken into account when considering the unloading from soil to column due to creep, e.g. Eq. C8).

(iii) The method necessitates an iterative solution technique; a closed-form solution would be more practical from a designer’s point of view.
The Madhav et al. (2009, 2010) formulation also predicts a soil overconsolidation effect due
to creep. However, the FE analyses in this thesis (which account for plasticity) suggest that
the soil overconsolidation effect will not benefit the combined soil-column system because
the surplus load transferred to the stone column, which has already reached its active state /
yield point, induces additional yielding.

The FE output in this chapter has been used to develop an empirical design approach to
incorporate creep into the vibro-replacement design process. The approach, developed for
end-bearing columns, is closed-form, and can be used in conjunction with any pre-existing
primary settlement design method. However, analyses carried out as part of the research have
indicated that the methods derived by Castro & Sagaseta (2009) and Pulko et al. (2011)
consistently predict n values that are in good agreement with FE output (Sexton et al., 2013)
and allow for the consideration of significantly more input data, so these methods are
recommended.

The data represented in Section 9.5 is presented as a ratio of \( \frac{n_{\text{CREEP}} - 1}{n_{\text{PRIMARY}} - 1} \) versus
\( A/A_c \) in Figure 9.22. The ratio \( \frac{n_{\text{CREEP}} - 1}{n_{\text{PRIMARY}} - 1} \) is used instead of \( \frac{n_{\text{CREEP}}}{n_{\text{PRIMARY}}} \)
to ensure that the value of \( n_{\text{CREEP}} \) predicted using Eq. 9.2 will always be greater than 1. The
ratio is lowest at \( A/A_c = 3 \) (i.e larger differences between \( n_{\text{PRIMARY}} \) and \( n_{\text{CREEP}} \)) and increases
as \( A/A_c \) increases. In general, Eq. 9.2 (superimposed on Figure 9.22) provides a good and
slightly conservative match to the FE data in the majority of cases; the deviation at \( p_a =
50\text{kPa} \) occurs due to the absence of yielding. In the interest of simplicity, the formula
developed for \( n_{\text{CREEP}} \) is a function of \( A/A_c \) only. This design equation will only be applicable
to creep-prone soils (\( \lambda^* \geq 0.8 \), see Table 9.1), for which the influence of modular ratio (i.e.
\( \lambda^* \)) is small.

\[
\frac{(n_{\text{CREEP}} - 1)}{(n_{\text{PRIMARY}} - 1)} = 0.225 + 0.01(A/A_c) \tag{9.2}
\]
Having established an expression for \( n_{CREEP} \), \( n_{TOTAL} \) can be calculated as a weighted average of \( n_{PRIMARY} \) as predicted by a pre-existing settlement design method and \( n_{CREEP} \) (from Eq. 9.2), e.g. Eq. 9.3:

\[
    n_{TOTAL} = n_{PRIMARY} \cdot w_1 + n_{CREEP} \cdot w_2
\]

(9.3)

where \( w_1 \) and \( w_2 \) are weighting factors dependent on the percentages of primary and creep settlement in the untreated profile. The percentages can be worked out using the 1D compression formulae outlined in Section A4.6.
9.7 Application to the multi-layer Bothkennar profile

As a check on the validity of Eq. 9.2, it has been used in conjunction with Castro & Sagaseta’s (2009) $n_{\text{PRIMARY}}$ predictions to predict $n_{\text{CREEP}}$ values for the multi-layer Bothkennar profile, analysed in Chapters 7 and 8, see Figure 9.23. The predictions are in good agreement with the FE output (note that these $n_{\text{CREEP}}$ values have not been presented hitherto), despite the fact that the profile for which the empirical equation was developed is different to the multi-layer Bothkennar profile. The slight overprediction at $A/A_c = 3$ occurs because Castro & Sagaseta’s (2009) predictions are larger than the corresponding FE $n_{\text{PRIMARY}}$ values, e.g. Figure 8.7; accordingly, this feeds through to the $n_{\text{CREEP}}$ prediction.

Using Eqs. A4, A5, and A6, it is estimated that primary and creep settlements contribute to 60% and 40% of the total settlement for the multi-layer profile after 100 years respectively (i.e. $w_1 = 0.60$ and $w_2 = 0.40$). Based on these relative proportions, $n_{\text{TOTAL}}$ values for the multi-layer profile can be calculated using Eq. 9.3 in conjunction with $n_{\text{PRIMARY}}$ from Castro & Sagaseta (2009) and $n_{\text{CREEP}}$ from Eq. 9.2. The predictions, shown in Figure 9.24, are in good agreement with the FE output for the different models/scenarios, confirming the ability of the expression to capture the reduced $n$ value due to creep very well.
Figure 9.24 \( n_{TOTAL} \) values for the multi-layer Bothkennar profile

9.8 Comparison of Eq. 9.2 with Moorhead (2013)

To the author’s knowledge, there are no field measurements with which to test the applicability of Eq. 9.2. In Figure 9.25, Eq. 9.2 is compared with Moorhead’s (2013) laboratory measurements (see Section 2.6.3.4) for the raft loading scenario. While the laboratory results exhibit significant scatter, the predictions are in moderate agreement for \( n_{PRIMARY} > \sim 1.5 \), although there are some doubts as to the consistency of the laboratory data. The high values of the ratio \( (n_{CREEP} - 1)/(n_{PRIMARY} - 1) \) at low \( n_{PRIMARY} \) values (normally consolidated condition) arise because the \( n_{PRIMARY} \) values calculated by Moorhead (2013) reduce dramatically with increasing load whereas the \( n_{CREEP} \) values either decrease marginally or remain constant. For the isolated foundation loading scenario modelled by Moorhead (2013), the initial conditions for the sleech layer were different for the untreated and treated cases (as explained in Section 2.6.3.4), leading to sources of error in calculating settlement improvement factors. Accordingly, the data are not included in Figure 9.25.
The influence of creep on stone columns: applicability to other clays

### 9.9 Summary

The influence of a practical range of soil parameters on $n_{\text{primary( SSC, } \mu^* = 0)}$ and $n_{\text{total( SSC)}}$ has been studied in the first half of this chapter to assess the applicability of findings heretofore to worldwide clays. The influence of the different parameters on the average vertical effective stresses in the soil and column for $A/A_c = 3, 6,$ and $10$ has also been studied, as has their effect on profiles of stress in the soil with depth after 100 years at $A/A_c = 3$. The outcomes may be summarised as follows:

- Regardless of the parameters adopted, the presence of creep gives rise to lower $n$ values than if only primary consolidation was considered.
- For the lower $K_0$ value considered in this study, the radial stresses in the soil for the ‘with creep’ and ‘without creep’ cases are similar in magnitude; the additional shear planes for the ‘with creep’ case are therefore caused by the additional vertical stress (additional yielding) transferred to the stone column and not by radial stress reductions in the soil.

For the higher $K_0$ value considered in this study, the radial stresses in the soil for the case with creep are lower than those for the case without creep; comparable to the conclusions in Chapter 7 for the multi-layer Bothkennar profile (similarly high initial $K_0$ values). The columns appear to trigger viscoplastic creep strains that reduce the larger horizontal stresses that arise due to the $K_0$ increase.
- The magnitude of vertical stress transferred from soil to column due to creep is more pronounced in soils with higher $\mu^*$ values; additional yielding is induced in the columns, resulting in more radial stress oscillations (shear planes) and lower $n_{\text{TOTALSSC}}$ values.
- The amount of stress transferred from soil to column due to creep is independent of $\lambda^*$, $\kappa^*$, and $\phi'$. 
- The stress transfer due to creep is more significant at the lower load level considered in this study because full yielding of the granular material is not induced for the ‘without creep’ case and so there is more scope for stress transfer from soil to column due to creep. Additionally, the stress transfer process is more significant closer to the surface.

The FE output has been used to develop a simple empirical approach to incorporate creep into the vibro-replacement design process. The approach is applicable to end-bearing columns in soft creep-prone soils and can be used in conjunction with any pre-existing primary settlement design method. It has been validated using the FE output for the multi-layer Bothkennar profile analysed in Chapters 7 and 8.
10. Development of a novel procedure to evaluate the influence of stone column installation on settlement improvement factors for an infinite grid

10.1 Introduction

In this chapter, a novel two-stage procedure based on Cylindrical Cavity Expansion (CCE) has been used to model the stone column installation process. This two-stage procedure has not been used for the analyses in Chapters 6-9; the influence of creep on the settlement performance of stone column foundations is being investigated numerically for the first time and so additional complexity was avoided. The analyses carried out in this chapter are intended as a development on some of the aforementioned work by Debats et al. (2003), Guetif et al. (2007), and Castro & Karstunen (2010).

This chapter is subdivided as follows:

(i) The two-stage procedure is introduced in Section 10.2.
(ii) The steps involved in using CCE to model stone column installation (i.e. Stage 1) are described in Section 10.3.
(iii) The results for Stage 1 are presented in Section 10.4. The PLAXIS 2D output is compared with Gibson & Anderson’s (1961) analytical solution and also with previous field measurements to provide context.
(iv) Initial stress generation for the second stage (i.e. Stage 2) is described in Section 10.5.
(v) The results for Stage 2 are presented in Section 10.6.

For both stages, the influence of creep has been examined by comparing the SSC model output with and without creep (i.e. $\mu^* \approx 0$). This is based on a similar idea to Approach C (see Section 6.5). The $n_{\text{primary(SSC, } \mu^* = 0)}$ and $n_{\text{total(SSC)}}$ values obtained using this two-stage procedure are compared in Section 10.6 as an additional check on the conclusions made heretofore.
10.2 Two-stage procedure

The numerical modelling work herein is split into two distinct stages.

10.2.1 Stage 1

Firstly, CCE is used to approximate post-installation $K$ values (and hence a post-installation stress-regime in the ground) arising due to the lateral expansion imposed by the vibrating poker as columns are installed in a soft clay ($K_0$ denotes the at-rest earth pressure coefficient prior to installation). The approach used to establish the post-installation $K$ values is described in Section 10.3. As for the earlier work in this thesis, a comparison of the SSC model output with and without creep will give an indication of the possible effect of column construction on $K$ values surrounding columns in creep-prone soils.

The analyses in this chapter have been carried out for the single-layer 5m long unit cell described in Sections 4.3.1 and 5.4 ($R_c = 0.3m$). The clay, with soil stiffness, $E_s$ (see Table 5.3), has been modelled as an undrained material (PLAXIS Undrained A approach), while the stone has been modelled as a highly permeable drained material (see Table 5.4).

