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A Study of Microstructure-Sensitive Crack Nucleation and Wear in Fretting

by

Oliver McCarthy

B.E., National University of Ireland, Galway, 2009.

A thesis submitted to the National University of Ireland as fulfilment of the requirements for the Degree of Doctor of Philosophy

Mechanical Engineering
NUI Galway
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Dr. Patrick McGarry
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Abstract

This thesis presents the development of a computational and experimental methodology for predicting crack initiation and wear in fretting of engineering materials.

A bridge-type fretting fatigue test rig is designed and developed to characterise the fretting fatigue behaviour of 316L stainless steel (SS). The plain and fretting fatigue testing of the material demonstrated a significant reduction in fatigue life due to fretting, as well as a significant effect of normal load.

A cylinder-on-flat crystal plasticity (CP) finite element (FE) fretting model is developed to simulate the experimental test. A $J_2$ cyclic plasticity model of the fretting test rig is also developed for comparative purposes. The CP, based on a UMAT user subroutine, and $J_2$ material constants are calibrated against published cyclic plasticity data for 316L SS. A microstructure-sensitive crack initiation prediction methodology is implemented based on accumulated crystallographic plastic slip as a fatigue indicator parameter (FIP). This accumulated plastic slip parameter is calibrated against the experimental plain fatigue crack initiation data for subsequent application to fretting fatigue crack initiation. Significant non-uniform (inhomogeneous) micro-plasticity effects are predicted by the microstructure-sensitive model due to the representation of inhomogeneous material behaviour at the micro-scale.

A mixed mode short crack growth propagation methodology is implemented for the CP model. The CP predicted total fretting fatigue lives are shown to be in agreement with the experimental data and, more specifically, give improved predictions over a critical-plane Smith Watson Topper (SWT) approach for the $J_2$ model. An experimental fretting life reduction factor due to fretting is reasonably
well captured by the CP model. Furthermore, a novel fretting wear prediction methodology was developed using the microstructure-sensitive crack initiation FIP and the CP model. This method was shown to give reasonable correlation to the measured wear scars and published wear coefficients.

A CPFE model of the cylinder-on-flat fretting wear test rig at the University of Nottingham was developed. CP dual phase and single phase material models were developed and implemented for Ti-6Al-4V within a UMAT user subroutine. Published experimental data based on a laboratory fretting wear test rig is used to validate the microstructure-sensitive prediction methodology. The constitutive behaviour of individual phases of the Ti-6Al-4V material are calibrated against experimental cyclic stress-strain curves (CSSC) from the literature to obtain phase-specific CP material constants. The accumulated crystallographic slip parameter is shown to capture the low cycle fatigue (LCF) response of the material vis-à-vis the published Coffin-Manson relationship. A crystallographic random orientation study demonstrated the ability of the methodology to generate scatter in fatigue life for plain fatigue and fretting situations, akin to measured experimental fatigue scatter. The microstructure-sensitive prediction methodology successfully captured crack initiation lives, crack locations and orientations, when compared to the published experimental data from the University of Nottingham. The microstructure-sensitive fretting damage methodology based on accumulated plastic slip is implemented to distinguish between fretting wear and cracking. This technique was used to predict wear scar profiles which gave reasonable correlation with experimental measurements, and also allowed for the estimation of a wear coefficient for Ti-6Al-4V, that is consistent with the previously published values for the same test rig and Ti-6Al-4V material.
List of publications

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1 Introduction

1.1 General

The current global recession has had a dramatic effect in shaping the mentality of the world’s population regarding efficient and sustainable economic growth. The financial boom that drove the world’s economy has dramatically subsided, leaving behind it a demand for innovative technology at an economic price. Industrial services, especially the engineering sector, are under pressure to keep up with the world's demand for engineering goods and technology at a cost effective price. This demand leads to novel and pioneering methods for reducing cost at every stage of the manufacturing process. Consumers are becoming more concerned with the quality of their purchases and either directly or indirectly the efficiency of the engineering practices is having a major impact on the cost of these products and services. All mechanical devices, from household appliances to gas turbine engines, are comprised of moving parts. These parts are typically subject to a combination of static and cyclic loading conditions, requiring careful design with regards to the desired function. Most appliances or machines have multi-purpose functionality and are subjected to complex loading scenarios and histories which result in a demand for correct part design. This, coupled with a call for reduction in cost and an increase in part life, leads to the need for sophisticated engineering analysis. Machine parts come in contact with each other through mechanical couplings and are typically susceptible to wear and surface damage. A particularly detrimental form of wear is fretting.

Fretting typically occurs where two materials come in contact under a normal (clamping) load and are subjected to a cyclic tangential displacement e.g. due to
vibration. It manifests itself in a wide range of industries and, in fact, based on its simple definition above can be present in any machine where materials in contact are subjected to the relevant loading conditions. Fretting typically has a detrimental effect on component fatigue life. Premature crack initiation and material removal due to wear are typical effects of this phenomenon which need to be fully understood for accurate design solutions to be developed.

1.2 Engineering applications

Engineering is a broad term that encompasses a wide variety of categories. However, regardless of the individual discipline, the same logic and systematic approach to problems solving is practiced. Similar problems need to be overcome within each discipline and many share comparable predicaments. Fretting is a common problem that has been shown to occur in a variety of engineering disciplines. Depending on the application it can have a more catastrophic result. For example in the aerospace industry problems have been reported due to the occurrence of fretting in gas turbine engines. The US air force deems fretting in gas turbine engines to be one of the costliest sources of damage [1]. These turbofan engines, see Figure 1-1, have revolutionised the commercial airline industry as we know it. Their advantages over conventional combustion engines include greater power for a given weight and size. However these technologies requires constant improvement and development. Failure on a gas turbine engine component during operation could have disastrous consequences. For this reason great emphasis is placed on understanding and minimising the risk of such an occurrence. Two of the main areas that are affected by fretting are turbine root connections and spline couplings.
The turbine blades within a gas turbine engine are connected to the shaft and fan disc via dovetail joints. These dovetail joints are typical locations where fretting occurs. A simple schematic shown in Figure 1-2 gives an idea of the loading conditions experienced by a turbine blade; when combined with the complex loading conditions imposed by take-off and landing, as well as in-flight manoeuvres, one can appreciate the importance of these connections. Unfortunately fretting is a typical problem that arises in the contact areas highlighted in Figure 1-2.

Spline couplings are used to transmit torque between parallel in line shafts within the turbine engine. They consist of external and internal splined shafts that slide into each other. This results in a set of contacting surfaces between the internal and external spline teeth. The complex loading conditions experienced by spline coupling shafts and the contact surfaces between the two components make it a prime candidate for the presence of fretting.

Fretting is not solely restricted to larger engineering applications. Biomedical applications are affected by its presence in a number of different situations such as hip replacements and osteosynthesis plates and screws [2]. With about 2 million total hip arthroplasty (THA) surgeries every year in Europe and the USA, THA is one of the most common implantable prosthetics available. However regardless of it being one of the most innovative and successful medical surgeries of the 20th and 21st centuries nearly 10% of THA cases require reintervention [3]. To limit the amount of unsuccessful surgeries significant work has been carried out on improving and further developing the materials and implant designs. One such area that has had significant work is fretting fatigue, also referred to as fretting corrosion. Fretting damage has been known to occur between contacting surfaces at the taper hip joint and between the bone and hip stem, as seen in Figure 1-3, and between bone-plates.
and screws. This is caused by the cyclic loading induced by walking, for example. The material removal process can release toxic particles into the bloodstream that are typically in a chemically inert state on the implant surface. Once this passivation layer has been removed, new metal will be exposed to the aggressive environment within the body. Such a problem occurred with Stryker Rejuvenate and ABG II modular hip stems when a nationwide voluntary recall of implants took place during the summer of 2012. It was reported that fretting corrosion had the potential to occur at the modular-neck junction. In some cases the stem of the hip implant is bolted directly to the femur instead of using bone cement. The small surface area between the screw head and plate leads to high contact pressures; this, combined with the relative displacement induced by walking results in conditions susceptible to fretting.

Offshore oil and gas companies are highly motivated to understand and better design against fretting damage. Marine flexible risers, as shown in Figure 1-4 (a), are designed to transport oil from the seabed to the surface, while undergoing large deformations from combined torque and bending loads. The flexible riser is comprised of numerous different layers each designed to perform a different function. The helically wound interlocking layers of the pressure armour are responsible for structural integrity during these loading situations. However this is a prime location for fretting damage to occur [4], see Figure 1-4 (b). Work carried out by Zhang [5] and co workers deals with a specific example of the complex loading situation found within the pressure armour.

In large structures such as cable bridges or transporting mechanism such as lifts, cranes etc., high tension cables are used for structural support. These cables have excellent mechanical properties and are typically made up of multiple strands helically wound around a central wire. Figure 1-5 shows a simple schematic of the
wire rope construction. Due to the cyclic loading resulting from traffic and wind or the sudden increase or decrease in wire tension, both cable bridges and transport mechanism cables, are subjected to varying fatigue loads. These large loads result in small relative displacements within the helically wound strands and typically fretting fatigue is induced. A second cause of damage results when the wire ropes run over a pulley, for example, and the variable tension causes a change in contact pressure and displacement amplitude within the individual wire strands [6].

1.3 Materials

While it is impossible to cover all materials used in engineering applications, some are more widespread due to their attractive properties e.g. density, corrosion resistance, cost and mechanical properties. Two metals that incorporate such qualities, and are used in applications prone to fretting, are 316L SS and Ti-6Al-4V.

- 316L SS is a widely used austenitic stainless steel. Its corrosion resistance properties make it an ideal candidate for biomedical implants and marine environments. It is easily machine able due to the low carbon content and still maintains excellent toughness properties. Due to its versatility it is used in a wide selection of areas that are prone to fretting and is one of the two materials studied in this work.

- Ti-6Al-4V is one of the most widely used titanium alloys; it accounts for roughly 50% of the titanium used in the world [7]. It is most notably used in aerospace applications, for high strength, low weight, heat and corrosion resistance. These superior qualities over other structural materials such as aluminium, nickel and ferrous alloys result in it being an accepted material for military and commercial jets. Titanium is used in numerous aerospace
applications where composite or aluminium materials would be ineffective; for example, Ti-6Al-4V is a suitable material of choice for airplane and rocket manufacture because of its high heat resistance. Titanium is galvanically compatible with polymer matrix composites (PMC) which is another advantage over conventional structural materials [8]. Ti-6Al-4V is a dual phase alloy made up of α and α-β phases. The α phase consists of a HCP microstructure while the α-β phase combines both BCC and HCP slip systems in a lamellar colony. These phases represent the crystallographic structure of the alloy and give it beneficial properties suitable for a number of different structural applications. The dual phase combination gives the alloy the strength of the α phase and the ductility of the α-β phase.

1.4 Aims and objectives

The aim of this thesis is to develop and experimentally validate a microstructure-sensitive methodology to predict fretting crack nucleation and wear. The main objectives are:

1. To design and develop an experimental fretting fatigue test capability.
2. To develop an FE-based microstructure-sensitive crack initiation and wear prediction methodology for fatigue and fretting fatigue.
3. To experimentally characterise 316L SS microstructure and plain and fretting fatigue behaviour.
4. To develop an FE microstructure-sensitive frictional contact fretting model based on the experimental test rig.
5. To predict and validate the number of cycles to crack initiation and propagation against experimental data via a microstructure-sensitive crack
Chapter 1

initiation methodology and a mixed mode micro-crack propagation methodology based on short crack growth.

6. Investigate an extension of the microstructure-sensitive crack initiation method for wear prediction.

1.5 Scope of thesis

Chapter 1 gives an insight into the general motivation and understanding behind the research topic of the thesis. It describes the application areas where fretting occurs and the impact that it has on the general population. It provides details of the main objectives as well as the main methodologies implemented within the work.

Chapter 2 presents a detailed account of the current literature associated with experimental and computational modelling of fretting damage. It contains a detailed description of the first analytical contact mechanics equations based on elastic solids as well as rate-dependent crystal plasticity (CP) theory. It highlights the importance of experimental testing and some approaches for capturing fretting damage through experimental (laboratory) methods. This chapter also gives an overview of the current prediction methodologies employed for life estimation under fretting conditions.

Chapter 3 is concerned with the development of a fretting fatigue test capability using a fretting bridge-type arrangement. Plain and fretting fatigue specimens are manufactured from 316L SS and a number of material characterisation methods are described. Fretting fatigue and plain fatigue tests are carried out, using a cylinder-on-flat contact situation for the former, and the results are compared to assess the effect of fretting for 316L SS. Microscopy of the failed
specimens resulted in SEM images of fracture, wear scar profiles and evidence of fretting corrosion.

Chapter 4 describes the development of a FE methodology for microstructure-sensitive plasticity and crack initiation in fretting for 316L SS. The approach adopted is based on a previously-published method [9] for microstructure-sensitive (uniaxial) fatigue life prediction, which proposes the use of a unit cell CP model to identify the critical value of accumulated plastic slip associated with crack initiation. This approach is successfully implemented here, using an FCC unit cell CP model, to predict the plain low-cycle fatigue behaviour of a stainless steel. Furthermore, a CP frictional contact model for stainless steel is developed for microstructure-sensitive fretting analyses. A methodology for microstructure-sensitive fretting crack initiation is presented, based on identification of the number of cycles in the fretting contact at which the identified critical value of accumulated plastic slip is achieved. Significant polycrystal plasticity effects in fretting are predicted, leading to significant effects on evolution of contact pressure, fatigue indicator parameters (FIP) and microstructural accumulated slip. The CP fretting predictions are compared with J2 continuum plasticity critical plane SWT predictions. The microstructural accumulated plastic slip parameter is a powerful technique to unify the prediction of wear and fatigue crack initiation (see Chapter 5), leading in some cases, e.g. gross slip, to wear, via a non-localised distribution of critical crystallographic slip, and in other cases, e.g. partial slip, to fatigue micro-crack initiation, via a highly-localised distribution of critical crystallographic slip with preferred orientation (cracking locations and directions).

In Chapter 5, the microstructure-sensitive fatigue and fretting fatigue approach is applied to Ti-6Al-4V, which is a complex material with attractive
mechanical properties used in areas involving intricate contact loading conditions. The micro-structure of Ti-6Al-4V is modelled as a duplex material with individual micro-mechanical material properties for each phase. Previously published experimental data on the characteristic wear and crack nucleation behaviour of Ti-6Al-4V is employed for validation of the microstructure-sensitive predictions. A CPFE model is used to predict the crack nucleation life under a cylinder-and-flat fretting arrangement. Furthermore the microstructure-sensitive methodology is used to distinguish between different fretting damage regimes, e.g. cracking or wear. A new methodology for wear scar prediction based on the microstructure-sensitive crack initiation method is presented. Wear scar profiles produced by the critical plastic slip parameter provide good comparison with the experimental data, while excellent agreement is obtained between predicted and measured wear coefficients.

Chapter 6 is concerned with the validation of the new micro-mechanical methodology for prediction of fretting fatigue crack nucleation life against the fretting tests of Chapter 3. A short crack propagation methodology for the CP fretting model is also developed to facilitate comparison with the experimental results which is total life data. CP unit cell models of the FCC microstructure are again employed for calibration of constitutive and crack nucleation parameters against the measured 316L SS fatigue data from this thesis. A CP friction contact model of the fretting test rig is developed to predict the micro scale surface fretting damage, crack nucleation and estimated micro-crack propagation. The new method is shown to be superior to a combined $J_2$ plasticity and critical-plane FIP method.

Chapter 7 presents the conclusions of the thesis and makes recommendations for future work.
1.6 References


1.7 Figures

Figure 1-1. Image of a typical gas turbine used in aerospace.

Figure 1-2. Turbine dovetail connection highlighting loading conditions and contact regions responsible for fretting.
Figure 1-3. Locations of fretting damage in a hip replacement situation.

Figure 1-4. Simple schematic of a marine flexible riser highlighting the main areas of fretting damage.
Figure 1-5. Simple construction of wire rope [6].
2 Literature Review

2.1 Introduction

The following literature review covers the main topics associated with understanding fretting wear and fatigue. A summary of relevant knowledge and previous work is presented below. The subsequent sections cover a range of engineering topics that are key to understanding and analysis of fretting. Initially the problem of plain fatigue and fracture mechanics is introduced without the complexity of the contact load. Subsequently, a summary of contact mechanics and tribology are presented which leads to a detailed account of fretting damage (fatigue and wear). The review also details the existing relevant experimental and microstructural modelling research in this field. A summary of the key developments of this work with respect to fretting damage and crack prediction is also presented.

2.2 Fatigue

2.2.1 Introduction

Fatigue can be defined as the accumulation of damage typically at a microstructural level as a result of cyclic loading. This is an important phenomenon that occurs over a wide range of loading situations, but more importantly it can occur at load levels below the observed monotonic elastic limit of the material. Fatigue is normally categorised as either low or high cycle fatigue depending on the number of cycles to failure, $N_f$. Total life $N_f$ can subsequently be broken into crack initiation, $N_i$, and propagation, $N_p$, life where $N_f = N_i + N_p$. This topic shall be presented later in greater detail highlighting subcomponents of different crack growth regimes.
2.2.2 High cycle fatigue

Metal fatigue is not a new subject for discussion. Work first published in the mid 1800's has characterised the occurrence of this phenomenon. Pioneers in the field such as Wöhler have shaped our understanding of the current problem. One of the first documented studies including fatigue testing is attributed to Wöhler who began testing railway axles in the 1850's. In a global or macroscopic sense a material can be cyclically loaded numerous times without any observable damage even to the point of where sudden fracture occurs. The damage is localised and occurs at a microscopic scale. Even if the cyclic loading is below that of the elastic limit, fatigue damage can still occur. Metals are made of multiple crystal lattices (metallic grains), in a repeated unit of an atomic structure [1]. Depending on the material in question the atomic structure, and therefore the metallic grains, have a certain number of slip planes and slip directions. Combined together these result in slip systems where slip or dislocation flow can occur. Until Taylor [2], in 1934, made the discovery of dislocations within the crystal lattice, it was not fully understood that slip could occur in a metal at shear stress values much lower than that required to break a perfect crystal.

Each metallic grain has an orientation assigned to it which results in the possibility of a slip system orientated favourably in the direction of loading. This, combined with the presence of dislocations, results in localised microscopic damage at load levels less than the yield stress of the material. Wöhler initially characterised metal fatigue via an S-N curve. This shows the number of cycles to failure $N_f$ as a function of stress amplitude, $\sigma_{ap}$, or max stress, $\sigma_{max}$. Using this curve a fatigue limit can be observed, below which no damage occurs. This limit is normally taken as above $10^7$ cycles.
According to Lemaitre [3] high cycle fatigue is defined as \( N_f > 10^5 \) cycles and poses difficulty in accurately determining the number of cycles to failure as a result of plastic strain dissipation on a microscale level. For that reason a stress life approach is normally used. The Basquin equation is based on a log-log relationship between stress amplitude and number of cycles to failure, as follows:

\[
\frac{\Delta \sigma}{2} = \sigma_f' \left( 2N_f \right)^b
\]  

(2.1)

where \( \Delta \sigma \) is the stress range, \( \sigma_f' \) is the fatigue strength coefficient and \( b \) is the fatigue strength exponent. The Basquin equation is relevant to situations where macroscale plastic strains are small, i.e. elastic behaviour is dominant. However, it is well known that micro-plasticity exists in metallic materials under macroscopically elastic conditions. Hence, crack nucleation sites are created from the formation of persistent slip bands from local plastic deformation.

2.2.3 Low cycle fatigue

Low cycle fatigue is defined as \( N_f < 10^4 \) cycles, where plastic strain is the dominant failure parameter. This occurs once the (maximum) stress exceeds the elastic limit. As a result a second method for quantifying fatigue life is required based on or incorporating plastic strain. The Coffin-Manson equation expresses \( N_f \) as a function of plastic strain range, \( \Delta \varepsilon_p \), as follows:

\[
\Delta \varepsilon_p = \varepsilon_f' \left( 2N_f \right)^c
\]  

(2.2)

where \( \varepsilon_f' \) and \( c \) are material constants, the fatigue ductility coefficient and ductility exponent, respectively.

2.2.4 Smith-Watson-Topper (SWT)

The Smith-Waston-Topper (SWT) parameter [4] is an example of a fatigue indicator parameter (FIP), which is normally considered to be a total life prediction parameter.
An FIP quantifies the severity of fatigue damage within a loading situation based on stress, strain or a combination of the two. These FIPs are typically based on constants obtained from experiments showing failure cracks in the region of 1 mm [5], since it is typically easier to quantify the number of cycles to failure than the number of cycles to crack initiation. In choosing an FIP for a given application it is important to note the suitability of each parameter for the particular stress state and cracking behaviour [6] and [7]. Socie [5] makes recommendations for each mode of failure and corresponding FIP; for example, for Mode I (tensile), SWT is recommended, while for Mode II failure (shear), the Fatemi-Socie parameter (see below) is recommended. The SWT parameter is a combination of the Coffin-Manson and Basquin equations for low and high cycle fatigue, respectively, multiplied by a \( \sigma_{max} \) term to account for mean stress effects as follows:

\[
SWT = \frac{\sigma_{max}}{2} \frac{\Delta \varepsilon}{2} = \left( \frac{\sigma'_f}{E} \right)^2 (2N_f)^{2b} + \sigma' f \varepsilon'_f (2N_f)^{b+c}
\]

(2.3)

where \( \sigma'_f \) is the fatigue strength coefficient, \( \varepsilon'_f \) is the fatigue ductility coefficient, \( b \) is the fatigue strength exponent, \( c \) is the fatigue ductility exponent and \( E \) is Young's modulus. Acknowledging the fact that crack growth is a complex problem and that cracks can initiate and grow at any angle, this parameter has been adopted within a critical plane approach to allow multiaxial effects to be incorporated [8]. This allows SWT to predict the "damage" per cycle on any given orientation. In the implementation of Sum et al. [9] the critical plane approach calculates the SWT parameter combining the peak normal stress \( \sigma_{max} \) and the strain range \( \Delta \varepsilon \) within one cycle for a range of plane orientations at 5° intervals through a 180° range. SWT has been used to produce realistic and accurate results compared to experimental data for
fretting fatigue of both Ti-6Al-4V [10], [11] and CrMoV aero-engine steel [9], [12] and in other work such as Araujo and Nowell [7] and Szolwinski and Farris [13].

2.2.5 Fatemi-Socie

Fatemi-Socie (FS) is an alternative FIP based on shear strain range [6], $\Delta \gamma$, which also includes the effect of mean stress as follows:

$$FS = \frac{\Delta \gamma_{\text{max}}}{2} \left(1 + \alpha_{FS} \frac{\sigma_{\text{max}}}{\sigma_y}\right) = \frac{\tau_c}{G} (2N)^{b_y} + \gamma_c (2N)^{c_y}$$  \hspace{1cm}  (2.4)

where $\Delta \gamma_{\text{max}}$ is the maximum shear strain range, $\alpha_{FS}$ is the normal stress sensitivity, $\sigma_{\text{max}}$ is the maximum normal stress, $\sigma_y$ the material yield stress, $G$ the shear modulus and $\tau_c$, $b_y$, $\gamma_c$ and $c_y$ are parameters similar to the ones used in the SWT equation where

$$\tau_c \approx \frac{\sigma_f}{\sqrt{3}}, \quad \gamma_c \approx \varepsilon_f \sqrt{3}$$  \hspace{1cm}  (2.5)

$$b_y \approx b, \quad c_y \approx c$$  \hspace{1cm}  (2.6)

The advantage of SWT and FS is the ability to consider the effects of mean stress on crack initiation and propagation. A negative or zero mean stress has a beneficial effect on crack growth while a positive mean stress increases the fatigue damage per cycle. Also, a further benefit is the ability to span both the (low cycle fatigue) LCF and high cycle fatigue (HCF) range.

2.2.6 Dang Van

The Dang Van [14] criterion hypothesises that cracks initiate within grains where plastic shakedown has occurred. Within a macroscopically elastic stress state localised plasticity will occur in favourably orientated grains, i.e. orientations that are aligned with the loading direction. The criteria hypothesises that crack initiation occurs when
\[ f(\sigma) > 0 \] (2.7)

where \( f(\sigma) \) is defined by the following equation:

\[ f(\sigma) = \tau + a_5 P^* - b_T \] (2.8)

where \( P^* \) is the hydrostatic tension and \( \tau \) is the shear stress. \( a_5 \) is related to the sensitivity of the material to the hydrostatic stress and \( b_T \) is the fatigue strength determined from a torsion test. More detail can be found in Araujo et al. [15]. An alternative method is seen in Dick and Cailletaud [16] where the Dang Van parameter can be used with the von Mises stress range \( R \) as described by:

\[ f(\sigma) = R + aP^* - b_T \] (2.9)

Dang Van differs from other fatigue parameters as it is ostensibly based on microscale considerations.

2.2.7 Plain fatigue testing

Since Wöhler there have been numerous advancements in testing equipment and testing procedures. Universal standards from both ASTM and ISO have created benchmark fatigue test methods that ensure validity and repeatability of experiments. However a large number of factors influence the behaviour of fatigue life. Mean stress has a significant effect; for example, a tensile stress mean stress is detrimental to fatigue life while a compressive mean stress is beneficial [17]. The effect of stress amplitude and mean stress on fatigue limit has been represented as constant life lines in the stress region \( (\sigma_a - \sigma_m) \), as shown in Figure 2-1, where \( \sigma_a \) and \( \sigma_m \) are the amplitude and mean stress respectively.

The modified Goodman equation assumes a linear effect of mean stress with static failure corresponding to the UTS as follows:

\[ \frac{\sigma_{app}}{\sigma_e} + \frac{\sigma_m}{\sigma_{UTS}} = 1 \] (2.10)
The Soderberg equation also assumes a linear relationship between the mean stress effect and the static yield stress of the material as follows:

\[
\frac{\sigma_{app}}{\sigma_e} + \frac{\sigma_m}{\sigma_y} = 1
\]  

(2.11)

The Gerber equation assumes a non-linear (parabolic) effect of mean stress, as follows:

\[
\frac{\sigma_{app}}{\sigma_e} + \left(\frac{\sigma_m}{\sigma_{UTS}}\right)^2 = 1
\]  

(2.12)

where, \(\sigma_e\), \(\sigma_{UTS}\) and \(\sigma_y\) are the fatigue limit, UTS and yield stress respectively.

Other important factors influencing the stress-life behaviour of a metal are microstructure size effects, loading frequency and surface finish. The importance of the microstructure and size effects will be discussed later on in Section 2.2.8 Microstructural modelling.

Frequency effects are complicated but for the most part negligible for standard fatigue tests. Generation of heat due to cyclic loading can cause a significant effect in large specimens as a result of the volume of material that is being stressed. In general if the frequency of the test is less than 200 Hz there is an insignificant frequency effect [17]. Surface finish can have a pronounced effect on fatigue life and typically result in stress concentrations. Surface deformities and microstructural anomalies are the usual location for cracks to initiate; therefore added surface roughness or damage from manufacturing can significantly affect the fatigue life of a component [18]. Obviously due to cost and time constraints not all engineering materials can be ground or polished to a fine finish. However a design surface factor, \(k_s\), exists that represents the effect of surface finishes on fatigue life. Materials with higher UTS values are more influenced by surface finish, because a
greater percentage of cycles to failure is spent in the nucleation stage and are hence affected more significantly by surface defects.

2.3 Fracture mechanics

2.3.1 Linear elastic fracture mechanics

Linear elastic fracture mechanics (LEFM) is concerned with the stress required to grow cracks through a structure based on elastic isotropic assumptions. In practical engineering it is impossible to achieve a perfect smooth surface free of defects. It is possible however to detect cracks up to a certain measurable crack length. Using this initial crack length, growth equations can be employed to estimate the fatigue life of the component. One of the most important advancements in crack growth analysis came about in the 1960's through the Paris equation:

\[
\frac{da}{dN} = C (\Delta K)^m
\]  

(2.13)

where \(C\) and \(m\) are material constants, \(da/dN\) is the change in crack length per cycle (crack growth rate), \(a\) is the crack length and the stress intensity factor \(\Delta K\) is

\[
\Delta K = \Delta \sigma Y \sqrt{\pi a}
\]  

(2.14)

with \(\Delta \sigma\) equal to \((\sigma_{max} - \sigma_{min})\) and \(Y\) is a geometrical factor. Plotted on a log plot it is possible to see the linear relationship between crack growth rate, \(da/dN\), and stress intensity, \(\Delta K\). However two additional non-linear regimes of crack growth exist above and below the Paris regime [18], as seen in Figure 2-2. Regime 1 represents short crack growth which occurs at a much faster rate than in the Paris regime. \(\Delta K_{th}\) is the threshold stress intensity factor below which crack arrest occurs. Once \(\Delta K\) increases beyond the threshold value, \(\Delta K_{th}\), cracks propagate at a much faster rate until Regime 2. Regime 2 shows a linear relationship between crack
growth rate and stress intensity factor, the Paris regime. Regime 3 highlights the catastrophic nature of crack growth as $\Delta K$ approach as the fracture toughness of the material, $\Delta K_{fc}$. It is obvious from Figure 2-2 that the Paris equation cannot accurately predict the propagation rates of short cracks. Experimental work carried out by Obrtlik et al. [19] on short crack growth in 316L SS highlights the accelerated crack growth rates of short cracks compared to long crack growth for a given stress intensity factor.

