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Assessing the Empirical Performance of DSGE Models in the lead up to the Great Recession

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Abstract

The global financial crisis has sparked renewed debate over the state of macroeconomic modeling, particularly in the lead up to the 2008/2009 Great Recession. The standard workhorse of macroeconomic modeling, the Dynamic Stochastic General Equilibrium (DSGE) model, has been subject to intensive scrutiny. Over the past decade, there has been significant increase in the use of DSGE models by central banks for policy analysis, forecasting and prescriptions. The majority of central banks from developed countries have established DSGE models, including the Federal Reserve Bank, the European Central Bank, the IMF and the Bank of England. Given their prevalence among central banks coupled with their use by policy makers for analysis and forecasting, the objective of this research paper is to assess the behavior and forecasts made by these DSGE models in the run up to a financial crisis. A DSGE model is estimated for the United States for the pre-crisis period Q1.1947 to Q4.2007. An empirical verification of the data is undertaken, whereby forecasts made by the DSGE models are compared with the observed post-crisis data. We find that the DSGE model does a poor job of forecasting the Great Recession, and gives no indication that a downturn is imminent in the economy. Within the current paradigm, there is no role for financial frictions. As such, we suggest that the building blocks of DSGE models are too simplistic to effectively model key dynamics within the economy. We use the role of debt accumulation by US households as a means of illustrating this.
**Section 1: Introduction**

The global financial crisis has sparked renewed debate over the state of macroeconomic modeling, particularly in the lead up to the 2008/2009 Great Recession. Beginning as a crisis of debt and asset price inflation, the 08/09 crisis lead to the simultaneous decline of almost every asset category - leaving investors with nowhere to hide. Global per capita output saw the biggest contraction the world had experienced since WWII. What is perhaps even more significant is that the mainstream macroeconomic models adopted by the most influential central banks across the world, seemed pretty inept at forecasting such a crisis. The standard workhorse of macroeconomic modeling, the Dynamic Stochastic General Equilibrium (DSGE) model in particular, has been subject to intense scrutiny.

DSGE models represent a broad class of macroeconomic model which combines elements of the standard neoclassical growth model of King, Plosser, and Rebelo (1988), the Real Business Cycle theory of Kydland and Prescott (1982), with the introduction of real and nominal frictions within a New Keynesian framework, as popularized by Christiano, Eichenbaum, and Evans (2005), and Smets and Wouters (2003, 2007). These real and nominal frictions can take the form of sticky prices and wages, habit formation in the consumption equation and adjustment costs in the investment equations and are included in an attempt to capture real aspects of the data, Harrison, and Oomen (2010). DSGE models are built upon microeconomic foundations, whereby the decision rules of economic agents are derived with respect to their preferences, technologies, and the current monetary policy regime, by solving intertemporal optimization problems. Exogenous stochastic shock processes are present in the model which cause shifts in, for example, the nominal interest rate set by the Central Bank, or total factor productivity. As a result, the decision rules of economic agents are subject to uncertainty. The DSGE model can be evaluated using several different methods, from calibrating the model, as in Kydland and Prescott (1982), to GMM estimation of equilibrium relationships, as in Christiano and Eichenbaum (1992), to the more popular Bayesian estimation techniques of for example, Christiano, Eichenbaum, and Evans (2005), An and Schorfheide (2007), Smets and Wouters (2003, 2007) and, Del Negro and Schorfheide (2012). The DSGE model used in this paper is empirically estimated using Bayesian estimation techniques. This method allows the likelihood function to transform the prior distribution for the structural parameters of the model into a posterior distribution. This posterior distribution is then used for model inference and policy decision-making, Geweke & Whiteman (2006).

Over the past three decades, there has been significant increase in the use of DSGE models by central banks for policy analysis, forecasting and prescriptions. The majority of central banks from both developed and emerging economies have established DSGE models, including for example, the Federal Reserve Bank’s “SIGMA” model which is used for policy analysis, the European Central Bank’s “NAWM” model which is used for ‘broad macroeconomic projection exercises’, the Bank of England’s “BEQM” model which is used ‘in preparing the Monetary Policy Committee (MPC) quarterly economic projections’, the IMF’s “GIMF” model and more recently, the Central Bank of Ireland’s “HERMES” model. Given the prevalence of DSGE models among central banks, one must wonder why they have become so popular. According to Tovar (2010), DSGE models can identify
sources of fluctuations, perform counterfactual experiments, can forecast and predict the effects of policy changes and answer questions about structural change within the economy.

Given their prevalence among central banks coupled with their use by policy makers for analysis and forecasting, the objective of this research paper is to assess the output produced by the prototypical DSGE model in the run up to the Great Recession, so as to understand how useful DSGE models are for policy analysis and forecasting. We find that monetary policy shocks within the model, do not appear to be an important source of business cycle fluctuation. Price and wage mark up shocks are the main drivers of fluctuations in inflation, the interest rate and the real wage, while consumption and investment shocks are driven primarily by risk premium and investment specific shocks respectfully.

To assess the forecasting performance of the DSGE model, four-quarter ahead forecasts are computed using data up to 2007.4. The forecasts are then evaluated on an absolute basis by comparing them with the observed data. The results show that the forecasting performance of the DSGE model in the run up to the current crisis was poor. We examine the relative performance of the DSGE model by computing forecasts made by a Bayesian VAR model using the same dataset over the same period. The results indicate that the BVAR model does a better job in predicting an economic downturn, however only marginally. This is verified by the calculation of the root mean squared errors (RMSE) for the two models. We suggest that the foundations upon which DSGE models are built are too limited to cope with crisis periods. Given the financial nature of the Great Recession, the exclusion of financial frictions within the baseline model meant that key fundamentals within the economy were not modeled. As such, their impact on the real economy could not be assessed. We provide evidence of this by examining the role of debt accumulation by households in the run up to the Great Recession.

The paper proceeds as follows. Section 2 outlines the DSGE model of Smets and Wouters (2003, 2007). This model represents the benchmark DSGE model available at present. Section 3 discusses Bayesian estimation techniques. This includes information on the importance of prior distributions in the model, the derivation of the likelihood function using the Kalman filter, details of the posterior distribution which is obtained using a Metropolis-Hastings algorithm and finally, methods for evaluating DSGE model forecasts. Section 4 provides information regarding the dataset, while Section 5 examines the results of the model estimation. Section 6 outlines the limitations of the current DSGE paradigm, with a particular emphasis on the role of debt accumulation and it’s potential impact on the model’s results.
Section 2: The Prototypical DSGE Model

The DSGE model used is that of Smets and Wouters (2007). Within this model, we have three economic agents: households, firms and a monetary authority. The basic structure of the model is as follows: there exists a continuum of homogenous households who maximise their utility function over an infinite life horizon. The household derives utility from consumption and disutility from working. Households have monopoly power over wages and rent capital services to firms. Sticky wages and prices à la Calvo (1983) are present in the model and introduce nominal frictions within the model. Intermediate good producers rent capital and labour to manufacture intermediate goods. Final goods producers aggregate these intermediate goods into final goods. There exists a monetary authority which follows a generalised Taylor rule by gradually adjusting the policy controlled interest rate in response to inflation and the output gap.

2.1.1 Households

Household (j) maximizes the following utility function:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1 - \sigma_c} (C_{t+j}^{h}(j) - \lambda C_{t+j-1}^{h})^{1-\sigma_c} \right] \exp \left( \frac{\sigma_c - 1}{1 + \sigma_f} L_{t+j}^{h}(j)^{1+\sigma_f} \right)$$

(1)

where $C_t^{h}(j)$ is consumption, $L_t^{h}(j)$ denotes hours worked, $\beta \in [0,1]$ is the factor, $\sigma > 0$ is the inverse of intertemporal elasticity of substitution and $\lambda \in [0,1]$ determines the degree of habit persistence in consumption.

