Title	An investigation into the dynamics of financial catastrophe
Author(s)	Raghavendra, Srinivas
Publication Date	2012
Publication Information	Bhaduri, Amit; Guttal, Vishwesha (2012) An Investigation into the Dynamics of Financial Catastrophe. Working Paper
Publisher	Economics Working Paper
Item record	http://hdl.handle.net/10379/3791

An Investigation into the Dynamics of Financial Catastrophe

Amit Bhaduri*, Srinivas Raghavendra† and Vishwesha Guttal‡

Abstract

The paper captures the dynamics of interaction between the real and the financial sector in a model of endogenous credit that exhibits multiple steady states and generates phase transitions with catastrophic abruptness. This resembles aspects of financial meltdowns witnessed in global capital markets in recent times. The paper also explains how financial crisis assume different characteristics under alternative monetary arrangements in regimes of profit- and wage-led growth and, concludes with linking the analysis to the recent financial crises in the United States and in the Eurozone.

JEL codes: G01, E44, E41, E51, E32, E12, C02, C63

Keywords: Financial crisis, Endogenous credit, Composition effect, Debt-Default, Systemic Illiquidity, Cusp Catastrophe, Abrupt change, Hysteresis

*Professor Emeritus, Jawaharlal Nehru University, New Delhi and Visiting Professor, Council for Social Development, New Delhi, India

†Department of Economics, J.E. Cairnes School of Business & Economics, National University of Ireland, Galway, Ireland [Corresponding author - s.raghav@nuigalway.ie]

‡Centre for Ecological Studies, Indian Institute of Science, Bengaluru, India

1 Introduction

Expectation about asset prices is like the Prince of Denmark without whom the story of Hamlet or, that of financial crisis, cannot be told. We tell this story in a starkly schematized form, for a closed economy without government. With the influence of external capital flows as well as monetary and financial policies ruled out by assumption, expected asset price (p) provides the crucial link between the real and the financial sector. Rising asset price and expectations of capital gain involve both borrowers and lenders. Higher asset prices create positive wealth effect for wealth holders and induce them to utilize their increased borrowing power against the rising value of their wealth. At the same time, banks and other financial firms enjoy a greater margin of safety due their improved balance sheets and expand their lending activities.[1] The result is debt financed expansion although rising obligation to service the stock of debt (D) might increasingly strain debt driven economic expansion.

For our present purpose a consumption function incorporating income, asset price and debt suffices :

$$C = C(Y, p, D), C_Y > 0, C_p > 0 \text{ and } C_D < 0 \tag{1}$$

Consumption expenditure (C) increases with current income (Y); higher expected asset price (p) increases the capacity of households to borrow more because they hope to service their debt more easily with expected capital gains that enhances their borrowing power. At the same time credit offered to them is expanded endogenously through various debt instruments by the modern financial sector to exploit every profitable lending opportunity and the accumulated obligation on borrowing as the stock of debt exerts a negative influence on consumption.

The effect of rising asset prices on investment is more complex involving investment either to 'make' or to 'buy' assets. A 'two price theory' of investment was proposed originally by Keynes to capture the interlocking between the real and financial sector (Keynes, J.M., 1964, (p. 95,150-51)) and (Minsky, H.P., 1975, (p. 119-20)). It compares the ratio of 'make' or replacement price (π) with the 'buy' or acquisition price (p_a), i.e. $\left[\frac{p_a}{\pi}\right]$. Acquisition merely transfers the ownership right of an asset through the secondary stock market in a manner similar to other second hand market without creating any new assets through real investment (Scitovsky, T., 1994). Under perfect arbitrage the ratio of acquisition and replacement value of an asset is unity indicating perfect alignment between the real and financial sector (Tobin, J., 1969). However, when the expected asset price (p) differs from its current acquisition price (p_a),

[1] During periods of tranquil expansion profit seeking financial institutions invent and reinvent "new" forms of money, substitutes for money in portfolios and financing techniques for various types of activities: "financial innovation is a characteristic of our economy in good times" (Minsky, H.P., 1986, (p. 178,186)).

opportunities for making capital gains arise to govern the choice of the investor between the two alternative avenues of investment. When the expected price is higher than the current acquisition price, the acquisition price (p_a) in effect gets reduced by an amount equal to expected capital gain ($p - p_a$); consequently, the attractiveness of acquisition increases over real investment with higher capital gain encouraging the composition of total investment to shift in favor of acquisition.[2].

However, since lending institutions with improved balance sheets are also more willing to lend general conditions for obtaining finance are more favorable. In these circumstances the cost of borrowing as well as the qualifications for credit worthiness are likely to be lower. As a result the volume of total real and financial investment tends to be higher. Other things being equal it would be reflected in a higher level of aggregate demand and economic activity (Y), although the composition of investment might change against real investment due to the possibility of capital gains through acquisition.

We capture this with a demand function for real investment (I),

$$I = I(Y, p, D), I_Y > 0, I_D < 0; I_p \lesseqgtr 0 \tag{2}$$

Depending on whether the composition effect is quantitatively less or more than the volume effect the sign of I_p would be positive or negative.

In a typical Keynesian short period characterized by equilibrium of only the flow variables, the stock of debt is taken as a given parameter. Equality between saving (S) and investment (I) determines income in the real economy. From Eqs. 2, 3:

$$(Y - C(Y, p, D)) = S(Y, p, D) = I(Y, p, D). \tag{3}$$

Total differentiation and rearrangement of terms yield the slope of the IS curve in (p, Y) plane as

$$\frac{dY}{dp} = \frac{I_p - S_p}{S_Y - I_Y} \quad \text{having signs} \quad \frac{\overbrace{(- \text{ or } +) - (-)}}{\underbrace{(+) - (+)}} \tag{4}$$

The usual condition for the convergence of the Keynesian income adjustment process in the single variable closed economy, namely ($S_Y > I_Y$) guarantees that the denominator of Eq. 4 is positive and, the sign of the numerator determines the sign of the slope of the IS curve.[3] From Eq. 4, higher expected asset

[2]Expectation however is never certain, but the uncertainty is underplayed during periods of tranquil expansion when a 'convention' gets established that the present can be extrapolated into the future (Keynes, J.M, 1937)

[3]For a more complicated stability analysis of the multiplier process involving income as well as a second variable like income distribution, see (Bhaduri, A., 2011a).