2.3.2 El Haddad short crack growth

The El Haddad approach [20] incorporates a threshold crack length, $a_0$, which represents the transition from short crack growth (SCG) to conventional crack growth, as can be seen from the Kitagawa and Takahashi [21] diagram in Figure 2-3. $a_{th}$ can be calculated from the following equation [12]

$$a_{th} = \frac{1}{\pi} \left( \frac{\Delta K_0}{\sigma_e} \right)^2$$  \hspace{1cm} (2.15)

where $\sigma_e$ is the fatigue limit and $\Delta K_0$ is the threshold stress intensity factor $\Delta K_{th}$ for $a > a_{th}$. The El Haddad approach is an empirical method to allow for the prediction of short crack growth, by substituting $(a + a_{th})$ for $a$ in the Paris equation when $a < a_{th}$ as follows:

$$\Delta K = \Delta \sigma Y \sqrt{\pi(a + a_{th})}$$  \hspace{1cm} (2.16)

where $Y$ is the geometry factor. This crack growth prediction methodology can be implemented within a weight function method [22] for mixed mode cracking analysis. Houghton et al. [12] implemented this technique in a frictional contact FE model for crack growth prediction in a multiaxial representative fretting test for spline couplings based on the work of Nicholas et al. [23] who analysed crack growth for a rounded punch-on-flat fretting test rig.
2.4 Contact mechanics

2.4.1 Introduction

Contact mechanics is the study of the deformation of solids in contact. Hertz (1882) is credited with being the first to explore the problem with the publication of his paper [24]. This is a well known analytical theory which is limited to smooth frictionless elastic materials but is still used extensively as a tool for contact problems; see [25] [26] [27], for example.

2.4.2 Hertzian elastic contact

The following formulae are derived for a two-dimensional cylinder-on-flat contact model. Figure 2-4 shows a simple schematic typical of two bodies in contact. The Hertzian stress distribution is plotted in Figure 2-5 and is given by:

\[ p(x) = \frac{2P}{\pi E^*} \left( a^2 - x^2 \right)^{\frac{1}{2}} \]  \hspace{1cm} (2.17)

where \( x \) is the horizontal distance from the centreline, \( a \) is the contact semi width and \( p_o \) is the maximum contact pressure defined by

\[ a = \left( \frac{4PR}{\pi E^*} \right)^{\frac{1}{2}} \]  \hspace{1cm} (2.18)

and

\[ p_o = \left( \frac{PE^*}{\pi R} \right)^{\frac{1}{2}} \]  \hspace{1cm} (2.19)

where \( P \) is the normal force and \( R \) is the relative curvature,

\[ R = \left( \frac{1}{R_f} + \frac{1}{R_c} \right)^{-1} \]  \hspace{1cm} (2.20)

where \( R_f \) and \( R_c \) are the radii of each contacting surface. \( E^* \) is the composite modulus between the two contacting materials defined as:
where $E_f, v_f$ and $E_c, v_c$ are the elastic moduli and Poisson's ratio for the flat and cylinder, respectively.

2.4.3 Elastic sliding of contacting surfaces

Once a tangential force, $Q$, is applied to a body in contact under a normal load, $P$, surface shear tractions will be introduced. The shear force required to cause sliding is $\mu P$, where $\mu$ is coefficient of friction (COF). When $Q$ is less than $\mu P$, the contact region will consist of two areas, a contact stick zone in the centre where no relative displacement occurs and a slip zone, surrounding the stick region, where small relative displacements occur. This is called a partial slip contact. With increased tangential force the stick region decreases in size until eventually gross slip occurs.

Coulomb friction assumes that; for $Q < \mu P$:

$$ q(x) \leq \mu p(x) $$

(2.22)

so that the shear traction $q(x)$ at any point $x$ is less than the product of $\mu$ and the contact pressure $p(x)$. When $Q = \mu P$, $q(x) = \mu p(x)$, according to Coulomb friction, so that for a cylinder-on-flat contact, when sliding is about to occur the shear traction is given by:

$$ q'(x) = \mu p_0 \left( 1 - \frac{x^2}{a^2} \right)^{\frac{1}{2}} $$

(2.23)

When $Q < \mu P$, the stick region width is $2c$, where

$$ c = a \left( 1 - \frac{Q}{\mu P} \right)^{\frac{1}{2}} $$

(2.24)

The shear traction in the stick region is given by
\[ q''(x) = -\mu \frac{c}{a} p_0 \left( 1 - \frac{x^2}{c^2} \right) ^{\frac{1}{2}} \]  

(2.25)

A condition of no slip must be present within the range of \(-c \leq x \leq c\) and a condition of \(q(x) = \mu p(x)\) is present when \(-c \leq x \leq a\). Adding both \(q^{(x)} + q''(x)\) results in the resultant shear traction plot of Figure 2-6 for partial slip. It is important to note that if \(P\) is kept constant and \(Q\) is increased slip begins immediately and increases inwards until the stick region \(2c\) decreases to a point. Any attempt to increase \(Q\) beyond \(\mu P\) results in sliding.

2.4.4 Inelastic contact

The normal load required to cause yielding is given by the following equation:

\[ P_y = \frac{\pi R}{E^*} (p_o)_{y}^2 \]  

(2.26)

where

\[ 1.8\sigma_y = (p_o)_{y} \]  

(2.27)

where \(\sigma_y\) is the yield stress of the material and \((p_o)_{y}\) is the peak pressure at yield.

2.5 Tribology

2.5.1 Introduction

Tribology is defined as the science and technology of interacting surfaces in relative motion [28]. It deals specifically with friction, wear and lubrication. It occurs as a result of surface asperities that are present at a microscopic level regardless of how flat or smooth the surface appears. These imperfections, along with molecular adhesion forces between the two surfaces, cause friction.
2.5.2 Friction

The study of tribology dates back to Leonardo da Vinci in the 16th; however it was Guillaume Amontons that is credited with the basic laws of friction (dry friction) as follows:

1. The force of friction is directly proportional to the applied load
2. The force of friction is independent of the apparent area of contact

Later, Coulomb verified these laws and proposed a 3rd law

3. The kinetic force is independent of the sliding velocity

The following equation explains the relationship of coefficient of friction to normal and tangential force:

\[ F_f = \mu F_n \]  \hspace{1cm} (2.28)

where \( F_f \) is the frictional force parallel to the surface and acts opposite to the applied force, \( \mu \) is the coefficient of friction and is dependent on the contacting materials in question and \( F_n \) is the normal force transmitted by the bodies in contact.

2.5.3 Wear

When two surfaces come into contact one or both surfaces will typically suffer wear. Wear can be defined as the removal of surface particles from an originally undamaged surface. There are many major variables that influence wear. Normal load, contact area, sliding speed and lubrication are some of the key ones. Wear can be categorised into multiple forms as listed below. Fretting wear however is explained in the following section as greater focus has been placed on this topic in Chapter 4 and Chapter 5. Adhesive wear occurs at an atomic level [29] when two bodies in contact undergo undesired relative displacement. This wear mechanism results in material removal from the softer material because of the strong adhesive forces between atoms. An important aspect of bodies in contact is the difference in
real and apparent area. In most contact mechanics or elastic analysis it is easier to assume that the contact bodies are smooth. In reality this is not the case as real material surfaces are rough with asperities and surface deformities. When two real materials come into contact the resulting contact area is a fraction of the apparent area [24]. This results in a nominally elastic macroscopic normal load having a significant effect at the microscale. Therefore, asperities are subjected to significant plastic deformation as a result of the small contact area which can lead to small fragments shearing and separating from the contacting bodies.

Abrasive wear is a more severe form of material surface damage which can easily be initiated by adhesion wear. The small particles that detach from the main body due to adhesion can become oxidised leading to harder particles which can slip and slide between the contacting bodies. These particles are responsible for scratching, gouging and further material removal. A second form of abrasion is where the hard particles are not detached but still remove or cause damage to material from the opposing body.

Surface fatigue is the process whereby cyclic loading induces wear on the material surface. Either natural surface deformities or favourably orientated grains cause multiple crack initiation sites on the material surface. The weakened micro-surface detaches as a result of micro-cracking and leads to a modified surface roughness and material wear at the surface.

Erosion wear is the process of hard particles striking a surface carried by the medium of gas or liquid. In the case of a liquid medium this is more commonly referred to as slurry erosion. A typical example would be piped water containing sand. If transported at high velocities the inside of the pipe is subjected to wear and
surface damage. Particle mass and impact velocity are the controlling factors when dealing with erosive wear.

When selecting materials for engineering applications, bulk material properties, availability and cost can be more important than the tribological properties of the material. Therefore multiple surface treatments are available to increase tribological properties. Surface treatment is the method of making small changes to the materials hardness through chemical infusion or heat treatment. For the sake of brevity, a small number of related studies are presented below. Laser surface modification (LSM) has shown success in increasing the hardness of tool steel AISI H13, for example [30]. The effects of both laser and shot peening on the fretting fatigue behaviour of Ti-6Al-4V was investigated by Liu and Hill [31]. Comparisons against as-machined material showed a significant increase in fretting fatigue life for both peening methods. In fact, the shot peening surface treatment outperformed laser peening for low stress amplitudes as a result of high tensile residual stress present at greater depths in the laser peening coupons. Plasma nitriding on plain and fretting fatigue coupons of AISI 304 stainless steel [32] resulted in a higher surface hardness, lower surface roughness and compressive residual stress at the surface. However, the nitrided coupons exhibited inferior plain and fretting fatigue behaviour compared to unnitrided samples. This is attributed to the weakening of the chromium at the grain boundaries of the plasma nitrided samples. However, nitrided 316L fretting fatigue samples [33] showed significant increases in fretting fatigue life compared to as-received samples. In some cases the fretting fatigue life of nitrided samples exceeded the plain fatigue life of the unnitrided samples. Surface coating is an alternative method where by a harder material such as nickel, chromium, cobalt or tungsten [29] is deposited onto the
surface to provide corrosion and wear resistance. Multiple processes exist such as electroplating and thermal spraying.

2.6 Fretting

2.6.1 Introduction

Fretting is a surface damage phenomenon that occurs in a wide range of industries such as offshore oil and gas, biomedical and aerospace. In numerous engineering applications, this damage occurs at size scales competitive with the material microstructure, e.g. 5 to 100 μm. Three main sliding regimes are commonly associated with fretting, namely, partial, gross and mixed slip, primarily dependent on normal load, displacement amplitude and COF [34]. Fretting is a phenomenon that occurs in a wide range of mechanical assemblies such as osteosynthesis plates and screws [35], bolted and riveted joints, steel wires [36] [37], blade-disc dovetail connections [38] and splined couplings in gas turbine engines [10] and [39].

2.6.2 Fretting damage

A large number of parameters are responsible for fretting, such as normal load, slip amplitude, surface roughness and micro-mechanical properties. The variables can be grouped into primary and secondary variables depending on their effect. The primary controlling variables in fretting fatigue, as defined by Dobromirski [34], are:

- Slip amplitude
- Contact pressure
- Coefficient of friction

Figure 2-7 illustrates the concepts of a stick region and partial and gross slip from a plan view of the contact region, as a result of an increase in slip amplitude. As the tangential load or the relative displacement between the two materials is increased
the initial stick region reduces and slip zones develop at the contact edges. Cracking is the predominant form of failure in partial slip. The stick-slip interface and the slip zone are generally the locations of new crack initiations sites [40]. With increasing relative displacement, mixed, and eventually gross, sliding is achieved. In this situation, wear is the predominant form of failure as material removal limits crack initiation and growth [40]. The running condition fretting map (RCFM) in Figure 2-8 (a) shows the relationship between normal load, displacement amplitude and different fretting regimes. The material response fretting map (MRFM) in Figure 2-8 (b) shows the typical damage response for materials [41].

Figure 2-9, from Vingsbo and Soderberg [42], depicts schematically the effect of slip regime on fatigue life and wear rate. At low slip amplitudes, in the stick regime, little or no surface damage, either wear or cracking, occurs. Once partial slip occurs, there is a significant reduction in fatigue life as a result of surface damage causing crack initiation and propagation which eventually leads to premature failure. Coinciding with this slip regime a low wear rate is present. With an increase in slip amplitude, gross slip occurs and a considerable increase in fatigue life is evident, concomitant with a significant increase in wear rate with increasing slip. Similar results have been observed experimentally by Jin and Mall [43], for example.

The effect of contact pressure on fretting is not as clearly defined as the effect of slip amplitude [44]. This generally results in other variables being indirectly varied as contact pressure is changed, e.g. slip amplitude. However, typically with an increase in contact pressure it has been observed that there is a decrease in fatigue life, as characterised by Nakazawa et al. [45], for example, where fretting fatigue experiments were carried out on 316L SS.
COF is one of the primary variables affecting fretting fatigue. Typically a decrease in surface roughness leads to a decrease in COF. However, results presented by Kubiak and Mathia [40] show an increasing COF for decreasing surface roughness and thus demonstrate the detrimental effect of a high COF on wear rates in the gross sliding regime. This is achieved through machining and abrasive processes where the surface roughness of the material was altered. Swalla and Neu [46] show the dramatic effect of COF via an elastic-plastic FE fretting fatigue damage prediction methodology incorporating the Fatemi-Socie-Kurath damage parameter, whereby a significant decrease in crack initiation life is predicted as COF increases. A 33% increase in COF resulted in an 80% decrease in predicted crack initiation life. In an effort to reduce the damaging effects of COF on life, numerous methods have been implemented to increase the surface roughness. In work presented by Volchok et al. [47] laser texturing of fretting specimens, for example, showed a beneficial effect on fretting fatigue life. The regular micro-topography obtained allowed for easier wear debris removal which improved fretting fatigue resistance.

There are two main forms of fretting damage, fretting fatigue and fretting wear. Fretting fatigue is typically associated with cases where a cyclic load is superimposed on one of the moving contacting bodies, which typically results in fatigue cracks propagating from the contact region, initiating from the stick-slip interface or slip zones for partial slip and at the contact edges for gross slip [7] [25] [48]. Fretting wear results in surface damage and eventually material removal. This can be considered as a fatigue micro-cracking phenomenon. It is quite often the case that both fretting fatigue and wear occur simultaneously as a result of the complex geometries and loading conditions in which fretting problems arise, e.g. see [39].
2.6.3 Fretting fatigue

Fretting fatigue is a common form of fretting damage as the majority of contact problems experience a variety of complex loading situations and are at some point subjected to fatigue loading. The term fretting fatigue is used to describe a situation where a reduction in the plain fatigue life of a material is explained by the relative slip between two or more contacting bodies [49]. Fretting fatigue is present in a wide number of industries and is a major concern in critical engineering situations such as cable transportation, aerospace and railway [36], [50] and [51]. Extensive modelling has been carried out by several groups in an attempt to (i) characterise the controlling parameters in fretting fatigue, as well as (ii) investigate the reduction in life compared to plain fatigue testing, (iii) implement life prediction methodologies and (iv) investigate the effect of microstructural modelling. The latter is a key subject of this thesis.

Since fatigue is closely related to fretting fatigue, many fatigue methodologies have been implemented within the context of fretting fatigue prediction. Lykins et al. [52] investigated a number of fatigue methodologies and compared the predictions against experimentally obtained data. The main parameters evaluated within the context of this work were SWT, FS and the Ruiz parameter. Ruiz et al. [38] proposed two parameters as follows:

\[ F_1 = (\sigma_T)_{\text{max}}(\tau \delta)_{\text{max}} \]  
\[ F_2 = (\sigma_T \tau \delta)_{\text{max}} \]

where \( (\sigma_T)_{\text{max}} \) is the maximum tangential stress, \( \tau \) is the surface shear traction and \( \delta \) is the relative slip amplitude. Lykins et al. [52] investigated both \( F_1 \) and \( F_2 \), since the maximum tangential stress and maximum frictional work, \( (\tau \delta)_{\text{max}} \), can occur at
differently positions. It was concluded that $F_2$ was ineffective at predicting the numbers of cycles to crack initiation but did predict crack locations that were reasonably consistent with experimental results. Other fatigue parameters such as SWT and FS have been found to be more effective at predicting initiation lives. For example, Araujo and Nowell [7] evaluated two critical plane implementations of SWT and FS, under a variety of contact pad radii to investigate contact size effects. Both parameters predicted accurate results for the large pad radii but overly conservative lives for the smaller pad radius. A limitation of both fatigue parameters however is their inability to capture the stress gradient effects. For that reason two averaging methods were implemented, namely critical depth and critical volume. Averaging methods hypothesise that a critical volume or depth of highly stressed material must be achieved before fatigue cracks initiate. Other authors have adopted the same or similar approach to rationalise the effects of steep stress gradients in reconciling predicted lives with measured results, e.g. Fouvry et al. [53]. However, there is no objective method for defining this critical volume/depth other than via empirical calibration against test data and the critical value may not be unique for a given material. Fatigue parameters are used to determine the location of initiation sites and subsequent averaging methods are thus applied at this region. The analytically determined location, at the trailing edge, correlates well with the experimental observations. Within the work of Araujo and Nowell [7] both averaging methods resulted in more accurate life predictions. Similar work is presented by Naboulsi and Mall [54] which corroborates, the use of these fatigue parameters within fretting fatigue situations for crack location and life prediction.

Since, and mentioned above, most fretting fatigue situations occur under complex loading scenarios, it is difficult to apply methodologies evaluated on simple
geometries or simplified test rigs to real-life service components. Leen and co-workers [39] [12] investigated fretting fatigue predictions within complex spline couplings for gas turbine aero-engines using the aforementioned fatigue methodologies. In order to deal with the small length-scales for fatigue crack initiation in fretting and the very fine FE mesh employed in simulations, Houghton et al. [12] made crack initiation predictions based on fatigue constants and crack lengths of 10 \( \mu \text{m} \) instead of 1 mm. Greater detail is available in Madge et al. [55], where Basquin fatigue constants for 10 \( \mu \text{m} \) cracks are back-calculated using LEFM and the El-Haddad correction for short crack growth, starting with Basquin constants corresponding to 1 mm long fatigue cracks. Houghton et al. [12] used the back-calculated SWT parameter to locate and quantify crack nucleation in a representative fretting fatigue test, designed to mimic the complex fretting condition between aero-engine spline teeth under combined torque and rotating bending moment. The El Haddad correction with the Paris crack growth equation, incorporating mixed mode fracture via a weight function method, was subsequently employed to simulate crack propagation through the substrate. The predicted and experimental fretting fatigue results correlated with each other.

2.6.4 Fretting wear

Fretting wear, as previously mentioned, is associated with the removal of material from the contacting bodies. The importance of wear in terms of fatigue cracking is evident in Figure 2-9 where a significant increase in wear rate is seen to occur concomitantly with an increase in component life on transition from partial slip to gross sliding. The significance of wear-induced material removal on fretting fatigue life has been simulated by Madge et al. [11], where SWT life predictions were presented incorporating the effects of wear using a FE methodology. Within the
partial slip regime, minimal material removal leads to only a comparatively small effect on fatigue life. The key conclusion of [11] was the successful prediction of the pickup in life in the gross sliding regime, as validated by comparison with experimental fretting tests. Consequently, the development of fundamental methods for simulating and predicting the interaction of wear and fatigue, particularly for crack nucleation, is critical. Micro-plasticity is argued here to be the key tool for unification of wear and fatigue crack nucleation via micro-cracking prediction in contact.

Archard's equation [56] is the most commonly used wear law. It is a relatively simple expression which has been shown to successfully predict sliding wear across a wide range of materials [28] and loading conditions. The equation is presented as follows:

\[
\frac{V}{S} = K \frac{P}{H}
\]  

(2.31)

where \(V\) is the wear volume, \(S\) is the total sliding distance, \(K\) is the wear coefficient, \(P\) is the contact pressure and \(H\) is the material hardness. However there are some limitations to the equation. It has been shown that the Archard equation only works for a constant COF value [57] which is not always the case in fretting [58]. McColl et al. [25] applied a modified version of the Archard equation to gross slip fretting wear of a high strength CrMoV aero-engine site alloy, with a reasonable degree of success in predicting wear-scar dimensions across a range of normal loads. As previously mentioned, the limitation of the Archard equation means it is successful within the gross slip regime, but, is less accurate within the partial slip regime when \(q(x) < \mu p(x)\).

Fouvry and co-workers, e.g. [57] have developed an alternative wear equation based on fretting energy dissipation. This integrates COF into the wear
analysis and is argued to be independent of load and stroke. This is achieved through an energy based wear volume calculation as follows:

$$V = \alpha \sum_{i=N} E(x, t)$$  \hspace{1cm} (2.32)

where $\alpha$ is the wear coefficient and $\sum E$ is the accumulated dissipated (frictional) energy, summed over the total number of fretting cycles, $N$, defined by:

$$E(x, t) = \int_{t=0}^{t} q(x, t) \, ds(x, t)$$  \hspace{1cm} (2.33)

where $q(x, t)$ is the instantaneous shear stress and $ds(x, t)$ is the relative slip at a time $t$ and position $x$. Hence, wear volume from sliding (gross slip) or partial slip experiments are calibrated against energy (measured) dissipated over the complete number of cycles, via a wear coefficient $\alpha$. This energy equation calculates the shear traction times the sliding distance. Since the shear force in sliding is equal to the instantaneous COF value times the normal load, the shear force changes accordingly and is dependent on the COF. Magaziner et al. [59], Fridrici et al. [60] and Fouvry et al. [57] have argued that the energy based wear methodology is superior to the Archard model because of the independence with respect to stroke.

Wear simulation methods have been introduced based on either Archard or the energy based wear methodology [61] [62]. McColl et al. [25] used a Fortran (wrap-around) program for Abaqus to incrementally update the mesh by moving the contact nodes on both surfaces, applying the Archard wear equation locally based on nodal contact pressures and relative slip. One approach within the Abaqus FE code is adaptive meshing via a user subroutine (UMESHMOTION). Madge et al. [61] implemented this approach, using the UMESHMOTION subroutine to simulate fretting wear via incremental removal of material with re-meshing of the contact area to account for the "worn" nodal geometry. To save on computational time a cycle
jumping factor, $\Delta N$, was incorporated which allows the user to assume that the amount of material wear occurring over a small number of cycles is constant.

A typical assumption in the modelling of wear is that the removed material simply disappears. This is clearly not realistic, as seen previously with abrasive wear, it is quite common for a third body layer to be created. It is hypothesised that this third body layer has two effects [63]:

1. The first is a positive effect where the third body debris forms an oxide layer that acts as a protective barrier from further first body contact.

2. The second is a detrimental abrasive effect that accelerates the rate of wear.

Ding et al. [64], used the idea of third body flow to model the effects of debris in fretting wear, a continuous thin layer of "debris" elements that formed on the top surface of the substrate. In contrast to this, Basseville et al. [65] modelled wear as individual rectangular bodies on the surface as a result of experimental observation of fragmented wear spots as opposed to a continuous third body layer of debris. This hypothesis, of third body flow, was originally introduced by Godet and colleagues [66] and [67], who state that it is not solely debris formation but elimination of wear particles that define a real wear process. The detached particles are transported outside of the contact region via third body flow. Ding et al. [64] introduces two new parameters, a formation rate, $\nu_f$, and an escape rate, $\nu_e$. Constant formation of debris is predicted throughout the analysis; however the rate of escape can be divided into three different regimes. The initial stage has zero escape rate, $\nu_e$, as all the debris remains trapped within the contact zone. In the intermediate stage a significant increase in the escape rate, $\nu_e$, is predicted as the contact width increases. In the latter stage the escape rate, $\nu_e$, approaches the formation rate but at a much slower rate. The methodology produces interesting results with respect to contact pressure.
distributions and wear scar profile, compared to simulations neglecting debris generation. The predicted wear scars become smaller in width and deeper in depth. The contact pressure acts over a smaller width with the peaks in maximum contact pressure predicted to occur at the contact edges.

2.7 Experimental

2.7.1 Introduction

Experimental work is an essential part of any engineering research topic. This section discusses some of the different experimental tests carried out to characterise fretting. Due to the nature of loading imposed on most fretting situations, plain fatigue testing is used as a comparative tool to assess the detrimental effect of fretting damage on component life as described in Section 2.6.

Fretting is a difficult phenomenon to replicate in a laboratory environment due to the small displacements required to cause crack nucleation and wear. Two experimental fretting arrangements are discussed in the following sections. They can be divided into fretting rigs and fretting fatigue rigs.

2.7.2 Fretting specimens

Great care must be taken in specimen preparation to avoid inaccurate or biased test data acquisition. Fretting specimens are manufactured according to the ASTM standards [68] as stated in the fretting fatigue testing standards [69]. Multiple geometries exist for fatigue testing, Figure 2-10 shows a selection of dog-bone specimens. Cylindrical specimens are more desirable than flat plate because of the continuous smooth surface. The rectangular cross section of the gauge length results in four corners where stress concentrations can manifest if inadequate surface machining occurs. However in a contact problem a cylindrical surface is impractical.
For this reason a specimen with a rectangular gauge length is used in the majority of fretting fatigue tests, e.g. Jin and Mall [43] [70]. An average surface roughness, $R_a$ value of 0.2 $\mu$m [68] is recommended to remove any unwanted machining marks that would eventually lead to crack initiation sites. Similarly the fretting pads must be treated with the same standard of manufacturing to ensure no early damage or wear on the substrate. Other substrate geometries can be seen in Ding et al. [58] and Fridrici et al. [60]. However these experiments are classed here as fretting wear tests. They have an induced contact displacement but do not require the specimen to be clamped in a fatigue testing machine.

2.7.3 Fretting wear rigs

This particular experimental apparatus involves controlled displacement of the fretting pads while in contact with the specimen substrate. Ding et al. [58] used an electromagnetic vibrator force transducer to oscillate a cylindrical fretting pad, as shown in Figure 2-11. A linear variable displacement transducer (LVDT) is used to measure the net displacement at the pad. It is important to note that the local displacement occurring at the surface of the specimen is different from the global applied displacement. The relative slip between the pad and substrate is an important controlling parameter in fretting life. This is as a result of experimental compliance within the test rig as seen in Jin and Mall [43], for example. The following equation has been developed to allow for the estimation of local slip, $\delta$, [71].

$$\delta = \delta_{global} \frac{C_{COF}}{C_{rig}}$$  \hspace{1cm} (2.34)

where $C_{rig}$ represents the effects of rig compliance and $C_{COF}$ represents the effect of COF on compliance. Other such methods of inducing controllable displacement can be seen in Fridrici et al. [60] where a tension-compression hydraulic machine
controls the applied displacement to the substrate. In this case an extensometer along with a compliance factor is used to ascertain the local contact displacement.

### 2.7.4 Fretting fatigue rigs

As mentioned earlier, it is quite difficult to recreate fretting fatigue conditions experienced by in-service components in simple laboratory tests. Instead simplified test rigs are developed to study particular loading parameters such as contact load or pad displacement.

Experimental testing to replicate fretting fatigue has been in existence since the 1960's. The initial investigations used bridge-type fretting pad arrangements as shown in Figure 2-12. This arrangement is simple in design and easily adapted to different materials, pad radii, contact pressures and displacements. Pape and Neu [72] and Goh et al. [73] both implemented bridge-type fretting fatigue rigs within their work. A variation of the bridge-type arrangement first introduced by Nishioka and Hirakawa [74] was employed by Jin and Mall [43] where by a single fretting pad clamp was employed, (see Figure 2-12 (a) and (b) for more detail). The bridge-type fretting pads consist of two feet on either side of the specimen that are clamped in place but are not in contact with the frame or rig. The relative displacement is due to the larger cyclic displacement in the specimen compared to the fretting clamps leading to friction induced at the contacting surfaces. In the case of the single clamp fretting rig the fretting pads are attached to the rig frame. Therefore the relative displacement is dependent on the compliance between the fatigue specimen and fretting pads. However as previously mentioned the presence of compliance makes it difficult to calculate the relative displacement. Wittkowsky et al. [75] proposed a method for calculating relative slip, \( \delta \). A simple schematic shown in Figure 2-13 illustrates the following formula:
\[
\delta = \delta_{AB} - (\delta_{\text{ext}} + \delta_{DC})
\]  
(2.35)

where

\[
\delta_{AB} = \frac{F - 2Q}{SE} l_{AB}
\]  
(2.36)

\[
\delta_{DC} = -\alpha l_{DC} Q
\]  
(2.37)

\(\delta\) is the relative slip, \(\delta_{AB}\) is the displacement between A and B, \(\delta_{\text{ext}}\) is the displacement measured by the extensometer and \(\delta_{DC}\) is due to the compliance of the fretting pad fixture system. \(F\), \(Q\), \(\alpha\), \(S\) and \(E\) is the normal force, tangential force, compliance constant, cross sectional area and Young's modulus, respectively. \(l_{AB}\) and \(l_{DC}\) are the distances between points A and B and D and C, respectively.