The household maximizes their utility function subject to the following budget constraint:

$$C_{t+j}^{h}(j) + I_{t+j}^{h}(j) + \frac{B_{t+j}^{h}(j)}{\varepsilon_t R^{t+j} P^{t+j}} - T^{t+j}$$

$$\leq \frac{B_{t+j}^{h}(j)}{P^{t+j}} + \frac{W^{h}(j) L_{t+j}^{h}(j)}{P^{t+j}} + \frac{R^{k} K_{t+j}^{h}(j) K^{t+j}^{h}(j)}{P^{t+j}} - a(Z_{t+j}^{h}(j)) K_{t+j}^{h}(j) + \frac{Div_{t+j}^{h}}{P^{t+j}}$$

(2)

where $I_t^{h}(j)$ is investment, $B_t^{h}(j)$ denotes bonds, $R_t$ is the nominal interest rate, $P_t$ is the price of the final good, $T_t$ represents lump-sum taxes, $W_t^{h}(j)$ is the nominal wage, $R^{k}$ denotes the rental rate on capital, $Z_t^{h}(j)$ is capital utilization, $K_t^{h}(j)$ represents capital, and finally $Div_t^{h}$ denotes dividends distributed by intermediate good producers and labour unions.

The capital accumulation equation is:
\[ K_i(j) = (1 - \delta)_{t-1}(j) + \epsilon_i \left[ 1 - S \left( \frac{I_i(j)}{I_{t-1}(j)} \right) \right] I_i(j) \]

where \(\delta\) denotes the rate of depreciation, and \(I_i(j)\) denotes the investment good. From Christiano, Eichenbaum, and Evans (2005) the function, \(S\), summarises the technology that transforms current and past investment into installed capital for use in the following period. Households choose the rate of capital utilization. The amount of effective capital that household’s can rent to firms is:

\[ K_i^j(j) = Z_i(j)K_{t-1}(j) \]

The income from renting capital services is:

\[ R_i^jZ_i(j)K_{t-1}(j) \]

while the cost of changing capital utilization is:

\[ P_i\alpha(Z_i(j))K_{t-1}(j) \]

The household’s optimal choices on consumption, hours worked, bonds, investment, capital and capital accumulation are as follows:

\[ \frac{\partial L}{\partial C_i} = 0 \Leftrightarrow (C_i - hC_{t-1})^{\gamma_C} \exp \left( \frac{\sigma_C - 1}{1 + \sigma_C} L_i^{1+\sigma_C} \right) \]

\[ \frac{\partial L}{\partial L_i} = 0 \Leftrightarrow \left[ \frac{1}{1 - \sigma_C} (C_i - hC_{t-1})^{\gamma_C} \right] \exp \left( \frac{\sigma_C - 1}{1 + \sigma_C} L_i^{1+\sigma_C} \right) (\sigma_C - 1) L_i^{\sigma_C} = \lambda_i \frac{W^h_i}{P_i} \]

\[ \frac{\partial L}{\partial B_i} = 0 \Leftrightarrow \lambda_i = \beta \epsilon_i R_i E_i \left[ \frac{\lambda_{i+1}}{P_{i+1}} \right] \]

\[ \frac{\partial L}{\partial I_i} = 0 \Leftrightarrow \mu_i \epsilon_i \left[ 1 - S \left( \frac{I_i}{I_{t-1}} \right) - S \left( \frac{I_i}{I_{t-1}} \right) \right] + \beta E_i \left[ \mu_{i+1} \epsilon_i S \left( \frac{I_{i+1}}{I_i} \right)^2 \right] \]

\[ \frac{\partial L}{\partial K_i} = 0 \Leftrightarrow \beta E_i \left[ \frac{R_i^k}{P_{i+1}} Z_{i+1} - a(Z_{i+1}) \right] + \mu_{i+1}(1 - \delta) \]

\[ \frac{\partial L}{\partial c_i} = 0 \Leftrightarrow \frac{R_i^k}{P_i} = a'(Z_i) \]
where $\lambda_t$ and $\mu_t$ are the Lagrange multiplier’s associated with the budget and capital accumulation constraints respectively. Tobin’s Q is defined as $\left( \mu_t / \lambda_t \right)$.

### 2.1.2 Household labour supply

Households supply homogenous labour services to an intermediate labour union who sets the wage subject to Calvo (1983). The intermediate labour union differentiates the labour supply and offers it to intermediate labour packers. Labour packers then combine the differentiated labour and sell it to intermediate good producers. Labour used by intermediate goods producers $L_t$ is a composite of differentiated labour services $L_t(i)$.

The labour packer maximizes profits as follows:

$$\max_{L_t, L_t(i)} W_t L_t - \int_0^1 W_t(i)L_t(i) di$$

subject to

$$\left[ \int_0^1 \frac{L_t(i)}{L_t} \epsilon^w_i \right] = 1$$

where $W_t$ is the price of composite labour services and $W_t(i)$ is the price of intermediate labour services. $H$ is a strictly concave increasing function characterized by $H(1)=1$, while $\epsilon^w_i \in (0, \infty)$ is an exogenous shock process. This exogenous shock process affects the aggregator function, which can result in changes in the elasticity of demand and the mark-up.

Combining the first order condition on $L_t$ and $L_t(i)$ gives:

$$L_t(i) = L_t H^{-1} \left[ \frac{W_t(i)}{W_t} \int_0^1 \frac{L_t(i)}{L_t} \frac{L_t(i)}{L_t} di \right]$$

The optimal wage set by the union, which is allowed to reoptimise its wages a la Calvo (1983) is as follows:

$$\max_{W_t(i)} \left[ \sum_{i=0}^{\infty} \beta^i \frac{\lambda_t(i)}{\lambda_t(i)+1} \left[ W_t(i) \left( \pi_{i+1}^{\epsilon} \pi_{i+1}^{\nu} \pi_{i+1}^{1-nu} - W_t^{h} \right) \right] L_{i+1}(i) \right]$$
\( \tilde{W}_t \) is the newly set wage and \( \xi^w \) is the Calvo probability that the household can re-optimize its wage. \( \pi_t \) is inflation which is defined as \( P_t / P_{t-1} \), \( \beta \) is the nominal discount factor and \( \iota_w \) is wage indexation.

The optimisation problem (16) is subject to
\[
L_{\pi_s}(i) = L_{\pi_s} H^{-1} \left( \frac{W_s(i)X^w_{i\pi_s}}{W_{i\pi_s}} \right) \tag{17}
\]
where \( \tau^w = \frac{1}{H} \int H \left( \frac{L_t(i)}{L_t} \right) \frac{L_t(i)}{L_t} di \) and
\[
X^w_{i\pi_s} = \begin{cases} 
1 & \text{for } s = 0 \\
\left( \pi^w_{i\pi_s} \gamma \pi^w_{i\pi_{s-1}} \tau^w_{i\pi_{s-1}} \right) & \text{for } s = 1, \ldots, \infty
\end{cases}
\]

Combining equations (16) and (17) gives the following first order condition for the optimal wage
\[
E_i \sum_{s=0}^{\infty} \frac{\beta^s \lambda_{t+s} P_{t+s}}{\lambda_t P_{t+s}} L_{\pi_s}(i) \left[ X^w_{i\pi_s} \tilde{W}_t(i) + \left( \tilde{W}_t(i) X^w_{i\pi_s} - W^h_{t\pi_i} \right) \frac{1}{H^{-1} \left( \tilde{z}_{i\pi_s} \right) H^{\prime} \left( X^w_{i\pi_s} \right) H^{\prime \prime} \left( X^w_{i\pi_s} \right) } \right] = 0 \tag{18}
\]
where
\[
X^w_i = H^{-1} \left( \tilde{z}_i \right) \quad \text{and} \quad \tilde{z}_i = \frac{W_t(i)}{W_t} \tau^w_i
\]

We can now obtain the aggregate wage index:
\[
W_t = (1 - \tilde{z}_i) \tilde{W}_t H^{-1} \left[ \frac{W_t \tau^w_t}{W_t} \right] + \tilde{z}_i \gamma \pi^w_{i\pi_i} \tau^w_{i\pi_i} W_t H^{-1} \left[ \frac{\gamma \pi^w_{i\pi_i} \tau^w_{i\pi_{i-1}} W_{i\pi_i} \tau^w_{i\pi_{i-1}}}{W_t} \right] \tag{20}
\]

2.2 Firms

2.2.1 Final Goods Producers:
The final consumption good, \( Y_t \), is a composite made up of a continuum of intermediate goods, \( Y_t(i) \), and is produced by a perfectly competitive, representative firm. The final goods producers buy intermediate goods, and package them into \( Y_t \), which they sell to consumers, investors and government in a perfectly competitive market.