price has the effect of expanding aggregate demand through depressing saving i.e. ($S_p < 0$) due to higher consumption, whereas higher expected asset price has an ambiguous effect on investment (sign of I_p) depending on the relative strength of the volume and the composition effect. If the composition effect is relatively weak, ($I_p > 0$), real investment expands and, the slope of the IS curve is unambiguously positive, $(\frac{dY}{dp}) > 0$. When the composition effect is sufficiently strong, $I_p < 0$; but the slope of IS may still be positive provided the stimulating effect of lower saving outweighs the depressive effect of lower real investment to cause aggregate demand and income to expand with higher asset price,$(\frac{dY}{dp}) > 0$. However the numerator in Eq. 4 turns negative and $(\frac{dY}{dp}) < 0$ when the composition effect is strong enough to divert enough investment from the real to the financial sector through devices like acquisition and merger or purchase of financial derivatives summarizes the crucial role played by expected asset price in the determination of income in the real economy and, captures an important aspect of the interaction between the real and the financial sector.

In line with existing literature (Bhaduri, A and Marglin, S., 1990), we may characterize the positive relation between income and asset price, $(\frac{dY}{dp}) > 0$ as a consumption-led regime where the stimulating effect of capital gains on consumption outweighs its depressive composition effect on real investment. However, this statement needs to be qualified in so far as real investment might actually be increasing even in a consumption-led regime due to the dominating positive volume effect over negative composition effect arising from asset price movement. On the other hand, if the composition effect is sufficiently powerful, aggregate demand and income decreases at higher asset price, $(\frac{dY}{dp}) < 0$. This characteries the investment-led regime in which expectation of higher asset price causes decline in real economic activity. The emergence of two regimes can easily be seen through the locus of shifting investment saving equality or the IS curve at different levels of capital gains.[4]

2 A Linear Model of Two Regimes

Assuming linear functions,

$$C = c_1 Y + c_2 p - c_3 D + E_1, \quad c_{i=1,2,3} > 0 \tag{5}$$
$$I = b_1 Y - b_2 p - b_3 D + E_2 \quad b_{i=1,2,3} > 0 \tag{6}$$

The saving investment equilibrium condition (3) reduces to,

$$Y = (1 - c_1 - b_1)^{-1}[(c_2 - b_2)p + E - (c_3 + b_3)D] = m(ap + H) \tag{7}$$

[4]See (Bhaduri, A., 2011b) for a preliminary graphical investigation along these lines.

In Eq. 7, $(1 - c_1 - b_1)^{-1} = m$ acts as a 'super multiplier' involving both consumption (c) and investment (b) propensities, together assumed to be a positive fraction less than unity for the income generation process to be convergent. The sign of the parameter $a = (c_2 - b_2)$ determines whether the regime is consumption or investment-led due to expected movements in asset price. With the two regimes taking simple linear forms, if $c_2 > b_2$, implying $a > 0$, the resulting economic regime is consumption-led regime; in the opposite case, $a < 0$ and the economy is investment-led. The coefficient $H = [E - (c_3 + b_3)D]$, where $E = (E_1 + E_2)$, is a time dependent variable whose sign depends on the service burden the stock of debt over time.

An important implication follow from the stylization of the contrasting regimes. The relation between the real economy and the stock market depends on the nature of the regime. The stock market is seen as a reasonable indicator of the health of the real economy in consumption-led regimes because the level of real economic activity and expected stock price move in the same direction. However, in the strictly investment-led regime the composition effect on investment is powerful enough to drive economic activity in the real sector and stock price in the financial sector in opposite directions. Thus the stock market as an indicator of the health of the real economy becomes misleading. In general, the use of stock prices as an index of the performance of the real economy is particularly misleading in the investment-led regime; over time asset prices continue to be driven up by a sufficiently strong composition effect in favor of financial investments to the detriment of the real economy.

3 A Banking Theory of Debt Cycles

Since the level of fresh loan ($\frac{dD}{dt}$) from lending institutions usually depends on the increased value of wealth of the borrowers ($\frac{dW}{dt}$) including expected capital gains, this may be captured by a simple creditworthiness criterion,

$$\frac{dD}{dt} = v\left[\frac{dW}{dt}\right] = vp, \quad v > 0 \tag{8}$$

where in Eq. 8 v represents the extent of credit worthiness of the borrower from the point of view of the lender. The increase in wealth ($\frac{dW}{dt}$) depends on the state of expected asset price represented by an increasing function $\phi(p)$. For simplicity of exposition, we assume $\phi(p) = j.p$ (see equations 10 and 11 in the text) and set the constant $j = 1$, and rewrite on the extreme right handside of Eq. 8 in the simplest form. Inserting Eq. 8 in Eq. 7 the stock and flow mechanism of debt driving income becomes,

$$Y = \left[\frac{ma}{v}\right]\left[\frac{dD}{dt}\right] + m[E - (c_3 + b_3)D]. \tag{9}$$

Over successive short periods, the flow of new credit accumulates to a larger stock of debt with higher debt service obligations which exert a depressive effect on demand. However in the consumption led regime, the flow of new credit stimulates demand. This interaction between the contradictory effects of the stock and flow of debt on on aggregate demand has the possibility of generating fluctuations in income in a manner familiar from some endogenous Keynesian or Kaleckian business cycle models. (e.g.(Goodwin, R. M., 1951; Goodwin, R.M., 1981; Kaldor, N., 1960; Kalecki, M., 1971))

Although debt-driven fluctuations resulting from Eq. 9 could be modelled in various ways depending on specific assumptions, their common feature require debt to grow stimulating expenditure until at some threshold level the flow of new lending is switched off. This brings about the 'turning point' of the cycle and, shifts the explanation to defining the thresholds of 'ceiling' and 'floor' in the level of debt. The typical underlying assumption is that that the lending authorities are in a position to set the ceiling and the floor on the stock of debt implying their control over the supply of credit money. In effect, this becomes a theory of crisis in which changes in the credit policy of the monetary authority is the driving force behind fluctuations.(Friedman, M and Schwartz, A., 1963) It assumes control over exogenous supply of money as a policy variable in the Monetarist tradition. However, this view of exogenous money is untenable in a modern financialized economy where various private credit instruments, some of them not even indirectly guaranteed by the monetary authority, are created by private business for exploiting profitable opportunities arising from the demand for credit.(Bhaduri, A., 2011a)

4 Endogenous Credit Expansion: Background to Financial Collapse

'Money' in the narrowest sense (of legal tender) is the liability of the monetary authority which controls how much liability it wishes to crate vis-a-vis the public. To the extent, the wider banking system is obliged to use this liability as the reserve base for its credit creation, 'money' remains under the indirect control of the monetary authority. And, so long as the financial system is controlled indirectly by the monetary authority, money supply can be treated broadly as exogenous, because monetary authority has the freedom to decide its base level irrespective of the demand for credit.

