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<th>Imprecise Empirical Ontology Refinement: Application to Taxonomy Acquisition</th>
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1 INTRODUCTION

This paper builds on a novel representation of uncertain knowledge in the scope of automatic ontology acquisition, which was introduced in (Nováček and Smrž, 2006). The main objective of the ontology acquisition platform OLE is to implement a system that is able to automatically create and update domain-specific ontologies for a given domain. Ontologies are used for many different tasks in the Semantic Web – mainly for annotation of the web content, formal description of specific domains and reasoning on them.

As the amount of data on the Internet is vast and dynamically growing and changing, we emphasise an empirical approach to the ontology construction by means of bottom-up acquisition of concepts from the domain-relevant resources (documents, web pages, corpus data, etc.). The acquisition process is incrementally boosted by the integration with the knowledge already stored in the ontology.

The ontology engineering process is a difficult task. Manual efforts of collaborative ontology design (Gomez-Perez et al., 2004; Zhdanova et al., 2005) lead to development of relatively precise and complex ontologies, however, it is infeasible to cover data intensive domains (e.g. medicine or computer science) using only this approach to knowledge engineering.

Therefore, automatic techniques (ontology learning (Buitelaar et al., 2005; Staab and Studer, 2004)) are needed to be applied in line with the collaborative efforts. But they have another drawback – they are not 100% correct, though they are generally broad in coverage of the domain. There is obvious need for tools that can refine the possibly incorrect statements in such ontologies before presenting them to users. One way is to incorporate uncertainty into the learned ontologies and select only the most important parts according to the adopted uncertainty measure.

Besides the simple threshold-based refinement of the learned ontologies, there are also important cognitive motivations of the utilisation of uncertainty in our empiric ontologies that led us to the proposal of a novel ANUIC (Adaptive Net of Universally Interrelated Concepts) framework for representing uncertain knowledge. This format can be easily applied in very simple, yet effective refinement of the results of ontology acquisition methods. The main
contribution of this paper is the presentation of initial results of application of the ANUIC-based refinement by integration to taxonomy acquisition.

The structure of rest of the paper is as follows. We briefly recall the ANUIC by integration to taxonomy acquisition. We go on describing the progress in our current research in the meaning of new taxonomy acquisition techniques implemented and more elaborate results achieved (Sections 3 and 4). Section 5 briefly resumes related work. We conclude the paper in Section 6.

2 Data-driven Assignment of Fuzzy Relevance Measures in ANUIC

Uncertain, and especially fuzzy semantics is considered as very important for the future development of the Semantic Web (Sheth et al., 2005; Sanchez, 2006). We work on development of such (formal) semantics model within the ANUIC framework. Currently, a very initial proof of concept concerning the (automatic) assignment of reasonable fuzzy measures has been implemented for our experiment in taxonomy acquisition.

The implementation is based on a function that assigns the relevance measure to a relation between terms according to the frequency of the particular relation in input data. The function definition is described as follows.

Fuzzy appropriateness of a relation's $R$ element $(c_1, c_2) \in R$, where $c_1, c_2$ are respective terms, is given by a special function $\mu$ (derived from standard sigmoid):

$$\mu((c_1, c_2) \in R) = \frac{1}{1 + e^{-s f_r((c_1, c_2) \in R) - \beta}}$$

where $f_r((c_1, c_2) \in R) = \frac{|O(c_1, c_2)\in R|}{\sum_{c \in V} |O(c)\in R|}$ is the relative frequency of relation observations in input data\(^2\), $s$ is a parameter regulating the "steepness" of the function and $\beta$ influences the placement of the inflexion point. The domain of the function is real interval $(0, 1)$ (but only rational numbers obviously appear as an input). The range is real interval $(0, 1)$.

This function maps relative frequencies of respective observations in input data to the fuzzy appropriateness measure of the relation. It can model various natural characteristics of human mind like conservativeness, open-mindedness (in the meaning of influence of major or minor observations to the overall conviction) and so forth\(^3\).

The function is continuous and thus can be implemented in a very straightforward way. However, it can easily imitate discontinuous jumps in the shape of the curve, which is also very useful. Examples showing shapes of the conviction function are displayed in Figure 1\(^4\).