In recent years more realistic experimental rigs have been developed that accurately portray the loading scenarios imposed on in-service components. Wavish et al. [76] present an experimental methodology for the combined cyclic loading and torque of aero-engine spline couplings. Conner and Nicholas [77] and Golden [78] [79] both implemented a dovetail fretting fatigue test rig to quantify the fretting fatigue damage on Ti-6Al-4V turbine blades. Golden [80] improved on previous designs by developing an unique dovetail fretting fatigue rig which simulated the centrifugal forces and elevated temperatures that are applied to a turbine blade, the specimen in this case, within engine operation. This rig is similar to the apparatus designed by Ruiz et al. [38] which allows for two turbine blades to be tested simultaneously while mounted in a central disk specimen.
2.8 Microstructural modelling

2.8.1 Introduction

With increased computational capabilities, recent trends are aimed towards advancing the understanding of the microstructure in crack nucleation and propagation. Better understanding of the early stages of crack nucleation will lead to advanced engineering materials that will be better suited to fatigue resistance. For this to be achieved an advanced material model capable of realistically simulating the material microstructure needs to be employed, for example, CP theory. CP theory models the deformation of metallic grains at a microscopic level. Crystallographic slip occurs as a result of dislocation movement along individual slip systems within a crystal lattice. CP theory typically uses a rate dependent, physically based, viscoplastic formulation where dislocation movement is modelled within a continuum framework.

2.8.2 Microstructure

The microstructure of a material plays a critical role in material deformation. Lattice composition, grain size, orientation and texture significantly affect material behaviour. Most metals can be classified into the following atomic structures [81], (see Figure 2-14).

- Face centred cubic (FCC)
- Body centred cubic (BCC)
- Hexagonal close packed (HCP)

Each structure has a unique number and orientation of slip planes and slip directions. Miller indices are used to describe the orientation of the atomic structure. 316L SS, for example, has a FCC structure consisting of 4 slip planes (111) and 3 slip...
directions [110] and, hence, 12 slip systems in total. Dislocation movement occurs along these slip systems within the atomic structure. Dislocations are imperfections within the crystal structure and explain how slip occurs in metallic materials at stress levels much lower than theoretically required to break a perfect crystal. Three types of dislocations exist: screw, edge and mixed dislocations. These dislocations move through slip planes to slide relative to each other through the breaking and re-bonding of atomic bonds, see Figure 2-15 for more details.

Schmidt's law, proposed in 1935 by Schmidt and Boas [82], allows calculation of the resolved shear stress, $\tau_r$, on a slip system within a single crystal.

$$\tau_r = \sigma \cos \phi \cos \lambda = \sigma (n \cdot t)(s \cdot t)$$

(2.38)

Figure 2-16 shows a schematic of the applied force, $F$, and the resolved shear stress, $\tau_r$, on the slip direction. Once the shear stress, $\tau_r$, exceeds the critical resolved shear stress, $\tau_{CRSS}$, slip will occur.

2.8.3 Size effects

Geers et al. [83] list the important known size effects within metallic structures. i) Intrinsic or microstructural size effects and (ii) statistical size effects. The former can be best explained via the Hall-Petch relationship [84] [85], which relates the yield strength of the material to the grain size as follows:

$$\sigma_y = \sigma_0 + \frac{k_y}{\sqrt{d}}$$

(2.39)

where $\sigma_y$ is the yield stress, $\sigma_0$ is the resistance of the crystal lattice to dislocation motion, $k_y$ is the strengthening coefficient and $d$ is the average grain diameter. This relationship states that as the grain size of the material decreases the yield strength increases. The grain size dependency of the yield stress is observed in work by Jain et al. [86]. Statistical size effects occur when the number of grains through the
thickness of the specimen decreases and the material's mechanical response becomes
dependent on the orientation of a small number of grains. This is observed in the
work of Murphy et al. [87], Savage et al. [88] and Keller et al. [89].

2.8.4 Grain size

Material grain size is dependent on manufacturing or heat treatment processes. Grain
size has a considerable effect on crack initiation and propagation. Miller [90] showed
that small grains inhibit crack initiation while large grains hinder crack propagation.
Hanlon [91] observed an increase in resistance to fatigue between ultra fine and nano
crystalline nickel grains. Mall et al. [92] [93] carried out fretting fatigue tests on both
Ti-6Al-4V and a nickel based super alloy IN100 for two different grain sizes.
Experimental observations concluded that the smaller grain size had increased crack
initiation resistance while the larger grains had greater crack propagation resistance.
Ding et al. [58] showed that initiation cracks for cylinder-on-flat fretting fatigue
experiments were of comparable length to that of the material (Ti-6Al-4V) grain
size, 20 µm in this case. Thus, initiation sites of this size can be significantly affected
by grain morphology. Mayeur et al. [94] highlights the influence of texture,
individual or groups of grains sharing similar orientation, on cyclic plastic
deformation caused by ratchetting. The work of Goh et al. [95] highlights a
considerable effect of texture on plastic strain accumulation and states that the role
of texture may be significant in resisting fretting damage.

2.8.5 Microstructure modelling in fretting

In an effort to develop better engineering materials and design practices for complex
fatigue problems such as fretting, it is important to understand the fundamental
principles behind the problem. In this case the problem is closely related to fatigue
crack nucleation which is heavily influenced by the microstructure. An excellent
review paper on the computational modelling of fatigue crack formation was presented by McDowell and Dunne [96].

Microstructural modelling has been previously implemented to study the effects of the microstructure on fretting fatigue and wear. McDowell and co-workers, [94], [95], [97] and [98], have implemented CP theory suitable for the modelling of HCP metals, in this case Ti-6Al-4V. A kinematic hardening variable is included in this theory to represent the Bauschinger effect based on the Armstrong-Fredrick rule. This results in a back stress that can be subtracted from the resolved shear stress on a slip system. Other such work, for example, Dunne [99], has dealt with isotropic hardening CP theory, arguing that random heterogeneity of metallic grains in a polycrystal model leads to the macroscopically observed Bauschinger effect. Goh et al. [95] [73] [100] employed the micro-mechanical material model along with conventional FIPs (SWT and FS) to determine plastic strain maps that gave qualitative reasonable agreement with experimentally observed crack locations and orientations for fretting fatigue testing of Ti-6Al-4V. In contrast, the continuum plasticity model showed less favourable results when compared against the experimental data. Cailletaud and co-workers [101] have also employed a CP material model within a 3D fretting analysis along with the Dang Van fatigue prediction methodology. Hence, although comparisons have been carried out in terms of crack location and orientation, no microstructure-sensitive life prediction methodology has been previously presented for crack initiation life and hence total life predictions.

Ti-6Al-4V is an advanced engineering material with excellent mechanical properties making it suitable for a wide number of engineering applications including complex loading and contact situations. As a result, this material has been
investigated by a number of research groups to quantify its fatigue and fretting behaviour. Ti-6Al-4V is a dual phase material consisting of a (HCP) α phase and (BCC and HCP) lamellar α-β phase. In the HCP α phase slip occurs on the basal, prismatic and pyramidal slip systems [102]. Within the work of Zhang et al. [103], the pyramidal slip systems are not included as a result of the significantly higher critical resolved shear strength required to activate them (3 to 4 times) compared to the basal or prismatic slip systems [104]. According to Philippe et al. [105], the material can be further simplified by neglecting the effect of twinning. The lamellar α-β phase contains BCC and HCP slip systems and can be modelled as a continuous phase with representative properties, as detailed in Morrissey et al. [102].

2.8.6 Microstructure-sensitive crack prediction methodology

A number of fatigue indictor parameters (FIPs) have been implemented within microstructural modelling. A modification of the SWT and FS parameters was incorporated in the work of [95] where a plastic strain replaces the normal and shear strain respectively. This was used to study the local plasticity effects within the contact region. Plastic strain maps originally based on work implemented by Ambrico and Begley [106] were developed by Goh et al. [100] to study the location of plastic ratchetting as plastic deformation plays an important role in crack formation. The CP material model was shown to produce greater accumulation and more realistic distributions of plastic ratchetting compared to the continuum model. However, these parameters are essentially macroscopic FIPs.

Manonukul and Dunne [107] proposed a new microstructure-sensitive parameter, accumulated plastic slip, $p$, as the key parameter controlling crack initiation for low and high cycle fatigue. $p$ is defined in terms of an effective plastic crystallographic slip rate as follows:
\[ \dot{p} = \left( \frac{2}{3} L^p : L^p \right)^{1/2} \quad ; \quad p = \int_0^t \dot{p} \, dt \]  

(2.40)

where the plastic velocity gradient \( L^p \) is defined by:

\[ L^p = \sum_{\alpha=1}^{n} \dot{\gamma}^\alpha s^\alpha m^{\alpha T} \]  

(2.41)

with \( s^\alpha \) and \( m^\alpha \) as the slip direction and normal vectors, respectively, for a given slip system, \( \alpha \), with \( n \) slip systems. \( \dot{p} \) and \( p \) are computed in the CP user subroutine.

The criterion for crack initiation presented in [107] is \( p = p_{\text{crit}} \). As pointed out in [107], with calculation of the maximum accumulated slip, together with knowledge of the experimentally determined number of cycles to failure, \( N_f \), for a particular test with known loading conditions, it is possible to determine \( p_{\text{crit}} \). Due to the quick stabilisation of the stress-strain response [107] it is possible to determine a stabilised maximum accumulated plastic slip per cycle, \( p_{\text{cyc}} \). The crack initiation criterion is then written as [107]:

\[ p_{\text{crit}} = N_i p_{\text{cyc}} \]  

(2.42)

The critical accumulated slip, \( p_{\text{crit}} \), was argued to be a fundamental quantity and was shown in [107] to be able to predict crack initiation for both LCF and HCF for C263, a FCC nickel alloy, and over a range of temperature.

### 2.9 Conclusions

This review summaries the main research that has been carried out to further the understanding of fretting wear and fatigue. It has been shown that fretting is a detrimental form of surface damage that can greatly reduce the fatigue life of a component and is present in a wide range of engineering applications. Significant work has been carried out since the 1960's in experimental, analytical and, more
recently, computational modelling of fretting situations. This chapter presents an overview of the fundamental knowledge that is required to understand the phenomenon as well as the engineering tools required to quantitatively assess the situation. Work has been carried out using FIPs to predict crack initiation and propagation lives based on macro-scale continuum mechanics material models. Due to the important role of microstructure in crack initiation, some research has been carried out on microstructure-sensitive material models that can evaluate the influence of microstructural parameters on fretting behaviour. However a microstructure-sensitive life prediction methodology has not been previously presented (and/or validated) for fretting. This thesis presents a microstructural sensitive fretting fatigue life prediction methodology. The method is validated against experimental fretting fatigue testing with a calibrated micromechanical material model that is implemented within a cylinder-on-flat fretting model. Furthermore, the microstructure-sensitive fretting fatigue crack initiation methodology presented is shown to be a key first step towards unifying predictions of fretting wear and fretting cracking.
2.10 References


2.11 Figures

Figure 2-1. Effect of mean stress on fatigue limit according to Soderberg, Gerber and modified Goodman equations. The lines shown, are constant life lines for a life corresponding to the endurance life, $\sigma_e$. 

[Diagram showing the relationship between stress amplitude and mean stress.]
Figure 2-2. Schematic highlighting the different cracking regimes during fatigue crack propagation.

Figure 2-3. A simple schematic of the Kitagawa and Takahashi diagram.
Figure 2-4. Cylinder-on-flat elastic contact model.

Figure 2-5. Hertzian pressure distribution for cylinder-on-flat, where $a_0$ is the initial contact semi-width.
Figure 2-6. Cylinder-on-flat contact highlighting stick and slip regions.

Figure 2-7. Plan view schematics of different slip regimes for cylinder on flat, (a) stick regime, (b) partial slip regime, (c) gross sliding regime.
Figure 2-8. Simple schematics of (a) running condition fretting map (RCFM) and (b) the material response fretting map (MRFM) for different combinations of displacement amplitude and normal load.

Figure 2-9. Schematic fretting map shows the effect of slip regime on fatigue life and wear rate [42].
Figure 2-10. Selection of fatigue specimens that can be used in fatigue testing.

Figure 2-11. Simple fretting test schematic of the arrangement used by Ding et al. [58].
Figure 2-12. Different fretting fatigue test configurations, (a) bridge-type, (b) single clamp.

Figure 2-13. Representative schematic of Jin and Mall [43] fretting fatigue rig.
Figure 2-14. Three atomic structures, (a) FCC, (b) BCC and (c) HCP.

Figure 2-15. Different stages of dislocation migration without the distortion of the crystal lattice.
Figure 2-16. The resolved shear stress, $\tau_r$, can be found via Schmid's law.
Chapter 3

3 Experimental Testing

3.1 Introduction

This chapter describes the development and implementation of a fretting fatigue test rig and the experimental testing of 316L SS under plain and fretting fatigue conditions. Methods used to characterise the mechanical properties and microstructure of 316L SS, as well as microscopy techniques used to observe and record fretting wear failure data, are described and the results are presented. The development, manufacture and implementation of a bridge-type fretting rig are presented as well as testing procedure. A comparative study on the detrimental effect of fretting fatigue compared to plain fatigue with respect to failure life, via S-N curves, is presented.

3.2 Specimen and material

3.2.1 Sample design

Fatigue coupons were manufactured from cold rolled austenitic 316L SS plate in accordance with ASTM standards [1] for force-controlled axial fatigue tests. The specimens were manufactured from the steel plate using electric discharge machining (EDM) to achieve accurate dimensions and tolerances. Figure 3-1 shows the dog bone specimen design and dimensions. Figure 3-2 shows the steel plate at the halfway stage of manufacture where the overall shape of the dog-bone specimen has been machined but the individual specimens have not been sectioned.

3.2.2 Material characterisation

A monotonic tensile test was carried out to characterise the yield stress, $\sigma_y$, of the material. Figure 3-3 shows the stress-strain curve of the material with focus on the
yield stress evaluated at 0.2% offset. The mechanical tensile behaviour was found to be consistent with the material data sheets provided by the supplier as seen in Table 3-1. Energy dispersive X-ray (EDX) spectroscopy was used to chemically characterise the material and confirm the composition. Table 3-2 presents the measured EDX results which are also consistent with the material data sheets. A surface roughness of 0.2 µm for the machined specimens was verified using both an atomic force microscope (AFM) and Mitutoyo Surftest-211 profilometer. These coupons were designed to be used in both plain and fretting fatigue tests. Additionally, Rockwell hardness (G-scale) tests were carried out on the material to ascertain the material hardness in MPa via the following ASTM standards [2], see Table 3-3 for more detail.

3.2.3 Microscopy

Chemical etching was performed on material samples cut from the specimens for microstructural characterisation. Samples underwent a grinding and polishing process using a range of diamond abrasive suspensions from 30 to 0.25 µm. A solution of 20% HCL and 80% distilled H₂O with an addition of 0.5-1.0 g of K₂S₂O₅ (potassium metabisulfite) per 100 ml was used to colour-etch the samples for between 30 and 120 s [3]. Multiple micrographs of the material microstructure were examined using this technique and sample grain areas were taken from each to obtain in an average grain area. This grain area was then transformed into square grain dimensions as seen in Figure 3-4, to give an average grain size of 19 µm.

3.2.4 Plain fatigue tests

Samples were cyclically tested in a servo-hydraulic Instron testing machine for a range of stress amplitude, σ_{amp}, values. The specimens were subjected to sinusoidal loading at a frequency of 7 Hz. Table 3-4 shows the range of loading conditions used
for the plain fatigue testing. All tests were conducted at a stress ratio, \( R \), of 0.1 defined as:

\[
R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}
\]

where \( \sigma_{\text{min}} \) is the minimum applied stress and \( \sigma_{\text{max}} \) is the maximum applied stress. Repeat test were conducted at each \( \sigma_{\text{amp}} \) to establish experimental scatter.

### 3.3 Fretting fatigue test rig

#### 3.3.1 Manufacture and design of fretting rig

Figure 3-5 shows a solid model of the bridge-type fretting arrangement designed here for experimental testing. The rig contact geometry is a cylindrical fretting pad configuration. Detailed drawings of the fretting rig components and assembly are included in Appendix 1.1. The purpose of the proving ring is to provide a clamping mechanism for clamping the fretting pads and the fatigue specimen with a fixed, calibrated normal load. The fatigue specimen is cyclically loaded and acts as a substrate for fretting to occur on. Also, it facilitates the propagation of fretting-induced cracks to cause early precipitation of fatigue cracks. An important requirement for the proving ring is that it does not plastically deform during loading or testing. With this in mind a suitable ring thickness was chosen that allows:

- Suitable number of threads through the thickness to stabilise the loading screws
- The ring deflections to stay within the elastic loading regime
- Suitable flexibility which guarantees accurate loading.

Thin-walled ring theory was used to determine the correct thickness for appropriate elastic stresses and strains [4]. Figure 3-6 shows a schematic of the
elastic horizontal and vertical ring deflections as a result of loading calculated with the following equations:

\[
\Delta D_h = 0.1366 \frac{WR^3}{EI} \tag{3.2}
\]

\[
\Delta D_v = -0.1488 \frac{WR^3}{EI} \tag{3.3}
\]

where \( W \) is the load, \( R \) is the mean radius, \( E \) is Young's modulus and \( I \) is the second area moment of inertia for the ring cross section. The maximum circumferential stress on the proving ring can be calculated from the following equation:

\[
\sigma_\theta = \frac{My}{Ahr} \tag{3.4}
\]

where \( y \) and \( r \) are the distance from the neutral axis and the radial position of the desired stress state, respectively, and \( A \) is the cross-sectional area. \( h \) is the distance from the centroidal axis to the neutral axis defined as:

\[
h \approx \frac{I_c}{RA} \quad \text{for} \quad \frac{R}{t} > 8 \tag{3.5}
\]

where \( t \) is the thickness of the vessel. The bending moment \( M \) is defined as:

\[
M = \frac{WRk_2}{\pi} \tag{3.6}
\]

where \( k_2 = 1 - \alpha \) and

\[
\alpha = 1 - \frac{2(a - b)}{(a + b)\ln{(a/b)}} \tag{3.7}
\]

where \( a \) and \( b \) are the inner and outer diameters, respectively. It was therefore determined that the proving ring would perform within the elastic loading regime with a factor of safety, \( \eta \), of 25, where \( \eta \) is defined by the following:

\[
\eta = \frac{\sigma_v}{\sigma_\theta} = \frac{205 \text{ MPa}}{8.2 \text{ MPa}} = 25 \tag{3.8}
\]
Ball bearings are used to transfer the force between the loading screws and the fretting pad supports to ensure self-alignment and consistent distribution of contact pressure. Four cylindrical pins are placed behind each fretting pad to ensure even transfer of contact load from the loading screws to the specimen surface eliminating any 3-point bending effects; see Figure 3-7 (a) for more detail.

The interchangeable fretting bridges were manufactured from the same grade of 316L SS as the fatigue coupons. The fretting bridges are designed to transfer normal load into the fretting contact pads and hence on to the fatigue specimen. The fretting contact pads were 6 mm in radius and had a surface roughness, \( Ra \), value of 0.18 \( \mu m \) when measured with a Mitutoyo Surftest-211 profilometer. The two fretting pads were placed symmetrically on either side of the gauge length of the fatigue coupon as shown in Figure 3-7 (b). The coupon is clamped between the fretting feet and is placed in an Instron testing machine, as shown in Figure 3-8. The proving ring was calibrated so that a known clamping pressure could be applied to the specimen gauge length via the loading screws. Calibration was performed by loading the proving ring in tension via the loading screws and acquiring a load-strain curve via strain gauges placed on the proving ring. Figure 3-9 shows the measured calibration curve. This allowed a measurable clamping pressure to be applied to the surface of the fretting fatigue specimen.

Sufficiently fine pitch threads on the loading screws allowed for small incremental adjustments to the clamping pressure during initial loading on the fatigue coupon. Thread stresses based on power screw theory were calculated to estimate the von Mises stress on the most heavily-loaded thread, i.e. the first thread. Empirical evidence indicates that 38% of the load \( F \) is carried by the first thread,
i.e. 0.38\(F\). The following equations are used as inputs to calculate the von Mises stress calculation;

\[
\sigma_b = \sigma_x = \frac{6(0.38F)}{\pi d_r n_t p} = 129.58\text{MPa} \tag{3.9}
\]

\[
\sigma = \sigma_y = \frac{4F}{\pi d_r^2} = -18.56\text{MPa} \tag{3.10}
\]

\[
\tau = \tau_{xy} = \frac{16T_r}{\pi d_r^3} = 6.82\text{MPa} \tag{3.11}
\]

where \(d_r\) is the thread root diameter, \(n_t\) is the number of threads, \(p\) is the thread pitch and \(T_r\) is the torque required to turn the screw. The following von Mises equation calculates a stress within the elastic regime = 140.3 MPa. Therefore, no plastic deformation will occur on the threads;

\[
\sigma_{eq} = \frac{1}{\sqrt{2}} \left[ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \right]^{1/2} \tag{3.12}
\]

The lock-nut is used in conjunction with the loading screws as part of the loading mechanism to ensure a constant normal load during testing. Once the loading screws and lock-nuts are tightened a contact pressure of 0.5\(P_y\), is present at the substrate surface, as described in equation 2.26 and 2.27 in Section 2.4. A Hertzian contact distribution shown in Figure 3-10 highlights the contact pressure required to cause yielding as well as the applied contact pressure of 0.5\(P_y\). The specimen was cyclically loaded at 1 Hz and the relative tangential movement between the fretting pads and the substrates induces fretting fatigue. Fretting fatigue tests were conducted using the same applied stress ranges as tabulated in Table 3-4 for plain fatigue.
3.4 Plain and fretting fatigue test results

3.4.1 Plain fatigue

Figure 3-11 shows the measured S-N curve from plain fatigue tests. A fatigue limit of 189 MPa, $\sigma_{amp}$, is evident when the results are graphically presented. Repeat tests carried out at the same applied stresses, $\sigma_{amp}$, showed reasonably consistent results with original data, highlighting the repeatability and validity of the experimental procedure. Table 3-5 shows the experimental failure lives for each stress range plus the averaged data used in the following computational work. All failed specimens fractured within the gauge length with no evidence of slip present at the clamps. Figure 3-12 shows an SEM image of a typical failure mode seen in all plain fatigue specimens. The distinction between the crack initiation and propagation region compared to the fast fracture region is evident. In this case, initiation began in the top left corner and propagated radially throughout the specimen until failure. It is typically observed that cracks nucleate at edges of rectangular gauge sections due to the unconstrained slip of grains at sharp corners which are not confined by neighbouring grains [1]. An average dimension of the propagation region for all plain fatigue specimens is used in the crack growth analysis later. Failure is deemed to have occurred when the propagation region has grown to 1.5 mm in length.

3.4.2 Fretting fatigue

Fretting fatigue tests were carried out until complete specimen failure occurred. All fretted specimens failed at a contact region between the fretting pad and gauge length similar to Figure 3-13. Cracks propagated from the surface between 70° - 110° resulting in a comparable fracture surface as seen in Figure 3-12 for the plain fatigue case. Fretting fatigue lives are plotted in Figure 3-14 against the plain fatigue results
and shows a significant reduction in life. Results for two different normal loads are presented in Figure 3-14 for $\sigma_{\text{amp}} = 225$ MPa. A significant effect of normal load is evident in Figure 3-14 and Table 3-5 where a normal load equal to $6P_y$ is presented and compared to $0.5P_y$. The total life is reduced by a factor of about 3 for the higher normal load. All further analysis was carried out using fretting fatigue test data obtained at $0.5P_y$. On average a life reduction factor of 3.5 is obtained, as tabulated in Table 3-5. SEM imaging carried out on the failed specimens show fretting wear scars and crack nucleation sites at the contact edges. Figure 3-15 shows the surface damage and debris typical of a fretting situation. Quantification of fretting wear scar depths and areas were carried out via profilometry and SEM. A Taylyprofile Surtronic 3+ was used to obtain wear scar profiles at contact regions where failure did not occur. Profiles were traced at regular intervals through the width of the wear scar, see Appendix 1.2; Figure 3-16 shows a representative image highlighting the U-shaped wear scar which is typical of most gross slip fretting conditions. To allow for more accurate microscopy, characterisation and validation of the wear scar, a technique proposed by Ding et al. [5], was implemented. Specimen gauge lengths were sectioned via a slow diamond cutter and mounted in an epoxy resin. After being mounted the specimens were subjected to plane grinding across a variety of silicon carbide paper and polished through a range of abrasive diamond suspensions finishing with a 3 µm paste. The finely polished surface allowed for quantification of the wear scar via microscopy techniques. Figure 3-17 shows the polished cross sectional area of a fretting wear scar. Good agreement exists between the two methods with regards shape, width and depth as seen in Table 3-6.

A red oxide layer was observed on the outer edges of the wear scar. EDX spectroscopy of three distinct regions was carried out on the specimen surface to
assess the levels of oxidation. Figure 3-18 shows the undamaged, worn and debris region of the gauge length where spectroscopy occurred. The highest levels of oxygen were present in region 2 and 3 where surface damage had occurred in the form of wear and debris build up. Oxygen levels compared to the undamaged surface had increased from 2% to 36% on average highlighting the presence of fretting corrosion; see Table 3-7 for more detail.

3.5 Discussion

Fretting was induced on fatigue specimens through the implementation of a bridge-type fretting rig. Experimental observations showing the presence of fretting fatigue is associated with: (i) the significant reduction in life compared to plain fatigue testing, (ii) the presence of a red oxide layer and (iii) wear scars at the contact regions on the specimen-pad contact surfaces. The two methods used to obtain measured wear scars show consistent results for scar depth and width. SEM imaging (Figure 3-12) shows crack nucleation sites at the trailing edge of contact, consistent with published work e.g. [6]. EDX carried out on sample points on the wear scar highlights the presence of a metallic oxide layer (Table 3-7). Fretting causes material surface damage which removes the corrosion resistant passive layer and allows for oxidation to occur on the material beneath the surface. This phenomenon typically results in debris which can have a higher hardness than the original material and accelerates the process of further material removal. An apparent experimental anomaly is observed for the test result at 202.5 MPa which deviates from the experimental trend. A much lower fatigue life is observed at this stress amplitude. Two explanations are presented below to rationalise this result. Fretting fatigue is a combination of (i) a multiaxial fatigue problem, where decreased stress amplitude
results in longer life, and a (ii) complex fretting problem, where decreased slip amplitude (in gross sliding) results in decreased life [7]. These two effects represent boundaries of an envelope of possible interactive fretting fatigue responses, with the more complex curve of Figure 3-19 showing another possible characteristic response, whereby life initially decreases with reducing stress amplitude, due to predominance of the fretting (surface) effects due to high relative slip, and then increases with further lowering of stress amplitude, due to dominance of fatigue. This more complex effect corresponds to that measured experimentally in this chapter for fretting fatigue. It is important to point out, however, that the FLRF continues to increase with decreasing stress amplitude, (increasing life) as illustrated in Figure 3-20 consistent with the HCF nature of fretting, i.e. the effect of fretting increasing significantly with increasing life.

However, it is evident from the plain fatigue testing that a significant amount of experimental scatter is present at the mid to lower stress amplitudes (213.75 to 202.5 MPa) i.e. higher cycle fatigue. It is therefore reasonable to assume that this irregularity can be explained by experimental fretting fatigue scatter.

### 3.6 Conclusion

- A bridge-type fretting fatigue rig was developed and applied to study the effects of fretting fatigue on 316L SS. Comparisons against plain fatigue tests demonstrated a significant reduction in life of the order of 3.5 due to the gross slip fretting action.
- The realistic microstructure grain size of the 316L SS material was identified through chemical etching and microscopy techniques
3.7 References


### 3.8 Tables

Table 3-1. Experimentally measured material properties for 316L SS.

<table>
<thead>
<tr>
<th>E (GPa)</th>
<th>YS (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>213</td>
<td>253</td>
</tr>
</tbody>
</table>

Table 3-2. Element composition of 316L SS from EDX.