However, the difficulty arises with this view of exogenous money because acceptance of liabilities between two private parties is largely a matter of mutual trust, like a financier reposing his trust in the borrower he considers creditworthy. When a borrower is considered creditworthy, financial institutions and corporations can devise ways of creating money through numerous technique of financing which entails promises and guarantees on debt instruments owned by them. The demand for credit is met with supply of various debt instruments, and, as soon as the commitments on the debt instrument are fulfilled by the borrower the 'money' so created is destroyed. In this way, profit seeking financial firms can devise techniques of financing based on mutual trust to exploit profitable lending opportunities making money supply or credit endogenously adjusting to the level of demand for credit.

Endogenous credit supply exploits with great flexibility all profitable lending opportunities to adapt to almost any level of demand for credit which it considers profitable as business.[5] Nevertheless this very flexibility that serves well in times of credit expansion turns out to be a double-edged weapon in times of stress; because it wounds badly the very financial system that forges it during period of tranquil expansion. During the process of adaptation to expanding demand various forms of credit instruments (e.g. securitization of debt, over-the-counter derivatives, debt default swaps and other forms of hedges) are created. It is a complex network of mutual debt obligations among private parties, but without the guarantee or the supervision of a lender of last resort, none of the individual participant has a comprehensive idea of the complex network of debt obligations and counter-obligations among parties participating in the network.[6] As a matter of fact that guarantee from monetary authority is often foregone deliberately with considerable accounting skill to avoid supervision!

A process of under-supervised endogenous credit expansion by private parties economizes heavily on generally acceptable liquid 'money' in the form of liabilities of the monetary authority. The result is a disproportionate expansion of the volume of credit in relation to liquid money, i.e. direct liabilities of the monetary authority making the financial system increasingly fragile. In stressful times of default on loans the fault lines open because default leads to erosion of private trust resulting typically in 'margin contraction' for the concerned lending institutions. Facing default from their borrowers and with the danger of their non-performing loans exposed to the market, they need to settle more stringent requirement with liquid money.

The inverted pyramid like structure of huge mutual private debt obligations with disproportionately narrow availability of liquidity within the financial system needed for default settlement crumbles easily if relatively small defaults begin to reverberate through the fragile financial system in a chain reaction of margin contractions among mutually indebted financial institutions. It raises suddenly the demand for liquid money.[7] A situation arises when everybody

[5] Perhaps a close analogy to credit creation in response to demand is devising private (or coded) languages to meet the need for particular communications. The limit to the number and scope of such privately coded languages would be set by the needs of the communication network and, innumerable innovations that are possible.

[6] It is a case of incomplete information; information is not only asymmetric between the buyers and sellers of securities but the sellers might become buyers and buyers the sellers in a complex financial network.

[7] See (Longworth, D., 2010) for some recent episodes.

wants to have liquidity; and yet liquidity is in desperately short in supply in the financial system. As a result every financial firm clings to whatever liquidity it has. A general credit freeze results and, the sheer abruptness of this freezing of internal lending among financiers themselves tends dramatically abrupt. An analytical model of financial crisis needs to capture this phase transition - catastrophic abruptness with which the phase of credit expansion turns suddenly into its opposite of sudden and massive credit contraction.

5 The Dynamics of Catastrophic Financial Collapse

A stark scheme providing the barest outline of the crucial interconnections causing such abrupt transition begins with a revision of the previous demand determined endogenous credit creation equation (Eq. 8). It is extended to include a constant term (k_0) which modifies Eq. 8 as [8]

$$\left[\frac{dD}{dt}\right] = B = k_0 + vp, \qquad k_0 > 0 \tag{10}$$

The credit worthiness criterion (v) applied by the lenders gets increasingly less rigorous and, v rises as higher expected asset price generates greater economic optimism based particularly on the assumption that debt can be repaid from higher capital gains.

$$v = k_1 p, \qquad k_1 > 0 \tag{11}$$

Substituting Eq. 11 in Eq. 10 we get,

$$B = k_0 + k_1 p^2 \tag{12}$$

However the default rate (u) also increases with easier credit condition reflected in a higher value of v but decreases with higher level of income Y,

$$u = k_2 v - k_3 Y, \qquad k_2 > 0, k_3 > 0 \tag{13}$$

Using Eq. 7 and Eq. 11

$$u = k_2 k_1 p - k_3 map - k_3 mH \tag{14}$$

so the total default on loan advanced becomes,

$$U = uB = [k_2 k_1 p - k_3 map - k_3 mH][k_0 + k_1 p^2]$$

[8]The constant term k_0 can be thought of as a function of income (Y) due to transaction demand for money. Since income is a function of asset price (p) and stock of debt (D) in Eq. 7, the whole model can be closed by making parametric variation in the level of debt. The above generalization is not pursued in this paper for the sake of expositional simplicity.

$$= (k_1^2k_2 - k_1k_3ma)p^3 - (k_1k_3mH)p^2 + (k_0k_1k_2 - k_0k_3ma)p - (k_0k_3mH) \quad (15)$$

Let the amount of generally acceptable liquidity for settling margin contraction due to default within the financial system be specified as L, which decreases as expected asset price rises to lure profit-seeking financial firms to substitute in favor of less liquid assets that promise capital gain at higher expected price. This is represented as,

$$L = L_0 - k_4p \quad (16)$$

We define,

$$\Phi = (U - L)$$

as the liquidity available with the financial sector after settlement required for default. Thus $\Phi > 0$ implies a deficit in available liquidity for default settlement.

Substituting the expressions for U and L from Eq. 14 and Eq. 15 we obtain,

$$\Phi = k_1hp^3 - kk_1Hp^2 + (k_0h + k_4)p - (k_0kH + L_0) \quad (17)$$
$$\text{where } k = k_3m,\ h = (k_1k_2 - ka).$$

From Eq. 7, $a < 0$ characterizes the investment-led regime with $h > 0$. However, in the consumption-led regime also $h > 0$ when the condition $[\frac{k_1k_2}{k_3m}] > a > 0$ is satisfied indicating a weakly consumption-led regime with respect to given values of the parameters. In contrast $h < 0$ with $a > [\frac{k_1k_2}{k_3m}] > 0$ indicates a strong consumption-led regime.