One of the key properties of the ANUIC format is that it allows to naturally merge ontologies from the same domain. When we have a large amount of ontologies gained from a vast number of domain resources, we can join them simply using their mutual insertion into one complex ANUIC structure. After proper configuration of the aforementioned function parameters $s, \beta$, we obtain qualitatively different representation of the domain – many formerly incorrect relations are mostly marginalised, whereas the empirically more valid relations obtain high relevance measures, signalling strong belief in their appropriateness.

After several experiments with configuration of ANUIC parameters, we have found that a very good heuristic for configuration of the function parameters is dynamic setting of the $\beta$ inflexion point value. The steepness parameter $s$ was set to 100, which performed best among various other settings.

The $\beta$ for a term $c$ and relation $R$ is set as:

$$\beta = \frac{1}{|\{r|c, r \in R\}|}$$

\(^1\)Which was considered as an open problem to some extent in (Sheth et al., 2005).
\(^2\)This is rather an abstract, yet intuitive notation — the $O(\text{Fact})$ expression stands for an observation of the $\text{Fact}$ in input data; the frequencies are absolute. The frequency measure is not generally symmetric, as the relations themselves do not have to be symmetric.
\(^3\)One can, for example, fix the meaning of a specific group of terms and allow meaning variations for another one.
\(^4\)With the relative frequency and relevance measure on the horizontal and vertical axes respectively.
Moreover, any relative frequency \( f \) higher than 0.5 is adjusted by modifying the \( \beta \) parameter with \( 1 - (f - 0.5) \) expression. Only thus we obtain, for example, natural conviction of (almost) 1 when we deal with a single relation instance. Thus we can discriminate very well between the relation instances with significant and insignificant frequencies due to the shape of the conviction function\(^5\).

The process of integration of newly coming facts is similar to the process of how people construct their conceptual representations – first they have almost crisp evidence of a relation between objects (for example that dogs have four legs). This opinion is strengthened by further observations of four-legged dogs, but one day they see a cripple dog having only three legs. So they have to add another instance of the “have-number-of-legs” relation, but with much more decreased relevancy (unless they keep seeing other and other different three-legged dogs). And this is a simplified example of what we call continuous meaning refinement of conceptual models and what is happening also when the new resources are integrated within the ANUIC framework.

3 Application of Empirical Refinement to Taxonomy Acquisition

The technical description of application of our framework in the taxonomy acquisition from English natural language texts is presented here\(^6\). We use the well-known pattern-based technique (Hearst, 1992) for creation of miniontologies from each input resource. These ontologies are ANUIC-integrated then in order to build a reference ontology that is exploited by the consequent steps of complex taxonomy acquisition. Section 3.1 elaborates our method based on approximate clustering that ensures significant enhancement in coverage. And eventually, the meta-algorithm of conceptual refinement is introduced in Section 3.2.

\(^5\)Supposing that the higher the relation frequency is with respect to the average relative frequency for relation edges coming from the \( e \) term, the more the relation is significant and vice versa.

\(^6\)We have concentrated on extraction of single general terms in the presented experimental settings, but other techniques of acquisition of more specific and multi-word terms (like those in (Ryu and Choi, 2005)) can be easily incorporated as well.

3.1 Clustering and Autonomous Annotation

The clustering is very tempting in the scope of acquisition of taxonomy of an ontology. The main idea of this approach is to gain clusters of similar terms and induce a hierarchical structure upon these terms somehow. However, these approaches usually do not offer a reliable automatic mechanism of annotation of the resulting clusters that naturally correspond to classes in an ontology.

We can obtain an initial domain ontology ontology by integration of ontologies gained from particular resources by pattern-based methods into the ANUIC format (see Section 3.2 for details). Thus we obtain a reference ontology that can be used for annotation of clusters obtained from the same resources. The contribution of such technique is obvious – we will dramatically increase the ontology coverage by incorporation of all significant terms from the resources. We have designed a method similar to \( k\)-means one-level clustering, tuned to suit our demands in order to identify rough clusters in the input data.

3.1.1 Preprocessing Specialties

We require our platform to be scalable even for extreme amounts of data. Therefore we are interested also in efficiency of the clustering. The clustering speed is significantly influenced by the dimension of the feature space.