For this ‘CCE’ stage, the external far boundary of the ‘unit cell’ is located 30m ($= 100R_c$) away from the axis of symmetry (to ensure that results are unaffected by boundary proximity). For the unit cell radii required for typical $A/A_c$ ratios encountered in practice, the boundary would have been too close (leading to numerical problems). Roller boundaries have been applied to all sides. The FE mesh (Figure 10.1), consisting of approximately 16,000 6-noded triangular elements has been refined in the region surrounding the cavity (largest strains in this region).
Development of a novel procedure to evaluate the influence of stone column installation on settlement improvement factors for an infinite grid

Secondly, the influence of the new $K$ values on $n$ values for an infinite grid has been established by carrying out two sets of axisymmetric ‘wished-in-place’ unit cell analyses under an applied load for (a) a soft clay assuming the clay properties to be unaffected by column installation (Case A, $K/K_0 = 1$) and (b) a soft clay with increased $K$ values calculated using the CCE analyses in Stage 1 (Case B, $K/K_0 > 1$). The procedure employed to obtain the $n$ values for Case B is novel because the FE mesh following the CCE stage is too distorted to use as a realistic starting point to study settlement behaviour.

Castro et al. (2013) (also published in ICIEGE (2013)) also employed a two stage process to incorporate installation effects into an axisymmetric unit cell model, in which the first step (Stage 1) is similar, but the second step (Stage 2) differs. They used advanced numerical techniques (based on a curve-fitting process) to match changes to the vertical, radial, and hoop stresses (all of which contribute to a post-installation $K$ value). The stress fields have been modified prior to initial stress development whereas in the present study, the post-installation $K$ values are incorporated into the initial stress generation phase. Additionally, they incorporate changes to the void ratio and OCR state parameters. They used the MCC, S-CLAY1, and S-CLAY1S models, none of which incorporate creep. As is the case with the work in this chapter, Castro et al. (2013) only consider the installation of a single column (overlapping stress changes caused by the installation of several columns are not considered).
Development of a novel procedure to evaluate the influence of stone column installation on settlement improvement factors for an infinite grid

10.3 Installation (CCE) stage for Case B

Simulating column installation in PLAXIS 2D using a cavity expansion technique involves three phases after initial stress generation (initial stresses have been generated using the \( K_0 \) procedure):

(i) Install ‘dummy material’ over the entire column length to a radial extent, \( a_0 \). Guetif et al. (2007) suggest modelling the cylindrical hole created by the poker using a ‘dummy material’ with a very low Young’s modulus, \( E = 20 \text{kPa} \).

(ii) Apply a prescribed displacement (undrained conditions) from an initial radius, \( a_0 \), to a final radius, \( a_f \) (Figure 10.1). Three different \( a_0 \) values of 0.10m, 0.15m and 0.20m have been used for which \( a_f \) values have been calculated according to Eq. 2.3 as \( a_f = 0.316 \text{m}, 0.335 \text{m} \) and 0.361m respectively.

(iii) This cavity expansion stage is followed by a consolidation phase to allow excess pore pressures to dissipate (to a maximum excess pore pressure of 1kPa) to establish the long-term stress changes in the ground caused by column installation.

This approach is described in detail by Castro & Karstunen (2010). The application of a prescribed displacement in this manner was deemed to be the best option for numerical stability purposes (Kirsch 2006, Castro & Karstunen 2010), as opposed to the application of a volume strain to an expanding soil cluster. Based on the increased horizontal stresses (after the consolidation phase), post-installation \( K \) values can be calculated using Eq. 10.1.

\[
K = \left( \frac{\sigma'_{xx} + \sigma'_{zz}}{2 \sigma'_{yy}} \right)
\] (10.1)

The Updated Mesh option (with subsequent updating of the water pressures) has been used to account for the large displacements that occur in the region closest to the expanding cavity. This results in a smoother variation of \( K/K_0 \) with \( r/R_c \) than if a normal mesh (with no updating of the nodal coordinates was used). Additionally, using an Updated Mesh yields marginally higher \( K/K_0 \) values. It has been verified that the \( K/K_0 \) values are independent of the amount of
lateral expansion and so the predictions hereafter pertain to an initial radius, \( a_0 = 0.15 \text{m} \); it has also been verified that the \( K/K_0 \) values are independent of depth (\( K_0 \) was set equal to \( 1 - \sin \phi'_s = 0.44 \) for the normally consolidated clay considered in this study).

As highlighted in Section 4.2.5, the SSC model requires a time interval to yield realistic predictions. As a result, the cavity expansion phase for the SSC model should have a time interval; a time interval of 0.01 days (~15 minutes, of the same order as the time to install a column in practice) was used. The influence of the duration of the expansion phase on the short-term (after expansion) and long-term predictions (after consolidation) is examined in Section 10.4.3.

10.4 Stage 1 Results

10.4.1 Comparison with Gibson & Anderson (1961)

The variations of radial and hoop (tangential) total stress with \( r/R_c \) (\( r \) is the radial distance from the axis of symmetry) at mid-depth (2.5m) after the cavity expansion phase predicted by SSC model with and without creep are compared to Gibson & Anderson’s (1961) undrained analytical solution in Figures 10.2 and 10.3. The Gibson & Anderson (1961) elastic-plastic method is based on a single soil stiffness whereas the SSC model requires the input of \( \lambda^* \) and \( \kappa^* \) (which can be related to \( E_{50} \) and \( E_{ur} \) using Eqs. 4.8 and 4.21); therefore two sets of Gibson & Anderson (1961) predictions are included in Figures 10.2 and 10.3 (for \( E_{50} \) and \( E_{ur} \)). Examination of these figures indicates:

- The Gibson & Anderson (1961) predictions of radial total stress (\( \sigma_{xx} \)) with \( r/R_c \) using \( E_{ur} \) are in good agreement with the PLAXIS output (Figure 10.2).
- The SSC and SSC (\( \mu^* \approx 0 \)) predictions are almost identical, as would be expected as viscoplastic strains should be negligible at 0.01 days.
- Using \( E_{50} \), Gibson & Anderson (1961) predict that the limit pressure should reach a value of 61.6kPa, whereas using \( E_{ur} \), Gibson & Anderson (1961) predict that the limit pressure should reach a value of 73.6kPa. For the SSC and SSC (\( \mu^* \approx 0 \)) analyses, the internal
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cavity pressures reach 71.9kPa and 71.7kPa respectively (Figure 10.2). These are in good agreement with Gibson & Anderson’s solution using $E_{ur}$.

- The variation of hoop (tangential) total stress ($\sigma_{zz}$) with $r/R_c$ predicted by the SSC and SSC ($\mu^* \approx 0$) analyses is smoother than that predicted using the simplified elastic-plastic analytical formulation (Figure 10.3).

- Examination of the distributions of $\sigma_{zz}$ with $r/R_c$ (Figure 10.3) indicates that the radii of influence predicted by Gibson & Anderson (1961) are $\sim 5R_c$ and $\sim 10R_c$ using $E_{50}$ and $E_{ur}$ respectively; the latter is in good agreement with the PLAXIS influence radii of $\sim 10R_c$ (the influence radii can be seen more clearly in Figure 10.8), despite the different shape (as would be obtained using the linear elastic perfectly plastic MC model).

![Figure 10.2 Radial total stress ($\sigma_{xx}$) versus $r/R_c$ after cavity expansion](image1)

![Figure 10.3 Hoop (tangential) total stress ($\sigma_{zz}$) versus $r/R_c$ after cavity expansion](image2)
10.4.2 $K/K_0$ versus $r/R_c$ (influence of creep)

The variations of $K/K_0$ with $r/R_c$ calculated using the SSC and SSC ($\mu^* \approx 0$) analyses after consolidation are compared in Figure 10.4. In both cases, the $K/K_0$ values plateau at $\sim 15R_c$. The predictions indicate that creep (viscoplastic strain) marginally lowers the values of $K/K_0$ (the short-term predictions, see Figures 10.2 and 10.3, were identical).

Also included in Figure 10.4 are field values of $K/K_0$ reported by Kirsch (2006) mostly beyond $r = 5R_c$. This study has been described in Section 2.5.2; the ‘Field 2’ data are more comparable to the situation modelled. The SSC and SSC ($\mu^* \approx 0$) predictions are in moderate agreement with the field data. The low $K/K_0$ values close to the column in the field (e.g. where the field values are below those predicted using PLAXIS) were attributed to remoulding and dynamic effects. The purpose of the field comparisons is simply to indicate that the PLAXIS 2D $K/K_0$ values (after consolidation) obtained in this study are in reasonable agreement with field predictions.

![Figure 10.4 $K/K_0$ versus $r/R_c$ after consolidation (influence of creep and comparison of predictions with field data)](image_url)
10.4.3 SSC model (influence of expansion phase duration)

As noted in Section 10.3, the predictions using the SSC model were obtained using a time interval of 0.01 days for the cavity expansion phase. The influence of the expansion phase duration on the short-term (undrained) and long-term (after consolidation) predictions for the case with a standard creep coefficient is examined in this section. This is a worthwhile exercise because the influence of creep (using the SSC model or any other soil model) on post-installation soil properties, with the exception of Castro et al. (2012), has received very little investigation to date.

The variations of radial and hoop (tangential) total stress with \( r/R_c \) after the cavity expansion phase for different time intervals are shown in Figures 10.5 and 10.6. The 0.01 day, 0.1 day, 1 day, and 10 day predictions are very close to one another. The predictions using a zero-time interval (unrealistic for the SSC model because the yield cap will not be able to move, e.g. Section 4.2.5) are much higher. In general, the cavity pressure decreases marginally as the time interval increases (attributable to the influence of viscoplastic strain), although the effect is almost negligible apart from the zero time interval case for which the cavity pressure appears overestimated; intuitively, it would be expected that the SSC model standard creep coefficient cavity pressure (using a zero time interval, Figure 10.5) would be comparable with the SSC (\( \mu^* = 0 \)) cavity pressure (0.01 day time interval, Figure 10.2).

![Figure 10.5 SSC model - Radial total stress (\( \sigma_{xx} \)) versus \( r/R_c \) after cavity expansion (influence of expansion phase duration)](image)
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Figure 10.6 SSC model - Hoop (tangential) total stress ($\sigma_{zz}$) versus $r/R_c$ after cavity expansion (influence of expansion phase duration)

Figure 10.7 shows that the influence of the duration of the expansion phase on the long-term $K/K_0$ predictions is minor. The predictions for 0.01 days, 0.1 days, 1 day, and 10 days collapse on one another, although the $K/K_0$ values using a zero time interval are out of kilter with the other predictions; owing to the initially overestimated cavity pressure.