Asset price declines as financial firms facing shortage of acceptable liquidity on account of default are forced to fire sale their assets to procure liquidity, often to save reputation.[9] If it occurs on a large enough scale caused by a chain reactions of more stringent margin requirement among mutually indebted financial firms it leads to falling actual asset price which gets transmitted to falling expected asset price, i.e.

$$-\left[\frac{dp}{dt}\right] = \theta[\Phi(p)] = \theta[k_1hp^3 - kk_1Hp^2 + (k_0h + k_4)p - (k_0kH + L_0)], \theta > 0 \quad (18)$$

The price adjustment equation (18) exhibits the impact of funding pressure on asset prices arising from shortage of systemic liquidity due to default. Without loss of generality, to simplify notations, we set $\theta = 1$ in the following discussion.

By setting $\left[\frac{dp}{dt} = 0\right]$, i.e. $\Phi(p) = 0$ the cubic power of equation (18) capturing this process suggest the possibility of multiple equilibria as rest points of

[9] (Bookstaber, R., 2008) narrates various episodes.

this process. In this generic form equation (18) covers not only all three different regimes that are investment led, weakly consumption led and strongly consumption-led, it also contains a time dependent variable H which changes with the level of accumulated debt.[10]

The Cardano discriminant $Q = q^3 + r^2$ provides information necessary about the nature of the roots Abramotwitz, A and Stegun, I. (1964); Poston, T and Stewart, I. (1978) where q and r are given by:

$$q = \left[\frac{k_0 h + k_4}{3k_1 h}\right] - \left[\frac{(kk_1 H)^2}{9(k_1 h)^2}\right]$$

and

$$r = \frac{1}{6}\left[\frac{(kk_1 H)(-k_0 h + k_4)}{(-k_1 h)^2} + 3\frac{(k_0 kH + L_0)}{k_1 h}\right] + \frac{1}{27}\left[\frac{kH}{h}\right]^3$$

We know that (Abramotwitz, A and Stegun, I. (1964))

$Q < 0 \Rightarrow$ all the roots distinct and real
$Q = 0 \Rightarrow$ all real roots with at least two equal
$Q > 0 \Rightarrow$ at least one real root

Since both q and r involving the time dependent variable H, for certain configuration of values of the coefficients of equation (17) the discriminant Q would change sign. The emergence of a state of 3 roots from 1 root means structural instability of Eq. 17 due to changes in the level of debt over time, as it alters qualitatively the dynamics in the phase plane allowing for bifurcation.

By taking recourse to algebraic representation common in Catastrophe Theory,(Poston, T and Stewart, I., 1978; Haken, H., 1983), a formal way of summarizing this argument would be to consider the integral of Φ on the r.h.s in Eq. 18 up to an arbitrary constant. It acts like a potential function $V(p)$ given as

$$V(p) = \frac{1}{4}\left[hk_1\right]p^4 - \frac{1}{3}\left[kk_1 H\right]p^3 + \frac{1}{2}\left[k_0 h + k_4\right]p^2 - \left[k_0 kH + L_0\right]p \qquad (19)$$

This function $V(p)$ can be imagined as an uneven surface on which the debt-driven economy moves like a ball and tends to settle abruptly in a minimum.

[10]Resulting parametric variation of H. Note H is a hybrid term which depends on the exogenously given components of consumption and investment expenditure as well as the cumulative outstanding obligations on borrowing which is affected by asset price (p) and other parameters like interest rate etc not explicitly considered. Thus H would be influenced by the trajectory of p over time, but also by other factors. In this model, we simply treat H as a parameter, influenced partly by the level of outstanding debt.

If there are more than one local minima of unequal depths in the multiple equilibria case, the global minimum is the most stable and the strongest attractor.[11]

Moreover, since H tends to be a continuous time-dependent variable due to accumulation or deccumulation of debt over time, a range of stable states (i.e. minima) may appear as 'branches' with abrupt transition from one branch to another. The derivative of the $\Phi(p)$ function with respect to p, i.e. $\left[\frac{d}{dp}\left(\frac{dp}{dt}\right)\right]$ represents equilibrium or rest points of p on a branch. This corresponds to a quadratic function,

$$\left[\frac{d}{dp}\left(\frac{dp}{dt}\right)\right] = -3k_1hp^2 + 2kk_1Hp - k_0h - k_4 = 0 \tag{20}$$

or,

$$p^2 - \left[\frac{2kk_1H}{2k_1h}\right]p + \left[\frac{(k_0h + k_4)}{3k_1h}\right] = 0$$

The roots are given by

$$\lambda_{1,2} = \left[\frac{2kH}{3h}\right] \pm \sqrt{\left(\frac{kH}{3h}\right)^2 - \left(\frac{k_0h + k_4}{3k_1h}\right)}$$

with the discriminant,

$$\Delta = \left(\frac{kH}{3h}\right)^2 - \left(\frac{(k_0h + k_4)}{3k_1h}\right)$$

.

The characteristic equation will have two real and distinct roots when $\Delta > 0$. The critical value of the level of debt (H_c) for which this condition is satisfied is given by[12]

$$H_c = \left[\frac{3h}{k}\sqrt{\left(\frac{k_0h + k_4}{3k_1h}\right)}\right] \tag{21}$$

Since H_c determines the sign of the discriminant, it determines in turn the possibility of abrupt transitions through bifurcation of the system. The bifurcation may be separated by two loci of stable situations at varying levels of debt. In this rather remarkable case of 'hysteresis'(Nicolis, G and Prigogine, I., 1977, p. 171-78) the economy lingers in one stable branch for some time and then jumps suddenly with catastrophic abruptness to another stable branch. This is analogous to several episodes of financial catastrophe. The various possibilities depend on particular constellation of parameters with plausible parametric values and, simulations are useful to capture the range of possibilities.

[11] However, such dynamics can not be taken for granted in a stochastic system with bistability where the noise can take the system from one minima to the other minima even in the case of unequal depths, and this is shown in the context of ecological systems (Vishwesha, G and Jayaprakash, C., 2007).