<table>
<thead>
<tr>
<th>Element</th>
<th>Weight %</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>12.30</td>
</tr>
<tr>
<td>O</td>
<td>2.32</td>
</tr>
<tr>
<td>Si</td>
<td>0.51</td>
</tr>
<tr>
<td>Cr</td>
<td>15.84</td>
</tr>
<tr>
<td>Mn</td>
<td>1.74</td>
</tr>
<tr>
<td>Fe</td>
<td>59.11</td>
</tr>
<tr>
<td>Ni</td>
<td>6.86</td>
</tr>
<tr>
<td>Mo</td>
<td>1.31</td>
</tr>
</tbody>
</table>

Table 3-3. Average results from the Rockwell hardness G-scale test and corresponding converted values.

<table>
<thead>
<tr>
<th>Rockwell G-scale</th>
<th>Vickers Indenter (HV/10)</th>
<th>(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>116</td>
<td>1138</td>
</tr>
</tbody>
</table>

Table 3-4. Experimental loading conditions for both plain and fretting fatigue experiments using $R = 0.1$.

<table>
<thead>
<tr>
<th>$\sigma_{max}$ (MPa)</th>
<th>$\sigma_{min}$ (MPa)</th>
<th>$\sigma_{amp}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>50</td>
<td>225</td>
</tr>
<tr>
<td>475</td>
<td>47.5</td>
<td>213.75</td>
</tr>
<tr>
<td>450</td>
<td>45</td>
<td>202.5</td>
</tr>
<tr>
<td>425</td>
<td>42.5</td>
<td>191.25</td>
</tr>
</tbody>
</table>
Table 3-5. Tabulated data of the experimental results for plain fatigue over a range of $\sigma_{amp}$ values including repeat results and the averaged plain fatigue response of the material.

<table>
<thead>
<tr>
<th>Test</th>
<th>Test type</th>
<th>$\sigma_{amp}$</th>
<th>$N_f$</th>
<th>$P/P_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PF</td>
<td>225</td>
<td>112108</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>PF</td>
<td>225</td>
<td>102284</td>
<td>N/A</td>
</tr>
<tr>
<td>3</td>
<td>PF</td>
<td>213.75</td>
<td>177862</td>
<td>N/A</td>
</tr>
<tr>
<td>4</td>
<td>PF</td>
<td>213.75</td>
<td>113796</td>
<td>N/A</td>
</tr>
<tr>
<td>5</td>
<td>PF</td>
<td>202.5</td>
<td>279580</td>
<td>N/A</td>
</tr>
<tr>
<td>6</td>
<td>PF</td>
<td>202.5</td>
<td>160816</td>
<td>N/A</td>
</tr>
<tr>
<td>7</td>
<td>PF</td>
<td>191.25</td>
<td>Run-off</td>
<td>N/A</td>
</tr>
<tr>
<td>8*</td>
<td>FF</td>
<td>225</td>
<td>17000</td>
<td>$6P_y$</td>
</tr>
<tr>
<td>9</td>
<td>FF</td>
<td>225</td>
<td>56669</td>
<td>$0.5P_y$</td>
</tr>
<tr>
<td>10</td>
<td>FF</td>
<td>213.75</td>
<td>59780</td>
<td>$0.5P_y$</td>
</tr>
<tr>
<td>11</td>
<td>FF</td>
<td>202.5</td>
<td>35091</td>
<td>$0.5P_y$</td>
</tr>
<tr>
<td>12</td>
<td>FF</td>
<td>191.25</td>
<td>245806</td>
<td>$0.5P_y$</td>
</tr>
</tbody>
</table>

Note: PF = Plain Fatigue; FF = Fretting Fatigue

* Test 8 was carried out $6P_y$. All other FF tests were carried out at $0.5P_y$.

Table 3-6. Comparison of wear scar measurement techniques as illustrated in Figure 3-16 and Figure 3-17.

<table>
<thead>
<tr>
<th>Wear scar 1 (Profilometer)</th>
<th>Wear scar 2 (Microscopy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape</td>
<td>U shaped</td>
</tr>
<tr>
<td>Width (µm)</td>
<td>349.0</td>
</tr>
<tr>
<td>Max Depth (µm)</td>
<td>9.46</td>
</tr>
<tr>
<td></td>
<td>U shaped</td>
</tr>
<tr>
<td></td>
<td>304.29</td>
</tr>
<tr>
<td></td>
<td>8.97</td>
</tr>
</tbody>
</table>

Table 3-7. Percentage weight of oxygen levels in three regions, undamaged, worn and debris from fretting fatigue specimen.

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>Region</th>
<th>Weight %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Undamaged</td>
<td>2.32</td>
</tr>
<tr>
<td>2</td>
<td>Worn</td>
<td>34.61</td>
</tr>
<tr>
<td>3</td>
<td>Debris</td>
<td>37.71</td>
</tr>
</tbody>
</table>
3.9 Figures

Figure 3-1. Dimensions of the plain fatigue specimen.

Figure 3-2. Picture of EDM of the fatigue specimens. The red line is highlighting the location of the wire used in the process.
Figure 3-3. 316L SS stress-strain curve, highlighting the yield point of the material.

Figure 3-4. Tint etching of 316L SS highlighting an average square grain size, $d$, of 19 µm.
Figure 3-5. Assembly schematic of the bridge-type fretting arrangement highlighting key design components.
Figure 3-6. Simple illustration of the loading, $W$, and the resulting horizontal, $\Delta D_h$, and vertical, $\Delta D_v$, elastic deflections of the proving ring. The dimensions of $t$ and $d$ are 6.4 mm and 104.8 mm, respectively.
Figure 3-7. (a) Illustrates the design method for eliminating three point bending effects during loading, (b) Schematic of the fretting pads clamped to the gauge length of the fatigue coupon.

Figure 3-8. Simple schematic highlighting the fatigue specimen and loading conditions placed between the fretting feet and clamped within the testing machine.
Figure 3-9. Calibrated load-strain curve highlighting the applied force required to give an applied normal load of \(0.5P_y\).
Figure 3-10. Hertzian contact pressure distributions for yielding, $P_y$, and $0.5P_y$. 
Figure 3-11. Experimental plain fatigue results including repeat tests also highlighting the material fatigue limit in terms of stress amplitude, $\sigma_{amp} = 189$ MPa.

Figure 3-12. SEM image of the plain fatigue specimen subjected to a $\sigma_{amp} = 202.5$ MPa highlighting the dimensions and size of the crack propagation region.
Figure 3-13. The bridge-type fretting rig is clamped in place surrounding the specimen gauge length. The outset shows a typically fretting fatigue failure at a contact zone.

Figure 3-14. Fretting fatigue lives compared to corresponding plain fatigue data.
Figure 3-15. Edge view of a gauge length of a fretting fatigue specimen where crack initiation has occurred at the edge of the wear scar. The specimen was subjected to a $\sigma_{amp}$ of 225 MPa and failed after 56,669 cycles.

Figure 3-16. A typical gross slip wear scar, tested at $0.5P_y$ normal load and $\sigma_{amp}$ of 225 MPa, obtained using a profilometer.
Figure 3-17. A wear scar profile for specimen 9 tested at 0.5$P_y$ normal load and a $\sigma_{amp}$ of 225 MPa obtained via microscopy techniques.

Figure 3-18. EDX was carried out in three different zones on a contact region of a fretting fatigue specimen highlighting the presence of fretting corrosion.
Figure 3-19. Schematic illustrating the effects of fretting, fatigue and fretting-fatigue interaction in terms of the characteristic stress-life material response.

Figure 3-20. Measured effect of stress amplitude on fretting life reduction factor.
Chapter 4

4 Microstructure-Sensitive Fatigue and Fretting

Methodology

4.1 Introduction

In this chapter a microstructure-sensitive methodology for fretting is presented. This approach is based on an existing method for microstructural sensitive plain fatigue life prediction from Manonukul and Dunne [1]. It is hypothesised that plastic slip is the controlling factor for crack initiation in both low and high cycle fatigue. Once a critical value for accumulated plastic slip, \( p_{\text{crit}} \), is reached, crack initiation is deemed to have occurred. This methodology is implemented within a CP material model to predict the LCF response of 316L SS via a unit cell model.

Two frictional contact models are developed, namely (i) a hybrid model consisting of a CP region, in the contact area of primary interest, surrounded by a non-linear kinematic hardening (NLKH) \( J_2 \) region, and (ii) a \( J_2 \) model, are developed to investigate the effects of key fretting variables, such as applied displacement and normal load. The CP model is calibrated against previously published cyclic stress-strain data for 316L SS and the microstructure-sensitive prediction method is also calibrated and validated against published LCF data for 316L SS. Number of cycles to fretting crack initiation, \( N_i \), are predicted using the microstructural-sensitive approach. Comparisons are made between the \( J_2 \) continuum and CP material models and significant effects are seen with regards to contact pressure, surface shear, relative slip and fatigue indictor parameter (critical-plane SWT), across a range of key fretting variables. It is shown that the accumulated plastic slip parameter \( p_{\text{crit}} \) has the ability to unify predicted wear and crack initiation. Characterisation of key...
microstructural parameters (e.g. grain orientation) on fretting performance is also investigated.

4.2 Methodology

4.2.1 $J_2$ NLKH theory

The $J_2$ plasticity formulation employed here incorporates non-linear kinematic hardening to model the Bauschinger effect. The plastic flow-rule, defining the plastic strain increment is given by the following equations:

\[
de^p = d\lambda \frac{\partial f}{\partial \sigma} = \frac{3}{2} dp \frac{\sigma'}{\sigma_e} \tag{4.1}
\]

\[
dp = \left(\frac{2}{3} d\varepsilon^p : d\varepsilon^p\right)^{\frac{1}{2}} \tag{4.2}
\]

where $f$ is the von Mises yield function, $dp$ is the increment in effective plastic strain, $\sigma_e$ is the von Mises equivalent stress, $\sigma'$ is the deviatoric stress tensor and $d\lambda$ is the plastic multiplier. $f$ is given as follows:

\[
f = \sigma_e - \sigma_y = \left(\frac{3}{2} (\sigma' - \chi') : (\sigma' - \chi')\right)^\frac{1}{2} - \sigma_y \tag{4.3}
\]

where $\sigma_y$ is the yield stress and $\chi'$ is the deviatoric backstress tensor. The translation of the centre of the yield surface in kinematic hardening is governed by the backstress tensor $\chi$, defined here via the Frederick-Armstrong non-linear hardening rule as follows:

\[
d\chi = \frac{2}{3} C d\varepsilon^p - \gamma dp \tag{4.4}
\]

where $C$ is the initial hardening modulus and $\gamma$ is the rate of decay of the modulus. The cyclic strain-hardening behaviour is given by the following equation [2]:
\[
\frac{\Delta \sigma}{2} - k = \frac{C}{\gamma} \tanh \left( \frac{\gamma \Delta \varepsilon_p}{2} \right)
\]  
(4.5)

where \(k\) is the cyclic (initial) yield stress. Identification of the NLKH parameters, \(k\), \(C\), \(\gamma\) is achieved using equation 4.5, as described in [2]. The NLKH constants are identified from a series of stabilised hysteresis loops of tension-compression strain controlled tests. The following steps can be used to obtain these constants:

1. \(k\) is approximately the average elastic proportion of the stabilised loops
2. \(C/\gamma\) is the asymptotic value of the measure \((\Delta \sigma/2) - k\) plotted against \(\Delta \varepsilon_p/2\)
3. \(C\) can be obtained from equation 4.5 by using the results from the previous steps.

### 4.2.2 Crystal plasticity theory

The CP theory [3] used in this work is a physically-based, rate-dependent crystallographic theory that models the deformation of a metallic crystal lattice. The total deformation in a crystal lattice can be described by the deformation gradient \(F\). The following equation describes (using standard tensor notation throughout) the decomposition of the deformation gradient \(F\) into its elastic \(F^*\) and plastic \(F^p\) parts:

\[
F = F^* \cdot F^p
\]  
(4.6)

where \(F^*\) typically represents the rigid body rotation and elastic deformation of the crystal lattice and \(F^p\) represents the plastic shear flow through the material. Figure 4-1 illustrates the deformation of a crystal in terms of the reference, intermediate and current configurations. However both the elastic and plastic deformation gradients may contain stretch and rigid body rotation. The velocity gradient \(L\) is defined through the following, where the dot represents a time derivative:

\[
L = \dot{F} \cdot F^{-1}
\]  
(4.7)
\[ \begin{align*}
\dot{\mathbf{F}} & = (\dot{\mathbf{F}}^* \cdot \mathbf{F}^p + \dot{\mathbf{F}}^p \cdot \dot{\mathbf{F}}) \mathbf{F}^{-1} \\
& = \dot{\mathbf{F}}^* \cdot \mathbf{F}^p \cdot (\mathbf{F}^p)^{-1} \cdot (\mathbf{F}^*)^{-1} + \dot{\mathbf{F}}^* \cdot \dot{\mathbf{F}}^p \cdot \mathbf{F}^p \cdot (\mathbf{F}^p)^{-1} \cdot (\mathbf{F}^*)^{-1} \\
& = L^* + L^p
\end{align*} \]

The elastic and plastic velocity gradients, \( L^* \) and \( L^p \), are thus defined as follows:

\[ L^* = \dot{\mathbf{F}}^* \cdot (\mathbf{F}^*)^{-1} \]  \hspace{1cm} (4.11)

\[ L^p = \dot{\mathbf{F}}^* \cdot \dot{\mathbf{F}}^p \cdot (\mathbf{F}^p)^{-1} \cdot (\mathbf{F}^*)^{-1} \]  \hspace{1cm} (4.12)

The velocity gradient \( \mathbf{L} \) can be decomposed into the deformation rate, \( \mathbf{D} \), and the spin tensor, \( \mathbf{W} \),

\[ \mathbf{L} = \mathbf{D} + \mathbf{W} \]  \hspace{1cm} (4.13)

where

\[ \mathbf{D} = \text{sym}(\mathbf{L}) = \frac{1}{2} (\mathbf{L} + \mathbf{L}^T) \]  \hspace{1cm} (4.14)

\[ \mathbf{W} = \text{asym}(\mathbf{L}) = \frac{1}{2} (\mathbf{L} - \mathbf{L}^T) \]  \hspace{1cm} (4.15)

\[ \mathbf{D} = \mathbf{D}^* + \mathbf{D}^p \]  \hspace{1cm} (4.16)

\[ \mathbf{W} = \mathbf{W}^* + \mathbf{W}^p \]  \hspace{1cm} (4.17)

Plastic slip is assumed to obey Schmidt's law [3], where the rate of plastic shear strain, \( \dot{\gamma}^\alpha \), for a particular slip system, \( \alpha \), is assumed to depend on the resolved shear stress, \( \tau^\alpha \), through the following power law:

\[ \dot{\gamma}^\alpha = \dot{\alpha} \left| \frac{\tau^\alpha}{g^\alpha} \right|^n \]  \hspace{1cm} (4.18)

where \( \dot{\alpha} \) and \( n \) are a reference slip rate and rate sensitivity exponent, respectively.

The resolved shear stress \( \tau^\alpha \) is defined as:

\[ \tau^\alpha = m^\alpha \cdot \tau \cdot s^\alpha \]  \hspace{1cm} (4.19)
where \( m^\alpha = F^{\alpha-1} \cdot m^\alpha \) and \( s^\alpha = F^\ast \cdot s^\alpha \). Material strain hardening is specified by the slip system strain hardness, \( g^\alpha \), which is given by the integral of the following equation:

\[
\dot{g}^\alpha = \sum_{\beta} h_{\alpha\beta} \dot{\gamma}^\beta
\]  

(4.20)

where \( h_{\alpha\beta} \) are the strain hardening moduli and \( \alpha \) and \( \beta \) represent particular slip systems. When \( \alpha = \beta \), \( h_{\alpha\beta} \) are the self hardening moduli and when \( \alpha \neq \beta \) they are the latent hardening moduli. Self hardening is due to slip on the current slip system and latent hardening is due to slip on adjacent slip systems. In this work Taylor isotropic hardening is assumed and self and latent hardening moduli are considered equal. \( g(\gamma_a) \) is the slip system strain hardness defined by the following hardness function [4]:

\[
g(\gamma_a) = g_0 + (g_\infty - g_0) \tanh \left( \frac{h_0 \gamma_a}{(g_\infty - g_0)} \right)
\]  

(4.21)

where \( h_0 \) is the initial hardening modulus, \( g_\infty \) is the saturation stress and \( g_0 \) is the initial slip system hardness. The hardening moduli can be found through differentiation of the above equation, as follows:

\[
h_{\alpha\beta} = h(\gamma) = h_0 \sech^2 \left( \frac{h_0 \gamma_a}{g_\infty - g_0} \right)
\]  

(4.22)

Figure 4-2 shows a simple schematic of the evolution of the hardening function. The accumulated slip, \( \gamma_a \) is defined as follows:

\[
\gamma_a = \sum_{\alpha} \int_0^t |\dot{\gamma}^\alpha| \, dt
\]  

(4.23)

The Jaumann stress rate is an objective stress rate, in terms of which the constitutive behaviour is calculated. It is defined as:
For the crystal plasticity theory implemented within this work the Jaumann stress rate on the material axes is given by the following:

\[
\dot{\sigma} = \dot{\sigma}^* - (W - W^*) \cdot \sigma + \sigma \cdot (W - W^*)
\]  

(4.25)

where \( \dot{\sigma}^* \) is the Jaumann stress rate on the crystal lattice axes, given by the following constitutive relationship [5]:

\[
\dot{\sigma}^* = K : D^* - \sigma (I : D^*)
\]  

(4.26)

where \( K \) is an elastic modulus tensor. Using the tensors \( \mu \) and \( \omega \) defined as:

\[
\mu_{ij}^\alpha = \frac{1}{2} \left[ s_i^{*(\alpha)} m_j^{*(\alpha)} + s_j^{*(\alpha)} m_i^{*(\alpha)} \right]
\]  

(4.27)

\[
\omega_{ij}^\alpha = \frac{1}{2} \left[ s_i^{*(\alpha)} m_j^{*(\alpha)} - s_j^{*(\alpha)} m_i^{*(\alpha)} \right]
\]  

(4.28)

The \( D^p \) and \( W^p \) are given by:

\[
D^p = \sum_\alpha \mu^\alpha \dot{\gamma}^\alpha
\]  

(4.29)

\[
W^p = \sum_\alpha \omega^\alpha \dot{\gamma}^\alpha
\]  

(4.30)

Substituting equations 4.17 and 4.26 into 4.25 gives the stress rate:

\[
\dot{\sigma} = K : D - K : D^p - W^p \cdot \sigma + \sigma \cdot W^p - \sigma (I : D) + \sigma (I : D^p)
\]  

(4.31)

This theory is implemented here in Abaqus 6.9-10 via a user-defined material (UMAT) subroutine, following the approach of [6]. Within the UMAT the stress increment is found by multiplying the Jaumann stress rate, \( \dot{\sigma} \), by the time increment.

A crystallographic slip parameter for capturing microstructure-sensitive fatigue cracking is implemented within the current UMAT. This is based on the work
of Manonukul and Dunne [1], who introduced a novel accumulated plastic slip parameter, \( p \), defined in terms of an effective plastic slip rate, as follows:

\[
\dot{p} = \left( \frac{2}{3} L^p : L^p \right)^{\frac{1}{2}} ; \quad p = \int_0^t \dot{p} \, dt
\]  

(4.32)

where the plastic velocity gradient \( L^p \) is defined by:

\[
L^p = \sum_{\alpha=1}^{n} \dot{\gamma}^\alpha s^\alpha m^{\alpha T}
\]  

(4.33)

with \( s^\alpha \) and \( m^\alpha \) as the slip direction and normal vectors in the current configuration, respectively, for a given slip system, \( \alpha \), with \( n \) slip systems. \( \dot{p} \) and \( p \) are computed in the CP user subroutine, see Appendix 1.3.

4.2.3 CP model calibration

316L SS has a face centred (FCC) crystalline structure which consists of 12 slip systems. The geometry within the crystal lattice is described in terms of 4 slip planes each with 3 slip directions described by the Miller indices, \( \{111\} \{110\} \), as shown in Figure 4-3. In this work, isotropic elasticity is assumed within the CP user subroutine, with a Young's modulus of 209 GPa and Poisson's ratio, \( \nu \), of 0.28. The unit cell model shown in Figure 4-4 (a) represents the metallic crystal grains at a microstructural level. This is the primary mesh used for the calibration of the CP constitutive model here. A grain size of \( d = 104 \, \mu m \), based on the 316L SS of [7] is modelled using an assumed regular hexagonal grain morphology, following the work of Savage et al. [8]. This permits inclusion of triple points, at grain boundary intersections, which are an important feature in microstructure modelling and CP deformation. Random crystallographic orientations are assigned to individual grains.

It is important to note that the two-dimensional geometry used throughout this work is a simplification of the real three-dimensional microstructure. However,
each grain is assigned a unique three-dimensionally oriented crystallographic structure (specified at each integration point, providing input data for the UMAT), which captures, to some extent, one three-dimensional aspect of the microstructure. Three-dimensional CP modelling of a microstructure is computationally demanding, e.g. see [9], for tensile loading of stent struts. Comparisons between the current two-dimensional approach and full three-dimensional analyses in [9] showed that the run-times for the latter were about fifty times those of the former. With the added complexity of a cyclic frictional contact problem, one can expect further significant increases in computational time. Nevertheless, some authors (Dick and Cailletaud [10] and Zhang et al. [11]) have previously modelled fretting problems using three-dimensional CP. One concern with two-dimensional CP modelling is that since it misrepresents the number of grains present within a problem, grain orientation may have a more significant effect than might be realistic. To address this, sensitivity studies on the effect of different distributions of randomly assigned grain orientations are presented, e.g. see Chapter 5.

The unit cell approach represents a repeating unit of a microstructure in a uniaxial fatigue specimen. The unit cell model consists of 42 uniform whole hexagonal grains and 22 other partial grains making up the grain boundaries. Four-noded, plane strain elements are used throughout this work. Symmetry boundary conditions are employed on the left and bottom edges of the unit cell and the right and top edges are constrained to remain straight. Figure 4-4 (b) shows a Voronoi tessellation mesh generated for the same average grain size which is used for comparative purposes. Figure 4-4 (c) shows a square grain mesh also employed here. The square grain mesh of Figure 4-4 (c) is particularly attractive for the contact region of the fretting model to allow better control of the contact region mesh, which
is critical for the accurate computation of contact tractions and associated substrate stresses and strain, as well as crystallographic slip and slip system shear stresses. The unit cell consisting of square grains was generated in a similar fashion to the hexagonal grains. Grain area, number of elements per grain, orientation and position were kept consistent between the two analysis as well as all boundary conditions and constraints. The position of the square grains allowed the modelling of triple points as seen between the hexagonal grains, which is characteristic of real microstructures.

An important issue in CP modelling is the identification of constitutive parameters. In this work, the CP parameters are identified using the macroscopic cyclic stress-strain curve (CSSC) of 316L SS, as represented by a NLKH material $J_2$ model, using the material data shown in Table 4-1 from [2].

The initial hardening modulus, $h_0$, saturation stress, $g_\infty$, and critical resolved shear stress, $g_0$, were identified using an iterative approach with respect to correlation of the unit cell model cyclic response to the macroscopic cyclic response, across a range of applied strain-ranges. Figure 4-5 shows the predicted stabilised hysteresis loops for the CP (hexagonal grain) unit cell model of Figure 4-4 (a) for different strain ranges. Figure 4-6 shows a comparison of the CP stabilised CSSC, using the identified CP constitutive constants, as given in Table 4-2, with the macroscopic (aggregate) CSSC for 316L SS; although there is not exact correlation, the fit is considered sufficiently accurate. $\dot{\varepsilon}$ and $n$ were set to 0.001s$^{-1}$ and 20, respectively, for this identification process and for subsequent use in this chapter. This also provided a reasonable convergence time for the analysis. Stabilisation (to within 2%) of the CP cyclic stress-strain loops occur within less than 20 cycles, as shown in Figure 4-7.
Figure 4-6 also shows the CSSC data points corresponding to CP analysis using the Voronoi tessellation mesh of Figure 4-4 (c) for 5 different random grain orientations. The vertical error bars represent the predicted scatter in cyclic stress from the different grain orientations. The predicted effect of random orientation is seen to be comparatively small and the uniform hexagonal mesh are satisfactorily close to those of the Voronoi tessellation mesh. Previous work by Savage et al. [8] presented a similar finding for a monotonic microstructure plasticity.

4.2.4 Microstructural sensitive crack initiation parameter

Manonukul and Dunne [1] proposed a new microstructural parameter, accumulated plastic slip, \( p \), defined in Equation 4.32 and 4.33 in Section 4.2.2, as the key parameter controlling crack initiation for low and high cycle fatigue. The criterion for crack initiation presented in [1] is \( p = p_{\text{crit}} \). As pointed out in [1], with calculation of the maximum accumulated slip, together with knowledge of the experimentally determined number of cycles to failure, \( N_f \), for a particular test with known loading conditions, is it possible to determine \( p_{\text{crit}} \). Figure 4-8 shows the predicted distribution of \( p \) for an applied strain range of 2%, illustrating the localised and inhomogeneous nature of the distribution. Due to the quick stabilisation of the stress-strain response [1] it is possible to determine a stabilised maximum accumulated plastic slip per cycle, \( p_{\text{cyc}} \). The crack initiation criterion can then be written as [1]:

\[
p_{\text{crit}} = p_{\text{cyc}} N_i
\]  \hspace{1cm} (4.34)

The critical accumulated slip, \( p_{\text{crit}} \), was argued to be a fundamental quantity and was shown in [1] to be able to predict the occurrence of crack initiation over a range of temperature and for both LCF and HCF for C263, a FCC nickel alloy. In general, it is not easy to determine \( N_i \), but it is relatively easy to determine \( N_f \). Hence, if
$N_i \approx N_f$, as was argued for the C263 material in [1], $p_{crit}$ can be determined from $N_f$. In the present work, it is argued that for uniaxial LCF, it can be assumed that $N_i \approx N_f$, i.e. $N_p$ is small relative to $N_i$ so that $p_{crit}$ can be determined from $N_f$ for a LCF test. This has been corroborated by short crack growth calculations which show that, in general, $N_p < 0.3N_f$. Therefore, the work does not assume that $N_i = N_f$ for fretting fatigue. Typically, fretting fatigue initiation life is short relative to propagation life. Hence, short crack propagation methods are generally used for the fretting fatigue simulations of this thesis. However, for the purpose of identification of the $p_{crit}$ from LCF data, it is assumed that $N_i \approx N_f$. For the 316L SS material considered here, the measured experimental LCF response of the material is encapsulated by the Coffin-Manson relationship between predicted number of cycles, $N_i$, and plastic strain range, $\Delta \varepsilon_p$, as follows:

$$N_i = \frac{(\Delta \varepsilon_p)^{ \frac{1}{c} }}{\varepsilon'_f^{1/c}} \quad (4.35)$$

where $\varepsilon'_f$ and $c$ are material constants given in Table 4-3. The $p_{crit}$ value, describing microstructural crack initiation for 316 L SS here, is then identified using the $N_i$ value corresponding to one particular value of plastic strain range, $\Delta \varepsilon_p$. The $p_{crit}$ value thus identified is 58.8. This identified $p_{crit}$ value is then validated by using it, (along with Equation 4.35), and the results of the unit cell model for a range of applied strain ranges, to predict the LCF response across the range of applied strain ranges. Figure 4-9 shows a comparison between the resulting CP unit cell LCF predictions and the Coffin-Manson relationship for stainless steel (using the constants of Table 4-3). It is clear that the identified $p_{crit}$ value gives good correlation with the measured Coffin-Manson relationship. This identified value of
$p_{crit}$ is subsequently applied to the prediction of fretting crack nucleation for 316L SS, for a cylinder-on-flat fretting wear configuration in the following sections.