Figure 10.7 SSC model - $K/K_0$ versus $r/R_c$ after consolidation (influence of expansion phase duration)
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### 10.4.4 \( p'/p'_0 \) versus \( r/R_c \) (influence of soil model)

The variations of \( p'/p'_0 \) (where \( p' \) and \( p'_0 \) are the final and initial mean effective stresses respectively) with \( r/R_c \) after consolidation predicted using the SSC and SSC (\( \mu^* \approx 0 \)) analyses are plotted in Figure 10.8. The radius of influence is \( \sim 10R_c \). The variation of \( p'/p'_0 \) with radius could be used to evaluate an increased Young’s Modulus in the region surrounding the cavity based on Eq. 5.1 (this idea has been adopted by Guetif et al. (2007)). However, for the current study, only the effect of \( K \) in the region surrounding the cavity has been investigated (the influence of \( E \) would be less pronounced, e.g. Section 6.4.2.4).

![Graph showing \( p'/p'_0 \) versus \( r/R_c \) after consolidation (influence of soil model)](image)

**Figure 10.8** \( p'/p'_0 \) versus \( r/R_c \) after consolidation (influence of soil model)

### 10.5 Initial stress generation for Stage 2

#### 10.5.1 Establishing an input \( K \) profile

The influence of the modified initial \( K \) profile on \( n \) values for an infinite column grid has been investigated for both SSC and SSC (\( \mu^* \approx 0 \)). The \( K/K_0 \) decay with \( r/R_c \) is approximated by a stepwise reduction as shown in Figures 10.9 and 10.10; average \( K/K_0 \) values have been interpreted at intervals of 0.1m representing concentric rings around the axis of symmetry. The \( K/K_0 \) values are used as input parameters for the unit cell (Case B) models; a different
initial $K/K_0$ profile has been used for the SSC and SSC ($\mu^* \approx 0$) analyses. Also demarcated on Figures 10.9 and 10.10 is the radial extent ($r/R_c = 3.16$) relevant to the largest unit cell modelled in this chapter ($A/A_c = 10$).

Figure 10.9 Adopted $K/K_0$ Profiles - SSC ($\mu^* \approx 0$)

Figure 10.10 Adopted $K/K_0$ Profiles - SSC

In the curve-fitting process used by Castro et al. (2013), they have ensured that equilibrium is fulfilled at all points in the stress field (see Eq. 10.2). Shear stresses have been neglected to minimise complexity.

$$\frac{\partial \sigma'_{xx}}{\partial r} + \frac{\sigma'_{xx} - \sigma'_{zz}}{r} = 0$$  \hspace{1cm} (10.2)
The initial stress generation procedure employed in the current chapter, based on stepwise changes to \( K \) values in concentric rings around the axis of symmetry, could lead to the development of shear stresses. To overcome this (and to ensure the equilibrium equations are upheld at all points in the FE model), a plastic nil-step has been used to restore any out-of-balance forces. Smaller concentric rings (< 0.1m) would result in less opportunity to violate equilibrium criteria because the variation of \( K \) with radius will be smoother but the 0.1m spacing was sufficient for the current study.

**10.5.2 Unit Cell Settlement Behaviour**

The axisymmetric unit cell model \( (R_c = 0.3m, L_c = 5m) \) has been described in Section 4.3.1. The analyses are performed for \( 3 < A/A_c < 10 \). The stages in each analysis are as outlined in Section 6.2. However, initial stress generation is different for Cases A and B; initial stresses in the clay for Case A are generated using a uniform \( K = 1 - \sin \phi' = 0.44 \) (mean effective stresses \( (p') \) are plotted in Figure 10.11a and are the same for the SSC and SSC(\( \mu* \approx 0 \)) analyses) while for Case B, the profile is generated from the CCE analyses, see Section 10.5.1 (Figures 10.11b and 10.11c for the SSC(\( \mu* \approx 0 \)) and SSC analyses respectively).

![Figure 10.11](image-url)

**Figure 10.11** SSC model, \( A/A_c = 10 \) (a) \( p' \) with depth for Case A, (b) \( p' \) with depth for Case B - SSC (\( \mu* \approx 0 \)), (c) \( p' \) with depth for Case B - SSC
10.6 Settlement Improvement Factors for Cases A and B

The $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$ values for Case A have been compared with Castro & Sagaseta (2009) and Pulko et al. (2011) in Figure 6.23c. Settlement improvement factors generated for Cases A and B are compared in Figure 10.12. The main points to note are:

- For both the SSC and SSC ($\mu^* \approx 0$) analyses, accounting for column installation (larger $K$ values) leads to a larger $n$ at all values of $A/A_c$, i.e. larger $K$ values indicate increased horizontal stresses in the soil; these provide more resistance to lateral bulging of the granular material.
- The $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$ values (Figure 10.12a) increase more than the $n_{\text{TOTAL}(SSC)}$ values (Figure 10.12b), as can be seen by comparing the hatched ‘areas’ between the Case A and B curves (the hatching is used for optical purposes to help highlight differences between the Case A and Case B $n$ values for two scenarios). This is to be expected because lower post-installation $K$ values are used for the case with creep (see Figures 10.4, 10.9, and 10.10), i.e. a different starting point. Additionally, as shown in Section 9.5.1, $K_0$ has a larger influence on $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$ than $n_{\text{TOTAL}(SSC)}$.
- Accordingly, the percentage differences between the $n_{\text{PRIMARY}(SSC, \mu^* \approx 0)}$ and $n_{\text{TOTAL}(SSC)}$ values for Case B are larger than those for Case A but the conclusions are consistent with the conclusions heretofore; $n$ values are lower when creep is considered.

![Figure 10.12](image-url) $n$ versus $A/A_c$ for Cases A and B; (a) SSC model ($\mu^* \approx 0$), (b) SSC model
10.7 Summary

In the first half of this chapter, CCE has been used to calculate cavity pressures (short-term) and post-installation $K$ values (long-term) following stone column installation in a soft clay. The influence of creep has been investigated by comparing the SSC model output with and without creep ($\mu^* \approx 0$). The results indicate that:

(i) The internal cavity pressure after the initial cavity expansion phase is unaffected by creep (based on a direct comparison of the SSC model output with and without creep), as would be expected for the short time interval used (a negligible creep effect would be expected).

(ii) However, creep (viscoplastic strains) lower the values of $K/K_0$ and $p'/p'_0$ after the consolidation phase (although the values of both $K/K_0$ and $p'/p'_0$ are nevertheless greater than 1.0).

In the second half of the chapter, the $K/K_0$ profiles generated using the CCE procedure for both cases have been imported successfully as input profiles for axisymmetric unit cell models. The modelling captures a variation of $K/K_0$ with radius not usually considered by other methods (e.g. global $K$ increases). Settlement performance has been analysed for two Cases (A and B) with $K/K_0 = 1$ and $K/K_0$ as predicted by CCE respectively.

The approach used in Case B is novel; the prescribed displacements in the CCE stage would cause both excessive heave and numerical problems (due to boundary proximity) for the size of unit cell required to give typical $A/A_c$ ratios ($<< 100R_c$). Additionally, the FE mesh post-installation becomes too distorted to use as a realistic starting point to study settlement behaviour, e.g. Castro et al. (2013).

The results show that the approach can be used as an improvement upon the conventional ‘wished-in-place’ column installation technique, with larger $n$ values predicted when installation (increased $K$) is taken into account. Regardless, the results obtained using this procedure are consistent with those in Chapters 6-9; the $n_{TOTAL(SSC)}$ values are lower than the $n_{PRIMARY(SSC, \mu^* \approx 0)}$ values.
11. Conclusions

11.1 Introduction

The vibro-replacement stone column technique is becoming increasingly used in geotechnical practice to improve the bearing capacity and settlement characteristics of soft cohesive (and/or organic) soil deposits, in which creep can make up a significant proportion of the total settlement. The majority of research to date has focused on the effectiveness of stone columns at reducing bearing capacity and/or primary consolidation settlement, with very little consideration given to the potential of stone columns to reduce long-term creep settlements.

In this thesis, a series of PLAXIS 2D axisymmetric unit cell analyses have been carried out with a view to assessing the effectiveness of using vibro-replacement stone columns in creep-prone clays. The majority of analyses have been carried out using the commercially available isotropic elasto-viscoplastic SSC model. Selected analyses have been repeated using the Creep-SCLAY1S model, which incorporates anisotropy, bonding, and destructuration.

The Bothkennar soft clay test site in Scotland, which consists of an overconsolidated crust overlying two layers of lightly overconsolidated Carse clay, has been used as a suitable soil profile for the FE analyses; its properties may be considered as a ‘mid-range’ of a host of soft clays encountered worldwide. Simplified single-layer profiles based on the Bothkennar Carse clay were used for preliminary numerical modelling purposes prior to analysing the full profile in order to remove the complexities associated with interpreting the results for a multi-layered soil deposit.

11.2 Research findings

11.2.1 Approaches to analyse the effectiveness of stone columns in creep-prone soils

The SSC model, based on the isotache concept (Hypothesis B), predicts creep occurring concurrently with primary consolidation. Accordingly, Casagrande's (1936) method cannot be used to calculate separate primary and creep settlement components (and corresponding
Conclusions

‘primary’ and ‘creep’ settlement improvement factors) under a given load increment. As a result of this, three different approaches (A, B, and C) were used to analyse the effectiveness of stone columns in creep-prone soils. The benefits and limitations associated with each approach are as follows:

- **Approach A**, based solely on SSC model output, will only hold in situations for which creep settlements occurring before 1 day are insignificant.

- **Approach B** is based on a comparison of the settlements and corresponding settlement improvement factors derived with the clay modelled using the commercially available SSC, SS, and HS models. ‘Total’ settlement improvement factors derived using the SSC model have been compared to ‘primary’ settlement improvement factors derived using the inviscid models. This approach is useful to establish the dependence of settlement improvement factor on soil model but the conclusions tend to be parameter dependent.

- **Approach C** is based on carrying out two separate sets of analyses using the SSC model on its own; one set using the standard creep coefficient for the soil (from which ‘total’ settlement improvement factors can be derived) and the other set using a very low creep coefficient to eliminate creep behaviour (from which equivalent ‘primary’ settlement improvement factors can be derived). Direct comparison between the two sets is used to isolate the impact of creep. This approach is based on a fair like-for-like comparison and is technically superior to approaches A and B.