[12] The condition H_c is not satisfied in the strong consumption-led regime with $h < 0$, as confirmed by subsequent simulations.

6 A Simulation Experiment

The qualitative algebraic argument of the preceding section is illustrated with a specific configuration of values of the relevant coefficients and parametric variations of H. Unless otherwise stated in all the following simulation exercises we have used the same values for parameters, viz., $k_0 = 0.5, k_1 = 1, k_2 = 1, k_3 = 1, k_4 = 2.5, m = 1, L_0 = 30$ and we used the values of $a = -0.05, -0.5, 1.5$ for investment-led, weakly consumption-led and strongly consumption-led cases respectively (see Eq. 17). The value of H lies in the interval $(-10, 10)$

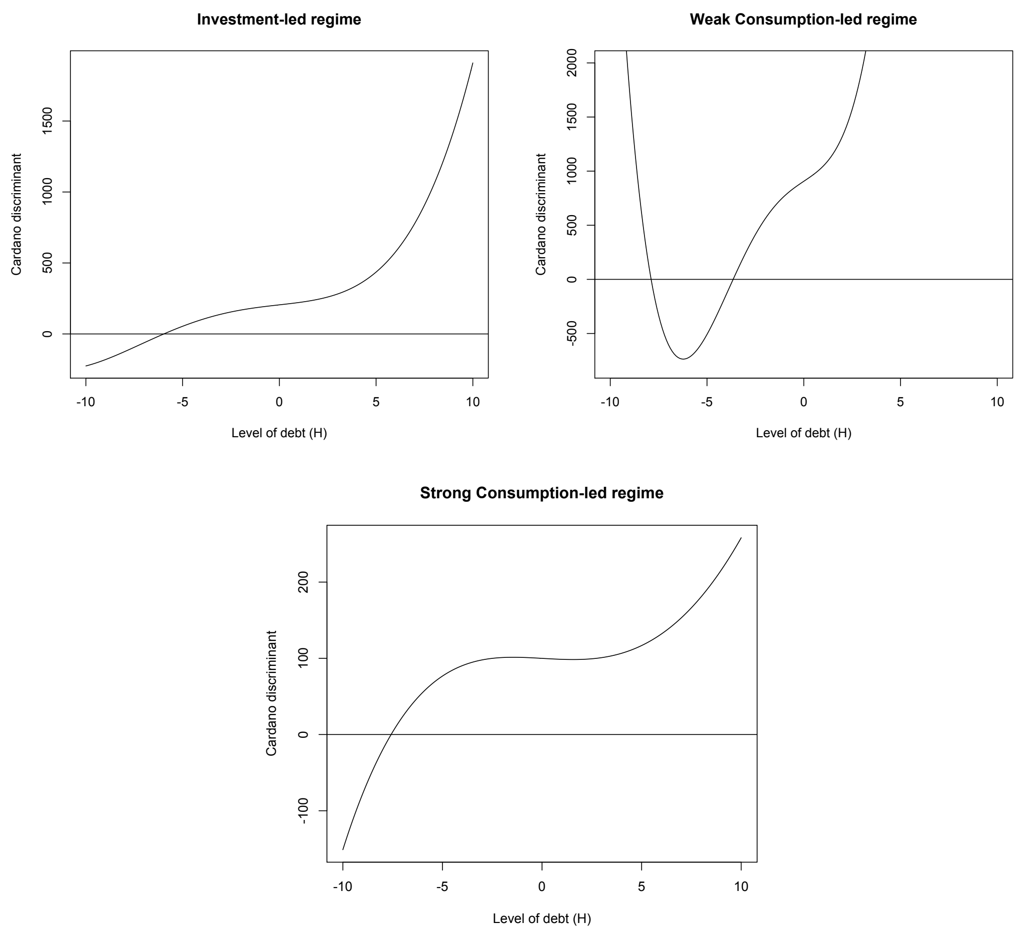

Figure 1: The Cardano discriminant for different values of the parameter h drawn against the different values for the level of debt H.

As seen from Fig. 1, the discriminant changes sign in all the economic regimes as the level of debt along with H varies bringing about transition from one to three real root case. Viewed in conjunction with Descartes's rule of sign, this guarantees bifurcation leading to the emergence of multiple economically meaningful equilibria with positive values of p.

For the same constellation of values, Fig. 2 plots the potential function $V(p)$ corresponding to equation (18).