We use quite a simple and standard metric of term similarity given by a cosine distance between vectors that represent their contexts. Given a set \( F = \{f_1, \ldots, f_n\} \) of features, the vector \( V_T = (v_1, \ldots, v_n) \) for term \( T \) is constructed this way:

- assign \( w^{d-1} \) to \( v_i \), if feature \( f_i \) is contained within a vicinity of \( \frac{w}{CS} \) from \( T \); \( w \in (0, 1) \) is a predefined weight, \( CS \) is the relevant context size and \( d \) is distance of the respective feature from \( T \);\(^7\);
- assign 0 to \( v_i \) otherwise.

Following the main idea of the meta-algorithm presented in Section 3.2, we extract the initial concepts from single resources of relatively small size and refine them further by empirical integration of the resulting ontologies.

\(^7\)When \( w = 1 \), the contexts are represented as bag of words. When \( w < 1 \), their distance from term \( T \) is projected into the vector characteristics. The context size was set to 14 – an average length of sentence in the resource sets. Lower or higher context sizes were tried as well, but without any significant contribution. All the other parameter settings used are further specified in Section 4.
Therefore we can select features only from the particular resources. We have tested several measures (like TF-IDF) for feature selection on the whole domain as well as on the isolated resources, but we have found out that a specific heuristics performs best for our method. We simply discard *hapax legomena* (terms with frequency equal to 1) from the resource’s dictionary (without application of a stop-list, because we would like to have the functional words in our contexts as well). Thus we obtain a feature space of dimension in range from 500 to 1,500 for most of the resources, which is satisfactory. The terms themselves are extracted in a similar way – a general English stop-list is applied and the terms with frequency above a given threshold are considered as terms. Frequency of 5 was found to be reasonable for it does not eliminate any domain-specific words and does not bring too many unwanted garbage-words in most cases.

### 3.1.2 Simplified Rough Clustering

Several variants of *k*-means clustering are discussed and briefly analysed for example in (Kanungo et al., 2002). General characteristic of all algorithms of this kind is that they find *k* points (centres) in the data space that minimise average square distance of all points in the data-set from these centres. Then the clusters are usually defined as a *k*-sized set of balanced groups of points that are nearest to particular centres.

Many algorithms find a local minimum for the problem in an iterative manner. We have found usual implementation of *k*-means clustering unsuitable for our reasons mainly due to their speed. We are not very interested in optimality of our clusters. Moreover, the points in our data space are quite uniformly distributed in most cases because of the restricted size of the feature space and characteristics of natural language that lay beyond the feature selection.

We have implemented a non-optimal (even locally), but very efficient technique that provides us with rough clustering of the initial resources that is further utilised within the refinement of ontologies gained from particular resources.\(^8\)

The method of simplified rough clustering is described\(^9\) in Algorithm 3 (given in Appendix).

\(^8\)The sub-optimality of clusters obtained by the technique is balanced by the efficiency and further empirical refinement in the resulting ontology model. However, the technique presented here could be used for preparing reasonable initial means and related reduction of iterations for the classical *k*-means clustering methods, if needed.

\(^9\)Certain parts of the algorithm are put rather informally due to simplicity of the description.

### 3.1.3 Annotation

The consequent annotation of the clusters using the reference ontology is sketched in Algorithm 1 (given in Appendix).

### 3.2 Refinement by Integration

The conceptual refinement idea lies in integration of small ontologies into bigger ones, smoothing many of the crisp and possibly incorrect relations by uncertain empirical evidence from large number of observations. It is inspired by a simple analogy of human mind and utilises the inherent dynamics and uncertainty of the ANUIC framework. Note that for the optimal performance of the ANUIC-based integration, it is needed to process at least tens of relevant resources of a sufficient size (hundreds or more words). We should not await reasonable refinement results when providing only few documents with size of a couple of sentences – even human learners cannot process such small amounts of data in order to create a valuable opinion about the domain’s conceptual structure.

Concrete application of the above mechanism to the taxonomy acquisition is quite straightforward and conforms to the abstract description in Algorithm 2 (given in Appendix).

We can easily update the domain ontology when keeping the track of how a relation was obtained. Thus we can still identify the more precise “reference” relations in the domain ontology *D*. We add the new resource by processing it first by the pattern-based technique. Then we integrate the result into the domain ontology and process the resource again by clustering-based method, annotating the classes using reference subset of *D*. The result of this step in then integrated in *D* as well – the new resource is completely covered then.