11.2.2 Settlement improvement factors

Examination of the evolution of settlement improvement factor with time, using PLAXIS 2D, has indicated that the inclusion of creep leads to a lower improvement factor than would be obtained for primary consolidation settlement alone. This effect is more pronounced in situations where creep settlements account for a greater proportion of the total settlement. Furthermore, percentage differences between ‘primary’ and ‘total’ settlement improvement factors tend to be larger when improvement factors are larger to begin with.

Additionally, the PLAXIS 2D output has been used to derive separate ‘primary’ and ‘creep’ settlement improvement factors. The ‘primary’ settlement improvement factors, which were found to be in relatively good agreement with a selection of pre-existing analytical
formulations (which pertain to primary settlement only), tend to be much larger than the ‘creep’ settlement improvement factors. Nevertheless, the latter factors are greater than unity, suggesting that columns help reduce creep settlements. The relative proportions of primary and creep settlement will dictate how significantly the lower ‘creep’ improvements factors affect the ‘total’ settlement improvement factors (determined by a weighted average of the primary and creep components).

### 11.2.3 Vertical, radial, and hoop stresses (with and without creep)

Examination of the variations of vertical, radial, and hoop stress (with and without creep) has indicated that as the soil creeps, vertical stress is transferred from soil to column; the amount of vertical stress transferred increases with depth. The additional stress transferred to the column induces additional yielding; this causes significant shear-plane formation, especially for closely-spaced columns. The additional yielding reduces the efficacy of the column and contributes to lower ‘total’ settlement improvement factors. The impact of creep on the stress transfer process for floating columns is similar to that for end-bearing columns; the soil is unloaded and the magnitude of this stress transfer (unloading) increases with depth.

Additionally, the radial and hoop stresses in the soil (both with and without creep) for the treated case are lower than those for the untreated case. These stresses are constant after EOP for the ‘without creep’ case. With creep, the stresses continue to reduce after EOP. The hoop stress reductions are larger than the corresponding radial stress reductions; both are more pronounced at closer spacings. The radial stress reduction leads to additional bulging of the granular column material (giving rise to more settlement) and a lower load-carrying capacity for the ‘with creep’ case. The hoop stress reduction is caused by the additional plastic deformation for the ‘with creep’ case, which, in general, leads to the dissipation of energy. The hoop stress reductions appear to be a secondary effect, dependent on strain, rather than vice versa.

### 11.2.4 Influence of chosen constitutive model (Creep-SCLAY1S model analyses)

The analyses on the multi-layer profile have been repeated using the Creep-SCLAY1S model, which requires implementation into the PLAXIS FE code as a user-defined model.
Conclusions

The model incorporates anisotropy, bonding, and destructuration, all of which can be ‘switched off’ by adjusting the input parameters. Emphasis was placed on assessing whether using the Creep-SCLAY1S model instead of the SSC model had any influence on the conclusions made heretofore; the influence of anisotropy and bonding/destructuration have then been studied by ‘switching on’ the relevant features. The main findings were as follows:

- Settlement improvement factors differ for each model/scenario; the largest values arise for the isotropic Creep-SCLAY1S model analyses (overpredicted horizontal stresses) and the smallest for the case incorporating bonding and destructuration. In the latter case, stone column presence triggers bond degradation.
- Regardless of the model/scenario, ‘total’ settlement improvement factors are lower than their ‘primary’ counterparts.
- The ratios of ‘total’ to ‘primary’ settlement improvement factors are similar for the isotropic case with both models, suggesting the findings are relatively independent of model type. The findings are also independent of whether anisotropy is taken into account or not.
- For the analyses incorporating bonding and destructuration, 'primary' settlement improvement factors are lower because columns destroy some of the in-situ bonding and 'total' settlement improvement factors are lower because column presence triggers creep strains which result in further bond degradation (especially if close to the preconsolidation pressure). Because the ‘primary’ settlement improvement factors for this scenario are lower to begin with, the effect of creep on the weighted average is less visible and so a smaller ratio is observed between ‘total’ and ‘primary’ settlement improvement factors. A lower 'intrinsic' creep coefficient has also been used for these analyses; this also contributes to a smaller 'total' to 'primary' ratio. Destructuration will be a more important factor in highly sensitive clays.
- For the Creep-SCLAY1S model, shear plane formation is more significant for the 'anisotropic' case than for the 'isotropic' case. For the isotropic case, the horizontal stresses in the soil are overpredicted and so additional resistance is provided against lateral column bulging, resulting in the formation of less shear planes in the column.
11.2.5 Applicability of findings to other soft clays

Various soft clays from around the world were reviewed with a view to identifying a range of parameters encountered for a parametric study using the SSC model. A ‘customised’ profile was developed for investigation purposes so that the effect of changing a single parameter could be isolated; this was preferred to analysing each profile individually. The main findings were:

- The magnitude of vertical stress transferred from soil to column due to creep is more pronounced in soils with higher creep coefficients. This leads to additional yielding in the stone columns (additional shear planes) and lower ‘total’ settlement improvement factors.
- In soils with lower $K_0$ values, the reduction in radial stress in the soil due to creep is almost non-existent; this clarifies that the vertical stress transfer from soil to column due to creep is the main reason for additional shear-plane formation and lower ‘total’ settlement improvement factors. In soils with higher $K_0$ values, radial stress reductions in the soil due to creep occur because the columns appear to trigger viscoplastic creep strains that reduce the larger horizontal stresses that arise due to the $K_0$ increase.
- Regardless of the adopted parameters, the columns reduce the hoop stresses in the soil in comparison with the untreated case (plastic deformation causes energy dissipation); additional hoop stress reductions occur for the ‘with creep’ case.
- The vertical stress transfer due to creep is more significant at the lower load level considered in this study because full yielding of the granular material is not induced for the ‘without creep’ case and so there is more opportunity for stress transfer from soil to column due to creep. Additionally, the stress transfer process is more significant closer to the surface.

11.2.6 Novel procedure to model stone column installation

A novel procedure which uses Cylindrical Cavity Expansion (CCE) theory in conjunction with the conventional 'wished-in-place' installation technique to account for column installation was used for a selection of analyses. A two-step process was implemented: (i) CCE theory was used to work out post-installation $K$ values caused by the lateral expansion of the vibrating poker when stone columns are installed in soft clay, and (ii) the new $K$ values
were incorporated in a standard axisymmetric unit cell model to establish their influence on settlement improvement factors for an infinite column grid.

The results for the first stage have been used to examine the influence of creep on cavity pressures (short-term) and post-installation $K$ values (long-term) by comparing the SSC model output with and without creep. The results indicate that the short-term cavity pressures are unaffected by creep but that the long-term $K/K_0$ predictions are lower when creep is considered. The procedure employed to obtain the settlement improvement factors for the second stage is novel and is needed because the FE mesh following the CCE stage is too distorted to use as a realistic starting point to study settlement behaviour.

The results have shown that the approach can be used as an improvement upon the conventional ‘wished-in-place’ column installation technique, with larger settlement improvement factors predicted when installation (increased $K$) is taken into account. The findings are consistent with those theretofore; settlement improvement factors are lower when creep is considered.

### 11.3 Implications for stone column design

The majority of the existing analytical settlement design methods pertain to primary settlement only, and in the absence of further guidance, designers will tend to apply the same settlement improvement factor to creep settlements as they have estimated for primary settlements. However, for practical purposes, the findings in this thesis indicate that the design of stone columns ignoring creep appears unconservative. If considerable creep is present, lower settlement improvement factors should be applied by practitioners or the amount of stone replacement should be increased to maintain the desired settlement improvement factor.

To overcome this, an approach which can be used to incorporate creep into the design of granular columns has been developed based on a parametric study carried out using PLAXIS 2D with the SSC model. The parametric study investigated the effect of a range of different soil parameters on ‘primary’, ‘total’, and ‘creep’ settlement improvement factors; the influence of load level was also considered.
In the interest of simplicity, the expression developed to predict ‘creep’ settlement improvement factors is a function of reciprocal area-replacement ratio \( \frac{A}{A_c} \) only and provides a reasonably conservative estimate for ‘creep’ settlement improvement factors for the majority of the FE output. The empirical design approach, developed for end-bearing columns in creep-prone soils, is closed-form, and can be used in conjunction with any primary settlement design method, although it is suggested to use one that captures all key features of primary settlement behaviour, e.g. Castro & Sagaseta (2009) or Pulko et al. (2011).

11.4 Recommendations for future research

In this thesis, the influence of creep on the settlement performance of stone column foundations has been investigated numerically for the first time and so the emphasis of the work initially was on obtaining practical estimates of the likely behaviour of stone columns in creep-prone soils using the commercially available SSC model; the influence of using a more advanced constitutive model was investigated thereafter. Recommendations for future research in this area are as follows:

- It would be useful to investigate the influence of creep on the performance of small column groups using a 3D FE program, as columns are often deployed in small groups to support foundations. This would involve a convergence of the work of this thesis with that of Killeen (2012).
- Laboratory-scale studies, supplementary to those carried out by Moorhead (2013), would also be beneficial with a view to investigating the effects of creep on the performance of stone columns in different soil types and at different column spacings. These tests could be carried out using simpler apparati, e.g. enlarged oedometers or Rowe cells.
- It would also be beneficial to monitor the long-term performance of a stone-column-supported structure in the field with a view to identifying if the manner in which settlement improvement factors evolve with time is in agreement with the FE output in this thesis.
- While settlement behaviour was the primary focus of the work in this thesis, the influence of creep on the bearing capacity of the composite soil-column system should also be investigated. It is well established that the undrained shear strength of an untreated soil
increases as a result of the void ratio reduction that occurs due to creep. However, the bearing capacity increase of treated ground due to creep has not been investigated; accounting for this may allow for a more economical design.

- While a simple procedure which aims to account for installation effects has been suggested, the procedure was not used for a model that incorporates both creep and bonding/destructuration. Natural clays possess an inherent degree of bonding; some of which may be destroyed by stone column installation (e.g. Castro & Karstunen, 2010), thus negating some of the beneficial installation effects. Additionally, the break-down of bonding in the field may trigger the onset of premature creep behaviour (rendering the soil near normally consolidated), which will be most significant at or close to the preconsolidation pressure. This could be a significant problem in creep-prone soils and its effect could be investigated.