Final good producers maximize profits:
\[
\max_{Y_t} P_t Y_t^d - \int_0^1 P_t Y_t d\epsilon_t
\]

subject to:
\[
\left[ \int_0^1 G \left( \frac{Y_t(i)}{Y_t} \epsilon_t^p \right) d\epsilon_t \right] = 1
\]

where \( P_t \) is the price of the final good and \( P_{it} \) is the price of the intermediate good. \( G \) is a strictly concave and increasing function, which is characterized by \( G(1) = 1 \), while \( \epsilon_t^p \in (0, \infty) \) which is an exogenous shock process. This exogenous shock process affects the aggregator function and can affect the elasticity of demand and the price mark-up.

Combining the FOC for \( Y_t \), and \( Y_t(i) \) to obtain the final goods producers optimal decision gives:
\[
Y_t(i) = Y_t G^{-1} \left[ \frac{P_t(i)}{P_t} \int_0^1 G \left( \frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} d\epsilon_t \right]
\]

### 2.2.2 Intermediate Goods Producers

Intermediate goods producers (i) produce differentiated goods, using the following production technology:
\[
Y_t(i) = \epsilon_t^a K_t^i(i)^\alpha [\gamma^t L_t(i)]^{1-\alpha} - \gamma^t \Phi
\]

where \( 0 < \alpha < 1 \), and \( \epsilon_t^a \) denotes total factor productivity. \( K_t^i \) represents capital services used in production and \( L_t(i) \) denotes composite labour input. The labour augmenting deterministic growth rate in the economy is \( \gamma^t \) and \( \Phi \) denotes fixed costs.

The firm’s profit is given by:
\[
P_t(i) Y_t(i) - W_t(i) L_t(i) - R_t^k K_t(i)
\]

where, as before, \( W_t \) denotes the aggregate nominal wage and \( R_t^k \) is the rental rate on capital. The first order conditions for the intermediate goods producers with respect to labour and capital are as follows:
\[
\frac{\delta L}{\delta L_{t(i)}} = 0 \iff W_t = \lambda_t(i) \gamma^{1-\alpha}(1 - \alpha)\epsilon_t^a K_t^i(i)^\alpha L_t(i)^{\alpha - 1}
\]
\[
\frac{\delta L}{\delta K_{t(i)}} = 0 \iff R_t^k = \lambda_t(i) \gamma^{1-\alpha} \alpha \epsilon_t^a K_t^i(i)^{\alpha - 1} L_t(i)^{1-\alpha}
\]
Combining the first order conditions and noting that the capital-labour ratio is equal across firms implies:

\[ K_t^i = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t^k L_t} \]  

(28)

Using this, we can find the real marginal cost by setting the level of labour and capital equal to the requirements of producing one unit of good, which gives:

\[ MC_t^i = \alpha^{-\alpha} (1 - \alpha)^{\alpha} W_t^{1-\alpha} R_t^{\alpha} T^{(1-\alpha)\alpha} (e_t^u) \]  

(29)

This implies that marginal cost is also equal across all firms, and is independent of the intermediate good produced.

The optimal price set by firms that are allowed to reoptimise their prices as a result of Calvo pricing with partial indexation, is given by the following optimization problem:

\[
\max_{P_t(i)} \sum_{s=0}^{\infty} \xi_{t+1}^p \beta^s \left[ P_t(i) \left( \pi_t \gamma \tau_{t+1} - MC_{t+1}^i \right) \right] Y_{t+s}(i)
\]  

(30)

where \( P_t \) is the newly set price and \( \xi_{t+1}^p \) is the Calvo probability that the firm can reoptimise its price.

As with the optimal wage equation, \( \pi_t \) is inflation which is defined as \( P_t / P_{t-1} \). \( \beta^s \left[ \frac{\lambda_{t+1} P_t}{\lambda_t P_{t+s}} \right] \) is the nominal discount factor for firms and \( t \) is price indexation.

The optimization problem (30) is subject to the following constraint:

\[ Y_{t+s}(i) = Y_{t+s} \frac{G_{t+s}^{-1}}{P_{t+s}^i} \left( \frac{P_t(i) X_{t+s}^{-1}}{P_{t+s}^i} \right) \]  

(31)

where \( \tau_t^p = \int_{0}^{G_t} \frac{Y_t(i)}{Y_t} Y_t(i) di \) and

\[ X_{t+s} = \begin{cases} 1 & \text{for } s = 0 \\ \left( \pi_t \gamma \tau_{t+s} \right) & \text{for } s = 1, \ldots, \infty \end{cases} \]  

(32)
Combining equation (30) and (31) gives the following first order condition for the optimal price set by firms:

$$E_i \sum_{i=0}^{\infty} \frac{\beta \lambda_{i,t}}{\lambda_{i+1}} Y_{t+1}(i) \left[ X_{t,i} \tilde{P}_i(i) + \left( \tilde{P}_i(i) X_{t,i} - MC_{t+1} \right) \frac{1}{G^{i-1}(z_{t+1})} G'(x_{t+1}) \right] = 0 \quad (33)$$

where $x_t = G^{-1}(z_t)$ and $z_t = (P_t(i)/P_t)\tau_t$

We can now obtain the aggregate price index which is as follows:

$$P_t = (1 - \xi_p)P_t(i)G^{-1} \left[ \frac{P_t(i)\tau_t}{P_t} + \xi_p \pi_{t+1}^{i-p} \pi_{t+1}^{1-p} G^{i-1} \left( \frac{\pi_{t+1}^{i-p} \pi_{t+1}^{1-p} P_{t+1} \tau_t}{P_t} \right) \right] \quad (34)$$

### 2.3 Monetary Authority

Following Taylor (1983), the central bank follows a nominal interest rate rule by adjusting its instrument in response to deviations of inflation and output from their respective target levels. The interest rate rule is as follows:

$$\frac{R_t}{R^*} = \left( \frac{R_{t-1}}{R^*} \right)^{\rho} \left[ \left( \frac{\pi_t}{\pi^*} \right)^{\gamma} \left( \frac{Y_t}{Y^*} \right)^{\gamma} \right]^{1-\rho} \left( \frac{Y_t}{Y_{t-1}} / \frac{Y^*}{Y^*} \right)^{1-\rho} \epsilon_{t}^{r} \quad (35)$$

where $R_t$ is the interest rate set by the central bank, $R^*$ is the steady state nominal interest rate, $\rho$ is the degree of interest rate smoothing and $\epsilon_{t}^{r}$ is a monetary policy shock.

The government budget constraint is as follows:

$$P_tG_t + B_{t-1} = T_t + \frac{B^*}{R_t} \quad (36)$$

### 2.4 Market Clearing Conditions

The aggregate resource constraint is

$$C_t + I_t + G_t + a(Z_t)K_{t-1} = Y_t \quad (37)$$

which is obtained by integrating the budget constrain across households and combining with the government budget constraint and the dividends of intermediate goods producers and labour union. Equations (1) to (37) complete the full set of equilibrium conditions for this model. The model is
detrended using the deterministic trend $γ$. The non-linear system is then log-linearised around its steady state. The log-linearised equations are provided in the appendix to this paper.