4.2.5 Total life prediction

As mentioned in the Introduction section, previous work on prediction of fretting crack initiation has focussed on the use of a class of parameters referred to here as FIP, examples of which are SWT, FS, Dang Van and Walker. Within a fretting context, these parameters are commonly evaluated on the basis of a critical-plane approach, as described below, to deal with the multiaxial stresses and strains in fretting. FIPs are generally considered to be total life methods, since they are generally predicated on predicting a number of cycles (or reversals) to a 1 mm surface crack, e.g. see Socie [12]. In this work, it is argued that, for crack initiation prediction, it is necessary to use a microstructural parameter, such as the $p_{crit}$ parameter described above. The choice of which FIP to use, according to Socie, should be based on consideration of material (fatigue) cracking behaviour, viz. Mode I (tensile) cracking or Mode II (shear) cracking. For Mode I failures, Socie recommends the SWT parameter and for Mode II failures the FS parameter. As pointed out by Socie [12], in relation to materials such as 304 SS (assuming 316L SS can be similarly classed), which mode pertains depends on stress state and cyclic strain amplitude. Socie points out that Mode I failures occur for all strain amplitudes in tensile loading and for low strain amplitudes in shear loading. On the assumption that fretting contact regions for 316L SS experience a combination of tensile and low strain shear cyclic loading, it is assumed here that the SWT parameter can be used to provide estimates of total life, $N_f$, defined here in the context of FIP constants corresponding to a 1 mm surface crack. The SWT parameter and life prediction equation is expressed in Section 2.2.4. The values of these constants for 316L SS,
tabulated in Table 4-3, are obtained from Lemaitre and Chaboche [2]. The SWT approach spans across both LCF and HCF, including mean stress effects and also additional strain hardening associated with out-of-plane loading, as displayed by materials such as stainless steel [12]. A critical-plane approach is adopted for evaluation of the SWT parameter acknowledging the fact that cracks can initiate and grow on planes at any angle to the loading axis. SWT predicts the 'damage' per cycle on any given plane orientation for a given loading cycle. The parameter combines the peak normal stress, $\sigma_{\text{max}}$, within one cycle and the strain range, $\Delta \varepsilon$, which is defined as the difference between the maximum and minimum strain within a cycle. The critical-plane approach involves maximisation of the SWT parameter with respect to potential cracking plane, using $5^\circ$ intervals over a range of $180^\circ$. A more detailed description of the methodology implemented here can be found in Sum et al. [13]. The SWT critical plane approach has produced realistic predicted results compared to experimental results for plain and fretting fatigue of a high strength CrMoV aero-engine steel [13], [14] and for fretting fatigue of Ti6Al4V [15], [16]. In this work, since the primary purpose of the FIP is to provide total life estimates in fretting, against which to compare predicted initiation lives for fretting, the use of alternative FIPs would be expected to give broadly similar results and trends.

4.2.6 Cylinder-on-flat fretting model

Figure 4-10 shows a schematic of the crossed cylinder-on-flat fretting rig modelled in the present study, which is the University of Nottingham fretting test rig described in more detail in previous work, e.g. [17]. The cylindrical specimen is pressed against the flat specimen under a dead weight and oscillated tangentially under displacement (stroke) control. This differs from fretting fatigue tests which have a (cyclic) fatigue load applied to the fretted substrate. It was argued in [18] that the substrate fatigue
load is an additional and unnecessary complication in terms of identification and decoupling of crack nucleation from crack propagation, so that a fretting wear configuration is more fundamentally suited to identification of fretting crack initiation/nucleation. For this purpose the microstructure-sensitive methodology is validated against simplified fretting wear tests in Chapter 4 and 5, before application, in Chapter 6, to the more complex fretting fatigue test arrangement of Chapter 3. This arrangement has been successfully used by McColl et al. [17] to experimentally characterise the fretting wear behaviour of a high strength, CrMoV aeroengine steel and thence to validate a novel wear simulation technique under different normal loads and stroke combinations. It has also been used, more recently, by Ding et al. [18], to characterise the fretting wear and crack initiation behaviour of Ti-6Al-4V, e.g. see Figure 4-11 and Chapter 5, across different fretting slip regimes, and as mentioned in the introduction, to validate $J_2$-based, SWT predictions of crack initiation, including the effects of wear simulation, i.e. material removal.

In this chapter, following the approach of previous studies, e.g. [17], this experimental arrangement is modelled using a 2D plane strain, cylinder-on-flat frictional contact model, as shown in Figure 4-12, consisting of a 6 mm radius ($R$) 316L SS pad on a $5 \times 10$ mm 316L SS substrate. The bottom surface of the substrate is constrained in the X and Y directions. Linear constraint equations are specified between a single node on the top surface of the pad and the other top surface nodes, to constrain the top surface to have uniform displacements in the vertical and horizontal directions. The normal load $P$ is applied in a first step and held constant thereafter, to simulate the experimental application of the dead-weight load; subsequently, a cyclic tangential displacement $\delta_{app}$ is applied to the pad top surface. The frictional contact modelling approach employed follows that presented in
previous work, e.g. [18]. In the unlubricated fretting of metallic surfaces, such as steel or titanium, the COF commonly starts off low, at about 0.3 and increases with number of fretting cycles to a stable value between 0.7 to 1.0 or higher, depending on many factors including normal load, stroke, material compositions etc. Consequently, in the present study, a representative COF value (μ) of 0.8 is adopted.

The Lagrange multiplier frictional contact algorithm is employed with Abaqus to enforce an exact sticking constraint between the two surfaces when the shear stress (τ) is less than the critical shear stress, i.e. τ < μp, where p is the local (nodal) contact pressure.

Two different material modelling approaches are adopted here in the fretting model as follows:

(i) a $J_2$ NLKH material model (referred to hereafter as the ‘$J_2$ fretting model’) throughout all of the substrate, using the material properties defined in Table 4-1.

(ii) a hybrid material model (referred to hereafter as the ‘CP fretting model’), consisting of a CP material model in the contact region (see Figure 4-12 (b)), using the material properties defined in Table 4-2, and $J_2$ NLKH material model outside of this region, using the properties of Table 4-1.

The CP fretting model is designed on the basis of square grains in place of hexagonal grains, for better control of the mesh design in critical contact region. Comparison between both square and hexagonal grains was made using the unit cell approach. Figure 4-13 shows a comparison of the predicted accumulated plastic slip ($p$) distributions for the two different grain geometries at an applied (cyclic) strain range of 2%. The contour plots show almost identical locations of localised plastic slip and maximum values of $p$. Furthermore, the predicted hysteresis loops from the square
grain and hexagonal grain microstructure unit cell models were identical across the range of strain ranges.

In any methodology involving FE modelling, the design of mesh is important. In the analyses of the steep stress gradients associated with fretting problems, the mesh design is critical to accuracy and reliability of predictions. In the model of Figure 4-12, a dense mesh is used in the contact regions for optimum accuracy where significant stress gradients are anticipated, leading to plasticity under tangential loading, while a coarser mesh is used away from the contact regions to optimise with respect to computational overhead; this is particularly onerous in the case of the CP fretting analyses. A mesh refinement study has been conducted to establish the optimum degree of mesh refinement in the contact regions with respect to plasticity prediction. This study was based on the use of the $J_2$ fretting model and total life prediction using the critical plane SWT approach, across a range of applied tangential displacements, for successively increasing mesh refinement in the contact regions. An initial coarse mesh of 13 µm square elements was used followed by further refinement, using 8.67 µm, 6.5 µm and 3.25 µm square elements. Figure 4-14 shows (i) the decreasing total life with increasing applied displacement and (ii) the decreasing total life with increasing mesh refinement. Based on these results, a 6.5 µm square element size is employed here as a good compromise between accuracy and computational expense. The same mesh design is used for both the $J_2$ fretting model and the CP fretting model. The CP region of the CP fretting model contains a $7 \times 33$ mesh of square grains (i.e. 231 grains in total), each of side length 104 µm. Each grain thus contain 256 square elements.

As mentioned above, normal load is a key variable in fretting and has been shown in previous work to be particularly important in plasticity analyses of fretting,
4.2.7 Fretting model results

Two of the key (running condition) loading variables controlling fretting wear and fatigue are normal load and applied stroke (tangential displacement). Two normal loads are considered in the fretting analyses presented here, an ‘elastic’ case with \( P = 0.5P_y \) and a ‘plastic’ case with \( P = 2P_y \). For each of these, a range of applied displacements are investigated, between 0.5 \( \mu \)m and 2 \( \mu \)m, typically spanning from partial slip to gross slip, for both fretting models. The objective is to present a comparative assessment of plasticity predictions from both material models, with particular emphasis on the implications for prediction of crack nucleation and initiation. Previous work has highlighted the importance of contact pressure evolution on wear \([17]\) and fatigue damage \([15]\) accumulation in fretting. Shear stress plays a key role in fatigue crack initiation and in fretting, this means contact shear stresses. Relative slip (with shear and contact pressure) in contact is a distinguishing variable for fretting surface damage, which undoubtedly leads to premature crack initiation relative to plain fatigue situations. Therefore, the effects of material modelling on the predicted distributions, trends and values of these variables are central to the development of novel methods for prediction of crack initiation and wear in fretting.

Figure 4-15 and Figure 4-16 show the predicted evolutions of contact pressure within the first twelve tangential loading cycles for both fretting models and across a range of applied displacements. Table 4-4 and Table 4-5 summarise the predicted slip regimes and evolution of contact widths. A number of observations can be made from these results as follows:
1. For the $J_2$ model, the initial $N = 0$ distributions for $0.5P_y$ (Figure 4-15 (a), (c) and (e)) correspond identically to the Hertzian distributions. Even for $0.5P_y$, it is clear that the NLKH material model predicts a significant effect of tangential friction-induced plasticity on the contact pressure distribution and contact width, resulting in as much as a 25% drop in peak pressure and as much as 25% increase in contact width, for gross slip cases. Comparative analyses for $\mu = 0$ and 0.3 confirmed that the plasticity is tangentially induced by frictional effects (results not shown here). Under partial slip, e.g. $\delta_{app} = 0.5 \, \mu m$, negligible plasticity effects are predicted for the $0.5P_y$ normal load.

2. For the $J_2$ model, with $2P_y$ (Figure 4-16 (a), (c) and (e)), the predicted effect of plasticity on contact pressure and contact width increases with increasing stroke, with up to 40% increase in contact width and up to 25% drop in peak pressure. In this case, the initial ($N = 0$) ‘plastic’ distribution of pressure is clearly non-Hertzian.

3. The predicted effect of frictionally-induced plasticity is more complex and more significant for the CP fretting model. First of all, even for the ‘elastic’ case of $0.5P_y$, the $N = 0$ distribution is non-Hertzian (Figure 4-15 (b), (d) and (f)) and, although the predicted initial contact widths are the same as the $J_2$ and Hertzian models, the predicted peak pressures are up to 10% lower. Note that in this case, the contact width is only about one grain wide (viz. about 100 $\mu m$). For the $2P_y$ case, the initial ($N = 0$) contact pressure distribution (Figure 4-16 (b), (d) and (f)) is now significantly different to both the Hertzian and the corresponding $J_2$ model. Also, the contact widths are now no longer precisely symmetric due to material inhomogeneity. The
key differences are (i) the significantly more non-smooth (inhomogeneous) distribution, (ii) the significantly larger (~27%) contact width, and (iii) the significantly lower (~15%) peak pressure. The occurrence of (i) is attributed to the development of a natural surface roughness (see also [21]) in the material, due to differential yielding across the different grains in the contact width. In this case, the contact width is about two grains wide.

4. The CP-predicted contact widths, under frictionally-induced plasticity, increase significantly by up to 94% (i.e. almost doubling in width), with significantly larger predicted increases for the $0.5P_y$, normal load (see Table 4-5).

5. Furthermore, the CP pressure distributions are predicted to (i) reduce significantly by up to 60%, the effect increasing with increasing stroke, (ii) become more inhomogeneous with increasing plasticity and (iii) become, in a gross sense, more uniform.

All of these trends in terms of contact pressure evolution for the CP model are remarkably similar to the predicted effects or wear-induced material removal, e.g. see [17]. Figure 4-17 and Figure 4-18 show the corresponding evolutions of contact shear distributions. The $N = 0$ curve corresponds to normal loading only while the $N = 1$ curve corresponds to the first tangential displacement cycle. Some key observations related to these are as follows:

1. The $J_2$ shear tractions for $0.5P_y$ (Figure 4-17 (a), (c) and (e)) show (i) that the predicted contact width increase in general occurs within the first tangential displacement cycle, i.e. $N = 1$, (ii) the characteristic partial slip shear traction distribution for a Hertzian contact for $\delta_{app} = 0.5 \mu m$, which is unaffected by plasticity, consistent with the corresponding pressure
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distribution evolution and (iii) that the effect of plasticity on the gross slip shear traction distributions of the 1 µm and 2 µm δ_{app} cases follows the corresponding contact pressure trends of Figure 4-15 (c) and Figure 4-15 (e); it is worth noting that the NLKH J_2 model predicts shear redistribution (including reducing peak shear) even between cycles 6 and 12. Clearly, the shear tractions are predicted to increase with increasing stroke, saturating with respect to slip once the partial to gross slip threshold is exceeded.

2. The J_2 shear tractions for 2P_y (Figure 4-18 (a), (c) and (e)) clearly show partial slip characteristic distributions for 0.5, 1 and 2 µm δ_{app} values; for the 2 µm case, the peak shear traction, corresponding to the stick-slip interface, is predicted to move outwards with increasing cycles, with the final peak occurring at x \approx \pm a_0, the initial (N = 0) contact edge, whereas for the 0.5 and 1 µm cases, the stick-slip interface is unchanged after the first tangential cycle (N = 1);

3. For the 0.5 µm case, the J_2 predicted peak shear is seen to be higher for the lower normal load (0.5P_y); for the 1 µm case, this is also true but with a much smaller difference. For the 2 µm case, the higher normal load value (2P_y) is higher.

4. The CP shear tractions (Figure 4-17 (b), (d) and (f) and Figure 4-18 (b), (d) and (f)) are, in general quite different to the J_2 distributions. Some of the key differences are as follows: (i) they generally have lower peak values, (ii) the shear is distributed over a larger contact width, consistent with the pressure distributions, (iii) the distributions show significantly more fluctuations with respect to horizontal contact position, with significantly more localised peak
values, attributed to inherent microstructural, material inhomogeneity, and
(iv) in general, the initial \((N = 1)\) high peaks, attributed to the naturally-
predicted surface roughness, are re-distributed by localised plastic
deformation, leading to final \((N = 12)\) broadly-speaking smoother
distributions.

Figure 4-19 and Figure 4-20 show the predicted effects of plasticity on the critical-
plane SWT distributions along the surface for the \(J_2\) and CP fretting models and
Table 4-6 and Table 4-7 show corresponding key summary data. For both models,
plasticity is predicted to have a significant effect on this FIP and hence on predicted
\(N_f\). The \(J_2\) model predictions for the \(0.5P_y\) (Figure 4-19 (a), (c) and (e)) case show
that the final \((N = 12)\) maximum SWT increases with \(\delta_{app}\) and saturates at a
maximum value at \(\delta_{app} = 1\ \mu m\), so that the predicted \(N_f\) levels off at that
displacement also; for the \(2P_y\) (\(J_2\) model) case (Figure 4-20 (a), (c) and (e)) the SWT
values are lower than for \(0.5P_y\) for \(\delta_{app} \leq 1\ \mu m\), but become greater for higher
\(\delta_{app}\) values and the predicted \(N_f\) does not level off, but continues to decrease, due to
the non-occurrence of gross slip for \(\delta_{app} \leq 2\ \mu m\). Hence, the \(J_2\) model predicts
significantly lower \(N_f\) for \(2P_y\) for \(\delta_{app} \geq 1.5\ \mu m\). In general, the \(J_2\) model predicts
the peak FIP values at or very close to the contact edge, for both partial and gross
slip cases.

It is also of interest to use the CP model to predict \(N_f\) using the critical-plane
SWT (FIP) approach (Figure 4-19 (b), (d) and (f) and Figure 4-20 (b), (d) and (f)).
The CP model predicts very similar SWT \(N_f\) values to the \(J_2\) model but the location
is in general different and typically can be away from the contact edge, e.g. for
\( \delta_{app} = 0.5, 1 \) and 2 \( \mu m \) for \( 0.5P_y \), or at the contact edge, e.g. \( \delta_{app} = 1.5 \ \mu m \) for \( 2P_y \).

Figure 4-21 (a) and (b) show the predicted variation of \( N_f \) with \( \delta_{app} \) for \( 0.5P_y \) and \( 2P_y \). Both models show broadly similar trends, with the CP model showing some scatter relative to the \( J_2 \) model, as expected, given the specified random crystallographic orientations of the grains and hence yield strengths. For Figure 4-20 the CP model shows some dramatically higher initial \((N = 1)\) SWT values which are re-distributed with plastic deformation. These are attributed to the development of a natural surface roughness, alluded to earlier, and associated significant local peaks in pressures and shears and associated substrate stress components.

Figure 4-22 shows the CP predicted surface distributions of \( p_{cyc} \). These distributions are, in general, broadly similar to the FIP distributions with localised peak values generally occurring away from the central contact region. However, unlike the FIP distributions, the peak values for \( 0.5P_y \) always occur well away from the contact edge, under the contact, while the peak values for \( 2P_y \) are closer to the contact edge, except for low \( \delta_{app} \). Significant changes (reductions) in \( p_{cyc} \) are predicted between the 6\textsuperscript{th} and 12\textsuperscript{th} cycles. \( p_{cyc} \) is predicted to saturate also for gross slip cases but still has a random (inhomogeneous) nature, leading to different values even for nominally saturated (gross slip) conditions. \( p_{cyc} \) is predicted to be higher for the lower normal load when \( \delta_{app} \leq 1.25 \ \mu m \) (see Table 4-8 and Table 4-9).

Figure 4-23 shows the predicted contour plots of \( p \) after 12 tangential cycles for \( 0.5P_y \) and \( 2P_y \) and different \( \delta_{app} \) values. It can be deduced from Figure 4-23 (and Figure 4-22) that two types of distributions of the crack initiation parameter, \( p \), can be identified. The first (Type 1), as displayed by \( \delta_{app} = 0.5 \ \mu m \) and 1 \( \mu m \) for
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$2P_y$ (Figure 4-23 (b) and (d)), is characterised by localised contours of $p$ (and $p_{cyc}$), aligned with preferred directions, making acute angles of $\approx 48^\circ - 53^\circ$, consistent with typical fretting crack directions (e.g. see Figure 4-11; experimental data shows cracks to occur at similar angles between $40^\circ$ to $60^\circ$ [18]) and of length about one or two grains, again consistent with observed short cracks in fretting. The second type (Type 2) of distribution of crack initiation parameter, $p$ (and $p_{cyc}$), is significantly more uniform over a region resembling a typical wear scar shape, either (i) over a small region of about one grain length by one quarter of a grain length, e.g. $\delta_{app} = 0.5 \, \mu m$, 1 and 2 $\mu m$ for $0.5P_y$ (Figure 4-23 (a), (c) and (e)), or (ii) over a much larger region of two or three grain lengths by about one grain depth, e.g. $\delta_{app} = 2 \, \mu m$ for $2P_y$ (Figure 4-23 (f)). It is argued here that these two types of distribution demonstrate that the microstructural crack initiation parameter, $p_{crit}$, has the ability to predict differentiation between localised crack nucleation leading ultimately to fatigue cracking (Type 1) and or uniform micro-cracking leading to wear (Type 2), which may or may not ultimately lead to fatigue cracking, depending again on substrate or superimposed loading (additional to fretting loads). In this respect, it is superior to more conventional FIPs used for fretting e.g. Figure 4-19 (a), partial slip, and Figure 4-19 (e), gross slip, show little discernible difference in terms of SWT trends.

Figure 4-24 shows the $p_{crit}$ predicted life for $0.5P_y$ and $2P_y$ over a range of $\delta_{app}$. Comparison between the two normal loads shows (i) a higher predicted life at small $\delta_{app}$ for $2P_y$, (ii) decreasing life with increasing $\delta_{app}$ for both loads and (iii) higher predicted life at large $\delta_{app}$ for $0.5P_y$. The levelling out of predicted life for $0.5P_y$ at the higher $\delta_{app}$ is consistent with the change in slip regime from partial slip
to gross sliding which has not occurred for $2P_y$. Figure 4-25 shows comparisons of the predicted numbers of cycles to crack initiation (based on $p_{crit}$) and to total life (based on critical-plane SWT with CP fretting model) for both the low and high normal loads, over the range of tangential displacements. Of course, these predicted total lives are only indicative, since they do not include a number of factors as follows:

1. they are based on the assumption of no wear, ignoring the possibility of crack arrest due to (i) wear-induced pressure and stress re-distribution [15] or (ii) wear-induced material removal of the cracked material [16].
2. they do not account for precisely the type of initiation prediction encapsulated by $p_{crit}$, viz. microstructure-sensitive crack nucleation, which is the principal subject of this chapter,
3. they do not account for short crack growth effects, e.g. see [16], including, for example, possible crack arrest due to lack of sufficient crack driving force.

With this caveat, it can be seen that the predicted $N_i$ values are less than 8000 for the $0.5P_y$ normal load and are relatively insensitive to applied displacement above the transition to gross slip (levelling off at ~5000 after gross slip onset), due to effective levelling-off of $p_{cyc}$. In contrast, the $N_i$ values for the $2P_y$ load decrease from about 27,000 to about 3500 continuously with increasing displacement, due to increasing $p_{cyc}$ value. A key point to note is that the predicted initiation lives are from 0.4% to 1.4% of the predicted total lives for $0.5P_y$ and from 1% to 11% for $2P_y$ for these fretting analyses. Golden and Calcaterra [22] have reported similar proportions of between 1% to 10% of total life for fretting of Ti-6Al-4V alloy. The calculated numbers of cycles to initiation in that case were between 5,000 and 50,000, which are also consistent with the fretting initiation lives presented here.
4.3 Conclusions

The key conclusions from the analysis of the present chapter are as follows:

- A previously-published method for microstructure-sensitive fatigue crack initiation prediction is successfully applied to 316L SS to (i) identify the critical microstructural accumulated slip for crack initiation and (ii) predict the low-cycle fatigue behaviour.

- FE modelling of a fretting wear cylinder-on-flat configuration for stainless steel has demonstrated the key role of frictionally-induced cyclic plasticity in fretting, for both a $J_2$ continuum and a CP model, for normal loads lower and higher than the (normal) yield load. The key effects predicted are a significant increase in contact width within the first 12 cycles and an associated significant reduction in peak contact pressure.

- Compared to $J_2$, the CP fretting model predicts (i) more non-uniform distributions of fretting and fatigue variables across the contact, due to material inhomogeneity (random crystallographic orientation), and (ii) significantly larger effects of plasticity on contact width and contact pressure.

- A microstructure-sensitive crack nucleation methodology for fretting is presented, based on a combination of (i) CP unit cell identification of CP material constants, via comparison with aggregate (bulk material) cyclic stress-strain response, (ii) the identified critical microstructural accumulated slip for crack initiation, and (iii) an FE CP model of fretting under specified loading conditions and tribological data (COF), for identification of critical microstructural location (grain) and value of accumulated plastic slip, and hence life.
• Application of the latter methodology to fretting crack nucleation prediction for 316L SS for two different normal loads and across a range of applied displacements, spanning across partial slip and gross slip conditions, has demonstrated trends consistent with those of a previously-validated critical-plane SWT (total life) approach and thence with widely accepted trends.

• The predicted numbers of cycles to initiation were found to be less than $10^4$ for the low normal load ($P = 0.5P_y$) and less than $3 \times 10^4$ for the higher normal load ($P = 2P_y$). In the $0.5P_y$ case these were between 0.4% and 1.4% of the corresponding CP predicted (SWT) total lives of between $3.1 \times 10^5$ and $2.8 \times 10^6$, while in the $0.5P_y$ case these were between 1% and 11% of the predicted total lives, consistent with previously published fretting data.

• The proposed fretting crack initiation method is more consistent with the length-scales associated with fretting-induced cracks, which is of the order of the grain size, and thus incorporates associated material inhomogeneities for this length-scale, e.g. crystallographic orientation.

• The predicted CP distributions of accumulated plastic slip can be used to distinguish between, and hence unify prediction of, wear and fatigue crack initiation. Specifically, in the present work, the gross slip distributions of plastic slip exhibit patterns more consistent with wear, i.e. no clear preferred direction and a reasonably uniform distribution over a given region similar in size to typical fretting wear scars, whereas the partial slip distributions show patterns more consistent with crack nucleation, i.e. localised distributions of plastic slip and preferred growth of plastic slip along a discrete direction consistent with the experimentally-observed angles of typical (partial slip) fretting cracks. The CP-predicted evolutions of contact pressure are also
remarkably similar to the predicted evolutions of contact pressure from previously published studies of fretting wear simulations.
4.4 References


4.5 Tables

Table 4-1. Material constants, including non-linear kinematic hardening data, for 316L SS [2].

| $E$  | 209 GPa |
| $\nu$ | 0.28    |
| $k$  | 300 MPa |
| $C$  | 30000 MPa |
| $\gamma$ | 60 |

Table 4-2. Identified CP constitutive constants for cyclic behaviour of 316L SS.

| $h_0$ | 60 MPa |
| $g_\infty$ | 250 MPa |
| $g_0$ | 70 MPa |
| $\dot{\varepsilon}$ | 0.001s$^{-1}$ |
| $n$ | 20 |

Table 4-3. High and low cycle fatigue constants for 316L SS [2].

| $\sigma_f'$ | 3280 MPa |
| $\varepsilon_f'$ | 0.34 |
| $b$ | -0.175 |
| $c$ | -0.483 |

Table 4-4. $J_2$-predicted slip regimes and contact width $a_f/a_o$ ratios across a range of $\delta_{app}$, where $a_f$ is final contact width (after 12 cycles).

<table>
<thead>
<tr>
<th>$\delta_{app}$ (µm)</th>
<th>$P/P_y = 0.5$</th>
<th>$P/P_y = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slip regime</td>
<td>$a_f/a_o$</td>
</tr>
<tr>
<td>0.5</td>
<td>Partial slip</td>
<td>1</td>
</tr>
<tr>
<td>0.75</td>
<td>Partial slip</td>
<td>1.25</td>
</tr>
<tr>
<td>1</td>
<td>Gross slip</td>
<td>1.25</td>
</tr>
<tr>
<td>1.25</td>
<td>Gross slip</td>
<td>1.25</td>
</tr>
<tr>
<td>1.5</td>
<td>Gross slip</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>Gross slip</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Note: $a_o = 0.052$ mm for $P/P_y = 0.5$; $a_o = 0.097$ mm for $P/P_y = 2$. 
Table 4-5. CP-predicted slip regimes and contact width $a_f/a_0$ ratios across a range of $\delta_{app}$, where $a_f$ is final contact width (after 12 cycles).

<table>
<thead>
<tr>
<th>$\delta_{app}$ (µm)</th>
<th>$P/P_y = 0.5$</th>
<th>$P/P_y = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slip regime</td>
<td>$a_f/a_0$</td>
</tr>
<tr>
<td>0.5</td>
<td>Partial slip</td>
<td>1.376</td>
</tr>
<tr>
<td>0.75</td>
<td>Partial slip</td>
<td>1.690</td>
</tr>
<tr>
<td>1</td>
<td>Gross slip</td>
<td>1.879</td>
</tr>
<tr>
<td>1.25</td>
<td>Gross slip</td>
<td>1.880</td>
</tr>
<tr>
<td>1.5</td>
<td>Gross slip</td>
<td>1.942</td>
</tr>
<tr>
<td>2</td>
<td>Gross slip</td>
<td>1.882</td>
</tr>
</tbody>
</table>

Note: $a_o = 0.052$ mm for $P/P_y = 0.5$; $a_o = 0.123$ mm for $P/P_y = 2$.

Table 4-6. $J_2$-predicted position and angle of maximum critical plane SWT value along the substrate surface for different displacement amplitudes for $P/P_y = 0.5$.

<table>
<thead>
<tr>
<th>$\delta_{app}$ (µm)</th>
<th>SWT (MPa)</th>
<th>Failure location, $x/a_0$</th>
<th>Life</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.340</td>
<td>-0.818</td>
<td>1446630</td>
<td>165°</td>
</tr>
<tr>
<td>0.75</td>
<td>0.427</td>
<td>0.769</td>
<td>828854</td>
<td>160°</td>
</tr>
<tr>
<td>1</td>
<td>0.656</td>
<td>0.962</td>
<td>301880</td>
<td>165°</td>
</tr>
<tr>
<td>1.25</td>
<td>0.648</td>
<td>0.962</td>
<td>309436</td>
<td>165°</td>
</tr>
<tr>
<td>1.5</td>
<td>0.668</td>
<td>-0.943</td>
<td>288482</td>
<td>15°</td>
</tr>
<tr>
<td>2</td>
<td>0.684</td>
<td>-0.943</td>
<td>274759</td>
<td>160°</td>
</tr>
</tbody>
</table>

Table 4-7. $J_2$-predicted position and angle of maximum critical plane SWT value along the substrate surface for different displacement amplitudes for $P/P_y = 2$.