- In this thesis, the effect of creep on column installation has been examined using CCE in conjunction with the FEM. However, meshfree methods such as the Material Point Method (MPM, e.g. Jassim et al. (2013)) have potential for analyses involving large deformations; the MPM models large deformations as particles moving through a fixed mesh.
12. References


References


References


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Appendix A. Preliminary FE Checks

A1 Introduction

This Appendix contains the following material:

- Mesh sensitivity studies that have been carried out to ensure accurate FE output.
- Sensitivity studies to establish how different PLAXIS Iterative Control Parameters influence the FE results.
- Investigations to establish the most appropriate way to model stone column behaviour using the FEM. This includes parametric studies to ascertain how interfaces elements at the boundary between the soil and column, the choice of stone column soil model (LE, MC, or HS), and the influence of nil-step duration (used in conjunction with the ‘wished-in-place’ column installation technique) affect the FE output.
- Other considerations relevant to the FE modelling work, e.g. parametric studies to assess how clay permeability affects the results, the choice of modelling soft soil behaviour using drained or undrained behaviour, and the influence of using an Updated Mesh.
- 1D compression formulae predictions used to verify the PLAXIS 2D settlements without columns for the 5m, 10m, and 15m unit cells.

A2 Mesh Sensitivity and Iterative Control Parameters

The accuracy of the solution obtained using the FEM is dependent on the quality of the mesh used to discretise the domain of interest; the mesh adopted needs to be sufficiently refined to obtain an accurate solution without necessitating excessive computational time. This section describes the mesh sensitivity studies that have been carried out for the axisymmetric unit cell models. The influences of element type and element discretisation have been examined; also discussed are the influences of altering a selection of the default iterative control parameters used by the PLAXIS FE program.
A2.1 Element Type

Mesh generation in PLAXIS 2D is based on a robust triangulation procedure, resulting in the development of 'unstructured' meshes (numerically superior to regular structured meshes) consisting of either 15-noded or 6-noded triangular elements (Brinkgreve et al., 2011), e.g. Figure A1. 15-noded triangular elements provide a fourth order interpolation for displacements whereas 6-noded elements provide a second order interpolation (numerical integration involves twelve and three Gauss/stress points respectively). The displacements are calculated in the nodes and the stresses are calculated in the Gauss points. 15-noded elements result in longer calculation times and more memory consumption. In general, 6-noded elements produce sufficiently accurate results for standard deformation analyses (Brinkgreve et al., 2011).

![Nodes and stress points in soil elements](image)

**Figure A1** Nodes and stress points in soil elements (a) 6-node (b) 15-node - Brinkgreve et al. (2011)

The analyses described in Section 5.3 using PLAXIS 3D Foundation are based on 15-noded wedge elements (Figure A2) having quadratic interpolation functions. These 15-noded wedge elements are generated from 6-noded triangular elements (as used in a PLAXIS 2D Mesh) in the horizontal direction and 8-noded quadrilateral elements in the vertical direction.
Appendix A. Preliminary FE Checks

**A2.2 Iterative Control Parameters**

**A2.2.1 Automatic Time-Stepping Procedure (Critical Time Step)**

When carrying out consolidation analyses, the automatic time-stepping procedure is used to choose appropriate time steps. Smaller time steps are used for calculations involving increased amounts of plasticity. The first time step (overall critical time step) in a consolidation analysis is dependent on the element type, e.g. Eqs. A1a-b (1D consolidation), where $H$ is the element height and $\gamma_w$ is the unit weight of pore fluid (e.g. water).

\[
\Delta t_{\text{critical}} = \frac{H^2 \gamma_w (1-2\nu)(1+\nu)}{80k_y E(1-\nu)} \quad 15 - \text{node triangular} \quad (A1a)
\]

\[
\Delta t_{\text{critical}} = \frac{H^2 \gamma_w (1-2\nu)(1+\nu)}{40k_y E(1-\nu)} \quad 6 - \text{node triangular} \quad (A1b)
\]

It is possible to use time steps smaller than the overall critical time step default set by PLAXIS, although this can result in stress oscillations and rapidly decreasing accuracy. The selected time step should be the maximum value of the critical time steps for all individual elements in the mesh. The mesh sensitivity studies described in this appendix have been carried out based on the default setting so that the only variables affecting the accuracy are the type and size of element chosen.

Figure A2 Comparison of 2D and 3D soil elements - Brinkgreve *et al.* (2007)
As noted in Section 4.3.2, FE consolidation analyses can exhibit oscillating pore pressures which tend to increase when the time steps are reduced, e.g. Vermeer & Verruijt (1981). In contrast to most numerical calculations, for which the accuracy would increase for smaller steps, this may not be the case for consolidation when the soil is loaded at a drained boundary (comparable with the FE studies in this thesis), e.g. Vermeer & Verruijt (1981). This was also considered when deciding on the appropriateness of using either 6-noded or 15-noded elements.

**A2.2.2 Desired minimum and desired maximum**

The iterative parameters (desired minimum and desired maximum) can be altered to encourage the program to use larger or smaller steps. The desired minimum and desired maximum parameters are set by default to 6 and 15 respectively (specifying the desired minimum and maximum number of iterations per step respectively). In situations where the calculation program can solve a load step (converge) in less iterations than the desired minimum, it will use a load step twice the size whereas in situations where the program needs more iterations to converge, it uses load steps that are half the size.

In a consolidation analysis, water flows out of the soil during a load step. Each load step assumes a constant flow of water even though the flow will generally be higher at the beginning of the step than the end. The average flow during a time step is thus underestimated leading to a higher consolidation time. For this reason, it is important to ensure the desired maximum and minimum parameters are set accordingly. The default values have been adopted for the simulations in this research. Lower values should be used for simulating laboratory tests on highly permeable soils where the consolidation problem can be solved in very few time steps. The consolidation analyses described in this thesis take hundreds of time steps so that the overall influence on consolidation time is negligible. The computational saving that could be achieved using larger values for the iterative parameters was neglected for accuracy considerations.
**A2.2.3 Arc-length control**

Arc-length control is used in load-controlled calculations to accurately determine failure loads; it is not applicable for consolidation analyses. It works by determining how much of a specified load can be applied before failure occurs; it can be activated or deactivated manually, although failure loads tend to be over-estimated in situations where arc-length control is switched off. This option is relevant for the load-displacement analyses described in Sexton et al. (2013) used for appraising the reliability of different analytical settlement prediction methods. In some cases, it was necessary to switch off arc-length control due to failure of the load-advancement procedure. For these analyses, it was necessary to manually assess whether or not failure had occurred.

**A2.3 Element Discretisation**

The initial studies have been carried out using axisymmetric analysis techniques based on a 5m long unit cell (see Sections 4.3.1 and 5.4). The influence of mesh coarseness (i.e. number of elements, see Table A1) and element type (i.e. 6-node or 15-node) has been investigated by examining the FE output for coarse, medium, and fine meshes (the number of elements is effectively doubled in each case), e.g. Figures A3 and A4. The level of coarseness corresponding to each selection is calculated based on the outer geometry dimensions and shape of the FE model. The procedure is described by Brinkgreve et al. (2011).

![Figure A3 Coarse, Medium and Fine Meshes for the Untreated Case](image)
Appendix A. Preliminary FE Checks

Figure A4 Coarse, Medium and Fine Meshes (a) \( A/A_c = 10 \) (b) \( A/A_c = 6 \) (c) \( A/A_c = 3 \)

Predictions of maximum vertical displacement \((u_y)\) at the surface of the unit cell (same in soil and column) and average mean effective stresses \((p')\) in the column and soil at the end of the final phase of analysis have been examined to determine the suitability of the adopted mesh. The percentage differences between the maximum displacements and average stresses have been calculated with respect to the fine 15-noded mesh, e.g. Eqs. A2a-b. The same procedure has been adopted by Killeen (2012).

\[
\text{% difference} = \left( \frac{u_y(15-\text{nodeFine}) - u_y}{u_y(15-\text{nodeFine})} \right) \times 100 \quad (A2a)
\]

\[
\text{% difference} = \left( \frac{p'(15-\text{nodeFine}) - p'}{p'(15-\text{nodeFine})} \right) \times 100 \quad (A2b)
\]

As noted by Killeen (2012), stress converges slower than displacement with increasing mesh density (the stress distribution within an element is calculated using lower order equations than the displacement) making it prudent to examine stress convergence in addition to displacement convergence (although settlement behaviour is the primary focus of this thesis). The results for the SSC model simulations with soil stiffness, \( E_s \) (Table 5.3) are presented in Table A1 for the untreated case and for \( A/A_c = 10, 6, \) and \( 3 \). Time-settlement behaviour has also been examined to verify the accuracy of the FE output (Figure A5).
## Appendix A. Preliminary FE Checks

<table>
<thead>
<tr>
<th>Mesh Type</th>
<th>No. of elements</th>
<th>No. of nodes</th>
<th>Maximum $u_y$ (m)</th>
<th>$%$ diff.</th>
<th>Average $p'$ in soil (kPa)</th>
<th>$%$ diff.</th>
<th>Average $p'$ in column (kPa)</th>
<th>$%$ diff.</th>
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$A/A_c = 10$

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<th>No. of nodes</th>
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<th>$%$ diff.</th>
<th>Average $p'$ in soil (kPa)</th>
<th>$%$ diff.</th>
<th>Average $p'$ in column (kPa)</th>
<th>$%$ diff.</th>
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<td>532</td>
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<td>1033</td>
<td>0.2428</td>
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<td>1.89</td>
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<td>1.41</td>
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<td>2013</td>
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<th>Average $p'$ in soil (kPa)</th>
<th>$%$ diff.</th>
<th>Average $p'$ in column (kPa)</th>
<th>$%$ diff.</th>
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$A/A_c = 3$

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<th>Average $p'$ in soil (kPa)</th>
<th>$%$ diff.</th>
<th>Average $p'$ in column (kPa)</th>
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<td>30.51</td>
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<td>123.64</td>
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</tbody>
</table>

**Table A1** Mesh Sensitivity Results
Appendix A. Preliminary FE Checks

**Figure A5** Time-settlement curves (a) Untreated (b) $A/A_c = 10$ (c) $A/A_c = 6$ (d) $A/A_c = 3$
As is evident from Table A1, the percentage differences between the maximum $u_y$ and average $p'$ values are less than 2% for meshes consisting of ~2000 or more nodes (6-node Fine, 15-node Coarse, Medium, and Fine); as expected, finer meshes allow for more flexibility and thus a marginally greater final settlement, e.g. Brinkgreve et al., 2011. This indicates that fine meshes consisting of 6-noded triangular elements produce sufficiently accurate results in all cases for the simplified 5m long soil profile and so the analyses described in this thesis have been carried out using this meshing procedure. Similar mesh sensitivity studies have been carried out for the multi-layer profile; a fine mesh consisting of 6-noded elements was again deemed sufficient. Chen et al. (2014) also noted that the effect of using 15-noded elements rather than 6-noded elements was minimal. The adopted meshes have also been inspected to ensure there are no discontinuities at inter-element boundaries.