**Section 3: Bayesian Analysis**

The DSGE model is estimated using Bayesian estimation techniques. According to Bayes theorem, we can combine our prior beliefs, with the sample information contained in the likelihood function to obtain a new set of beliefs (posterior distribution), as follows:

$$
\pi(\theta | y^T, i) = \frac{p(y^T | \theta, i)(\pi(\theta | i))}{\int p(y^T | \theta, i)(\pi(\theta | i))d\theta}
$$

To estimate the DSGE model, we must first log-linearise the model equations around their steady state. In doing so, we obtain the Linear Rational Expectations (LRE) model, which we solve and write in state space form. Once the model is written in state space form, we can compute the likelihood function. The Kalman filter is used to evaluate the likelihood function. The likelihood function is combined with the prior distribution on the parameters to obtain the posterior density function, which is estimated using MCMC techniques. The Metropolis-Hastings algorithm is then employed to obtain a complete picture of the posterior distribution. From Schorfheide (2000), the Metropolis-Hastings algorithm proceeds as follows:

1. A numerical optimization routine is used to maximize the log posterior, which is given by $\ln p(Y | \theta) + \ln p(\theta)$. The posterior mode is denoted by $\hat{\theta}$.
2. Let $\sum$ be the inverse of the Hessian computed at the posterior mode $\hat{\theta}$.
3. The algorithm then draws $\theta^{(0)}$ from $N(\hat{\theta}, \hat{\sum})$, or directly from a specified starting value.
4. For $s = 1, \ldots, n_{\text{sim}}$, draw $\theta$ from the proposal distribution $N(\hat{\theta}, \hat{\sum})$. The jump from $\theta^{(s-1)}$ is accepted $\left(\theta^{(s)} = \theta\right)$ with probability $\min\left\{1, r\left(\theta^{(s-1)}, \theta | Y\right)\right\}$ and rejected $\left(\theta^{(s)} = \theta^{(s-1)}\right)$ otherwise. Here,

$$
r\left(\theta^{(s-1)}, \theta | Y\right) = \frac{p(Y | \theta) p(\theta)}{p(Y | \theta^{(s-1)}) p(\theta^{(s-1)})}
$$

**3.1 Parameter estimates**

Prior distributions\(^4\) allow for the inclusion of some nonsample information, gleaned from sources such pre-sample data, the use of data from other countries (for example, a prior for a DSGE model of the euro area is specified based on US data) or from micro level data (for example, panel studies on price

\(^3\)The model is estimated using Dynare 4.2. The posteriors are based on 250,000 draws of the Metropolis-Hastings algorithm
\(^4\)The prior and posterior parameter graphs as provided in the Appendix to this paper, along with the Metropolis-Hastings convergence diagnostics.
elasticities or labour supply behaviour), which does not directly enter the likelihood function. This nonsample information is independent of the sample used in the model estimation. Thus, the prior distribution adds weight to the likelihood function and can introduce curvature into the posterior density surface that facilitates the use of MCMC methods to derive the posterior distribution, An and Schorfheide (2007). In constructing the priors, the parameters may be grouped into three broad categories which include, steady state parameters, endogenous parameters, and exogenous parameters. Steady state parameters tend to be calibrated in DSGE models and reflect “great ratios”, Del Negro & Schorfheide (2008). In estimating the model, the parameters are calibrated in the same way as Smets and Wouters (2007). Taking the steady state parameters first, five parameters are calibrated. These include the depreciation rate which is assumed to be 10 percent annually ($\delta = 0.025$). The degree of monopolistic competition in the goods and labour market are assumed to imply mark-ups of 10 percent respectively. The exogenous spending-GDP is set at 18 percent and the steady state mark-up in the labour market is 1.5. The prior standard deviation is provided in column 2 of Table 1, along with the distribution. The list of parameters estimated in the model, including the prior mean, standard deviation, and distribution used are provided in Table 1.

We now turn to the endogenous parameters. As before, the parameters used are the same as those used by Smets and Wouters (2007). The prior parameters of the consumption and investment utility functions are as follows; both the elasticity of intertemporal substitution and the elasticity of the capital adjustment cost function are assumed to normally distributed with a prior mean of 4.0. The distribution used to model the habit parameter is the beta distribution. The prior mean of the habit parameter is set at 0.7 with a standard deviation of 0.1. The beta distribution is defined on the interval $[0, 1]$ and is used to model the behaviour of random variables limited to intervals of finite length. Regarding the monetary policy reaction function, the interest rate in response to changes in inflation parameter is assumed to be approximately 1.5 and normally distributed, while the degree of interest rate smoothing uses a beta distribution and is set at 0.75. The parameters representing the interest rate in response to the output gap and changes in the output gap are set at 0.125 and assumed to be normally distributed, respectively. The prior mean of the Calvo probabilities of price and wage stickiness are set at 0.5 each, which suggests average price and wage contracts of half a year and a standard deviation of 0.1. The Calvo probabilities use a beta distribution.

Finally, we turn to the exogenous parameters representing exogenous shock processes within the model, which are given in Table 3. The prior mean and standard deviation are set at 0.1 and 2.0 respectively, which is seen as a relatively loose prior. The distribution used for the priors of the exogenous shock processes is an inverse gamma distribution, which restricts the parameters of the model to be positive. The persistence of the AR(1) processes are restricted to a beta distribution, with a prior mean of 0.5 and standard deviation of 0.2.
Section 4: Dataset

The model is estimated using seven key variables over the period 1947.1 to 2007.4. Quarterly data is obtained from Federal Reserve Economic Data (FRED). The seven variables used in the estimation include real GDP, consumption, investment, real wage, hours worked, interest rate, and inflation. Real GDP is expressed in Billions of Chained 1996 dollars (GDPC96). Personal Consumption Expenditure (PCEC) and Fixed Private Investment (FPI) are used for the consumption and investment variables. The real wage is obtained using data from the NonFarm Business Sector (NFB): Hourly Compensation (PRS8006023). Hours worked is determined using data on NFB: Average Hours Worked (PRS85006103). The aggregate real variables are expressed per capita by dividing by the total population over 16 (LNS), and are deflated using the GDP deflator. The aggregate real variables are also expressed as 100 times log. The Implicit Price Deflator (GDPDEF) is used to model inflation and the short-term interest rate is the Federal Funds Rate. The inflation rate and the interest rate are expressed quarterly. All variables are seasonally adjusted. Figure 1 in the appendix depicts the transformed variables used in the estimation.