### 4 Selected Results of Taxonomy Acquisition

In the following we describe some of the experiments with taxonomy extraction in OLE and show their results. The improvement of the integration within the ANUIC knowledge representation format is then illustrated in Section 4.2.
4.1 Extraction Phase and Its Initial Results

We tested the taxonomy acquisition on a sample of 3,272 computer science articles, automatically downloaded from the web. The compound size of the resources was 20,405,014 words. For the approximate manual evaluation we randomly chose five ontologies for respective resources from the whole set for each run of a method.

Due to problems with evaluation of automatic ontology acquisition (as expressed, for example, in (Brewster et al., 2004)) we performed only a limited evaluation within the initial experiments. For each selected ontology corresponding to a resource, we computed precision as the ratio of "reasonable" relations compared to all extracted relations. The reasonability of a relation was judged by a committee of computer science experts and students after analysing the respective resources.

The coverage was computed as the ratio of number of extracted significant terms (nouns for the simple experimental settings) to all significant terms present in the resource. For all the measures of precision ($Pr$) and coverage ($Cov.$), an average value was computed. We present these results in Table 1, provided with respective average original resource size and number of all concepts extracted.

The $M1$ row contains results of pattern-based extraction. The $M2$ rows contain results of clustering-based method for respective parameter settings given in Table 2. The rows' headings present settings ID, in the columns there are values of the respective parameters. The cluster size is used for derivation of the $k$ parameter for clustering algorithm.

<table>
<thead>
<tr>
<th>Settings ID</th>
<th>Context size</th>
<th>Position weight</th>
<th>Cluster size</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>14</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>S2</td>
<td>14</td>
<td>1.0</td>
<td>5</td>
</tr>
<tr>
<td>S3</td>
<td>14</td>
<td>0.7</td>
<td>10</td>
</tr>
<tr>
<td>S4</td>
<td>14</td>
<td>0.7</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 2: Settings for clustering-based method

4.2 Improvement Obtained by Uncertain Conceptual Refinement

In order to produce reference ontology for the autonomous cluster annotation, we generated ontologies for each resource by pattern-based OLE module and merged them into one ANUIC structure. We used the heuristics described in Section 3.2 for configuration of the parameters.

Using the ANUIC-integration we gained a taxonomy with 5,538 classes, 9,842 individuals and 61,725 mutual is-a relations.

It is very hard to formally decide what is the representation’s exact improvement when compared to the knowledge stored in the former crisp ontologies. But we can again give at least a rough picture – when we considered only the relations with the highest fuzzy relevance for a particular concept, we can compute an approximate ratio of “reasonable” relations similar to the one presented in Section 4.1. We computed the ratio on a random sample of 50 relations from the whole merged ontology and obtained the value 84 %, which definitely shows an improvement.

The ontology gained by incorporation of results of pattern-based method into ANUIC was used as the reference for clustering-based method. The results of the ANUIC merge of the source crisp ontologies for both methods and various settings of the algorithms are in Table 3 below.

We used the same merging parameters and criteria of reasonability as for the creation and evaluation of the reference ontology. Only the relations with the highest conviction(s) were taken into account for evaluation. The precision computed on random sample of 50 relations from the merged ontology is given in the $Pr_{latter}$ column. The average crisp precision for respective source ontologies is in the $Pr_{former}$ column. The improvement (in percents) is in the $Improvement$ column.

Table 1: Selected results of initial taxonomy extraction

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10For the clustering-based acquisition only 50 randomly selected relations were evaluated for each ontology, because the average number of all relations was too high for manual evaluation.

11We empirically assume that a concept is an individual as long as it has no hyponyms.

12Which is by the way a very strong restriction, the range of possible interpretations of the concrete conviction values is much higher.
The ontology with the best characteristics (gained with S1 configuration) was experimentally merged with the reference ontology. Resulting ontology has much higher range than the reference one – it contains 1,584 classes and 30,815 individuals, interconnected by 1,293,998 relations in the taxonomy. The approximate precision of the 50 randomly selected relations with the highest conviction was 71.05%. It is of course slightly lower than the similar measure for the reference ontology, but this is not a big drawback when we consider the widely increased coverage of the domain.

A sample from the resulting extended uncertain domain ontology is given in Figure 2. The ovals represent classes, squares are individuals and arrows go from sub-concept to its super-concept, labelled by respective fuzzy relevance measures.