Meshes consisting of 15-noded elements require excessive calculation time because the unit cells are stressed to a significant depth. When the column material yields, a significant amount of plasticity is developed, especially in softer soils. Increasing plasticity results in increasing calculation time and so analyses carried out with 15-noded elements are cumbersome.

The maximum number of allowable steps in any given calculation phase is 10,000. For softer soils, it is necessary to split the consolidation phase into a number of phases to overcome this limit; it has been verified that this does not affect time-settlement behaviour. An alternative would be to use an increased tolerated error of 5% (the default setting in PLAXIS is 1%). The tolerated error parameter is used to ensure local and global equilibrium errors remain within acceptable limits. Inaccuracies may occur if the adopted value is too large and so it was deemed more appropriate to use the default setting (a small number of simulations carried out using an increased tolerated error indicate marginally higher/overpredicted settlements).

Additionally, the comparison of PLAXIS 2D (6-noded elements) with PLAXIS 3D Foundation in Section 5.3.2 further validates the appropriateness of 6-noded elements. For the multi-layer profile, the PLAXIS 2D analyses (using 6-noded elements) provide a smooth distribution of stress and strain with depth. The predictions are consistent with Killeen (2012), see Section 5.3.2.
A3 Modelling Stone Column Behaviour using the Finite Element Method (FEM)

This section describes three different studies that have been carried out prior to modelling stone column behaviour with the FEM, the first to assess the influence of using interface elements, the second to establish the effect of using either the LE, MC, or HS models for the granular material, and the third to determine if the influence of nil-step duration affects the final settlement.

A3.1 Interface Elements

Interface elements (assigned a 'virtual thickness') can be used to model soil-structure interaction (i.e. relative movement between soil and structures in contact with it) in PLAXIS 2D. The 'virtual thickness', which is used to assign material properties to an interface element, should be minimal in order to ensure little or no elastic deformations are generated (Brinkgreve et al., 2011). Interface properties are determined based on the associated soil properties with a strength reduction factor (\(R_{inter}\)) used to model the loss of strength at the interface. Interface elements used in conjunction with 15-noded elements consist of five nodal pairs while those used in conjunction with 6-noded elements are composed of three nodal pairs (see Figure A6). Brinkgreve et al. (2011) have highlighted that although the interface elements appear to have a finite thickness, the coordinates of each nodal pair are assumed to be identical so that the element has a zero thickness in the FE formulation.

![Nodes and stress points in interface elements](image)

**Figure A6** Nodes and stress points in interface elements (a) 6-node (b) 15-node - Brinkgreve et al. (2011)
The majority of numerical studies investigating stone column behaviour have not used interface elements at the boundary between the granular column material and the in-situ soil, e.g. Ambily and Gandhi (2007), Domingues et al. (2007a,b), Gäb et al. (2008). A notable exception is Guteif et al. (2007), who have assumed perfect bonding between the column and soil materials using interface elements with $R_{\text{inter}} = 1$. Omitting interface elements allows for interlock between the clay and the stone as the column bulges under the applied load. Ambily and Gandhi (2007) have noted that deformation at the interface between the column and soil tends to be mainly radial bulging with no significant shear possible. Additionally the interface zone tends to be a mixed zone with varying shear strength properties depending on the method of installation employed during column construction. The resulting ambiguity contributes to the omission of interfaces in the majority of past studies.

In this thesis, the columns have been modelled without the use of interfaces. However, one series of analyses have been carried out for the 5m long single-layer profile with the clay modelled using the SSC model (soil stiffness, $E_s$, see Table 5.3) in order to determine how interface elements may influence the settlement behaviour. The interface elements have been modelled as rigid using $R_{\text{inter}} = 1$. The results have indicated that using interface elements yields marginally larger final settlements (and thus slightly lower $n$ values) as they permit slip at the soil-column interface. However, as is evident from the time-settlement curves presented in Figure A7, their influence is negligible.

![Figure A7 Time-settlement behaviour with and without interface elements (Int. = with interface elements)](image-url)
A3.2 Choice of stone column soil model

In this thesis, the granular column material has been modelled using the HS model (see Section 5.5). However, some previous studies (e.g. Ambily & Gandhi, 2007) have used the MC model and so one series of simulations was carried out to investigate the bearing that the choice of material model for the column may have on the FE output. The study was carried out using the 5m long single-layer profile with the clay modelled using the SSC model (soil stiffness, $E_s$, see Table 5.3).

The results presented in Figure A8 indicate that for the case examined, the HS and MC models yield $n$ values which are in very good agreement with one another. However, $n$ values will be significantly overpredicted if using the LE model because column yielding will not be accounted for. Similar studies have been carried out for the simulations with the clay modelled using the advanced S-CLAY1S and Creep-SCLAY1S models. Again the choice of soil model (i.e. HS or MC model) for the column appears to have little influence on $n$.

![Figure A8 Influence of stone column soil model on $n_{TOTAL(SSC)}$](image)

A3.3 Column Installation (The Nil-Step)

The ‘instantaneous’ ‘wished-in place’ column installation technique may generate out-of-equilibrium stresses which need to be restored prior to load application. In this thesis, a plastic nil-step with a small time interval has been applied after column installation for this purpose. The nil-step procedure has been described by Brinkgreve et al. (2011) and involves
a ‘zero time interval’ calculation phase in which no additional load is applied, after which all stresses will obey the failure criterion.

Plastic nil-steps applied in conjunction with the SSC model require a time interval; otherwise the yield cap will not be able to move and out-of-balance plastic strains will not be accurately restored. The influence of the nil-step duration on the final settlements has been investigated; typical results are presented in Figure A9 (5m unit cell, soil stiffness, $2E_s$, see Table 5.3, $A/A_c = 7$). It has been verified that the nil-step duration (i.e. a relatively short time interval so as to ensure little or no creep settlements) does not impact upon the final settlement. Based on these analyses, a nil-step duration of $1 \times 10^{-3}$ days was selected for all analyses with the SSC model (a zero time interval nil-step was used for the analyses carried out using the SS and HS models).

![Figure A9 Influence of nil-step duration on time-settlement behaviour](image)

**Figure A9** Influence of nil-step duration on time-settlement behaviour

### A4 Other Considerations

This section describes a number of other checks that have been carried out prior to the axisymmetric unit cell modelling work described in the main body of the thesis.

#### A4.1 Permeability ($k$)

The influence of the permeability of the clay on $n$ values for the 5m unit cell has been examined in the range $3 < A/A_c < 10$. Time-settlement behaviour for the SSC model with the default soil properties documented in Table 5.3 (soil stiffness, $E_s$) is compared against time-
settlement behaviour for a series of analyses in which $k_x$ and $k_y$ were increased by a factor of 10, see Figure A10. As expected, larger permeabilities accelerate the rate of consolidation. However, the final settlements and thus the corresponding $n$ values are unaffected.

**Figure A10** Influence of permeability on time-settlement behaviour

### A4.2 Change of Permeability Index ($C_k$)

$C_k$ is used to account for the change of permeability during a consolidation analysis. The default setting ($C_k = 1 \times 10^{15}$) has been used for the single-layer profiles, resulting in no change of permeability with void ratio during consolidation. For the multi-layer Bothkennar profile, $C_k$ has been defined equal to $0.5 e_0$ as reported by Leroueil et al. (1992). Its influence on time-settlement behaviour with and without columns for the multi-layer profile is depicted in Figure A11. Using $C_k = 0.5 e_0$ results in a marginally slower rate of excess pore pressure dissipation (permeability reduces with void ratio during consolidation) in comparison with using the default value of $C_k = 1 \times 10^{15}$. The differences in the EOP consolidation times for the treated cases are almost non-existent; a slight difference is notable for the untreated case. However, the final settlements (and the corresponding $n$ values) are unaffected in all cases.
Appendix A. Preliminary FE Checks

A4.3 Modelling Soft Soil Behaviour

The checks in the preceding section relate to soil permeability and its effect on time-settlement behaviour and corresponding $n$ values. This is only relevant when the soil is modelled as an undrained material (more realistic for cohesive soils); the initial undrained loading phase is followed by a consolidation phase to allow for the dissipation of excess pore pressure. Soil behaviour can be modelled in two ways:

- **Drained**: No excess pore pressures are generated during loading. Drained behaviour is suitable for modelling the behaviour of highly permeable materials such as sand or gravel. Additionally, it can be used for very low rates of loading (no build up of excess pore pressure) and for simulating the long-term behaviour of cohesive soils without needing to model the precise loading history of undrained loading followed by consolidation (Brinkgreve et al., 2011).

- **Undrained**: This option is suitable for saturated low permeability soils which develop significant pore pressures under load, e.g. clays, silts, and peat. It can also be used in situations where the loading rate is very high (thus preventing quick excess pore pressure dissipation). Undrained behaviour can be modelled in terms of total or effective stresses. For effective stress approaches, geotechnical FE programs use a very large bulk modulus for the pore water to simulate the initially undrained soil behaviour. As a consequence, the volumetric strains during undrained loading are very small, mimicking the condition of little or no volume change exhibited by the soil under undrained conditions (Mar,

![Figure A11 Influence of $C_k$ on time-settlement behaviour](image-url)
Hence, all the stresses generated as a result of load application will be taken by the pore water. Consolidation is simulated by removing the extra bulk modulus which results in a gradual transfer of stress from the pore water to the soil skeleton.

The shear moduli for drained and undrained loading are identical whereas the Young’s moduli are related as shown in Eq. A3, where $E'$ and $E_u$ are the drained and undrained Young’s moduli respectively and $\nu'$ is the drained Poisson’s ratio.

$$E' = \frac{2}{3} (1 + \nu') E_u$$  \hspace{1cm} (A3)

There are three options for modelling undrained behaviour using PLAXIS:

- **Undrained A**: Undrained effective stress analysis with effective stiffness and effective strength parameters. This method gives a prediction of the excess pore pressures and can be followed by a consolidation analysis. The predicted undrained shear strength needs to be validated against a known shear strength profile.

- **Undrained B**: Undrained effective stress analysis with effective stiffness parameters and undrained strength parameters. In this case, the undrained shear strength is an input parameter. Excess pore pressure predictions are given but the undrained shear strength is not updated during a subsequent consolidation analysis.