<table>
<thead>
<tr>
<th>$\delta_{app}$ (µm)</th>
<th>SWT (MPa)</th>
<th>Failure location, $x/a_0$</th>
<th>Life</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2125853</td>
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<tr>
<td>0.75</td>
<td>0.394</td>
<td>1.039</td>
<td>1013391</td>
<td>15°</td>
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<tr>
<td>1</td>
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<td>-1.037</td>
<td>393459</td>
<td>15°</td>
</tr>
<tr>
<td>1.25</td>
<td>0.697</td>
<td>-1.104</td>
<td>262274</td>
<td>15°</td>
</tr>
<tr>
<td>1.5</td>
<td>0.842</td>
<td>-1.171</td>
<td>170617</td>
<td>15°</td>
</tr>
<tr>
<td>2</td>
<td>1.473</td>
<td>-1.324</td>
<td>50087</td>
<td>160°</td>
</tr>
</tbody>
</table>
Table 4-8. CP-predicted crack initiation life, $N_i$, location and angle for a range of displacement amplitudes for $P/P_y = 0.5$ based on $p_{crit}$.

<table>
<thead>
<tr>
<th>$\delta_{app}$ (µm)</th>
<th>$p_{cyc}$</th>
<th>Location $x/a_0$</th>
<th>$N_i$</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.0074</td>
<td>1.058</td>
<td>7946</td>
<td>N/A</td>
</tr>
<tr>
<td>0.75</td>
<td>0.011</td>
<td>1.347</td>
<td>5345</td>
<td>N/A</td>
</tr>
<tr>
<td>1</td>
<td>0.0115</td>
<td>1.443</td>
<td>5113</td>
<td>N/A</td>
</tr>
<tr>
<td>1.25</td>
<td>0.012</td>
<td>1.539</td>
<td>4900</td>
<td>N/A</td>
</tr>
<tr>
<td>1.5</td>
<td>0.012</td>
<td>1.539</td>
<td>4900</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>0.0125</td>
<td>1.539</td>
<td>4704</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 4-9. CP-predicted crack initiation life, $N_i$, location and angle for a range of displacement amplitudes for $P/P_y = 2$, based on $p_{crit}$.

<table>
<thead>
<tr>
<th>$\delta_{app}$ (µm)</th>
<th>$p_{cyc}$</th>
<th>Location $x/a_0$</th>
<th>$N_i$</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.0022</td>
<td>-1.174</td>
<td>26727</td>
<td>52.9°</td>
</tr>
<tr>
<td>0.75</td>
<td>0.0050</td>
<td>-1.292</td>
<td>11879</td>
<td>51.5°</td>
</tr>
<tr>
<td>1</td>
<td>0.0084</td>
<td>-1.344</td>
<td>7000</td>
<td>50°</td>
</tr>
<tr>
<td>1.25</td>
<td>0.0090</td>
<td>1.377</td>
<td>6497</td>
<td>48.7°</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0159</td>
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<td>3709</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>0.0169</td>
<td>-1.826</td>
<td>3469</td>
<td>N/A</td>
</tr>
</tbody>
</table>
4.6 Figures

Figure 4-1. Different stages of crystal deformation [3].
Figure 4-2. Simple schematic of the evolution of the hardness function [3].

Figure 4-3. (a) Atomic structure of a FCC crystal lattice and (b) illustration of the slip directions and corresponding slip plane for a FCC material.
Figure 4-4. Plane strain unit cell FE model for CP uniaxial simulations of 316L SS: (a) an assumed regular hexagonal grain shape and grain size dimension, $d$; see insert, (b) a Voronoi tessellation mesh of the microstructure, and (c) a square grain mesh.
Figure 4-5. CP predicted stabilised hysteresis loops for 316L SS.
Figure 4-6. Comparison of CP cyclic stress-strain curve with macroscopic (aggregate) cyclic stress-strain curve for 316L SS, as represented by data in Table 4-1 [2].

Figure 4-7. Stabilised loop for $\Delta \varepsilon = 1\%$ after 16 cycles.
Figure 4-8. FE predicted inhomogeneous distribution of $p$ for $\Delta \varepsilon = 2\%$.

Figure 4-9. Validation of identified $p_{\text{crit}}$ value for crack initiation in 316L SS, based on CP unit cell predictions, against measured Coffin-Manson relationship.
Figure 4-10. Schematic of University of Nottingham round-on-flat fretting wear test arrangement [18].

Figure 4-11. SEM images of partial slip crack initiation in Ti-6Al-4V [18], after 300,000 cycles under a normal load of 100 N/mm and a 50 μm stroke.
Figure 4-12. (a) Boundary conditions, loads, and displacements in the fretting model. (b) The CP region is highlighted in red in the model.

Figure 4-13. Comparison of CP predicted cyclic plasticity response under applied strain range, $\Delta \varepsilon$, of 2% for (a) hexagonal grain (deformed) mesh and (b) matching square grain (deformed) mesh; both grain morphologies predict almost identical maximum values of accumulated plastic slip of $p \approx 1.79$. 
Figure 4-14. The effect of mesh refinement on predicted total life ($N_f$), from critical plane SWT, as a function of applied displacement (the legend indicates square element side lengths).
Figure 4-15. $J_2$ and CP fretting model predicted evolutions of contact pressure under different tangential displacements ($\delta_{app}$) for $P/P_y = 0.5$. 

(a) $J_2$ model, $\delta_{app} = 0.5 \mu m$

(b) CP model, $\delta_{app} = 0.5 \mu m$

(c) $J_2$ model, $\delta_{app} = 1 \mu m$

(d) CP model, $\delta_{app} = 1 \mu m$

(e) $J_2$ model, $\delta_{app} = 2 \mu m$

(f) CP model, $\delta_{app} = 2 \mu m$
Figure 4-16. $J_2$ and CP fretting model predicted evolutions of contact pressure under different tangential displacements ($\delta_{app}$) for $P/P_y = 2$. 
Figure 4-17. $J_2$ and CP fretting model predicted evolutions of contact shear traction under different tangential displacements ($\delta_{app}$) for $P/P_y = 0.5$. 

(a) $J_2$ model, $\delta_{app} = 0.5$ µm

(b) CP model, $\delta_{app} = 0.5$ µm

(c) $J_2$ model, $\delta_{app} = 1$ µm

(d) CP model, $\delta_{app} = 1$ µm

(e) $J_2$ model, $\delta_{app} = 2$ µm

(f) CP model, $\delta_{app} = 2$ µm
(a) $J_2$ model, $\delta_{app} = 0.5 \, \mu m$

(b) CP model, $\delta_{app} = 0.5 \, \mu m$

(c) $J_2$ model, $\delta_{app} = 1 \, \mu m$

(d) CP model, $\delta_{app} = 1 \, \mu m$

(e) $J_2$ model, $\delta_{app} = 2 \, \mu m$

(f) CP model, $\delta_{app} = 2 \, \mu m$

Figure 4-18. $J_2$ and CP fretting model predicted evolutions of contact shear traction under different tangential displacements ($\delta_{app}$) for $P/P_y = 2$. 

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Figure 4-19. $J_2$ and CP fretting model predicted evolutions of SWT distribution under different tangential displacements ($\delta_{app}$) for $P/P_y = 0.5$. 

(a) $J_2$ model, $\delta_{app} = 0.5 \, \mu m$

(b) CP model, $\delta_{app} = 0.5 \, \mu m$

(c) $J_2$ model, $\delta_{app} = 1 \, \mu m$

(d) CP model, $\delta_{app} = 1 \, \mu m$

(e) $J_2$ model, $\delta_{app} = 2 \, \mu m$

(f) CP model, $\delta_{app} = 2 \, \mu m$
(a) $J_2$ model, $\delta_{app} = 0.5 \, \mu m$

(b) CP model, $\delta_{app} = 0.5 \, \mu m$

(c) $J_2$ model, $\delta_{app} = 1 \, \mu m$

(d) CP model, $\delta_{app} = 1 \, \mu m$

(e) $J_2$ model, $\delta_{app} = 2 \, \mu m$

(f) CP model, $\delta_{app} = 2 \, \mu m$

Figure 4-20. $J_2$ and CP fretting model predicted evolutions of SWT distribution under different tangential displacements ($\delta_{app}$) for $P/P_y = 2$. 
Figure 4-21. SWT-predicted $N_f$ as a function of displacement amplitude for $P/P_y = 2$ and $P/P_y = 0.5$, using (a) $J_2$-fretting model and (b) CP fretting model.
Figure 4-22. Predicted evolution of accumulated plastic slip per cycle, $p_{\text{cyc}}$, under different tangential displacements ($\delta_{\text{app}}$) and normal loads ($P$).
Figure 4-23. FE predicted contour plots of accumulated plastic slip $\delta$ under different tangential displacements ($\delta_{app}$) and normal loads.
Figure 4-24. $p_{\text{crit}}$-predicted $N_i$ as a function of displacement amplitudes for $P/P_y = 2$ and $P/P_y = 0.5$. 
Figure 4-25. Comparison of CP-predicted fretting-induced initiation ($N_i$) and total ($N_f$) lives as functions of applied tangential displacement for low and high normal loads of (a) $0.5P_y$ and (b) $2P_y$. 

(a) 

(b)
5 Micro-Mechanical Modelling of Fretting Fatigue Crack Initiation and Wear in Ti-6Al-4V

5.1 Introduction

A micro-mechanical modelling methodology is presented for predicting fretting fatigue crack nucleation in dual-phase Ti-6Al-4V. The methodology is based on a combination of (i) a unit cell CP model of Ti-6Al-4V, (ii) a frictional contact model of an experimental fretting wear configuration, and (iii) implementation of a microstructure-sensitive FIP, for prediction of crack nucleation. A novel micro-mechanical method for wear prediction is presented. A key finding is the important role of cyclic micro-plasticity in ostensibly elastic loading regimes in fretting wear, resulting in significant effects on distributions of salient fretting contact variables such as contact pressure.

5.2 Methodology

5.2.1 Micro mechanical model

Ti-6Al-4V consists of a dual phase microstructure, typically divided into 60% $\alpha$ phase and 40% $\alpha$-$\beta$ phase. $\alpha$ phase consists of normal HCP crystalline structure with plastic slip occurring on the basal $3\{\bar{1}2\bar{0}\}\{0001\}$, prismatic $3\{\bar{1}2\bar{0}\}\{10\bar{1}0\}$ and pyramidal $3\{\bar{1}2\bar{0}\}\{10\bar{1}1\}$ slip systems as seen in Figure 5-1. Due to the high resolved shear stress required for slip to occur on the pyramidal plane, three times that of the basal plane [1], pyramidal slip is not modelled here. The lamellar structure of the $\alpha$-$\beta$ phase is modelled by combining both HCP and BCC slip systems into a continuous homogeneous phase [2]. This phase is not modelled as individual
alternating layers of $\alpha$ and $\beta$ due to the small size scales of these layers. This would significantly increase mesh density and result in potentially detrimental computational overhead; instead, following the approach of [3], a continuous grain with representative properties is implemented. This consists of 12 BCC slip systems $\langle 111 \rangle \{110\}$ and the previously mentioned HCP slip systems together. Twinning is also neglected here for Ti-6Al-4V, based on the work of Dick and Cailletaud [4] and Morrissey et al. [2]. Appendix 1.4 presents the code developed for modelling HCP and BCC slip systems within the user subroutine.

5.2.2 Cylinder on flat contact model

An FE cylinder-on-flat fretting configuration based on the experimental work of Ding et al. [5] is implemented within this chapter. The loading conditions used within the FE fretting model are tabulated in Table 5-1, including the applied parameters and resulting slip regimes. The schematic of Figure 5-2 illustrates the experimental arrangement and loading history.

The FE model follows the approach adopted in Chapter 4 and by McColl et al. [6]. A two-dimensional plane strain FE model as seen in Figure 5-3 is developed with a 6 mm cylindrical pad in contact with a $5 \times 10$ mm substrate. Numerous authors have reported a COF for Ti-6Al-4V under fretting conditions. The current work is based on experimental observations by Ding et al. [5] where a COF of 0.9 is reported for gross sliding conditions. Furthermore, experimental work carried out by Jin and Mall [7] for fretting on Ti-6Al-4V have reported a similar COF value in the region of 1.1 for the same sliding regime. The region of contact is of greatest interest within this work; therefore the greatest mesh density is used in this area, giving a minimum element size of 2.5 µm. Within the fretting model uniform square grains are used. This facilitates better mesh control and more systematic analysis of results,
as mentioned in the previous chapter. The comparison study between square and hexagonal grains of Chapter 4 showed almost identical results in terms of $p$, see Figure 4-13.

As in Chapter 4 two different material models are used here, namely a $J_2$ NLKH model and a CP material model. There are also two fretting models. The first is a hybrid model (see Figure 5-3) similar to that of Figure 4-12, with a CP region positioned in the area of interest, the contact region, surrounded by a $J_2$ NLKH model. Within this CP region there are 210 square grains, $30 \times 7$, consisting of an element size of 2.5 µm; each grain is assigned a unique randomly generated grain orientation, in addition to a randomly assigned material phase. The second is a $J_2$ fretting model, which uses the $J_2$ NLKH material model for both contacting surfaces, i.e. the substrate specimen and the cylindrical pad. The material constants are listed in Table 5-2, taken from Benedetti et al. [8]. Compliance within the experimental rig, with respect to tangential force-displacement response, is modelled here using a horizontal spring element as shown in Figure 5-3. Figure 5-4 shows the comparison of the FE and measured tangential force displacement response for the calibrated spring stiffness.

As previously discussed in Chapter 4, normal load has a significant effect on fretting. The largest normal load applied within this work is 100 N/mm, which is equal to $0.15P_y$ [9], i.e. well within the nominally elastic regime.

5.2.3 Microstructure-sensitive fretting damage

The significant difference between partial and gross slip and the resulting surface damage has been previously mentioned. It is important particularly in the context of micro-crack nucleation, due to contact stresses, to develop a method for distinguishing between fretting wear and cracking. Figure 5-5 introduces two
different damage concepts via a simple schematic. Case A (cracking) shows localised discrete areas where large concentrations of accumulated crystallographic slip, $p$, greater than $p_{\text{crit}}$ have developed. Case A represents a typical partial slip situation with cracking concentrated in the slip zones. Case B (wear) illustrates a simply connected, more evenly distributed region of accumulated crystallographic slip, $p$, greater than the critical value, $p_{\text{crit}}$. It is important to note here that Sweeney et al. [10], for example, have shown that cyclic micro-plasticity occurs in situations where loading is ostensibly elastic (at a macro scale). This distributed micro-cracking process, effectively wear, is characteristic of gross slip situations where micro-damage occurs throughout the entire contact region. The stabilised accumulated plastic slip per cycle parameter, $p_{\text{cyc}}$, is implemented here, within a numerical methodology to predict wear depth, based on the assumption that any material with $p(x, y, N) \geq p_{\text{crit}}$ has been worn away after that number of fretting cycles, $N$. Figure 5-6 illustrates the process, as follows:

1. Firstly, distributions of $p$ versus depth (at integration points) are computed for different horizontal locations $x_i$ across the contact width, by assuming that the stabilised (integration point) values of accumulated plastic slip, $p_{\text{cyc}}(x, y)$, from early fretting cycles, can be extrapolated to large numbers of fretting cycles, using the equation $p(x, y, N) = p_{\text{cyc}}(x, y)N$.

2. For a given $p_{\text{crit}}$, corresponding to micro-crack initiation, it is then possible to identify, by interpolation with respect to integration point values of $p(x, y, N)$, for each value of $x_i$, the depth $d_i$ to which $p(x, y, N) \geq p_{\text{crit}}$ after a given number of cycles, $N$. This distribution of depth $d_i(x_i, N)$ is then taken as the predicted wear depth distribution for that number of cycles $N$. 

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Hence, the methodology provides wear depth distributions for different numbers of fretting cycles.

**5.3 Results**

5.3.1 *Crystal plasticity calibration process*

Two different CP models are investigated:

- a single-phase CP model, where one set of CP hardening constants is used to represent both the α phase and the α-β phases.
- a dual-phase CP model, where two sets of hardening constants are used to separately model the α phase and the α-β phase.

A unit cell model consisting of a uniform hexagonal grain morphology is implemented, as shown in Figure 5-7, for calibration of the CP material and failure constants. The hexagonal morphology includes triple points which are an important feature in microstructural modelling and deformation. Individual random crystallographic orientations are assigned to each grain. A random number generator is used to arbitrarily assign grains as either α or α-β phase throughout the dual-phase model in the correct volume fraction proportions. The unit cell consists of 42 uniform size hexagonal grains and 22 partial grains making up the grain boundaries. 4200 four-noded, plane strain elements and symmetry boundary conditions are used; more details are given in Chapter 4. A grain size of 20 µm is used for both phases based on the micrographs of Ding et al. [5].

Individual CP material parameters \((h_0, g_\infty, g_0)\), see equation 4.18, 4.20, 4.21 and 4.22, are calibrated for each phase. This was achieved by identifying the CP parameters for the α phase from an iterative calibration process using α-phase Ti-6Al monotonic tensile data at different strain rates [11], as seen in Figure 5-8. In this case
the CP unit cell model was comprised entirely of \(\alpha\) phase grains. Benedetti [8] provides CSSC for Ti-6Al-4V. Keeping the identified \(\alpha\) phase CP parameters (see Table 5-3), the dual phase unit cell model (composed of randomly generated \(\alpha\)-phase and \(\alpha\)-\(\beta\) phase grains) is subjected to uniaxial strain-controlled, fatigue loading and compared to the CSSC for Ti-6Al-4V, to iteratively identify the CP material constants for the \(\alpha\)-\(\beta\) phase. In the case of the single phase CP model, the CSSC data for Ti-6Al-4V [8] was sufficient to calibrate the single set of hardening constants. Table 5-3 tabulates the CP hardening constants for both phases in the dual phase model and the single phase model. The effect of random orientation on the cyclic stress strain response is also investigated for the dual phase CP model. Five different random orientation sets are assigned to the microstructure and the results of the calibration process for the dual phase are presented in Figure 5-9, which compares the experimental results [8] and the FE predictions corresponding to the identified dual phase material constants.

5.3.2 Predicted low cycle behaviour of Ti-6Al-4V

Using a single \(N_t\) value and corresponding plastic strain range, \(\Delta\varepsilon_p\), a \(p_{crit}\) value of 126 is identified for micro-crack nucleation following the approach of Chapter 4 and [12] for the dual-phase CP model. Using this value of \(p_{crit}\) the entire LCF strain-life response of Ti-6Al-4V can be predicted. Figure 5-10 shows the correlation between the Coffin-Manson equation, based on fatigue constants presented in Table 5-4, and these microstructure-sensitive crack initiation predictions (CP response). The effect of random orientation on CP predicted life, as represented by the horizontal error bars, illustrates the ability of the CP model to represent fatigue life scatter due to microstructural inhomogeneity. This same methodology and identified \(p_{crit}\) value is subsequently applied to a dual-phase cylinder-on-flat fretting configuration, below,
to predict micro-crack nucleation and wear. A similar method is applied to identify the single phase $p_{crit}$ value of 68.

5.3.3 Single-phase CP fretting analysis

Figure 5-11 shows the predicted evolutions of partial slip contact variable distributions for the two different material models, $J_2$ and single phase CP. In general the trends are very similar to these of the Chapter 4 for 316L SS. The contact variables are plotted as a function of $x/a_0$, where $a_0$ is the contact semi width for the initial normal load, $N = 0$. The contact pressure distributions for the $J_2$ model and the initial ($N = 0$) contact pressure for the CP model agree with the analytical (Hertzian) solution (Figure 5-11(a) and 5-11(b)). However with increased fretting cycles it is clear that surface microstructural inhomogeneities leads to an inhomogeneous distribution of contact pressure. Goh et al. [13] described this as the development of a "natural surface roughness" in the contact wear scar. Hence, for $N > 1$ the CP pressure distribution follows the general Hertzian profile but with significant local variations about this mean Hertzian distribution, giving rise to locally (much) higher peak contact pressures. By $N = 6$, the distribution has effectively stabilised due to plastic shakedown. The characteristic partial slip surface shear distribution is evident from the $J_2$ model (Figure 5-11(c)), where the central stick zone has a contact shear value less than $\mu p(x)$. The CP predicted shear stress distribution ($N = 1$) also shows significant microstructure induced variations (Figure 5-11(d)) about the mean ($J_2$) partial slip distributions of Figure 5-11(c). The distribution has effectively stabilised by about $N = 12$ in Figure 5-11(d). In this case the local peaks are not predicted to exceed the peak partial slip values at the stick-slip interface of Figure 5-11(c). Similar trends are predicted for gross slip loading conditions, as shown in Figure 5-12. Comparison to the Hertzian solution shows
good agreement for both the $J_2$ model and the initial $N = 0$ cycle for the single phase CP model. Clearly frictional effects (shear) have a significant effect in terms of micro-plasticity on the distributions of contact pressure and shear. Significant micro-plasticity is evident in the case of the single phase CP model even in this nominally elastic loading situation. A significant widening of the contact area (11 to 18%) is predicted by the CP model especially for the higher normal load (partial slip case) as a result of tangentially-induced (frictional) plastic deformation. Note that the contact pressure distributions are sampled at the zero-stroke position while the shear traction distribution is sampled at the extreme stroke position.

Figure 5-13 shows the predicted distributions of accumulated crystallographic plastic slip per cycle, $p_{cyc}$, for the partial and gross slip cases. These distributions are calculated at the top layer of integration points on the substrate surface. For the partial slip case, Figure 5-13 (a), highly localised concentrations of $p_{cyc}$ are present in the slip zones with a zero $p_{cyc}$ region in the stick zone. A more uniform distribution is predicted for the gross slip condition in Figure 5-13 (b). It is proposed here that this more uniform distribution of $p_{cyc}$ be treated as a multiple micro-cracking process which eventually leads to material removal (Case B). The predicted crack initiation lives, based on the microstructure-sensitive methodology, for both regimes are presented in Table 5-5.

5.3.4 Dual phase CP fretting crack initiation

The dual phase model is used to compare against the experimentally observed crack initiation lives, locations and orientations from the work presented by Ding et al. [5]. Using this more realistic representation of the microstructure, the predicted contact variables for the partial slip case are presented in Figure 5-14, where a greater degree of inhomogeneity is evident in both the contact pressure and shear evolutions as
compared to the single phase case (see Figure 5-11 (b) and Figure 5-11 (d)). The peak surface tractions coincide with surface phase boundary locations. In the experimental fretting tests, cracks were observed at lengths ranging from 5 to 50 µm, after up to $10^5$ cycles. Figure 5-15 shows the effect of random grain orientations for a sample of orientation sets for contact pressure and shear. Clearly, the microstructure is predicted to have a significant effect on these distributions, both qualitatively, in terms of local peaks in normal or shear traction (note again that the contact pressure distribution is sampled at the zero-stroke position, while the shear traction is sampled at the extreme stroke position), and quantitatively, in terms of magnitudes of instantaneous peak pressures and shears, e.g. compare Figure 5-15 (a) to Figure 5-15 (e). Table 5-6 shows an average initiation life, of $5 \times 10^3$, which is consistent with the experimental observations. Microscopy techniques used in the experimental work quantified crack orientation and location. Comparisons are made in Table 5-6 between the predicted crack initiation lives of the different microstructures (random orientation sets). Figure 5-16 shows an inhomogeneous distribution of contact slip, corresponding with the distributions of contact pressure and shear of Figure 5-11, which, as mentioned above, are indicative of the development of "natural surface roughness" [13]. An apparent stick region is observed in Figure 5-16 showing the distribution of the contact slip for the 12th cycle for a typical partial slip situation. However on closer inspection the "stick-slip interface" is not at a single point but is distributed over a region where both sticking and slipping are predicted. This region is a typical location for cracking to occur, as can be seen from the SEM image shown in Figure 5-17 (a). It can also be seen that cracking is not limited solely to the stick-slip interface but can occur anywhere within the slip zone. Table 5-6 also shows the predicted crack locations for the five random orientation sets, in comparison with the
corresponding experimental crack location range. Comparing the CP model in Figure 5-17 (b) to the SEM image of experimental crack initiation sites in Figure 5-17 (a) [5] it is possible to qualitatively evaluate the micro-crack locations. The five experimental micro-cracks are labelled via roman numerals and the corresponding CP predicted locations are illustrated in a similar fashion. These comparisons are taken after 300,000 cycles for a partial slip situation.

5.3.5 Wear scar predictions and wear coefficients

Applying the microstructure-sensitive fretting wear extrapolation technique of Figure 5-6, wear scars are predicted for the distributions of $p_{cye}$ presented for the gross slip CP model in Figure 5-18. Damage is predicted across the entire contact region, corresponding to Case B (Wear) of Figure 5-5, and resulting in a "U shaped" wear scar. Figure 5-18 and Table 5-7 compare the experimentally measured wear scar to the FE predicted wear profile after 100,000 cycles. An Archard wear coefficient is computed from the predicted wear scar using the equation:

$$k = \frac{V}{PS} \quad (5.1)$$

where $k$ is the wear coefficient, $V$ is the measured wear volume, $P$ the normal load and $S$ is the total sliding distance. Using the wear scar presented in Figure 5-19 a comparison is given in Table 5-8 between the measured experimental and FE predicted wear coefficient after 300,000 cycles.

5.4 Discussion

The accumulated plastic slip per cycle parameter, $p_{cye}$, presented in Figure 5-13, gives distinct differences between the two sliding regimes. While the predicted crack initiation lives are similar, the form of $p_{cye}$ distribution, allows differentiation
between the resulting modes of fretting damage. For Case A (cracking), localised peaks in $p_{cyc}$, and hence micro-cracking, are predicted across the contacting surfaces and, therefore, this will lead to localised crack propagation and premature failure, upon fatigue loading of the substrate. In contrast, Case B (wear) predicts a more uniform micro-cracking and micro-damage occurring across the entire contact region. This uniform micro-cracking process will inevitably lead to asperity removal, debris, stress redistribution and eventually wear. This form of fretting damage will result in an increase in number of cycles to fatigue failure compared to the partial slip regime, in situations involving substrate fatigue loading [14].

Considerable phase boundary effects are evident from the contact variable distributions of the dual phase CP model that were not captured by the single phase CP model, (Figure 5-14 and Figure 5-15). These effects are due to the variations in yield stresses between the "harder" $\alpha$ phase and "softer" $\alpha$-\(\beta\) phase grains. The effect of random orientation is investigated for the partial slip model to compare against the experimental results. The CP model can capture fatigue scatter as illustrated in Figure 5-9 and Figure 5-10. This benefit is further evident in Table 5-6 where different initiation lives are predicted based on random orientation effects. Ding et al. [5] observed crack nucleation lengths from 5 to 50 µm and stated that all cracks nucleated within $10^5$ cycles. The hybrid CP model predicts cracks nucleating at length scales less than 2.5 µm and predicts initiation lives that compare well with the experimental data, given the shorter length-scales of the CP-predicted "cracks".

Using the dual phase CP model, reasonable correlation is evident for predicting crack locations throughout the contact region. The $p_{cyc}$ distribution of Figure 5-17 (b) can be compared against the SEM image (Figure 5-17 (a)) after 300,000 cycles. Cracks are shown to occur not only at the trailing edge of contact or
the stick-slip interface, as conventional FIPs predict [5], but throughout the slip zone, depending on microstructural effects or deformities. The dual phase CP model has the ability to model these microstructural effects and highlights similar cracking locations (Table 5-6). These crack locations are heavily influenced by the position of phase boundaries and favourably orientated grains. Although the majority of crack initiation sites correlate with the experimental results, orientation SET-4 predicts a nucleation site at the contact centre. While this is an unusual location for cracking under partial slip conditions, it corresponds to a phase boundary, coinciding with a favourably orientated grain, which causes a significant build-up in crystallographic slip. The strong mismatch between α and α-β phase properties, coupled with the high coefficient of friction, results in significant predicted discontinuities of contact pressure, shear (Figure 5-14 and Figure 5-15) and associated subsurface stresses and crystallographic slips. This in turn has led to convergence difficulties for the gross slip (wear) analyses; hence, the gross slip analyses in the present work employ the single phase CP material model.

The FE predicted gross slip wear scar (Figure 5-18) gives good correlation with the experimental for maximum wear depth; however the predicted width is significantly less than the measured width. The wear methodology presented here does not include material removal, e.g. Madge et al. [14], or wear debris effects, e.g. Ding et al. [15]. After 100,000 cycles there is significant widening of the experimental contact zone as a result of material removal, debris and ploughing effects. It is also important to note that the FE model is a two-dimensional representation of a 3D fretting wear scar sampled along a particular transverse location across the specimen width. Sampling along different transverse locations
could be expected to result in, at least slightly, different experimental wear scar profiles. This type of effect requires three-dimensional CP modelling.