5 Related Work

Our work is to some extent similar to the one presented in (Haase and Völker, 2005) paper on uncertainty handling in Text2Onto (Cimiano and Völker, 2005). Text2Onto also utilises initial automated knowledge extraction methods and integrates them into so called Learned Ontology Model. It incorporates uncertain rating annotations of the gained relations. A DL-consistent model is selected then as a subset of the statements in the learned model according to these annotations. On the other hand, we deal with the inconsistencies internally and allow to reason generally among all the gained knowledge under different well-defined perspectives. This provides us with very valuable option of contextualised inferences, among other things.

The reasoning perspectives of our paper are related to the work on fuzzy extension of OWL, which is presented in (Stoilos et al., 2005). However, our current research in automated ANUIC-based reasoning with learned ontologies is somehow different from this logical approaches and is motivated rather by the more general AI paradigms of analogical (Paritosh, 2006; Hobbs and Gordon, 2005) and heuristic (Kokinov and French, 2003) reasoning. The transformations of ANUIC into the current standard and fuzzy ontology representation formats (namely OWL (Bechhofer et al., 2004) and fuzzy-OWL (Stoilos et al., 2005) have to be studied in more detail before we can make proper conclusions and proceed with comparisons with traditional reasoning paradigms.

6 CONCLUSIONS AND FUTURE WORK

The main contribution of this paper rests with the presented boost of initial ontology acquisition methods by very simple application of the ANUIC-based empirical integration of learned knowledge. In this experiment, we have provided an initial proof of concept and groundwork for a novel ontology learning and reasoning paradigm we are currently working on. The full implementation of the related framework aims at extension of ontology learning with robust, though heuristic reasoning routines, that would enable rich and meaningful inferences even for large learned ontologies. Thus we can help to shift on-line knowledge acquisition, management and decision support in data-intensive domains (such as medicine) to a qualitatively different level.

Note that we have also implemented SOLE – a web interface demonstrating the basic functionalities of the current state of our OLE ontology acquisition framework. It processes documents (in plain text, HTML, PDF or PostScript) uploaded by users and creates (fuzzy) ontologies from them. It uses the techniques described here. Users can also define their own patterns for extraction of different semantic relations than the taxonomical ones. The system can be accessed at URL: http://nlp.fi.muni.cz/projects/ole/web/. One can download a brief manual for the system there. The preview credentials for a testing public account with few pre-defined relation patterns are test for user-name and test for password.

Our future work will focus on incorporation of results of another extraction methods to increase the recall and number of kinds of extracted relations. A formal development and validation of a specific calculus for ANUIC-based reasoning engine is needed then. The mutual correspondence and transformation possibilities between ontologies in ANUIC format and formats like OWL or fuzzy-OWL must be examined.

### Table 3: Results of merging for clustering-based method

<table>
<thead>
<tr>
<th>Settings ID</th>
<th>$P_{true}$</th>
<th>$P_{false}$</th>
<th>Improvement</th>
</tr>
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<tr>
<td>M1</td>
<td>61.73</td>
<td>84.0</td>
<td>136.08</td>
</tr>
<tr>
<td>M2/S1</td>
<td>45.78</td>
<td>65.52</td>
<td>143.12</td>
</tr>
<tr>
<td>M2/S2</td>
<td>33.11</td>
<td>65.38</td>
<td>197.46</td>
</tr>
<tr>
<td>M2/S3</td>
<td>46.13</td>
<td>63.16</td>
<td>136.92</td>
</tr>
<tr>
<td>M2/S4</td>
<td>41.52</td>
<td>64.07</td>
<td>154.31</td>
</tr>
</tbody>
</table>
as well, in order to thoroughly evaluate and compare the framework to other similar tools and provide an inter-operation layer by means of the Semantic Web standards.

Acknowledgements
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REFERENCES


Algorithm 1 Cluster annotation with super-class term

Require: \( C \) — set of clusters

Require: \( \theta \), hypernymy confidence threshold, 0.7 is reasonably discriminative

Require: \( R \), reference ontology

Require: \( \text{hyper}(t, R, \theta) \) — function that returns all hyperynms of \( t \) in \( R \) with hypernymy relation relevance higher than \( \theta \)

Require: \( \text{onto}(C) \) — functions that returns an internal ontology representation equivalent to the annotated clusters