- **Undrained C**: Undrained total stress analysis with all parameters undrained. No excess pore pressure predictions are attainable and the method is thus not suitable for subsequent consolidation analyses.

In this thesis, the majority of the simulations have been carried out using the Undrained A option for the soil (the shear strength profile has been validated in Section 5.3.3). Using the Undrained A approach, the initial loading phase is followed by a consolidation phase to allow the excess pore pressures to dissipate after which the final settlement can be evaluated. However, the analyses carried out with the HS model in Sexton et al. (2013) for the purposes of appraising settlement prediction methods applicable to vibro-replacement design have been performed using fully drained conditions to analyse load-settlement behaviour. Similar results are achieved modelling the clay as an undrained material with a follow-up
consolidation period, e.g. Figure A12. The drained approach predicts a marginally higher final settlement than the undrained approach for both the untreated and treated cases. The combined effect is that the resulting \( n \) values are relatively unaffected, e.g. Killeen (2012).

\[ \delta_{\text{Drained}} \approx \delta_{\text{Undrained} + \text{Consolidation}} \]

(a) Untreated (b) \( A/A_c = 6 \)

### A4.4 Updated Mesh Analyses

The analyses described in this thesis for the single-layer profiles are carried out assuming no mesh geometry change as settlement proceeds. However, the analyses carried out for the multi-layer Bothkennar profile have been carried out using the Updated Mesh option to account for large displacements. The Updated Water Pressures option has also been used so that water pressures are continuously recalculated based on the updated nodal coordinates. Brinkgreve et al. (2011) have suggested using the Updated Mesh option in situations where axial strains exceed 30\%. The Updated Mesh calculation procedures are based on an Updated Lagrangian formulation (e.g. McMeeking & Rice 1975, Bathe 1982). As noted by Brinkgreve et al. (2011), the Updated Mesh procedure involves more than just updating nodal coordinates as the calculation proceeds:
Appendix A. Preliminary FE Checks

- Additional terms are added to the stiffness matrix to account for the effects of large deformations on the FE equations.
- Procedures to model stress changes occurring due to finite material rotations are included, i.e., a definition of stress rate to include rotation rate terms is adopted. The co-rotational rate of Kirchhoff stress is adopted in PLAXIS.
- The FE Mesh is updated as the calculation proceeds.

The influence of using an Updated Mesh on \( n \) has been examined for the single-layer 5m unit cell using the SSC, SS, and HS models for \( E_c/E_s = 5, 10, 20 \) and 40. As expected, the final settlements are less when using the Updated Mesh option. The differences rise from about \( \sim 1.5\% \) for \( E_c/E_s = 5 \) to \( \sim 7.0\% \) for \( E_c/E_s = 40 \) (softer soil). However, in all cases, \( n \) is the same regardless of whether the Updated Mesh option is used or whether an undeformed geometry is assumed. The results for a typical case (SSC model, 5m unit cell, soil stiffness, \( E_s \), see Table 5.3) are presented in Figure A13 (the predictions collapse on one another).

![Figure A13](image)

**Figure A13** Influence of the Updated Mesh option on \( n_{TOTAL(SSC)} \) for the single-layer 5m profile

The \( n_{TOTAL(SSC)} \) values for the multi-layer Bothkennar profile are also relatively unaffected by the use of the Updated Mesh option (Figure A14). However, the magnitude of the differences in the final settlements in this case tends to range from 10-20% (owing to the presence of the very soft layer of Carse clay) and as such the Updated Mesh option was deemed more appropriate for the multi-layer profile analyses.
Figure A14 Influence of the Updated Mesh option on $n_{TOTAL(SSC)}$ for the multi-layer Bothkennar profile

A4.5 Validating SSC model input parameters using oedometer simulations

Oedometer simulations using the SSC model were carried out as preliminaries to the numerical modelling work described in the main body of the thesis. The parameters adopted correspond to those for the Bothkennar lower Carse clay listed in Table 5.2. These simulations were used to confirm that the input values of $C_c$ and $C_a$ (i.e. $\lambda^*$ and $\mu^*$) are in agreement with the output values. The influence of using an Updated Mesh has been investigated, as has the effect of the preconsolidation pressure on $\mu^*$.

A4.5.1 Test set-up

The oedometer tests (75mm x 20mm) were modelled in axisymmetry using 1192 15-noded triangular elements. Standard boundary conditions were applied to the mesh (Figure A15) with the base fixed in all directions and the vertical boundaries restrained laterally. The water table is located at the top of the soil layer. The vertical boundaries were modelled as closed consolidation boundaries while the top and bottom layers of the model remained open. This setup was preferred to using the ‘Soil Test’ facility because scale effects and the use of the Updated Mesh option can be examined in full FE calculations. These simulations were
carried out at the beginning of the research and served as worthwhile exercises with regards learning to use PLAXIS.

![Figure A15 Oedometer dimensions and mesh](image)

The oedometer tests have been simulated in undrained conditions. Load steps are applied incrementally (initial load = 2.5kPa) with a follow-up one day consolidation period to allow for excess pore pressure dissipation (the permeabilities in Table 5.2 have been increased by a factor of 10 to ensure all excess pore pressures dissipate within 1 day). A LIR of 1.0 has been adopted.

**A4.5.2 Test results**

The stress-strain response is shown in Figure A16a. In the NC range, the backfigured value of 
\[ \lambda^* = \frac{\Delta e_{yy}}{\ln[(\sigma'_{yy} + \Delta \sigma'_{yy})/\sigma'_{yy}]} \]
was computed as 0.1662. The percentage difference (0.18%) between the input and output values validates the model response in 1D compression. Time-strain curves are plotted in Figure A16b for successive load increments. The slope at the end of these curves was used to back-calculate 
\[ \mu^* = \frac{\Delta e_{yy}}{\ln[t/t_0]} \]. The value of \( \mu^* \) for each increment in the NC range was found to lie consistently between 0.0067 and 0.0070; these values are in agreement with the input value of \( \mu^* = 0.0066 \).
Figure A16 Test results: (a) Stress-strain response, (b) Time-strain response

The stress-strain response predicted using an Updated Mesh is included in Figure A16a for comparison purposes. In an Updated Mesh analysis, the updated geometry is used to calculate the strains, e.g. as the test progresses, the strain is calculated using a reduced sample height so that a stiffening effect is observed (Brinkgreve et al., 2011) and the slope of the line in the NC range is no longer constant. Plotting the results in terms of natural strain as opposed to logarithmic strain restores the linearity to the plot. The time-strain response for an Updated Mesh analysis results in $\mu^*$ values that progressively reduce in each load increment (similar behaviour to the stress-strain response).

A4.5.3 Effect of preconsolidation pressure

The effect of the preconsolidation pressure on $\mu^*$ has been examined by using a POP of 50kPa so that the transition from OC to NC behaviour can be clearly identified. The preconsolidation pressure for the previous simulations was much lower with the majority of load increments in the NC range. In this case, additional load increments of 30kPa, 50kPa, 60kPa and 70kPa have been used so that there is a clearer transition from the OC to the NC range.
Appendix A. Preliminary FE Checks

The stress-strain and time-strain curves are plotted in Figure A17. The transition from OC to NC behaviour occurs at approximately 50kPa (Figure A17a); the $\lambda^*$ value in the NC range is again consistent with the input value. In the OC range, the $\mu^*$ values are almost zero while in the NC range, the values correspond to the input value (Figure A17b).

**Figure A17** Effect of preconsolidation pressure: (a) Stress-strain response, (b) Time-strain response

**A4.5.4 Scale effect**

The influence of scale was also investigated by performing oedometer-type simulations on a larger specimen measuring 7.5m by 2.0m (i.e. scaled up by a factor of 10). The objective was to see how the larger dimensions (closer to those of the unit cell models) influence the values of $\lambda^*$ and $\mu^*$. The consolidation phase following the application of each load increment was run until a maximum excess pore pressure of 1kPa was reached. The conventional 1 day consolidation phase is no longer sufficient to satisfy the longer drainage path length. The results are presented in Figure A18; the calculated values of both $\lambda^*$ and $\mu^*$ are consistently in agreement with their respective inputs.
Figure A18 Scale effect: (a) Stress-strain response, (b) Time-strain response

A4.6 One-dimensional compression behaviour validation

The analyses described in Chapter 6 incorporate three different soil models (SSC, SS, HS), three different unit cell lengths (5m, 10m, 15m), and four different soil stiffnesses (denoted $0.5E_s$, $E_s$, $2E_s$, $4E_s$). In this section, the PLAXIS 2D predictions (for the untreated case) are validated against 1D compression theory using Eqs. A4 (for $\sigma'_0 + \Delta \sigma < \sigma_p$), A5 (for $\sigma'_0 + \Delta \sigma > \sigma_p$), and A6 (creep component), where $\Delta \sigma$ denotes the load and $H$ is the layer thickness.

$$
\delta = H \frac{C_s}{1 + e_0} \log \left( \frac{\sigma'_0 + \Delta \sigma}{\sigma'_0} \right) \quad \text{(A4)}
$$

$$
\delta = H \frac{C_s}{1 + e_0} \log \left( \frac{\sigma'_0}{\sigma'_0} + \frac{\sigma_p}{\sigma'_0} \right) + H \frac{C_c}{1 + e_0} \log \left( \frac{\sigma'_0 + \Delta \sigma}{\sigma_p} \right) \quad \text{(A5)}
$$

$$
\delta = H \frac{C_a}{1 + e_0} \log \left( \frac{t}{t_0} \right) \quad \text{(A6)}
$$

In each case, the profile is divided into 20 sub-layers ($\Delta H = H/20$) with $\sigma'_0$ calculated at mid-depth of each layer. The PLAXIS 2D predictions are compared to the predictions obtained
using Eqs. A4, A5, and A6 in Table A2. The percentage differences are less than 4% in all cases, giving good confidence in the FE predictions for the untreated case. Note that for all three soil models, the minimum allowable preconsolidation stress ($p_p$) is set equal to 1kPa in order to give the yield ellipse a minimum size in the $p'$-$q$ plane (e.g. Section 4.2.4). This ensures that there will not be excess settlement at the upper surface of the single-layer profiles (i.e. since the profile is defined using a uniform OCR value throughout, a zero effective stress at the surface would imply a zero preconsolidation stress). Profiles with a crust (defined using a POP) will not experience this problem.