The good correlation of predicted wear coefficient, of approximately $3 \times 10^{-9}$ MPa$^{-1}$, with the experimental value of approximately $8 \times 10^{-9}$ MPa$^{-1}$, gives confidence in the ability of CP modelling to capture the important effects of micro-plasticity, and hence micro-fatigue, in the wear process for Ti-6Al-4V. A key next step is the development of a material removal wear simulation methodology based on the use of $p_{cyc}$, instead of the Archard or energy based [14] [16] wear methods. This would give a more scale-consistent, and hence fundamental approach, which could also achieve unification of crack initiation for fatigue and wear.

This chapter has assumed a regular microstructure (grain shape and size). Clearly, the real microstructure has a statistical distribution of grain size and grain shape, which in turn affects micro-cracking (wear) and fretting crack nucleation. Of course, texture can have an important role on fretting behaviour. The work of Goh et al. [17], for example, highlights a considerable effect of texture on plastic strain accumulation and states that the role of texture may be significant in resisting fretting damage. The experimental work of Ding et al. [5] however indicated no significant texture within the material tested here. Since grain size has a significant effect on fatigue crack initiation and propagation [18], length scale effects (viz. strain gradient plasticity [19]), also need to be incorporated into future CP modelling.

### 5.5 Conclusion

A microstructure-sensitive crack initiation predictive methodology was implemented for fretting of Ti-6Al-4V. Significant effects of micro-plasticity on the evolutions of fretting contact tractions are demonstrated for an ostensibly elastic fretting loading
situation. The microstructure-sensitive approach of Chapter 4 was implemented for Ti-6Al-4V and (i) predicted partial slip cracking (multiple sites, number of cycles to crack initiation, crack angles and crack locations), corresponding with experimental data, and (ii) was adopted to facilitate a novel wear prediction methodology, which predicted a wear coefficient and a two-dimensional wear scar reasonably consistent with experimental data for gross slip cases.
5.6 References


### 5.7 Tables

Table 5-1. Loading parameters for experimental fretting tests [5].

<table>
<thead>
<tr>
<th>Regime</th>
<th>Normal load (N/mm)</th>
<th>Stroke (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross slip</td>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td>Partial slip</td>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 5-2. Material data, including non-linear kinematic hardening data, for Ti-6Al-4V [8].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>116 GPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.342</td>
</tr>
<tr>
<td>$k$</td>
<td>840 MPa</td>
</tr>
<tr>
<td>$C$</td>
<td>8976 MPa</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>102</td>
</tr>
</tbody>
</table>

Table 5-3. Identified CP constitutive constants for cyclic behaviour of Ti-6Al-4V for both CP models.

<table>
<thead>
<tr>
<th></th>
<th>Single phase</th>
<th>Dual phase</th>
<th>Dual phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_0$ (MPa)</td>
<td>170</td>
<td>220</td>
<td>128</td>
</tr>
<tr>
<td>$g_\infty$ (MPa)</td>
<td>270</td>
<td>650</td>
<td>175</td>
</tr>
<tr>
<td>$g_0$ (MPa)</td>
<td>165</td>
<td>265</td>
<td>75</td>
</tr>
<tr>
<td>$\dot{a}$ (s$^{-1}$)</td>
<td>0.0023</td>
<td>0.0023</td>
<td>0.0023</td>
</tr>
<tr>
<td>$n$</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 5-4. Fatigue constants for Coffin-Manson equation for Ti-6Al-4V [14].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_f'$</td>
<td>0.841</td>
</tr>
<tr>
<td>$c$</td>
<td>-0.688</td>
</tr>
</tbody>
</table>

Table 5-5. $p_{crit}$ based fretting micro-crack initiation predictions.

<table>
<thead>
<tr>
<th>$p_{crit}$</th>
<th>Partial slip</th>
<th>Gross slip</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of cycles, $N_i$</td>
<td>$1.6 \times 10^3$</td>
<td>$1.5 \times 10^3$</td>
</tr>
</tbody>
</table>
Table 5-6. Comparisons between experimental and CP (dual phase model) predicted crack initiation, crack orientations and crack locations.

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Crack angle$^\circ$</th>
<th>Location (x/a$_0$)</th>
<th>$N_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>&lt; 30</td>
<td>0.4 - 0.9</td>
<td>&lt; 100,000</td>
</tr>
<tr>
<td>Predicted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SET-1</td>
<td>6</td>
<td>0.65</td>
<td>3,100</td>
</tr>
<tr>
<td>SET-2</td>
<td>21</td>
<td>0.67</td>
<td>3,200</td>
</tr>
<tr>
<td>SET-3</td>
<td>30</td>
<td>1.24</td>
<td>2,200</td>
</tr>
<tr>
<td>SET-4</td>
<td>3</td>
<td>0.02</td>
<td>11,000</td>
</tr>
<tr>
<td>SET-5</td>
<td>29</td>
<td>1.1</td>
<td>7,100</td>
</tr>
<tr>
<td>Average</td>
<td>17.8</td>
<td>0.736</td>
<td>5,200</td>
</tr>
</tbody>
</table>

Table 5-7. Comparison of wear scar dimensions for the measured experimental and FE predicted wear profile after 100,000 cycles.

<table>
<thead>
<tr>
<th></th>
<th>Maximum wear depth (µm)</th>
<th>Wear scar width (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>23</td>
<td>1100</td>
</tr>
<tr>
<td>Predicted</td>
<td>26</td>
<td>237</td>
</tr>
</tbody>
</table>

Table 5-8. Comparison between the measured experimental and the FE predicted wear coefficient after 300,000 cycles for the gross slip case.

<table>
<thead>
<tr>
<th></th>
<th>Wear coefficient (MPa$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>$8.2 \times 10^{-9}$</td>
</tr>
<tr>
<td>Predicted</td>
<td>$2.89 \times 10^{-9}$</td>
</tr>
</tbody>
</table>
5.8 Figures

Figure 5-1. Simple schematic of the crystallographic slip systems for α and α-β phase, where the red arrows represent the slip directions and the hatched lines represent the slip planes.

Figure 5-2. Simple fretting test schematic plus the load and displacement history of the experimental arrangement.
Figure 5-3. FE frictional contact model highlighting the CP region in the contact zone.

Figure 5-4. Comparison between the measured [5] and FE predicted tangential force displacement hysteresis loops.
Figure 5-5. Distributions of accumulated plastic slip greater than the critical value (a) Partial slip (cracking) and (b) gross sliding (wear).

Figure 5-6. Simple schematic of the microstructure-sensitive fretting wear predictive methodology for a given number of fretting cycles $N$. 
Figure 5-7. Unit cell model highlighting the random distributions of \( \alpha \) and \( \alpha-\beta \) phases.

Figure 5-8. Comparison between experimental [11] and FE monotonic uniaxial loading response for iteratively identified \( \alpha \) phase constants of Table 5-3.
Figure 5-9. Comparison between the CP cyclic response corresponding to identified dual phase material constants and the experimental CSSC for Ti-6Al-4V [8]. The error bars correspond to the effect of different distributions of random crystallographic orientation on the predicted cyclic response.

Figure 5-10. Comparison of the dual-phase CP based life predictions compared to the Coffin-Manson relationship including the effect of random orientation.
Figure 5-11. Contact variable distributions for $J_2$ and single phase CP model under partial slip conditions where $P = 100$ N/mm and $\delta_{app} = 80$ $\mu$m, ($a_0 =$ initial contact semi width).

Figure 5-12. Predicted contact variable distributions for $J_2$ and single phase CP model under gross slip conditions $P = 50$ N/mm and $\delta_{app} = 50$ $\mu$m.
Chapter 5

Figure 5-13. Predicted evolution of accumulated plastic slip per cycle, $p_{cyyc}$, for two different fretting regimes for the single phase material model.

(a) CP model, $\delta_{app} = 100 \, \mu m$, $P = 100 \, N/mm$

(b) CP model, $\delta_{app} = 80 \, \mu m$, $P = 50 \, N/mm$
Figure 5-14. Dual phase CP model contact variables with the top layer of the Ti-6Al-4V microstructure highlighting the effect of phase boundary interface on stress heterogeneity for the following partial slip conditions, where $P = 100$ N/mm and $\delta_{app} = 80$ µm.
Figure 5-15. The effect of random orientation on contact variables (pressure and shear) for the dual phase microstructure for a sample of three different orientation sets of Table 5-6: (a) and (b) for Orientation Set 3; (c) and (d) for Orientation Set 4 and (e) and (f) for Orientation Set 5.
Figure 5-16. Contact slip distribution for 12\textsuperscript{th} cycle highlighting the occurrence of a natural surface roughness within the CP model.
Figure 5-17. (a) SEM image of a partial slip case after 300,000 cycles [5], (b) $p_{cyc}$ based crack prediction for partial slip case after 300,000 cycles.

Figure 5-18. Comparison of the FE predicted and the experimentally measured [5] wear profile for gross sliding ($P = 50$ N/mm, $\delta_{app} = 80$ µm) after 100,000 cycles.
Figure 5-19. Extrapolated wear scar after 300,000 cycles, (a) smoothed wear scar of $p(x, y, N)$ and (b) calculated wear profile, for gross sliding ($P = 50 \text{ N/mm}$, $\delta_{ap} = 80 \mu m$) using $p_{cyc}$.
6 Validation of Microstructure-sensitive Crack Nucleation Methodology for Fretting Fatigue of 316L Stainless Steel

6.1 Introduction

This chapter is concerned with the development of a micro-mechanical methodology for prediction of fretting fatigue crack nucleation life and short crack propagation for 316L SS. The methodology employs critical accumulated plastic slip as a FIP for microstructure-sensitive crack nucleation. The methodology is validated against the fretting fatigue tests described in Chapter 3 following the approach presented in Chapters 4 and 5. CP unit-cell models are employed for calibration of constitutive and crack nucleation parameters and a CP frictional contact model of the fretting test rig is developed to predict the micro-scale surface fretting damage, crack nucleation and estimated micro-crack propagation. The microstructure-sensitive method is shown to be superior to a combined $J_2$ plasticity and critical-plane FIP method.

Fatigue life, $N_f$, can be broken up into three different regimes:

$$N_f = N_i + N_{scg} + N_p$$  \hspace{1cm} (6.1)

where $N_i$ is crack initiation of a known crack dimension, typically less than 10 µm, $N_{scg}$ is the number of cycles within the SCG regime and $N_p$ is the number of cycles of crack propagation until specimen fracture. Crack initiation is difficult to experimentally detect due to the microscopic length scales involved. In fatigue problems dislocation motion along persistent slip bands cause a saw tooth surface roughness profile. This surface damage eventually leads to the nucleation of small cracks as a result of microstructural defects, grain boundaries or weak grain
orientations. For accurate modelling of slip system deformation a micromechanical model based on CP theory is used as described previously in Chapters 4 and 5.

6.2 Methodology

6.2.1 Crystal plasticity calibration

Isotropic elasticity is assumed within the CP user subroutine for the 316L SS material, with a Young's modulus of 213 GPa and Poisson's ratio, $\nu$, of 0.34. An important aspect of modelling the deformation on individual slip systems within a metallic grain is the calibration process used to identify the constitutive constants that relate the resolved shear stress to the shear strain. A unit-cell polycrystalline model of uniform hexagonal morphology is developed here (Figure 6-1) following the work of Savage et al. [1]. The approach of this chapter is almost identical to that of Chapter 4, except that a grain size of 19 $\mu$m, as measured, is employed here, where as a value of 104 $\mu$m is used in Chapter 4. Random crystallographic orientations are assigned to each grain using a material grain size of 19 $\mu$m, as seen in Figure 6-2 which shows an optical image of the microstructure of the 316L SS material presented in Chapter 3. In this work the CP constants are identified via calibration with respect to the macroscopic cyclic stress strain curve of 316L SS, as represented by a NLKH$f_2$ material model, using the material data shown in Table 4-1, from [2]. The initial hardening modulus, $h_0$, saturation stress, $g_\infty$, and critical resolved shear stress, $g_0$, thus identified by matching the stabilised cyclic response of the unit-cell model of Figure 6-1, to the macroscopic cyclic response, across a range of applied stress-ranges are listed in Table 6-1. Figure 6-3 shows the comparison between the cyclic stress strain response of the CP unit-cell model, using the
identified constants, and the macroscopic CSSC of 316L SS, using the NLKH constants of Table 4-1.

6.2.2 Microstructure-sensitive crack initiation parameter

As mentioned in previous chapters, work on modelling of fretting and fretting fatigue has employed macroscopic FIP e.g. [3] and [4]. However, as argued in Chapters 4 and 5, it is necessary to employ a microstructure-sensitive FIP for scale consistency when using CP modelling, e.g. see also Sweeney et al. [5]. Hence, the FIP used here is the accumulated plastic slip, \( p \), defined by Manonukul and Dunne [6], described previously in Chapter 4. The approach adopted here, following Chapters 4 and 5 and [6], is to identify \( P_{crit} \) from a specific LCF data point for the material.

6.2.3 Fracture mechanics

\( N_p \) can be calculated by the integration of the Paris equation to calculate the number of cycles for a small crack of a known size to propagate as described in equation 2.13 - 2.14. In this work the geometrical factor \( Y \) is based on a fatigue test specimens experimentally tested in Chapter 3 and modelled here in Chapter 6. (see below) this is taken to be that of an edge crack under uniaxial loading given by

\[
Y = 1.12 - 0.281 \left( \frac{a}{w} \right) + 10.55 \left( \frac{a}{w} \right)^2 - 21.72 \left( \frac{a}{w} \right)^3 + 30.39 \left( \frac{a}{w} \right)^4
\]

(6.2)

where \( w \) is the specimen thickness. Short cracks have been observed to propagate at a faster rate than long cracks [7]. However self-arrest occurs if \( \Delta K \) is below a threshold value, \( \Delta K_{th} \). The El-Haddad approach [8] incorporates a threshold crack length, \( a_{th} \), which represents the transition from SCG to conventional crack growth, as can be seen from the Kitagawa and Takahashi diagram in Figure 6-4. \( a_{th} \) can be
found from equation 2.15. The El-Haddad correction is an empirical approach which allows for the prediction of SCG by substituting \( (a + a_{th}) \) for \( a \) in the SIF equation, when \( a < a_{th} \) (equation 2.16). The stress-life relationship of a material can be described through Basquin's equation for HCF as presented in equation 2.1. The fatigue strength coefficient, \( \sigma' \), and the fatigue strength exponent, \( b \), are obtained from HCF experimental data. Since \( N_f \) in this case is the summation of a number of different cracking regimes it is important to note that the Basquin constants are for macroscopic cracks in the region of 1 to 2 mm. Using the method described previously the number of cycles for crack propagation, \( N_p^{exp} \), is subtracted from the experimental plain fatigue total life, \( N_f \), leaving crack initiation, \( N_i^{exp} \), data. Crack initiation based fatigue constants can subsequently be extracted from this data via equation 2.1.

6.2.4 Fretting fatigue crack growth

Crack propagation under fretting fatigue conditions is a complex issue. Houghton et al. [9] implemented a weight function method, based on the work of Nicholas et al. [10] which analysed the mixed mode cracking of Ti-6Al-4V to successfully predict multiaxial fretting fatigue in a simplified representative fretting fatigue test for spline coupling teeth. Houghton et al. used back calculated fatigue constants with a critical plane SWT approach to animate crack initiation at a length scale of 10 \( \mu \)m. Once crack location was established local stress ranges were identified for mode I and mode II weight functions, \( h_I \) and \( h_{II} \) respectively, as seen in Fett and Munz [11]:

\[
h_I = \sqrt{\frac{2}{\pi \alpha}} \frac{1}{\sqrt{1 - \rho}} \left[ 1 + \sum_{\nu,\mu} \frac{A_{\nu \mu} \sigma^{\mu}}{1 - \alpha} (1 - \rho)^{\nu + 1} \right]
\] (6.3)
\[ h_{II} = \sqrt{\frac{2}{\pi a}} \frac{1}{\sqrt{1 - \rho (1 - \alpha)^{1/2}}} \left(1 - \alpha\right)^{1/2} \]

\[ + \sum_{\nu,\mu} A_{\nu\mu} (1 - \rho)^{\nu+1} \alpha^\nu \]  

(6.4)

where \( \rho = x/a \) and \( \alpha = a/W \). \( a \) is the crack length, \( W \) the specimen width and \( A_{\nu\mu} \) are the influence coefficients [11] for each weight function. These weight functions are implemented within the following equations for mode I and II stress intensity factors.

\[ \Delta K_I = \int_0^a \Delta \sigma_{xx}(x) h_1(x, a) dx \]  

(6.5)

\[ \Delta K_{II} = \int_0^a \Delta \sigma_{xy}(x) h_2(x, a) dx \]  

(6.6)

where \( x \) is the independent variable representing direction along the crack and \( \Delta \sigma_{xx} \) and \( \Delta \sigma_{xy} \) are local FE-predicted normal and shear stresses during crack growth in model I and mode II, respectively. This approach allows incorporation of the effect of the contact-induced stress gradients on crack growth. An effective stress intensity factor is then defined as:

\[ \Delta K_{eff} = \sqrt{\Delta K_{Ieff}^2 + \Delta K_{II}^2} \]

(6.7)

where

\[ \Delta K_{Ieff} = \Delta K_I (1 - R)^{(1-n)} \]

(6.8)

where \( R > 0 \) and \( n \) is a material constant where \( 0 < n < 1 \). In this work \( n \) is chosen to have a value of 0.5. If \( R < 0 \), \( \Delta K_I = \Delta K_{max} \). This is as a result of the dependency
of crack propagation rate on stress ratio $R$ expressed by Walker [12] based on empirical results. $R$ is defined as:

$$R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \quad (6.9)$$

where $\sigma_{\text{min}}$ is the minimum applied stress and $\sigma_{\text{max}}$ is the maximum applied stress.

The increment of crack growth $\Delta a$ is defined by:

$$\Delta a = C(\Delta K_{\text{eff}} - \Delta K_{\text{th}})^m \Delta N \quad (6.10)$$

where $\Delta N$ is a cycle jumping factor. The process is repeated until the crack length, $a$, propagates through the specimen width.

6.2.5 Fretting fatigue modelling

The FE model is based on a quarter segment of the experimental bridge-type fretting rig described in Chapter 3. The experimental arrangement is modelled as a 2D plane strain cylinder-on-flat fretting model. A 6 mm radius cylinder is held in contact with a 4.5 x 13.5 mm substrate under a fixed normal (clamping) load. The substrate represents one quarter of the gauge length with symmetry boundary conditions on the bottom and right edges. The substrate is subjected to the same cyclic loading conditions as in the experiments, see Table 6-2. Linear equation constraints are defined on the left edge of the substrate to ensure uniform nodal displacements in the horizontal $X$ direction between the master and slave nodes when applying cyclic loading. The same method is used on the fretting pad to enforce uniform nodal displacements in the vertical $Y$ direction when applying a normal force $P$. The normal load $P$ is applied in the first step and held constant throughout the analysis.

In the second and subsequent steps the cyclic loading is applied to the substrate to simulate the experimental conditions as shown in Figure 6-5. A COF of 0.8 is used throughout this work based on unlubricated metallic contact as seen in McColl et al.
A more detailed description of the FE frictional contact methodology is presented in Chapter 4. Figure 6-6 illustrates the hybrid CP-\(J_2\) fretting model whereby a CP contact region is embedded within a NLKH \(J_2\) plasticity bulk model. This CP region is 20 grains wide and 10 grains deep. Again, square grains are used to give better mesh control. An element size of 2.5 µm is used in the contact region with a decreasing mesh density further away from the CP region. Mesh refinement studies carried out in Chapter 4 have shown that a 2.5 µm contact element size is more than sufficient for accurate results while the coarser mesh at the outer edges allows faster overall model run-times. A sensitivity study on orientation effects of grains within the CP contact region was conducted. Five different sets of random orientations were studied across all test stress amplitudes, to study the dependency of crack initiation life on crystallographic orientation.

Using the same mesh design a second fretting fatigue FE model was generated based on a NLKH \(J_2\) plasticity formulation [2]. This allows for direct comparison of conventional life prediction methodologies (SWT) and plasticity formulations against the microstructure-sensitive model. Furthermore a plain fatigue \(J_2\) FE model, representing the specimen gauge length, was developed without the added complexity of a contacting fretting pad. Implementing the SWT critical plane approach with this model, a plain fatigue damage parameter is determined for different \(\sigma_{app}\) values.

### 6.2.6 Smith Watson Topper (SWT)

The SWT critical plane approach is applied within this chapter for comparative purposes against the microstructure-sensitive crack initiation methodology. Greater detail can be found in Chapter 4. As previously mentioned however, constants for this parameter are normally calculated for 1 mm cracks. Madge et al. [14]
highlighted an inconsistency with regards the use of the SWT approach for quantifying crack nucleation life and location. Since most crack initiation, \( N_i \), occurs at length scales of 10 \( \mu m \) or below, and since element integration points are typically within 10 \( \mu m \) or so from the surface it is inconsistent to use SWT constants calibrated at 1 mm. Therefore, back-calculated 10 \( \mu m \) constants were obtained here by implementing a modified Paris law equation incorporating SCG through an El Haddad approach. A similar approach is used in this work to analytically back-calculate the crack initiation life from experimentally obtained total life, \( N_f \), data for fretting fatigue, except in this case, the FE-predicted local stress distributions are employed with the short crack growth prediction methodology of Section 6.2.4, see Appendix 1.5.

6.3 Results

6.3.1 Computational

The number of cycles to crack initiation, \( N_i \), for the plain fatigue test data were found by implementing the Paris and El Haddad methodology for short and long crack growth. Using a \( \Delta K_{th} = 5.81 \text{MPa m}^{0.5} \) [7] a threshold crack length, \( a_{th} \), of 59.5 \( \mu m \) is calculated. Incorporating this transitional crack length with the Paris equation constants \( C \) and \( m \), 2.0 \( \times 10^{-10} \) and 1.9, respectively, from [16] based on short crack growth testing of 316L SS, an "experimental" number of cycles for propagation \( N_p^{exp} \) is calculated, where \( N_p^{exp} \) is defined as the number of cycles for a 1.2 \( \mu m \) crack to propagate to failure, in this case to a length of 1.5 mm. Hence it is possible to infer an "experimental" number of cycles to initiation, as follows:

\[
N_i^{exp} = N_f - N_p^{exp}
\]
$N_p^{exp}$ is relatively small, about 15-30%, compared to $N_t^{exp}$ as shown in Table 6-3. Using $N_t^{exp}$ it is possible to deduce a value of critical accumulated plastic slip parameter, $p_{crit}$, for one test data point. Figure 6-7 shows the predicted CP crack initiation response plotted against the predicted $N_t^{exp}$ data, based on the identified value of $p_{crit} = 37.73$.

Using the averaged predicted $N_t^{exp}$ data obtained from the plain fatigue experimental results, Basquin fatigue constants can be fitted via the Basquin equation as shown in Figure 6-8. The critical plane SWT was applied within the plain fatigue $J_2$ FE model to obtain SWT values across the range of stress amplitudes. The $\varepsilon'_f$ and $c$ values were chosen iteratively to achieve a good fit of SWT to the plain fatigue response, $N_t^{exp}$. The identified SWT constants are listed in Table 6-4. These were subsequently used to provide fretting fatigue crack initiation predictions for the $J_2$ FE model.

As previously stated the microstructure-sensitive methodology was implemented within the hybrid FE fretting fatigue model to predict the fretting fatigue crack initiation life of 316L SS under the simulated experimental conditions. These microstructure-sensitive crack initiation lives, calculated at a depth of 1.2 $\mu$m, were predicted for the different applied stresses via the accumulated plastic slip per cycle, $p_{cyc}$, values predicted by the hybrid FE models in equation 4.34. The result of the crystallographic orientation study carried out using five sets of randomly generated orientations assigned to each grain in the FE model are used to quantify scatter. Error bars signify this scatter of predicted fretting fatigue crack initiation lives. Table 6-5 tabulates the data for comparative purposes. Figure 6-9 compares the two methods of predicting crack initiation lives, viz, (i) critical plane SWT for a
continuum $J_2$ plasticity model and (ii) a CP microstructural sensitive approach, against experimental fretting fatigue failure lives.

Contact variable distributions are presented in Figure 6-10 and Figure 6-11 which show the evolutions of contact pressure and shear for both plasticity formulations. The predicted evolutions of the SWT and $p_{cyc}$ damage parameter are presented in Figure 6-12. The position of maximum stabilised $p_{cyc}$ is the predicted localisation of crack initiation. In all cases, $p_{cyc}$ has stabilised after 6 cycles. The distinct localised cracking of the $p_{cyc}$ parameter is evident in contrast to the more uniform distributions of SWT as seen in Figure 6-12 (a), (c) and (e) and in [3], for example. The gross slip fretting condition is evident in both FE models from the contact slip distributions presented in Figure 6-13. In Figure 6-12 (b), (d) and (f) crack initiation is predicted to occur at the trailing edge of contact which correlates well with experimental observations. Chapter 3 provides an SEM image (Figure 3-15) of a typical location for crack initiation at the trailing edge of contact, further corroborating the FE predictions.

Using the $J_2$ and the hybrid FE crack initiation methodology, described previously, in conjunction with the weight function method for mixed mode crack propagation, to calculate total life for the fretting fatigue cases, comparisons are made against experimental total life fretting fatigue data in Figure 6-14. The $J_2$ crack location was determined via the SWT approach and assumed to propagate in a direction normal to the substrate surface. This assumption is validated against experimental observations of crack growth in Chapter 3, where cracks were shown to propagate at angle of between $70^\circ$ and $110^\circ$ to the substrate surface. Similarly for the hybrid model the location of crack initiation is found using the position of the max $p_{cyc}$ value. The process was implemented automatically within a computer program.
which incrementally calculated crack length until the crack had propagated through the substrate to a critical failure length. This critical length, based on experimental observations, was taken to be 1.5 mm. The predicted initiation, propagation and total lives for the $J_2$ SWT approach and the CP microstructure-sensitive predictions for one specific random orientation set are given in Table 6-6. An experimental fretting life reduction factor (FLRF) of 3.5 is expressed in Chapter 3 as the ratio between the plain and fretting fatigue total life. A similar ratio can be observed between the CP fretting fatigue total life results, as seen in Table 6-6, and a combination of the (i) CP plain fatigue crack initiation life and (ii) the Paris-El Haddad experimental plain fatigue propagation life presented in Table 6-3. This results in a reasonably similar FSRF of about 2.15. Similarly, the $J_2$ SWT approach produces a FSRF value of 1.5.

An experimental Archard wear coefficient was obtained by combining the experimental wear volumes presented in Chapter 3 for $\sigma_{amp} = 225$ MPa and the CP predicted contact slip distributions shown in Figure 6-13 via the Archard equation previously described in Section 5.3.5. A wear coefficient of $5 \times 10^{-8}$ MPa$^{-1}$ is calculated, which is compared in Table 6-7 to the wear coefficient presented in Hutchings for stainless steel [17], using the material hardness obtained in Chapter 3.

Based on the fretting wear numerical methodology explained in Chapter 5 a CPFE-predicted wear-scar profile and wear coefficient is calculated for the same loading situation as mentioned above, $\sigma_{amp} = 225$ MPa. Table 6-7 shows the CPFE-predicted wear coefficient of $0.5 \times 10^{-8}$ MPa$^{-1}$. Figure 6-15 shows the predicted wear-scar after 56,669 cycles with associated dimensions presented in Table 6-8, compared against the experimental results.
6.4 Discussion

Fretting was induced on fatigue specimens through the implementation of a bridge-type fretting rig. An FE model of the experimental test rig was generated to study the effects of the microstructure under experimental fretting fatigue conditions. The initial Hertzian and FE-predicted contact semi-widths are presented in Table 6-9. Due to the low contact pressure of \(0.5P_y\), both the CP and \(J_2\) plasticity models give the same initial contact width. With further fretting cycles, significant differences are predicted between the contact pressure and shears of the CP and \(J_2\) models.

The CP model predicts significant widening in the early fretting cycles combined with non-uniform distributions of both contact pressure and shear, as shown in Figure 6-10 (b), (d) and (f) and Figure 6-11 (b), (d) and (f). The significant widening of the contact area is the result of the micro-plastic deformation of the substrate microstructure during cyclic loading. The applied loads to the bridge-type fretting rig resulted in gross slip fretting conditions which is evident from the experimental wear scars presented in Chapter 3. The uniform U-shaped scar is a distinguishing feature of a gross slip situation. The CPFE plasticity model predicts similar fretting conditions which is clear from the contact slip distributions seen in Figure 6-13 (b), (d) and (f). The CP model results in contact slip distributions that are non-uniform in nature. This again, is caused by material inhomogeneity leading to differential yielding across different grains. During plain fatigue testing crack propagation made up a small proportion of total life (15-30 %) with the majority of cycles spent initiating a surface crack, especially for the lower stress range. In contrast, for the fretting fatigue specimens, a significant proportion of specimen life involved crack propagation. This is attributed, obviously, to surface fretting damage.
significantly shortening crack initiation life. Highly localised concentrations of plastic slip are predicted in Figure 6-12 (b), (d) and (f). This is explained by severe localised plasticity and varying yield strengths due to inhomogeneous grain orientations [18].