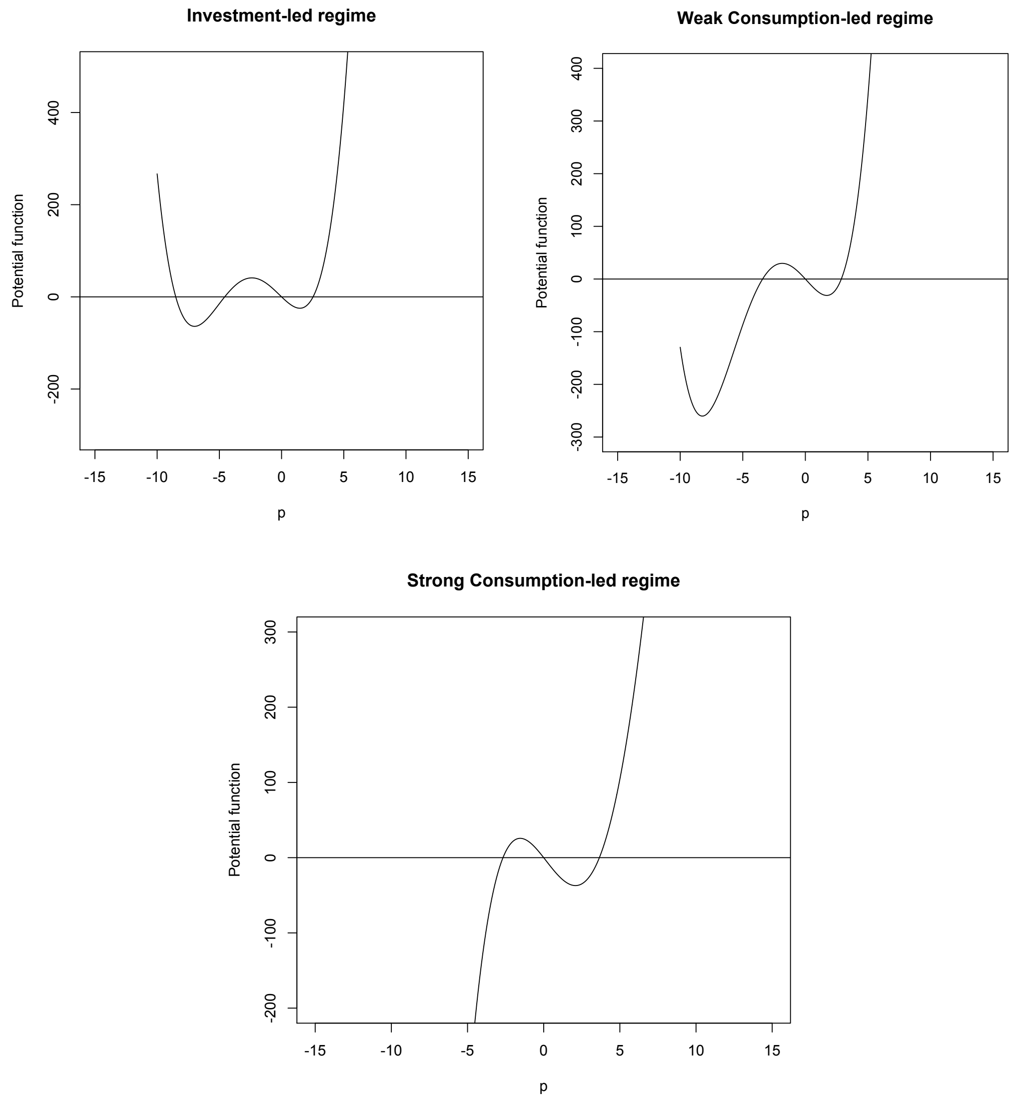

Figure 2: The potential function $V(p)$ is drawn against p for different regimes with the values of the parameters h and H correspond to the cases in Figure. 1.

The extrema, i.e. maxima and minima of the potential function $V(p)$ in (18) are linked with the stability of the roots of the equation $\frac{dp}{dt} = \Phi(p) = 0$. In Fig. 3 a one dimensional phase diagram of $\frac{dp}{dt}$ is plotted against p. The odd roots at the negatively sloped intersection points are stable (minima) and the even roots at positively sloped intersection points are unstable (maxima).

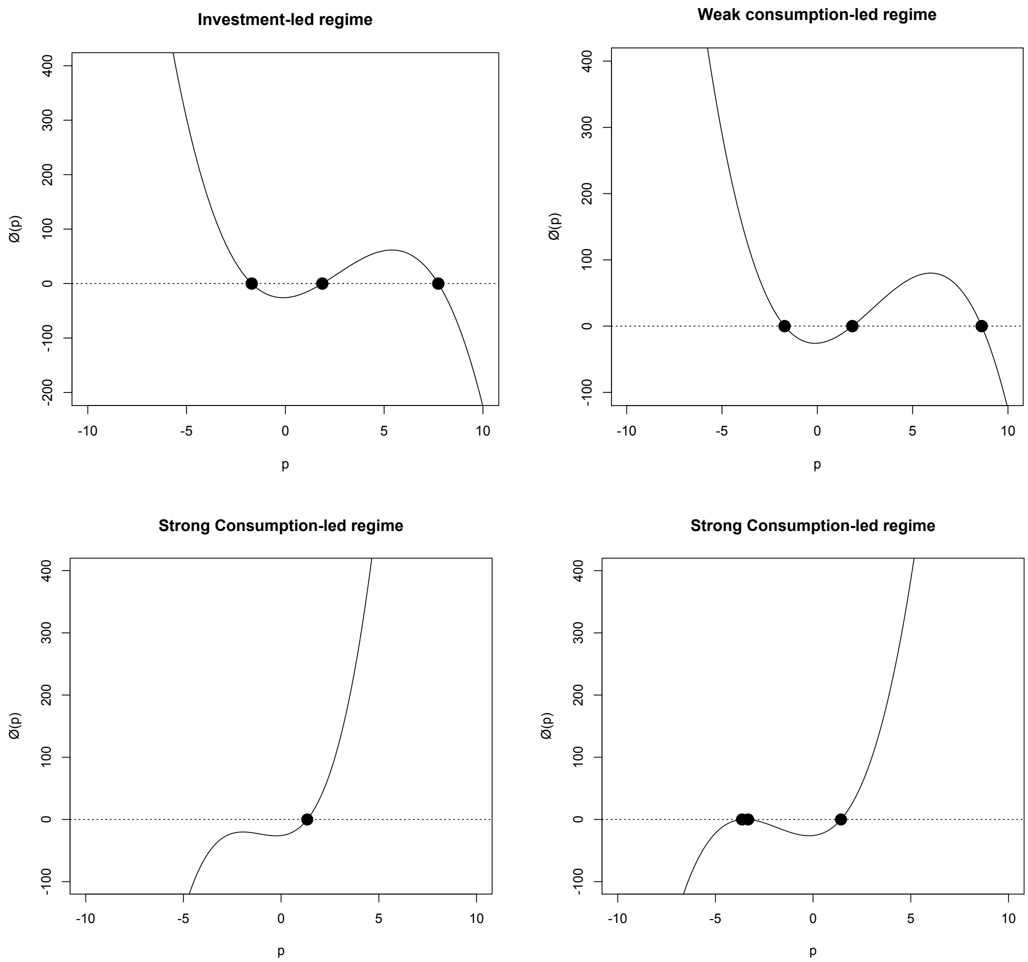

Figure 3: $\Phi(p)$ function drawn against p for different values of h and H. With the same values of a chosen for investment-led, strong and weak consumption-led regimes, and all other parameter values remaining the same. However, the parameter a was varied in the strong consumption-led regime to locate the cases of single and three roots. The numerical values of the roots are indicated by black dots.

In the investment-led regime the odd roots (first and third) are seen to be stable (meaning two minima) and the middle root appears to be unstable (maximum). Similarly, the weak consumption led regime also exhibits two minima of unequal depths and one maxima with the deepest global minimum as the stronger attractor. The strong consumption-led regime does not have stable rest point.

We compute values of the stationary points of p via numerical simulation that characterize the regimes at different values of H i.e. the stationary points of Eq. 20 as $\left[\frac{d}{dp}\left(\frac{dp}{dt}\right)\right] = 0$ for different values of H. The results of the simulation are shown in Fig. 4. It clearly shows that in the investment led regime (and in the weak consumption led regime) we get bistability and hysteresis. However, the strong consumption led regime does not exhibit the property of bistability and hysteresis as expected from our earlier analysis of the regime.