Measurement equations are added to the model to allow us to link model variables with observable quarterly time series. The measurement equations used in this model follow Smets and Wouters (2007) and are as follows:

\[
\begin{bmatrix}
  dl\text{GDP}_t \\
  dl\text{CONS}_t \\
  dl\text{INV}_t \\
  dl\text{WAG}_t \\
  l\text{HOURS}_t \\
  dl\text{P}_t \\
  FED\text{FUNDS}_t
\end{bmatrix}_t =
\begin{bmatrix}
  \gamma \\
  \gamma \\
  \gamma \\
  \gamma \\
  \gamma \\
  \pi \\
  r
\end{bmatrix}
\begin{bmatrix}
  y_t - y_{t-1} \\
  c_t - c_{t-1} \\
  i_t - i_{t-1} \\
  w_t - w_{t-1} \\
  \ell_t \\
  \pi_t \\
  \ell_t
\end{bmatrix}
\]

where \( l \) and \( dl \) represent log and log difference respectively. \( \gamma = 100(\gamma - 1) \) is the common quarterly trend growth rate to real GDP, consumption, investment and wages. \( \pi = 100(\pi - 1) \) is the quarterly steady-state inflation rate and \( r = 100(\beta^{-1}C \lambda \pi - 1) \) is the steady-state nominal interest rate. Finally, \( \ell \) is the steady-state hours worked, which is normalised to be equal to zero.
Section 5: Results

5.1 Prior and Posterior Distributions

The prior and posterior parameters estimated by the model are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior Mean</th>
<th>Prior Standard Deviation</th>
<th>Posterior Mean</th>
<th>Posterior 5%</th>
<th>Posterior 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of Capital Adjustment Cost Function</td>
<td>csadcost</td>
<td>4</td>
<td>1.5</td>
<td>7.1978</td>
<td>5.6514</td>
<td>8.7303</td>
</tr>
<tr>
<td>Elasticity of Intertemporal Substitution</td>
<td>csigmoid</td>
<td>1.5</td>
<td>0.375</td>
<td>1.2857</td>
<td>1.0802</td>
<td>1.4924</td>
</tr>
<tr>
<td>Habit Parameter</td>
<td>chabb</td>
<td>0.7</td>
<td>0.1</td>
<td>0.8128</td>
<td>0.749</td>
<td>0.8778</td>
</tr>
<tr>
<td>Degree of Wage Stickiness</td>
<td>cprobw</td>
<td>0.5</td>
<td>0.1</td>
<td>0.7817</td>
<td>0.7273</td>
<td>0.8376</td>
</tr>
<tr>
<td>Elasticity of labour supply w.r.t. Real wage</td>
<td>csigl</td>
<td>2</td>
<td>0.75</td>
<td>2.3801</td>
<td>1.5019</td>
<td>3.2023</td>
</tr>
<tr>
<td>Degree of Price Stickiness</td>
<td>cprodp</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5489</td>
<td>0.5</td>
<td>0.5928</td>
</tr>
<tr>
<td>Wage Indexation</td>
<td>cindw</td>
<td>0.5</td>
<td>0.15</td>
<td>0.5373</td>
<td>0.3864</td>
<td>0.6757</td>
</tr>
<tr>
<td>Inflation Indexation</td>
<td>cindp</td>
<td>0.5</td>
<td>0.15</td>
<td>0.2508</td>
<td>0.1269</td>
<td>0.3662</td>
</tr>
<tr>
<td>Elasticity of the Capital Utilisation Adjustment Cost Function</td>
<td>ccap</td>
<td>0.5</td>
<td>0.15</td>
<td>0.6065</td>
<td>0.483</td>
<td>0.7349</td>
</tr>
<tr>
<td>Share of Fixed Costs in Production</td>
<td>cfc</td>
<td>1.25</td>
<td>0.125</td>
<td>1.6888</td>
<td>1.5714</td>
<td>1.8224</td>
</tr>
<tr>
<td>Policy Controlled Interest Rate in Response to Inflation</td>
<td>cpix</td>
<td>1.5</td>
<td>0.25</td>
<td>1.8963</td>
<td>1.6601</td>
<td>2.1347</td>
</tr>
<tr>
<td>Degree of Interest Rate Smoothing</td>
<td>crr</td>
<td>0.75</td>
<td>0.1</td>
<td>0.8908</td>
<td>0.8562</td>
<td>0.9051</td>
</tr>
<tr>
<td>Policy Controlled Interest Rate in Response to Output</td>
<td>cry</td>
<td>0.125</td>
<td>0.5</td>
<td>0.1238</td>
<td>0.0852</td>
<td>0.163</td>
</tr>
<tr>
<td>Policy Controlled Interest Rate in Response to the Output Gap</td>
<td>cropy</td>
<td>0.125</td>
<td>0.5</td>
<td>0.1014</td>
<td>0.0723</td>
<td>0.1311</td>
</tr>
<tr>
<td>Quarterly Steady State Inflation Rate</td>
<td>constepinf</td>
<td>0.625</td>
<td>0.1</td>
<td>0.7149</td>
<td>0.6182</td>
<td>0.815</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>constbeta</td>
<td>0.25</td>
<td>0.1</td>
<td>0.2344</td>
<td>0.0741</td>
<td>0.415</td>
</tr>
<tr>
<td>Steady-state Hours Worked</td>
<td>constlab</td>
<td>0</td>
<td>2</td>
<td>0.5566</td>
<td>-0.7065</td>
<td>1.7783</td>
</tr>
<tr>
<td>Common Quarterly Trend Growth Rate</td>
<td>ctrend</td>
<td>0.4</td>
<td>0.1</td>
<td>0.4894</td>
<td>0.4599</td>
<td>0.5191</td>
</tr>
<tr>
<td>Share of Capital in Production</td>
<td>capla</td>
<td>0.3</td>
<td>0.05</td>
<td>0.2795</td>
<td>0.2231</td>
<td>0.3311</td>
</tr>
</tbody>
</table>

Table 1: Prior and Posterior Distributions

The parameters characterising output are the steady state growth rate and the exogenous spending shock. In the model, the prior mean is 0.4. The posterior mean estimate by the model is 0.49. The parameters characterising consumption are the elasticity of intertemporal substitution and the habit parameter. The elasticity of intertemporal substitution represents the degree to which people prefer a
stable rate of consumption relative to a higher rate of consumption in the future. It can be measured by the response of the rate of change in consumption to changes in the expected real interest rate. When the elasticity of intertemporal substitution is lower than the prior mean, then it is assumed that there is a lower sensitivity of consumption to changes in the real interest rate. In the model, the prior mean is set at 1.5. The resulting posterior mean is 1.28. The prior mean for the habit parameter is 0.7. The posterior mean is estimated to be higher at 0.81. According to Harrison & Oomen (2010), households derive utility from keeping consumption close to its previous aggregate level as well as higher levels of lifetime consumption. A higher degree of habit persistence will reduce the interest rate elasticity of consumption for a given elasticity of substitution. The rate of consumption is therefore more affected by consumers’ habits than movements in the real interest rate.

The parameters characterising investment are the elasticity of the capital adjustment cost function and the discount factor. From Groth & Khan (2006), the capital adjustment cost function represents the cost of changing the level of investment. Investment adjustment costs induce inertia in investment causing it to adjust slowly to shocks. As in CEE (2005), a higher elasticity of the cost of adjusting capital reduces the sensitivity of investment to the real value of capital stock. In our model, the prior mean is 4. The posterior mean is higher at 7.19. Investment is therefore less sensitive to changes in the real value of capital stock. The prior and posterior mean on the discount factor are similar at 0.25 and 0.23, respectively. We next examine the parameters regarding nominal rigidities. The prior mean on both price and wage stickiness is 0.5. The posterior mean for the wage stickiness is higher at 0.78, while the posterior mean for the price stickiness is only marginally higher than the prior at 0.54. This suggests that wages are reset less frequently than initially assumed. The posterior mean for the degree of price indexation and wage indexation is 0.25, and 0.53, respectively.

Finally, we examine the parameters related to the monetary policy reaction function. According to the Taylor rule, monetary authorities adjust the policy controlled interest rate in response to inflation and the output gap. The posterior mean of the long-run reaction coefficient to inflation is estimated to be relatively high at 1.80. There is also considerable degree of interest rate smoothing as the mean of the coefficient on the lagged interest rate which is estimated to be 0.88. Policy does not appear to react very strongly to the output gap or changes in the output gap at 0.12 and 0.10, respectively.

In Table 2, the prior and posterior distributions for the shock processes are presented. The autoregressive coefficients on the shock parameters suggest that productivity shocks, exogenous spending shocks and price and wage mark-up shocks are highly correlated, with all coefficients over 0.95. Monetary policy shocks and risk premium shocks are the least persistent, with values of 0.21 and 0.15, respectively.
5.2 Model Evaluation

Variance Decomposition

The variance decomposition allows us to identify the contribution each shock has in generating fluctuations in endogenous variables. The results are given in Table 3 below.