1: for \( c \in C \) do
2: \( H \leftarrow \emptyset \) (* hypernymic relation “stubs” *)
3: for \( t \in \text{c} \) do
4: \( h \leftarrow \text{hyper}(t, R, \theta) \)
5: \( H \leftarrow H \cup h \)
6: end if
7: end for
8: annotate all terms in \( c \) with the hyperynms from the set \( H \)
9: end for
10: return \( \text{onto}(C) \)

Algorithm 2 Empirical refinement

1: process the resources by the pattern-based method and produce a set of ontologies \( S_p \)
2: merge the ontologies in \( S_p \) into one ontology \( R \)
3: process the resources by the clustering-based method (Alg. 1 and Alg. 2) using \( R \) as a reference ontology in Alg. 2 and produce set of ontologies \( S \)
4: merge the ontologies in \( S \), and produce ontology \( C \)
5: join the \( R \) and \( C \) in order to produce domain taxonomy in ontology \( D \)
6: return \( D \)

Algorithm 3 Simplified rough clustering

Require: \( V \) — set of feature-vectors mapped to respective terms

Require: \( k \) — number of desired clusters

Require: \( r \) — number of optimisation repeats, value 5 was found to be sufficient

Require: \( \text{centroid}(V) \) — function that computes centroid of the vector set \( V \)

Require: \( \text{dist}(u,v) \) — cosine distance of two vectors \( u,v \)

Require: \( \text{pickBal}(d, V) \) — abstract (due to simplicity of the description) function, which pops a subset \( S \) from set \( V \), \( S \) is characterised by these conditions: (1) all \( v \in S \) all the closest possible vectors to \( d \), and (2) all the sets picked from \( V \) are balanced in size after a sequence of \( \text{pickBal()} \) applications that makes \( V \) empty

1: \( M_{\text{init}} \leftarrow \text{random } v \in V \) (* initial means *)
2: \( V_{\text{init}} \leftarrow V 
3: \text{repeat}
4: c \leftarrow \text{centroid}(M_{\text{init}})
5: v \leftarrow u \text{ such that } \text{dist}(u,c) \text{ is maximal for } u \in V_{\text{init}}
6: M_{\text{init}} \leftarrow M_{\text{init}} \cup \{v\}
7: V_{\text{init}} \leftarrow V_{\text{init}} \setminus \{v\}
8: \text{until } |M_{\text{init}}| < k
9: \text{FACT} \leftarrow \{\} (* \text{empty map } *)
10: V_{\text{init}} \leftarrow V
11: j \leftarrow 0
12: for \( d_i \in M_{\text{init}} \) do
13: \( S_{\text{balanced}} \leftarrow \text{pickBal}(d_i, V_{\text{init}}) \)
14: \( j \leftarrow j + 1 \)
15: \( \text{FACT}[j] \leftarrow S_{\text{balanced}} \)
16: end for
17: \( C \leftarrow \emptyset \)
18: for \( j \in \text{FACT.keys()} \) do
19: \( C \leftarrow C \cup \text{centroid}(\text{FACT}[j]) \)
20: end for
21: \( \text{VECT2SCORE} \leftarrow \{\} (* \text{empty map } *)
22: for \( v \in V \) do
23: \( \text{VECT2SCORE}[v] \leftarrow \{c_0, 0, \ldots, (c_{k-1}, k-1)\} \) such that \( \{c_0, \ldots, c_{k-1}\} \) is a sequence of centroids from \( C \) ordered by the increasing distance from \( v \)
24: end for
25: \( C_{\text{clus}} \leftarrow \emptyset \) (* clustering structure *)
26: \( S \leftarrow \{\} (* \text{empty map } *)
27: for \( j \in \{1, \ldots, r\} \) do
28: \( S_{\text{CLUS}} \leftarrow \text{random shuffle of } V \)
29: initialize clustering \( c_j \) with clusters given by pivotal centroids from \( C \)
30: sequentially process \( S_{\text{CLUS}} \) and assign each vector to the nearest available cluster from \( c_j \), keeping the clusters as balanced in size as possible
31: compute the score \( S[j] \) for the obtained clustering by summing up the numbers pointed by respective centroids in \( \text{VECT2SCORE} \) for each vector in each cluster in \( c_j \)
32: \( \text{CLUSTR} \leftarrow \text{CLUSTR} \cup c_j \)
33: end for
34: return \( c_{\text{clus}} \in C_{\text{clus}} \) with lowest score \( S[j], s \in \{1, \ldots, r\} \) associated with it