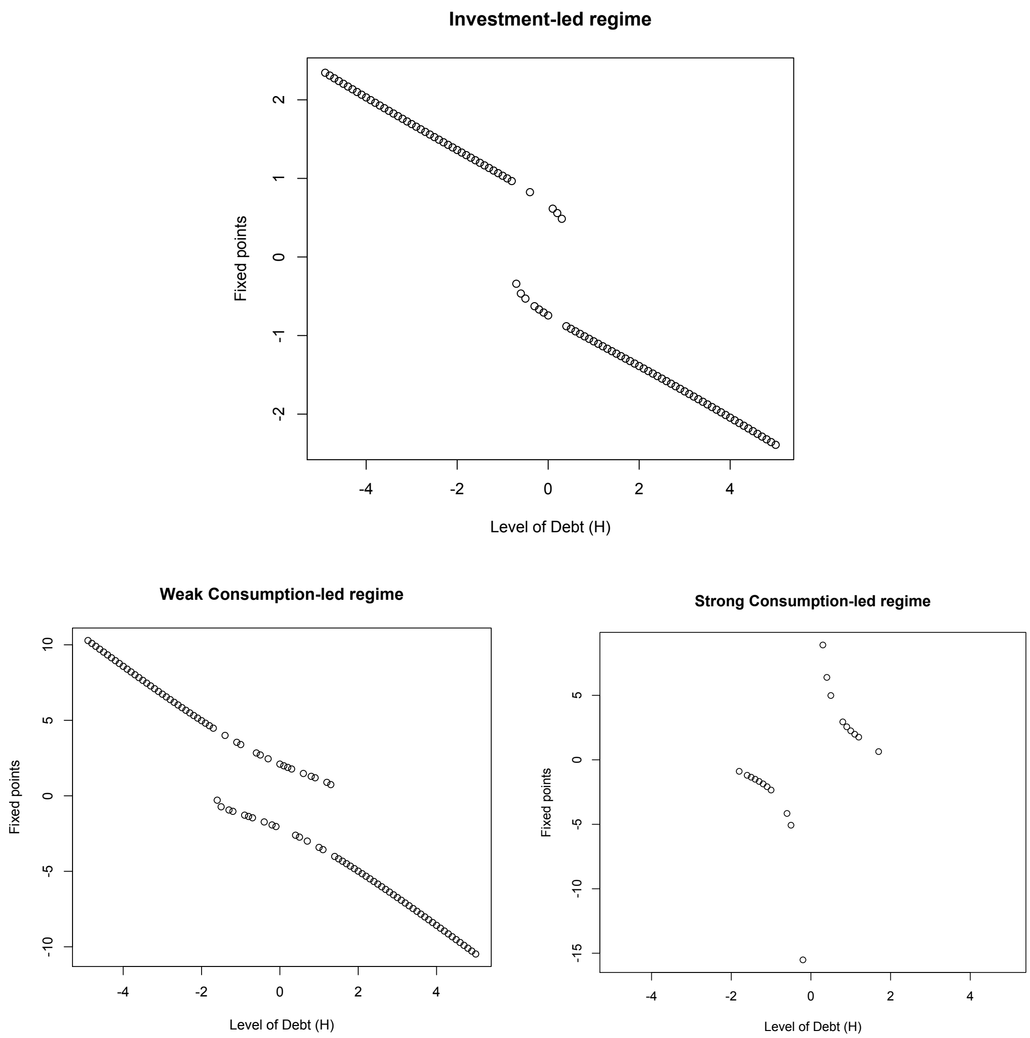

Figure 4: Fixed points of Eq. 20 i.e. $\left[\frac{d}{dp}\left(\frac{dp}{dt}\right)\right] = 0$ are drawn for different values of H. The H values are drawn from the $[-10, 10]$ and the initial conditions for p values are drawn randomly using uniform distribution probabilities from the same interval. The values of a chosen for the investment-led, weak consumption-led and strong consumption-led regimes are (-0.05),(0.5),(1.5) respectively, and all other parameter values remain the same as in Figure.1 through the current one. The bifurcation parameter H is given in the x-axis
.

From Figure. 4 it is clear that abrupt transitions occur in the investment-led (and in the weak consumption-led regime) regime as the value of the level of debt H changes. Both the upper and lower branches of the diagram are stable branches with an unstable middle branch connecting them. For a certain range of values of H the system resides in one stable state on the upper branch until at the critical value of H_c the system abruptly transits from the upper branch to the lower branch of this bistable system. Similarly, if the system is initially seen in the lower branch it will discontinuously transit to the upper branch when the system hits the critical value of H_c, in other words within the context of the model, as the reduction in the level of debt hits the left critical value H_c, the system discontinuously transits to the upper stable branch, and vice versa.

Such a transition is analogous to potential energy of physical systems settling to its one of the minima, in the case of multiple stable states. Within the context of the present economic model it is akin to the system settling to a minimum of liquidity loss within the financial system as it transits from one equilibrium to another equilibrium. It results in catastrophic jump with an abruptness typical of a sudden credit freeze.

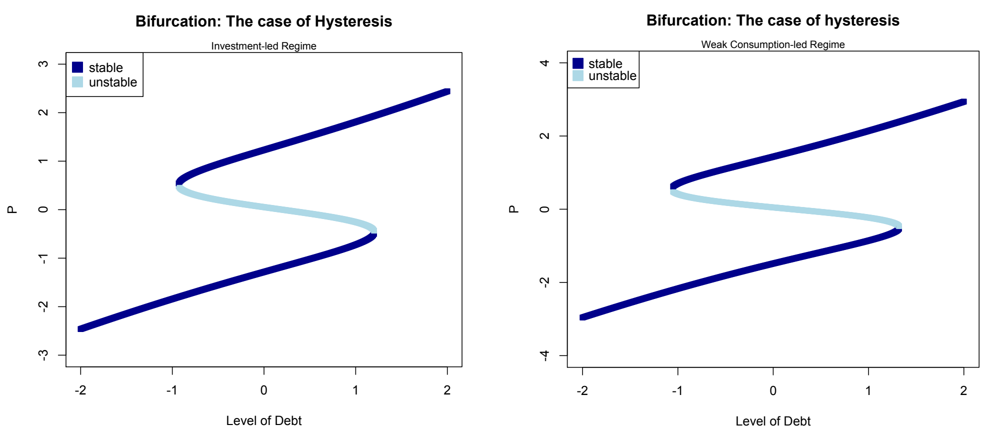

Figure 5: Bifurcation diagram: p versus H.