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<th>Modular Ratio ($E_c/E_s$)</th>
<th>Analytical $\delta_{\text{PRIMARY}}$ (m)</th>
<th>HS model $\delta_{\text{PRIMARY}}$ (m)</th>
<th>% diff.</th>
<th>SS model $\delta_{\text{PRIMARY}}$ (m)</th>
<th>% diff.</th>
<th>Analytical $\delta_{\text{TOTAL}}$ (m)</th>
<th>SSC model $\delta_{\text{TOTAL}}$ (m)</th>
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Table A2 PLAXIS 2D settlement predictions vs. one-dimensional settlement predictions
Appendix B. Additional results for the single-layer profiles

B1 Approach B Results - End of primary (EOP) consolidation

**Figure B1** EOP Consolidation Times for Untreated Profile - $L = 10m$ - SSC model

**Figure B2** EOP Consolidation Times for Untreated Profile - $L = 15m$ - SSC model
Appendix B. Additional results for the single-layer profiles

**B2 Approach B Results - Settlement-log(time) plots**

![Settlement-log(time) plots](image)

**Figure B3** Settlement vs. log(time); SSC model, $L=10$ m (a) $E_s/E_s = 5$ (b) $E_s/E_s = 10$ (c) $E_s/E_s = 20$ (d) $E_s/E_s = 40$
Appendix B. Additional results for the single-layer profiles

Figure B4 Settlement vs. log(time); SSC model, $L=15\text{m}$ (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$
Appendix B. Additional results for the single-layer profiles

**B3 Approach B Results - Evolution of settlement improvement factor time**

Figure B5 Evolution of $n$ with time - $L = 10m$, $E_c/E_s = 5$

Figure B6 Evolution of $n$ with time - $L = 10m$, $E_c/E_s = 10$
Appendix B. Additional results for the single-layer profiles

Figure B7 Evolution of $n$ with time - $L = 10m$, $E_c/E_s = 20$

Figure B8 Evolution of $n$ with time - $L = 10m$, $E_c/E_s = 40$
Appendix B. Additional results for the single-layer profiles

Figure 3B9 Evolution of $n$ with time - $L = 15$m, $E_c/E_s = 5$

Figure B10 Evolution of $n$ with time - $L = 15$m, $E_c/E_s = 10$
Appendix B. Additional results for the single-layer profiles

**Figure B11** Evolution of $n$ with time - $L = 15$m, $E_c/E_s = 20$

**Figure B12** Evolution of $n$ with time - $L = 15$m, $E_c/E_s = 40$
B4 Approach B Results - Comparison of 'primary' and 'creep' settlement improvement factors with analytical solutions

Figure B13 Analytical comparisons - $L = 10\,\text{m}$, (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$
Appendix B. Additional results for the single-layer profiles

Figure B14 Analytical comparisons - $L = 15$ m (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$ (c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$
Appendix B. Additional results for the single-layer profiles

**B5 Approach B Results - Effect of $(\lambda^*-\kappa^*)/\mu^*$**

Figure B15 Influence of $(\lambda^*-\kappa^*)/\mu^*$ - SSC model; $L = 5m$, $E_e/E_s = 10$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

Figure B16 Influence of $(\lambda^*-\kappa^*)/\mu^*$ - SSC model; $L = 5\text{m}$, $E_c/E_s = 40$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

B6 Approach C Results - Dependence of ‘primary’ settlement improvement factors on soil model

Figure B17 ‘Primary’ settlement improvement factors - $L = 10\text{m}$ (a) $E_c/E_s = 5$ (b) $E_c/E_s = 10$
(c) $E_c/E_s = 20$ (d) $E_c/E_s = 40$
Appendix B. Additional results for the single-layer profiles

**Figure B18** ‘Primary’ settlement improvement factors - \( L = 15 \text{m} \) (a) \( E_c/E_s = 5 \) (b) \( E_c/E_s = 10 \) (c) \( E_c/E_s = 20 \) (d) \( E_c/E_s = 40 \)
Appendix B. Additional results for the single-layer profiles

B7 Approach C Results - Evolution of settlement improvement factor with time

**Figure B19** Approach C - $L = 5m$, $E_c/E_s = 5$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

Figure B20 Approach C - $L = 5\text{m}$, $E_c/E_s = 10$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

Figure B21 Approach C - \( L = 5\text{m} \), \( \frac{E_{c}}{E_{s}} = 40 \) (a) \( \frac{A}{A_{c}} = 10 \) (b) \( \frac{A}{A_{c}} = 6 \) (c) \( \frac{A}{A_{c}} = 3 \)
Appendix B. Additional results for the single-layer profiles

Figure B22 Approach C - $L = 10m$, $E_c/E_s = 5$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

Figure B23 Approach C - $L = 10m$, $E_c/E_s = 10$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

Figure B24 Approach C - $L = 10m$, $E_c/E_s = 20$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Figure B25 Approach C - $L = 10\text{m}$, $E_c/E_s = 40$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

Figure B26  Approach C - $L = 15m$, $E_c/E_s = 5$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

Figure B27 Approach C - $L = 15\text{m}$, $E_c/E_s = 10$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

**Figure B28** Approach C - $L = 15\text{m}$, $E_x/E_y = 20$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix B. Additional results for the single-layer profiles

**Figure B29** Approach C - $L = 15m, E_c/E_s = 40$ (a) $A/A_c = 10$ (b) $A/A_c = 6$ (c) $A/A_c = 3$
Appendix C. Design Equations for Shahu et al. (2000) and Madhav et al. (2009, 2010)

C1 Formulation of Shahu et al. (2000)

The soil and column are assumed to undergo equal vertical strain under a uniform load (applied through a granular mat in this case). The mat is assumed rigid and smooth. The unit cell is discretised into a given number of elements. Equilibrium of vertical stresses in each element $i$ gives Eq. C1.

$$p_a = \sigma_c \cdot (A_c / A) + \sigma_s \cdot (1 - A_c / A)$$  \hspace{1cm} (C1)

Elastic theory is used to calculate the settlement of any element of the granular column ($\Delta S_c$), where $\Delta h$ is the thickness of the $i$th element, e.g. Eq. C2.

$$\Delta S_c = \frac{\sigma_c}{E_c} \Delta h$$  \hspace{1cm} (C2)

Conventional $e$ - log $\sigma$ theory is used to calculate the settlement of the $i$th layer of the soil (which is assumed normally consolidated), where $\sigma_0$ is the initial overburden stress at the centre of the $i$th element, e.g. Eq. C3.

$$\Delta S_s = \frac{C_c}{1 + \epsilon_0} \Delta h \log \left( 1 + \frac{\sigma_s}{\sigma_0} \right)$$  \hspace{1cm} (C3)

In each element, the vertical displacement of the column is equal to the vertical displacement of the soil ($\Delta S_c = \Delta S_s$), thus giving Eq. C4 after equating and rearranging Eqs. C2 and C3. The method does not consider radial displacement.

$$\sigma_c = \frac{C_c}{1 + \epsilon_0} E_c \log \left( 1 + \frac{\sigma_s}{\sigma_0} \right)$$  \hspace{1cm} (C4)
An iterative procedure can then be used to solve for $\sigma_c$ and $\sigma_s$ in each element by combining Eqs. C1 and C4. The total displacement is then obtained by summing the displacements in each layer. The authors suggest that the number of elements used should be greater than 20 to ensure the convergence of shear stresses in each layer.

C2 Formulation of Madhav et al. (2009, 2010)

The method developed by Madhav et al. (2009, 2010) incorporates creep into the design of granular columns based on a stress transfer process from soil to column during creep. The EOP state is calculated using the formulation by Shahu et al. (2000), assuming primary consolidation is finished before creep begins (i.e. Hypothesis A). Then, as the soil creeps, part of the stress acting on it is transferred to the granular column (which does not creep). The stress increase in the column is denoted $\Delta \sigma_c$, while the stress reduction in the soil is denoted $\Delta \sigma_s$.

Equilibrium of vertical forces in the $i$th element during creep is given by Eq. C5 or Eq. C6.

$$ p_a = (\sigma_c + \Delta \sigma_c)(A_c / A) + (\sigma_s - \Delta \sigma_s)(1 - A_c / A) \quad \text{(C5)} $$

$$ 0 = \Delta \sigma_c(A_c / A) - \Delta \sigma_s(1 - A_c / A) \quad \text{(C6)} $$

The settlement of the $i$th layer of the granular column is given by Eq. C7.

$$ \Delta S_c = \frac{\sigma_c + \Delta \sigma_c}{E_c} \Delta h \quad \text{(C7)} $$

The settlement of the $i$th element of the soil is given by Eq. C8, where the net settlement during creep is calculated as the creep settlement minus the rebound due to unloading ($e_p$ denotes the EOP void ratio, $t_0$ denotes the time at which creep begins and $t$ denotes the end time).

$$ \Delta S_s = \frac{C_s}{1 + e_0} \Delta h \log \left(1 + \frac{\sigma_s}{\sigma_0}\right) + \frac{C_a}{1 + e_p} \Delta h \log \left(\frac{t}{t_0}\right) - \frac{C_s}{1 + e_p} \Delta h \log \left(\frac{\sigma_s}{\sigma_s - \Delta \sigma_s}\right) \quad \text{(C8)} $$
Using the compatibility condition ($\Delta S_c = \Delta S_s$), the additional stress carried by the column during creep is calculated using Eq. C9.

$$
\Delta \sigma_c = \left[ \frac{C_c}{1+e_0} \log \left( 1 + \frac{\sigma_s}{\sigma_0} \right) + \frac{C_a}{1+e_p} \log \left( \frac{t}{t_0} \right) - \frac{C_s}{1+e_p} \log \left( \frac{\sigma_s}{\sigma_s - \Delta \sigma_s} \right) \right] * E_c - \sigma_c
$$

(C9)

An iterative procedure can again be used to calculate $\Delta \sigma_c$ and $\Delta \sigma_s$ in each element by combining Eqs. C5/C6 and C9. During creep, the void ratio ($e_f$) of the $i$th element is calculated using Eq. C10. To calculate the EOP void ratio ($e_p$), the $C_a/C_c$ and $C_s/C_c$ terms should be omitted.

$$
e_f = e_0 - C_c \left[ \log \left( 1 + \frac{\sigma_s}{\sigma_0} \right) + \frac{C_a}{C_c} \log \left( \frac{t}{t_0} \right) - \frac{C_s}{C_c} \log \left( \frac{\sigma_s}{\sigma_s - \Delta \sigma_s} \right) \right]
$$

(C10)

The OCR during creep is calculated as the ratio of the stress carried by the soil during creep to the EOP stress, i.e. Eq. C11.

$$
OCR = \frac{\sigma_s}{\sigma_s - \Delta \sigma_s}
$$

(C11)