<table>
<thead>
<tr>
<th>Shock Processes</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Distribution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Total Factor Productivity</td>
<td>ea Inverse Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>Investment Specific Technology</td>
<td>eqs Inverse Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>Risk Premium</td>
<td>eb Inverse Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>Exogenous Spending</td>
<td>eg Inverse Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>Price Mark-Up</td>
<td>ep Inverse Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>Wage Mark-Up</td>
<td>ew Inverse Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>Monetary Policy Shocks</td>
<td>em Inverse Gamma</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2: Prior and Posterior Distributions, Shock Processes

<table>
<thead>
<tr>
<th>Autoregressive Parameters of the Shock Processes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Factor Productivity</td>
<td>crhoa Beta</td>
</tr>
<tr>
<td>Risk Premium</td>
<td>crhob Beta</td>
</tr>
<tr>
<td>Exogenous Spending</td>
<td>crhog Beta</td>
</tr>
<tr>
<td>Investment Specific Technology</td>
<td>crhogs Beta</td>
</tr>
<tr>
<td>Monetary Policy Shocks</td>
<td>crhons Beta</td>
</tr>
<tr>
<td>Price Mark-Up</td>
<td>crhopinf Beta</td>
</tr>
<tr>
<td>Wage Mark-Up</td>
<td>crhow Beta</td>
</tr>
</tbody>
</table>

Table 3: Variance Decomposition

It is clear that price and wage mark-up shocks are the biggest determinants of fluctuations in inflation, the interest rate and the real wage, which on aggregate account for 84.7%, 61.4%, and 85% respectively, over the sample period. Risk premium shocks are the main drivers of fluctuations in
consumption (70%) and to a lesser extent output (26%), which is equally affected by exogenous spending shocks (25%). Not surprisingly, investment specific shocks cause the biggest movements in investment at 78%. Monetary policy shocks do not appear to be an important source of business cycle fluctuation.

**Historical Shock Decomposition**

To further assess the importance of various shocks at particular points in time during the sample period, we can estimate the historical shock decomposition for each of the variables.

Risk premium shocks and exogenous spending shocks are the key driver of fluctuations in output for the US economy. Negative productivity shocks from 2006.3 onwards coincide with a (small) decline in output. Price mark up shocks turn negative between (2000.3 and 2003.3) and continue to do so over the remaining sample period. The decline in output just before 2007.4 is driven by negative investment shocks. Overall, output appears to be driven by many different shocks.

![Graphs showing historical shock decomposition for output and consumption](image)

**Figure 2: Historical Shock Decomposition – Output (a) and Consumption (b)**

The decline in consumption prior to 2000.3 was a result of negative risk premium shocks. Positive price mark up shocks offset the decline, however their impact is small. Again, the decline in consumption in the period preceding 2003.3 was due to negative risk premium shocks. Positive productivity and monetary policy shocks are small. Positive risk premium shocks allow for the increase in consumption proceeding the decline. Prior to 2007.4, consumption is marginally below the steady state, where monetary policy and productivity shocks are becoming more important.

Investment specific shocks result in fluctuations in investment. An analysis of the historical shock decomposition shows that negative investment shocks caused a decline in investment prior to 2003.3. Positive monetary policy shocks were too small to offset the decline, which continues for some time. It is only when investment shocks turn positive that we see an increase in investment. This is consistent over the entire sample examined. Analysis of the historical shock decomposition shows that negative wage mark up shocks keep inflation below the steady state level. While positive price mark up shocks are large, they do little in causing a change in inflation.
Fluctuations in the real wage are directly as a result of both positive and negative wage and price mark-up shocks. Movements in the hours worked variable are driven positively by wage mark-up shocks and negatively by a combination of exogenous spending, and price mark-up shocks. Monetary policy and productivity shocks become more influential towards the end of the sample, but still have little impact.
The decline in the interest rate from 2001 to 2007 coincides with negative wage mark up shocks and investment shocks. Price mark-up shocks contribute positively to the interest rate, over the sample shown. The increase in the interest rate prior to 2007.4 is as a result of both positive monetary policy shocks and price mark-up shocks.

**Impulse Response Functions**

The impulse response functions are given in Figures (3-9) in the Appendix. The risk premium shock causes an increase in all variables over the short to medium term, however the impact is less pronounced for the real wage, inflation and interest rate variables. The productivity shock causes an increase in output, consumption, investment, and the real wage, which lasts into the short-to-medium term. A decline in hours worked and inflation is observed, which lasts for the short term. The negative impact on the interest rate lasts into the medium term. Exogenous spending shocks cause a decline in consumption and investment, which lasts into the short-to-medium term. The positive impact on the real wage turns negative after one year and does not return to the steady state in the medium-to-long term. Output and hours worked increase, as does inflation and the interest rate, but only marginally. A monetary policy shock causes a decline in all variables considered, with the exception of the interest rate, which increases by 0.2. The impact of the shock lasts for the short to medium term.

A price mark-up shock results in a 0.3 increase in inflation, which lasts into the medium-to-long term. The interest rate also increases as a result of the shock. All other variables decline with the real wage variable experiencing the biggest fall. This effect dissipates quickly and the real wage returns to its steady state. The decline in hours worked becomes more pronounced in the medium term and does not return to its steady state. An investment specific shock causes an increase in all variables considered, with the impact on investment being the most significant. Inflation increases slightly initially, however this increase turns negative over the medium term. The wage mark-up shock causes the real wage, inflation and interest rate variables to increase. It causes very small declines in the remaining variables.

**Forecasting performance:**

The model is forecast for horizons $h \in (0, 1, 2, 3, 4)$. Figure 10 shows the forecasts of the key endogenous variables in the model along with the 90% confidence bands. The variables are as expressed in the model, i.e. subject to the measurement equations and in deviation from the steady state. The dashed line represents the forecast made by the DSGE model and the solid line represents the observed time series. The green lines represent the 90% confidence interval.

As is apparent in the graphs, DSGE models failed to foresee a downturn in the economy. The model predicts a steady to slight increase in output over the next four quarters. In fact, what we observe is a steady decline in output. The observed data falls outside the 90% confidence interval, as given by the model. Consumption is also forecast to remain steady over the coming quarters, when the observed data shows a significant decline in consumption. The steep decline in investment is apparent from the
graphs, however, the DSGE model forecasts an expected increase in investment over the next four quarters. As with output, both consumption and investment data are outside the 90% confidence intervals. Forecast predictions of inflation are good, for the short term at least, and are almost in line with observed inflation. In forecasting the interest rate the model does well in foreseeing an interest rate cut, however the magnitude is less than what the model expected. It is however, within the 90% confidence interval. The DSGE model does not do well in forecasting the sharp decline and subsequent increase in the real wage, which it forecasts to decrease slightly and then remain steady over the next year.

![DSGE Model Forecast](image)

Figure 10: DSGE Model Forecast

The solid line represents the actual time series observed and the dashed line is the DSGE model forecast. The green lines represent the 90% confidence intervals. The x-axis is the forecast horizon.

In a bid to assess the forecasting performance of the DSGE model with other model based forecasts, a Bayesian VAR model is estimated. The forecasts made by the BVAR model are provided in Figure 11 along with the DSGE model forecasts and the observed data. The solid black line is use to depict the forecasts made by the BVAR model. The dashed black line is the DSGE model forecast, and the solid red line is the observed data. The BVAR model does a better job of forecasting output, consumption and investment. We see that the BVAR model picks up a decline in output, however it does not accurately forecast the magnitude of the decline. Unlike the DSGE model which predicts an increase in investments, the BVAR forecasts a small decline in investment over the next year. As with the DSGE model, the BVAR model does well in predicting inflation. It over-predicts the increase in hours worked, and under-predicts the interest rate forecast, however it does a better job of forecasting the decline in the real wage.
To evaluate the forecast accuracy of the models, the root mean squared error (RMSE) of the DSGE model and the BVAR model is computed and reported in Table 4. The RMSEs are reported for each variable.