The abrupt transition from one stable branch to another shown in dark blue curve in Fig. 5. It stimulates the conjecture that the investment-led regime where such jumps are most likely to occur is especially prone to a 'lock-in' effect of positive feedback in asset prices. The dominant composition effect characterizing this regime (see discussion of equation (4)) implies funds are attracted away from real towards financial investment. This raises capital gains on financial assets to breed expectation of still higher capital gains. In other words, a strong sufficiently strong composition effect in favor of financial investment might lead to a declining share of real in total investment and stagnation of demand.[13] In the climate of optimism, the expectations of higher

[13]The empirical verification using real financial data along the line is our ongoing work. Initial explorations with OEX put/call ratio, a ratio of daily put volume divided by daily call volume of the options with the underlying Standard and Poor 100 index taken as a proxy for the speculative money, seem to be encouraging. For instance, the Epanechnikov kernel (and also the Gaussian kernel) density of the series for the period 2007-08 clearly shows bimodality, indicative of bistability. (see also (Zeeman, E.C, 1974; Barunik, J and

capital gains locks the pattern of investment funds in the financial sector over a stretch of time. As a result real investment may decline relative to financial investment during the period leading to a stagnating real sector coexisting with a prosperous financial sector.

7 Conclusion

The model presented above draws a sharp distinction between two different outcomes of debt driven economic expansion. The first type leads to more regular debt cycles analogous in many ways to a class of business cycle models based on various stock flow mechanisms (Eq. 8). However, in all such models 'turning points' are largely explained by an exogenous mechanism setting the thresholds of the ceiling and the floor on debt. When they are set by banks, the explanation reduces essentially to a banking theory of debt cycles in which the availability of credit drives the cycles. In turn it relies on some monetarist version of control over exogenous money supply by the financial authority or the banking sector as the crucial element of the mechanism. Similar argument could be presented on the demand side of credit also e.g. changes in expectations of the public as demanders of credit. However this hardly deepens understanding because the relevant thresholds of debt at which such sudden change in expectation occurs either of the public as demanders or, of banks as suppliers of credit remains unexplained. As explained at some length in Section 4, this is a false trail of argument because the idea of an exogenously controllable money supply is inapplicable in the modern complex system of finance which generates credit endogenously in various ways to exploit all profitable lending opportunities.

Therefore, an alternative theoretical explanation is sought in this paper. It assumes endogenous credit creation and, some other standard features of a sophisticated modern financial sector. to capture the mechanism of sudden collapse of the market. The critical element in this model is the interaction between the real and the financial sector, which rests fundamentally on the Keynesian notion of money as a store of value in an uncertain world. When the real sector performs well over a period of time, agents in the financial sector increasingly meet the growing demand for credit from the real sector by creating private debt instruments. In this way financial firms exploit every profitable lending opportunity through the substitution of liquid assets with less liquid assets usually with a higher prospect of capital gains (see Eq. 16). As a result of this profit seeking activity, increasing systemic illiquidity takes over to make the financial system fragile. It tends to break easily in the face of even a relatively small unanticipated increase in the demand for liquidity resulting from default (see Eq. 14 and 16). This results in distress sale of assets which tends to lower asset prices to deepen further a rapidly developing crisis (see Eqs. 17 and 18).

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The analytical model corroborated by simulations with illustrative values brings out a commonly observed characteristic of financial crisis characterized by sudden collapse in asset prices and credit freeze. This scenario highlights also the problematic role of a hyperactive financial sector which innovates various credit instruments as substitutes for money especially during periods of expansion. Since, these instruments have less wide acceptability particularly for default settlement in times of financial stress systemic illiquidity makes the system highly vulnerable to default. Paradoxically, while in good times such operations of an over active financial sector provides easy credit for purchase of more esoteric financial assets to expand the system, it alters the composition of the total investment against the real sector and makes the financial system to greater illiquidity and fragility. In such a case, a dramatic phase transition from expansion to drastic contraction of credit can occur, particularly in the investment-led regime to trigger hysteresis. In this case due to the lock-in effect of changing asset price the economy lingers in one state to transit suddenly to another state (Figs. 4 and 5).

Debt cycle models akin to business cycle models driven by the stock flow mechanism are insufficient to explain financial crises caused primarily by the capacity of largely unregulated financial sector to innovate money-like credit instruments endogenously. This allows the financial sector to ignore virtually any capital reserve requirements as private instruments, for example derivates substitute for both the guarantee and the regulation of a lender of last resort. In such situation the policy of pumping liquidity into the system to ease a financial crisis might merely strengthen the liquidity position of the financial institutions. However, so long as the real economy is not in a position to demand and absorb this liquidity, it strengthens further the composition effect caused by substitution in favor of financial assets issued by the financial sector with out impacting positively real investment. In extreme cases stagnating real sector which cannot increase its demand for credit coexist with a booming financial sector (see discussion of Eq. 4, 16 and 20); the improved balance sheet of financial firms does not improve conditions in the real economy. A policy of pumping liquidity to the financial sector combined with the austere fiscal policies restraining public investment merely exacerbate such problems to prolong and deepen the crisis.

The analysis resonates to an extent with the ongoing crisis both in the United States and in the Eurozone. In the Eurozone in particular the monetary authority, the European Central Bank, is pursing a monetary policy that increases the availability of liquidity into commercial banks; on the other hand the IMF as a member member of *Troika* along with the ECB, is pushing austerity policies in the real economy for solving the sovereign debt crisis. Their coordinated policy amounts to subordination of the real economy to the interests of the modern financial sector. At the same time, their resistance to accepting expansion of demand in the real economy as an essential condition for overcoming the crisis has increasingly become part of the problem, not of the solution.

Acknowledgement

We are grateful to Lance Taylor, Duncan Foley, Anwar Shaikh and Petri Piiroinen and Srikanth Iyer for their constructive suggestions on the earlier draft of the paper. We also gratefully acknowledge the support of Raghavendra Gadagkar, Govindarajan Rangarajan and the staff at the Centre for Contemporary Studies, Indian Institute of Science, Bengaluru, for this work. All remaining errors are ours.

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