<table>
<thead>
<tr>
<th>Root Mean Squared Error (RMSE)</th>
<th>DSGE Model</th>
<th>BVAR Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.9798</td>
<td>1.222</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.9927</td>
<td>1.6814</td>
</tr>
<tr>
<td>Investment</td>
<td>4.3921</td>
<td>2.6827</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.2755</td>
<td>0.2652</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.518</td>
<td>1.6587</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.8427</td>
<td>0.5783</td>
</tr>
<tr>
<td>Hours Worked</td>
<td>0.8897</td>
<td>1.8134</td>
</tr>
</tbody>
</table>

Table 4: RMSE for DSGE & BVAR Models

The reported output, consumption, investment and real wage RMSEs are smaller in the BVAR model. There is considerable difference between the RMSE reported for the investment variable in both models. The BVAR model estimates an RMSE of 2.68%, while the DSGE model reports a RMSE of 4.39%. However, the DSGE model does a better job of forecasting the interest rate and hours worked. Both the DSGE model and the BVAR model have a very similar RMSE for inflation.

These findings are consistent with those of Edge and Gurkaynak (2011), Del Negro and Schorfheide (2012) and Wickens (2012). These authors compare model based forecasts of the DSGE model and a BVAR model with those made by the Federal Reserve Board of Governors ‘Greenbook’ forecasts and Blue Chip consensus forecasts. They find that short run forecasts of DSGE models are not as good as
those of professional forecasts, however Del Negro and Schorfheide (2012) argue that in the medium to long term, forecasts of output and inflation become more competitive with the professional forecasts.

Section 6: Limitations of the Model

As stated in the introduction, DSGE models are used by academics and policy-makers alike for many reasons; policy analysis, to identify the source of business cycle fluctuations, counterfactual analysis, and forecasting. However, the output given by the model provides little evidence that a recession is imminent. DSGE model forecasts predict a continuation in output, consumption and investment growth into the short term. The reality is very different. After the peak of 2007.4, the Great Recession begins. The year on year percentage decline in real GDP and consumption is -4%. The year on year percentage change in investment is -20%. The Federal Reserve cut the interest rate to almost 0% where it remains until now and unemployment reaches 10%.

Since the onset of the Great Recession, DSGE models have been heavily criticized for failing to show any signs that a recession was imminent. Given the foundations upon which DSGE models are built, this should come as no surprise. Within the prototypical framework, agents are assumed to be homogenous and optimizing with rational expectations. No endogenous instability exists in the model. Perturbations in the economy are the result of exogenous shocks which cause the economy to deviate from its steady state. Moreover, money within the model is merely a unit of account and there is no role for financial intermediaries, since financial markets are assumed to be complete and perfectly frictionless. Perfectly frictionless markets assume that agents can borrow or lend at the risk free interest rate and there is no risk that they will default on their loan. This is clearly at odds with what we observe occurring in the real economy. Take for example, the role of consumer debt accumulation and it’s impact on US consumption. The accompanying graphs (Figures 17-22) are available in the appendix to this paper.

An increase in leveraging in the US began in the 1980’s when the use of credit cards and home equity lines of credit became available. By 2000, household debt had increased to 90% of disposable income. This figure would rise to 138% by 2007. According to Bauer & Nash (2012), this significant increase in household debt came from increases in mortgage liability. An increase in house prices coupled with attainable mortgage financing causes a surge in the uptake of housing. Credit innovation allowed for less attractive mortgage candidates to enter the market. As mortgage debt soared, household net worth was also increasing as a result of rising house prices and stock market gains, producing a household wealth effect. This increase in wealth fueled consumer spending. The amount of consumer and mortgage loans increased significantly in the US in the run up to the boom. Very low interest rate environment and house price appreciation would have contributed to an increase in the mortgage refinancing behaviour of households. Between 1995 and 2005, the personal saving rate of households fell from a peak of 6.1% to a low of 1.3%, indicating that consumption was increasing faster than income.
There was a consumption collapse which was caused by an increase in household’s debt and a decline in house prices. House price declines beginning in 2006 caused a sharp drop in household net worth and in the value of debt backed by households (Krainer, 2012). The 2012 IMF report ‘Dealing with Household Debt’ found that, in advanced economies, housing busts and recessions tend to be more pronounced and protracted when they are preceded by larger run ups in household debt. In the five years preceding the Great Recession of 2007, the debt-to-income ratio for the US rose 40% to 138%. This increase in household leverage, coupled with a reduction in house prices meant that they had very high levels of debt relative to their assets (negative equity). The ratio of household debt to disposable income fell significantly during the 2007-2009 period in the US as households defaulted on loans, paid down debt and reduced their uptake of new loans. This deleveraging effect may be constraining consumer spending, which is contributing to a decline in overall economic activity. Both Mian, Roa & Sufi (2011) and Dynan (2012) find that the decline in consumption is more severe when a household has a highly leveraged balance sheet. This implies that the role of debt is important in understanding the consumption collapse of the Great Recession. In order to deleverage their balance sheet, the household can either default on their repayments or they can begin active repayments of the debt principal. Since defaulting is costly, a household is likely to cut back heavily on consumption and to forego a substantial fraction of their savings. A report by McKinsey (2012) found that the US has the highest rate of deleveraging, with 66% of the debt reduction reflecting default.

Given the financial nature of the Great Recession, it is imperative that the new generation of DSGE models includes financial frictions within their models, and allows a clear role for money, debt, and the possibility of default, if these models are to be useful for policy analysis and forecasting.

**Conclusion**

The global financial crisis has sparked renewed debate over the state of macroeconomic modeling, particularly in the lead up to the 2008/2009 Great Recession. The objective of this paper was to assess the output produced by the prototypical DSGE model in the run up to the Great Recession, so as to understand how useful DSGE models are for policy analysis and forecasting. A DSGE model is estimated for the pre-crisis period up to 2007.4. The results indicate that the four-quarter ahead forecast made by the model did not predict any downturn in the economy. The BVAR model which was estimated using the same dataset performed marginally better than the DSGE model, however neither do a good job forecasting the Great Recession.

We suggest that the foundations of DSGE models, in their current form, are too simplistic to cope with assessing the onset of economic crisis periods since they omit the financial side of the economy. In order to account for the real and financial interlinkages which we observe in the economy, the new class of DSGE model must incorporate a role for financial frictions if they are to be useful for both policymakers and academics alike.
References


Koop, G (2004), Bayesian Econometrics. John Wiley & Sons, Hoboken, New Jersey


Appendix

The Linearised DSGE model:

The following linearised DSGE model has been taken from Smet’s and Wouters 2007 and represents the log-linearised version of a DSGE model which has been estimated using data from the US. All variables are log-linearised around their steady-state balanced growth path.

The aggregate resource constraint is given by:

1. \( \dot{Y}_t = c_y \dot{C}_t + l_y \dot{L}_t + z_y \dot{Z}_t + \epsilon_t^H \)

Output is composed of consumption, investment, capital-utilisation costs that are a function of the capital utilisation rate, and an exogenous spending disturbance term.

The consumption equation follows from the consumption Euler equation and is given by:

2. \( \dot{C}_t = \frac{h/y}{1+h/y} \dot{C}_{t-1} + \frac{1}{1+h/y} E_t \dot{C}_{t+1} + \frac{w_t (\sigma_c-1)}{\sigma_t} (I_t - E_t \dot{I}_{t+1}) - \frac{1-h/y}{(1+y)^\gamma} (\dot{P}_t - E_t \dot{P}_{t+1}) - \frac{1-h/y}{(1+y)^\gamma} \epsilon_t^B \)

Current consumption depends on a weighted average of past and expected future consumption, on expected growth in hours worked, the ex-ante real interest rate and a risk premium disturbance term.

The investment equation follows the investment Euler equation and is given by:

3. \( \dot{I}_t = \frac{1}{1+\beta_y (1-\sigma_c)} \dot{I}_{t-1} + \frac{\beta_y (1-\sigma_c)}{1+\beta_y (1-\sigma_c)} E_t \dot{I}_{t+1} + \frac{1}{\gamma \delta (1+\beta_y (1-\sigma_c))} \dot{G}_t + \epsilon_t^i \)

Give explanation of this equation here

The corresponding arbitrage equation for the value of capital is given by:

4. \( \dot{G}_t = \beta (1-\delta) y^{-\sigma_c} E_t \dot{G}_{t+1} - \dot{P}_t + E_t \dot{P}_{t+1} + (1-\beta (1-\delta) y^{-\sigma_c}) E_t \dot{P}_{t+1} - \epsilon_t^b \)

The current value of capital stock depends positively on its expected future value and the expected real rental rate on capital, and negatively on the ex-ante real interest rate and the risk premium disturbance from equation (2).

The aggregate production function is given by:

5. \( \dot{Y}_t = \phi (a \dot{K}_t + (1-\alpha) \dot{I}_t + \epsilon_t^a) \)
Output is produced using capital and labour services. The parameter $\alpha$ captures the share of capital in production, while the parameter is one plus the share of fixed costs in production, reflecting the presence of fixed costs in production.

The current capital services used in production equation is given by:

$$6. \quad \dot{k}_t^c = k_{t-1} + \dot{z}_t$$

Newly installed capital becomes effective with a one-quarter lag, current capital service used in production are a function of capital installed in the previous period and the degree of capital utilisation.

The degree of capital utilisation equation is given by:

$$7. \quad \dot{z}_t = \frac{1 - \psi}{\psi} \dot{r}_t$$

where $\psi$ is a positive function of the elasticity of the capital utilisation adjustment cost function and normalised between zero and one. Cost minimisation by households that provide capital services implies that the degree of capital utilisation is a positive function of the rental rate of capital.

The accumulation of installed capital equation is given by:

$$8. \quad \dot{k}_t = \frac{(1 - \delta)}{\gamma} \dot{k}_{t-1} + \left(1 - \frac{1 - \delta}{\gamma}\right) \dot{i}_t + \left(1 - \frac{1 - \delta}{\gamma}\right) \phi \gamma^2 (1 + \beta \gamma^{(1 - \sigma_C)} \epsilon_t^i)$$

The accumulation of installed capital is not only a function of the flow of investment, but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology disturbance term.

The price mark-up equation is given by:

$$9. \quad \beta_t^p = \alpha (\dot{k}_t^c - \dot{i}_t) + \omega_t^p + \epsilon_t^p$$

Regarding the monopolist competitive goods market, cost minimisation by firms implies that the price mark-up, defined as the difference between the average price and the nominal marginal cost, is equal to the difference between the marginal product of labour and the real wage. The second equality in equation (9), implies that the marginal product of labour is a positive function of the capital-labour ratio and total factor productivity.

The inflation equation is given by:

$$10. \quad \dot{p}_t = \frac{\beta y^{(1 - \sigma_C)}}{1 + \epsilon_p} \dot{E}_t \dot{p}_{t+1} + \frac{\epsilon_p}{1 + \beta y^{(1 - \sigma_C)}} \dot{r}_t - \frac{(1 - \beta y^{(1 - \sigma_C)} \epsilon_p^p (1 - \epsilon_p)}{(1 + \epsilon_p \beta y^{(1 - \sigma_C)} (1 + (\phi - 1) \epsilon_p^p) \beta_t^p + \epsilon_t^p$$

Due to price stickiness as in Calvo (1983) and partial indexation to lagged inflation of those prices, prices only adjust sluggishly to their desired mark-up. Profit maximisation by firms gives rise to the
above New-Keynesian Phillips curve, where inflation depends positively on past and expected future inflation, negatively on the current price mark-up and positively on a price mark-up disturbance.

The rental rate of capital equation is given by:
\[ \dot{r}_t^k = \dot{I}_t + \delta_t - \ddot{k}_t^e \]

Cost minimisation by firms implies that the rental rate of capital is negatively related to the capital-labour ratio and positively to the real wage.

The wage mark-up equation is given by:
\[ \dot{\mu}_t^w = \sigma_t - \gamma_t - \frac{1}{1 - \gamma} \left( \dot{\ell}_t - \frac{h}{\gamma \ell_{t-1}} \right) \]

In the monopolistically competitive labour market, the wage mark-up will be equal to the difference between the real wage and the marginal rate of substitution between working and consuming.

The real wage equation is given by:
\[ \dot{w}_t = \frac{\beta_t \gamma_t (1 - \sigma_t)}{1 + \beta_t \gamma_t (1 - \sigma_t)} (E_t \dot{w}_{t+1} + E_t \dot{\mu}_{t+1}) + \frac{1}{1 + \beta_t \gamma_t (1 - \sigma_t)} (\dot{w}_{t-1} - \ell_{t-1}) - \frac{1}{1 + \beta_t \gamma_t (1 - \sigma_t)} \dot{\mu}_{t-1} - \frac{(1 - \beta_t \gamma_t (1 - \sigma_t) \xi_t (1 - \xi_t))}{(1 + \beta_t \gamma_t (1 - \sigma_t)) (1 + (\lambda_{t-1}) \xi_t) \xi_t} \dot{\mu}_t^w + \epsilon_t^w \]

Due to nominal wage stickiness and partial indexation of wages to inflation, real wages adjust gradually to the desired wage mark-up. The real wage is a function of expected and past real wages, expected, current and past inflation, the wage mark-up and a wage mark-up disturbance.

Finally, the monetary policy reaction function is given by:
\[ \dot{\tau}_t = \rho \dot{\tau}_{t+1} + (1 - \rho) \left( r_t \dot{\pi}_t + r_y (\dot{y}_t - \dot{y}_t^e) \right) + r_{\Delta y} (\dot{y}_t - \dot{y}_t^e) - (\dot{y}_{t-1} - \dot{y}_{t-1}^e) + \epsilon_t^\tau \]

Monetary authorities follow a generalised Taylor rule, by gradually adjusting the policy-controlled interest rate in response to inflation and the output gap. The parameter \( \rho \) captures the degree of interest rate smoothing. There is also a short-run feedback from the change in the output gap.
Exogenous Shocks:
There are seven exogenous shocks in the model, which evolve as follows:

Total factor productivity shock:

1. $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$

Risk Premium shock:

2. $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$

Exogenous Spending shock:

3. $\varepsilon_t^\sigma = \rho_\sigma \varepsilon_{t-1}^\sigma + \rho_\sigma \eta_t^\sigma + \eta_t^\sigma$

Investment Specific shock:

4. $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$

Monetary Policy shock:

5. $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$

Price Mark-Up shock:

6. $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$

Wage Mark-Up shock:

7. $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$
Figure 1: Dataset
Impulse Response Functions

Figure 3: Total Factor Productivity shock

Figure 4: Risk Premium shock
Figure 5: Exogenous Spending shock

Figure 6: Monetary Policy shock
Figure 7: Price Mark-Up shock

Figure 8: Investment Specific shock
Figure 9: Wage Mark-Up shock
Prior and Posterior distributions

The following figures represent the prior and posterior distribution of the parameters and shock processes used in the model estimation. The grey line is the prior density and the black line is the posterior density.

![Figure 12: Estimated distribution of parameters](image-url)
Figure 13: Estimated distribution of parameters
Figure 14: Estimated distribution of parameters
Figure 15: Estimated distribution of parameters
Figure 16: Metropolis-Hastings Convergence Diagnostics