Figure 6-10 (a), (c) and (e) and Figure 6-11 (a), (c) and (e) show a more uniform distribution of contact pressure and shear for the $J_2$ model, as well as a significant widening of the contact area. Gross sliding is predicted by the $J_2$ model as shown by the uniform contact slip distributions of Figure 6-13 (a), (c) and (e). A critical plane SWT approach was used within a continuum plasticity FE fretting fatigue model for crack initiation prediction. These predictions were based on Basquin constants calculated for crack lengths of 1.2 $\mu$m. While this results in more accurate predictions than your typical fatigue constants based on 1 mm crack lengths, continuum plasticity does not capture the micromechanical behaviour of the microstructure. The CP theory implemented in this work, combined with the microstructure-sensitive methodology, allows for capturing of length scale effects that are in the same order of magnitude as the grain size as well as material inhomogeneities resulting from random crystallographic orientations. Therefore the crack initiation predictions based on the CP formulation give more realistic results than the SWT continuum plasticity based predictions.

Two different plasticity models were used to predict the crack propagation life for fretting fatigue across a range of stress amplitudes. For high load levels, the CP method, with combined crack initiation, $N_i$, and propagation, $N_p$, are very accurate. At lower load levels more conservative results are observed. Interestingly for $\sigma_{app} = 202.5$ MPa case, experimental life seems to be out of trend with respect to other results. The plain fatigue scatter for this case is significant and if assumed to
apply to the fretting fatigue case for this load, would encompass the predicted CP result. It is possible that this low experimental life is due to complex interaction of wear and crack nucleation. In this case, the fact that wear-induced material removal is not explicitly simulated here (e.g. see Zhang et al. [19]), can explain the fact that the CP predictions over estimate life, for this load level. Furthermore, nucleation life prediction trends will be affected by future damage accumulation effects, i.e. material degradation due to damage (e.g. void growth and micro-cracking). This has not been incorporated into the present work although an approach similar to that of Zhang et al. [19] could be adopted in further studies. Additionally, an absence of an initial surface roughness from the CP model will undoubtedly affect predicted crack initiation and propagation lives.

The $J_2$ plasticity model total life results, incorporating critical plane SWT and the short crack growth propagation methodology, are presented in Figure 6-14 (b). The $J_2$ model shows conservative results at the higher stress amplitudes, less accurate than the CP model, and significantly more conservative results at the lowest stress amplitude. However, the $J_2$ model does show a closer correlation to the anomalous experimental result at $\sigma_{amp} = 202.5$ MPa.

The predicted wear scar shown in Figure 6-15 has partial slip characteristics, i.e. two disconnected regions of damage. While this may be unusual for a gross slip situation it can be explained by the presence of a less favourably orientated grain within the wear scar region. This "harder" grain has not experienced the same amount of accumulated plastic slip as a result of slip system orientation. Furthermore, multiple experimental wear profiles taken at different transverse locations through the fretting specimen, presented in Appendix 1.2, show similar non-uniform distributions of wear for a gross slip situation for the same loading.
conditions. It is therefore possible to assume that if a three dimensional CP model was implemented within this work similar wear profile distributions would be evident at different transverse locations through the thickness of the FE substrate. This gives further evident to suggest that the microstructure-sensitive model has the ability to realistically capture the experimental behaviour of fretting fatigue over its continuum counterparts.
6.5 Conclusions

- A CPFE fretting fatigue cylinder-on-flat arrangement of the experimental bridge-type rig was implemented, incorporating the microstructure-sensitive crack initiation methodology within the contact region. Crack initiation life predictions were made across a range of stress amplitudes.
- A $J_2$-plasticity FE model of the fretting rig was also developed. A critical-plane SWT life prediction methodology was also investigated for this model.
- A mixed-mode short crack growth methodology is combined with (i) a microstructure-sensitive crack initiation methodology, and (ii) a $J_2$ critical plane SWT methodology, to produce total life predictions. Both methods are successfully compared to the experimental fretting fatigue failure lives.
- The CPFE model captures the measured effect of stress amplitude in fretting-induced damage in terms of crack initiation and total life. The $J_2$ model however, shows a negligible effect of stress amplitude.
- The CP total life predictions are accurate for the higher stress amplitudes and conservative for the lowest stress amplitude. In comparison, the $J_2$ total life predictions are less accurate for the higher stress amplitudes and are significantly more conservative the lowest stress amplitude.
- Both the CP and $J_2$ models capture the FSRF due to fretting, with the CP model giving a mean value of 2.1 and the $J_2$ model giving a mean value of 1.5. Thus, the CP predictions is closer to the mean measured FSRF of 3.5.
- The CPFE wear predictions give a wear-scar depth which is consistent with test data. However, the scar width is significantly underestimated. These
trends are similar to those of the Ti-6Al-4V fretting wear predictions in Chapter 5.

- The CPFE-predicted wear coefficient is non-conservative (underestimates wear) but is of the same order of magnitude as (i) published data for stainless steel and (ii) the estimated wear coefficient from the fretting tests of Chapter 3. It is a factor of 3 smaller than published value and a factor of 10 smaller than the estimated value. This again is similar to the findings for Ti-6Al-4V in Chapter 5.
6.6 References


### 6.7 Tables

Table 6-1. Identified CP constitutive constants for cyclic behaviour of 316L SS.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_0$</td>
<td>10.5 GPa</td>
</tr>
<tr>
<td>$g_\infty$</td>
<td>207.84 MPa</td>
</tr>
<tr>
<td>$g_0$</td>
<td>82.68 MPa</td>
</tr>
<tr>
<td>$a$</td>
<td>0.0023s$^{-1}$</td>
</tr>
<tr>
<td>$n$</td>
<td>30</td>
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</table>

Table 6-2. Experimental loading range for both plain and fretting fatigue experiments using $R = 0.1$.

<table>
<thead>
<tr>
<th>$\sigma_{\text{max}}$ (MPa)</th>
<th>$\sigma_{\text{min}}$ (MPa)</th>
<th>$\sigma_{\text{amp}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>50</td>
<td>225</td>
</tr>
<tr>
<td>475</td>
<td>47.5</td>
<td>213.75</td>
</tr>
<tr>
<td>450</td>
<td>45</td>
<td>202.5</td>
</tr>
<tr>
<td>425</td>
<td>42.5</td>
<td>191.25</td>
</tr>
</tbody>
</table>

Table 6-3. Calculated experimental plain fatigue data separated into crack propagation and crack initiations lives, based on average plain fatigue data, $N_f$, compared against the predicted CP crack initiation data.

<table>
<thead>
<tr>
<th>$\sigma_{\text{amp}}$ (MPa)</th>
<th>$N_t^{\text{exp}}$</th>
<th>$N_p^{\text{exp}}$</th>
<th>$N_t^{CP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>225</td>
<td>76340</td>
<td>30856</td>
<td>95182</td>
</tr>
<tr>
<td>213.75</td>
<td>111815</td>
<td>34014</td>
<td>111815</td>
</tr>
<tr>
<td>202.5</td>
<td>182505</td>
<td>37693</td>
<td>122033</td>
</tr>
</tbody>
</table>

Table 6-4. SWT fatigue constants for 1.2 $\mu$m crack length.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_f'$ (MPa)</td>
<td>869.4</td>
</tr>
<tr>
<td>$\varepsilon_f'$</td>
<td>0.54</td>
</tr>
<tr>
<td>$b$</td>
<td>-0.12</td>
</tr>
<tr>
<td>$c$</td>
<td>-0.495</td>
</tr>
</tbody>
</table>

Table 6-5. Average microstructure-sensitive fretting fatigue crack initiation lives for different crystallographic orientation sets.

<table>
<thead>
<tr>
<th>$\sigma_{\text{amp}}$ (MPa)</th>
<th>SET-1</th>
<th>SET-2</th>
<th>SET-3</th>
<th>SET-4</th>
<th>SET-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>225</td>
<td>9093</td>
<td>15481</td>
<td>2117</td>
<td>2008</td>
<td>5753</td>
</tr>
<tr>
<td>213.75</td>
<td>15350</td>
<td>34451</td>
<td>2368</td>
<td>2572</td>
<td>6927</td>
</tr>
<tr>
<td>202.5</td>
<td>27044</td>
<td>32762</td>
<td>2911</td>
<td>3602</td>
<td>8027</td>
</tr>
<tr>
<td>191.25</td>
<td>58989</td>
<td>35798</td>
<td>3212</td>
<td>4513</td>
<td>8412</td>
</tr>
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</table>
Table 6-6. Combined crack initiation and propagation lives for the microstructural sensitive model based on random orientations SET-1 and the $J_2$ SWT approach compared against the experimental fretting fatigue data.

<table>
<thead>
<tr>
<th>$\sigma_{amp}$ (MPa)</th>
<th>$N_i^{CP}$</th>
<th>$N_p^{CP}$</th>
<th>$N_f^{CP}$</th>
<th>$N_i^{SWT}$</th>
<th>$N_p^{SWT}$</th>
<th>$N_f^{SWT}$</th>
<th>$N_f$ (Experimental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>225</td>
<td>9093</td>
<td>41868</td>
<td>50961</td>
<td>10259</td>
<td>35014</td>
<td>45273</td>
<td>56669</td>
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<td>213.75</td>
<td>15350</td>
<td>49689</td>
<td>65039</td>
<td>11034</td>
<td>39730</td>
<td>50764</td>
<td>59780</td>
</tr>
<tr>
<td>202.5</td>
<td>27044</td>
<td>64617</td>
<td>91661</td>
<td>11469</td>
<td>45071</td>
<td>56540</td>
<td>35091</td>
</tr>
<tr>
<td>191.25</td>
<td>58989</td>
<td>72531</td>
<td>131520</td>
<td>11584</td>
<td>47484</td>
<td>59068</td>
<td>245806</td>
</tr>
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</table>

Table 6-7. Comparison between experimental and FE predicted wear coefficients for stainless steel.

<table>
<thead>
<tr>
<th>Wear coefficient (MPa$^{-1} \times 10^{-8}$)</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Experimental [18]</td>
<td>1.5</td>
</tr>
<tr>
<td>Experimental and FE</td>
<td>5</td>
</tr>
<tr>
<td>FE predicted</td>
<td>0.5</td>
</tr>
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Table 6-8. Comparison between the experimentally measured wear-scar and the FE-predicted profile.

<table>
<thead>
<tr>
<th>Maximum wear depth (µm)</th>
<th>Wear scar width (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>9</td>
</tr>
<tr>
<td>Predicted</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6-9. Initial contact semi widths for analytical and FE models.

<table>
<thead>
<tr>
<th>Contact semi width (µm)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Hertzian</td>
<td>33</td>
</tr>
<tr>
<td>CPFE ($N=0$)</td>
<td>36</td>
</tr>
<tr>
<td>$J_2$ ($N=0$)</td>
<td>36</td>
</tr>
</tbody>
</table>
6.8 Figures

Figure 6-1. Unit-cell model under stressed controlled loading showing a contour plot of accumulated plastic slip, $p$.

Figure 6-2. Tint etching of 316L SS highlighting an average grain size, d, of 19 µm.
Figure 6-3. Comparison of the CP predicted response against the $J_2$ macroscopic CSSC.

Figure 6-4. A simple schematic of the Kitagawa and Takahashi diagram.
Figure 6-5. Load history for the experimental and FE model.

Figure 6-6. A schematic of the fretting fatigue model based on experimental work highlighting the CP embedded region in the contact zone.
Figure 6-7. Comparison between experimental and CP based predictions for number of cycles to crack initiation, $N_i$, for plain fatigue.

Figure 6-8. Basquin constants acquired for 1.2 $\mu$m crack length based on crack initiation estimates derived from the Paris law equation and the averaged plain fatigue experimental data.
Figure 6-9. Comparison of experimental fretting fatigue failure lives versus microstructural sensitive crack initiation lives including the effect of random orientations and critical plane SWT continuum plasticity based crack initiation predictions.
Figure 6-10. $J_2$ and CP fretting model predicted evolutions of contact pressure under different stress amplitudes, $\sigma_{amp}$. Both the $J_2$ and CP fretting models have a contact semi width of 0.035 mm.
Figure 6-11. $J_2$ and CP fretting model predicted evolutions of contact shear traction under different stress amplitudes, $\sigma_{amp}$. 

(a) $J_2$ model, $\sigma_{amp} = 225$ MPa  
(b) CP model, $\sigma_{amp} = 225$ MPa 

(c) $J_2$ model, $\sigma_{amp} = 213.75$ MPa  
(d) CP model, $\sigma_{amp} = 213.75$ MPa 

(e) $J_2$ model, $\sigma_{amp} = 202.5$ MPa  
(f) CP model, $\sigma_{amp} = 202.5$ MPa
Figure 6.12. $J_2$ and CP fretting model predicted evolutions of SWT and $p_{cyc}$ distributions under different stress amplitudes, $\sigma_{amp}$. 

(a) $J_2$ model, SWT, $\sigma_{amp} = 225$ MPa 
(b) CP model, $p_{cyc}$, $\sigma_{amp} = 225$ MPa 

(c) $J_2$ model, SWT, $\sigma_{amp} = 213.75$ MPa 
(d) CP model, $p_{cyc}$, $\sigma_{amp} = 213.75$ MPa 

(e) $J_2$ model, SWT, $\sigma_{amp} = 202.5$ MPa 
(f) CP model, $p_{cyc}$, $\sigma_{amp} = 202.5$ MPa
Figure 6-13. $J_2$ and CP fretting model predicted evolutions of contact slip distributions under different stress amplitudes, $\sigma_{amp}$. 

(a) $J_2$ model, $\sigma_{amp} = 225$ MPa  
(b) CP model, $\sigma_{amp} = 225$ MPa  
(c) $J_2$ model, $\sigma_{amp} = 213.75$ MPa  
(d) CP model, $\sigma_{amp} = 213.75$ MPa  
(e) $J_2$ model, $\sigma_{amp} = 202.5$ MPa  
(f) CP model, $\sigma_{amp} = 202.5$ MPa
Figure 6-14. Total experimental failure lives compared against the total predicted lives based on (a) microstructural sensitive model and (b) $J_2$ SWT model.
Figure 6-15. FE predicted wear profile after 56,669 cycles at $\sigma_{amp} = 225$ MPa.
Chapter 7

7 Conclusions and Future Work

7.1 Conclusions

This thesis presents a combined microstructure-sensitive and experimental study of fretting fatigue and wear. The microstructure-sensitive approach is based on a crack initiation methodology previously published for uniaxial fatigue loading which has been adapted and validated for the multiaxial fretting problem studied here.

Validation is achieved against a bridge-type fretting fatigue rig which has been designed and developed to measure the fretting fatigue behaviour of 316L stainless steel (SS). Plain fatigue testing carried out on the same fatigue specimens allowed for the quantification of a fretting life reduction factor (FLRF) between plain and fretting fatigue life of about 3.5. Material characterisation via mechanical testing and chemical etching microscopy allowed for macroscale and microscale modelling of the 316L SS material within a crystal plasticity (CP) finite element (FE) model and a $J_2$ plasticity model.

A frictional contact cylinder-on-flat CPFE model was developed for an idealised 316L SS microstructure to study the effects of normal load and displacement in comparison with a $J_2$ plasticity fretting model. Two fatigue indicator parameters (FIPs) were employed to quantify fretting fatigue life: (i) the novel crack initiation methodology using an accumulated crystallographic plastic slip parameter, $p$, and (ii) a macroscale critical-plane SWT parameter. The SWT approach has been previously validated as a total life prediction method for fretting. The CPFE simulation results showed significant micro-plasticity effects even for an ostensibly elastic fretting situation, compared to the $J_2$ plasticity model. The predicted microstructure-sensitive crack initiation results gave significantly shorter lives, as
expected, than the SWT (total) lives for fretting. Also, the predicted ratios of crack initiation life to total life were similar to the experimental observations. The microstructure-sensitive methodology is a more fundamental and scale-consistent approach for the experimentally-observed length scales of fretting induced cracking, than conventional FIP approaches.

An important finding is the validation and proof of concept that the micro-mechanical crack initiation predictive methodology could be applied to the fretting fatigue problem and give results consistent with test data. An additional key finding is that the distributions of accumulated plastic slip can be used to distinguish between fretting crack initiation and fretting wear. This conceptual technique is initially demonstrated for 316L SS but is then compared directly to published results from an experimental fretting wear test for Ti-6Al-4V. The CP UMAT is modified to model the dual phase microstructure of Ti-6Al-4V, consisting of both HCP and BCC slip systems. The crack initiation predictions are shown to give results consistent with the published experimental data [1], in terms of numbers and locations of nucleated cracks, as well as orientations. A microstructure-sensitive numerical technique is developed to predict the wear-scar profile and wear coefficient and is applied to the Ti-6Al-4V data for comparison against experimental wear results. The predictive technique gives a reasonable prediction for wear coefficient and maximum wear depth but significantly underestimates wear scar width. The method relies on extrapolation of plasticity-driven results from the first few thousand cycles, during which it can be argued that the local surface plastic deformation plays a key role in wear. Hence, a key next step is the modelling of material removal, based on \( p_{\text{crit}} \), to allow better wear prediction to larger numbers of cycles. This technique successfully distinguishes between the resulting wear profiles of both partial and gross slip.
Due to the difficulties associated with measuring crack nucleation, to facilitate the direct comparison with the 316L SS experimental fretting fatigue lives, a micro-mechanical FE model based on the experimental bridge-type fretting rig is developed and a mixed-mode short crack propagation methodology was implemented with the microstructure-sensitive crack initiation methodology. The crack propagation method employed a weight function approach to deal with the steep stress gradients in the fretting contact damage regions. This resulted in the ability to predict total fretting fatigue lives based on a realistic microstructure, that correlates with experimental data. The resulting predicted CPFE total life FSRF was about 2, compared to the experimental value of about 3.5. The microstructure-sensitive numerical technique for predicting wear scar and wear coefficient, was again shown to correlate with the measured 316L SS wear depth from the fretting fatigue tests. The relative displacement and COF of the fretting fatigue pads were not measured. Hence, in order to estimate the wear coefficient for the 316L SS fretting fatigue tests, it was necessary to employ the CPFE-predicted slip with a representative COF. The CPFE-predicted wear coefficient gave good correlation with this latter estimated wear coefficient and was also shown to correlate well with typical values for stainless steel from the literature [2]

7.2 Future work

7.2.1 Fretting rig

The fretting fatigue rig described in Chapter 3 differs slightly from the detailed drawings presented in Appendix 1.1. The manufactured bridge-type rig is a simpler, more robust design. While this prototype was successful in generating fretting
fatigue test data, some future design changes could enhance the control and usability, as follows:

1. A load cell could easily be incorporated into the rig to give a more user-friendly method for accurately calibrating and monitoring normal load during testing.

2. Secondly, a series of Belleville springs is recommended to maintain a constant normal load on the fatigue specimen throughout testing. During the fretting process it is possible that the removed/worn material would result in a decrease in contact pressure. The unusual behaviour of these springs allows a constant load for a range of deflection typically between 90% to 100% of maximum deflection.

7.2.2 Grain size study

Miller [3] explains the typical effect of grain size on crack initiation and propagation. In simple terms small grains at the surface of the material resist crack initiation while large grains lead to greater resistance to crack propagation. Ideally, an optimum grain morphology would consist of small surface grains to inhibit initiation with larger subsurface grains to inhibit crack propagation. This could be achieved through the use of Laser surface modification (LSM) which produces very fine grains (nano-grains) at the substrate surface for a range of depths, i.e. typical depth for Ti-6Al-4V ranges between 20 µm to 50 µm [4], depending on the laser specifications. An interesting study would investigate the effect of such a material treatment on fretting-induced cracking. Using the initial un-treated 316L SS results additional experimental testing could be carried out on LSM specimens for the same fretting conditions. Subsequent micro-modelling, as detailed in previous chapters, would investigate the effect of fine surface grains using the microstructure-sensitive
prediction methodology. However, this would also require a strain gradient CPFE approach (e.g. Sweeney et al. [5]) to capture grain size effects.

7.2.3 Wear simulation

The numerical technique described in Chapter 5 is a novel attempt at unification of fretting crack initiation and fretting wear predictions. A key issue with this methodology is the assumption that no material is removed during the fretting cycles. Further work should simulate material removal due to wear via incremental spatial adjustment of contact nodes. This wear simulation tool has been previously implemented with a macroscale FE framework with Archard or energy-based wear equations. It is proposed here that future work implement the wear simulation technique based on the critical accumulated plastic slip parameter, $p_{\text{crit}}$. This would result in a more inhomogeneous, and therefore arguably more realistic, distribution of wear depth in contrast to the more uniform and symmetrical distributions typically simulated, e.g. see [1] and [6].

7.2.4 Anisotropic elasticity with CP

Within the current work isotropic elasticity is used within the CP model. However, due to the heterogeneity present within individual grains during plastic deformation it is unlikely that elastic deformation is isotropic. Therefore, future work would include a study on the effect of anisotropic elastic behaviour within fretting. Recent work by Sweeney et al. [5], for example, has predicted a significant effect of anisotropic elasticity on $p_{\text{cye}}$ distributions for a notched four-point bend specimen under plain fatigue, in relation to predicted location of crack initiation. As the loading increased the effect reduces with increased plastic deformation. However, it was shown to play a key role in the distribution and magnitude of accumulated plastic slip within the context of HCF crack initiation. Elastic mismatch between
grains will also lead to localised stress singularities of grain boundaries, e.g. triple points, as a result of elastic anisotropy arising from different grain orientations. Furthermore, mismatch will also result in stress heterogeneity similar to what has been observed within this work from differential yielding of grains. Anisotropic elastic behaviour could therefore have a significant effect on predicted location and life of crack initiation sites within fretting wear and fatigue.

7.2.5 Kinematic hardening

The CP theory implemented here is based on a fairly simple flow rule formulation (power-law) which does not explicitly address phenomena typically considered important in the cyclic plasticity behaviour of materials, e.g. non-linear kinematic hardening and isotropic (cyclic) hardening. An obvious next step would be the incorporation of kinematic hardening and yield stress effects, ideally based on a physically-based approach, e.g. see Sweeney et al.[5].

7.2.6 Length-scale effects

An important phenomena associated with microstructure modelling is the effect of length scale. Within a typical macroscopic loading situation it is intuitive to think that scaling the size and relative loads on a component will result in the same stress-strain material response. However, it has been observed by Fleck et al. [7], for example, that at small size scales for components, e.g. copper wires in tension, when grain size becomes competitive with component or structurally-important length-scales, the mechanical (stress-strain) response of the material is significantly affected by size. This has been explained and modelled based on the theory of strain gradients, vis à vis dislocation theory. These strain gradients result in geometrically necessary dislocations which inhibit the flow of statistically stored dislocations and contribute to the hardening of the material. This results in different microstructure
responses for different geometrical dimensions. To model the effect of grain size at small size scales it is therefore important to employ a strain gradient plasticity theory. One example of such an approach is the strain-gradient CP constitutive model of Dunne et al. [8]. A disadvantage of this approach is that it results in significant additional computational cost in an already cost-intensive computer simulation, which would be particularly detrimental in a CPFE simulation of fretting fatigue.

7.2.7 Surface roughness

Real materials do not consist of flat surfaces at the microscale. Material surface roughness is arguably an important variable in crack initiation under fretting fatigue conditions. Surface roughness is shown to have a significant effect on COF [9] and can therefore be a controlling factor in fretting. Hence, another important aspect for future work is the modelling of more realistic surface profiles. Mulvihill et al. [10] for example, have modelled single asperities with a view to predicting COF, based on FE models without the need for experimental characterisation. The interaction of these asperities could eventually lead to the formation of debris, which is important in fretting (e.g. see Ding et al. [11]). The latter showed that debris formation and removal rate can have an important effect on contact pressure redistribution and wear scar formation, thus affecting crack initiation and total life predictions.
Chapter 7

7.3 References


Appendix

Appendix 1.1

Detailed drawings of the fretting fatigue specimen and fretting fatigue test rig, described in Chapter 3, are presented at the end of the thesis in the following order:

- Fretting fatigue specimen
- Fretting fatigue rig assembly
- Fretting pad sub-assembly
- Fretting pad
- Loading block
- Load cell plate
- Loading bolt
- Reaction plate
- Proving ring
- Roller
Appendix 1.2

The following appendix contains additional experimental data from fretting fatigue testing of 316L SS.

Figure 8-1. SEM image of a fretting wear scar of an un-cracked surface.

Figure 8-2. SEM image of the failed fretting fatigue specimen.
Figure 8-3. SEM image of the failed fretting fatigue specimen and wear scar region.

Figure 8-4. SEM image of the close up view of a crack initiation site.
Figure 8-5. SEM image of the three-dimensional image of the wear scar.

Figure 8-6. SEM image of the fretting wear scar of un-cracked region.
Figure 8-7. Red oxide layer present within the fretting wear scar highlighting the presence of fretting corrosion.

Figure 8-8. Image of a fretting wear scar of a failed specimen highlighting the red oxide layer and the crack initiation site present at the edge of contact.
Figure 8-9. Wear scar profiles for fretting fatigue specimen subjected to $\sigma_{amp} = 225$ MPa at different transverse positions within the wear scar.
Figure 8-10. Wear scar profiles for fretting fatigue specimen subjected to $\sigma_{\text{amp}} = 225$ MPa at different transverse positions within the wear scar.
Figure 8-11. AFM surface plot of a fretting fatigue specimens showing a surface roughness, $R_\alpha$, value within the acceptable standards stated by the ASTM standards.

Figure 8-12. Micrograph of 316L SS microstructure after tint etching.
Figure 8-13. Micrograph of 316L SS microstructure after tint etching
Appendix 1.3

The following appendix presents the code developed and implemented by the author of this thesis to calculate the accumulated plastic slip parameter, $p$. This implementation is used with the user subroutine of Huang.

```fortran
DO L=1,NSLPTL
  DO J=1,3
    DO I=1,3
      SNTALPHA(3*(L-1)+J,I)=(SLPDIR(J,L)*SLPNOR(I,L))
    ENDDO
  ENDDO
ENDDO

C**** initialise all variables to zero
DO J=1,3
  DO I=1,3
    PVGRAD(I,J)=0
  ENDDO
ENDDO

DO L=1,NSLPTL
  DO J=1,3
    DO I=1,3
      PVGRAD(I,J)=PVGRAD(I,J)+(DGAMMA(L))*(SNTALPHA(3*(L-1)+J,I))
    ENDDO
  ENDDO
ENDDO

XLP=0
STATEV(144)=0.
DO J=1,3
  DO I=1,3
    XLP=XLP+(((2./3.)*PVGRAD(I,J))*PVGRAD(I,J))
  ENDDO
ENDDO
```

XLP
ENDDO
ENDDO

STATEV(144)=SQRT(XLP)

STATEV(145)= STATEV(145)+ABS(STATEV(144))

RETURN
END
Appendix 1.4

The following appendix presents the code developed and implemented by the author of this thesis to determine HCP and BCC slip systems for Ti-6Al-4V. This implementation is used with the user subroutine of Huang.

C**** initialise all variables to zero

IF ((ISPNOR(1)).EQ.3) THEN

NSLIP=0
SLPDIR(1,1)=0
SLPDIR(2,1)=0
SLPDIR(3,1)=0
SLPDIR(1,2)=0
SLPDIR(2,2)=0
SLPDIR(3,2)=0
SLPDIR(1,3)=0
SLPDIR(2,3)=0
SLPDIR(3,3)=0
SLPDIR(1,4)=0
SLPDIR(2,4)=0
SLPDIR(3,4)=0
SLPDIR(1,5)=0
SLPDIR(2,5)=0
SLPDIR(3,5)=0
SLPDIR(1,6)=0
SLPDIR(2,6)=0
SLPDIR(3,6)=0
SLPNOR(1,1)=0
SLPNOR(2,1)=0
SLPNOR(3,1)=0
SLPNOR(1,2)=0
SLPNOR(2,2)=0
SLPNOR(3,2)=0
SLPNOR(1,3)=0
SLPNOR(2,3)=0
SLPNOR(3,3)=0
SLPNOR(1,4)=0
SLPNOR(2,4)=0
SLPNOR(3,4)=0
SLPNOR(1,5)=0
SLPNOR(2,5)=0
SLPNOR(3,5)=0
SLPNOR(1,6)=0
SLPNOR(2,6)=0
SLPNOR(3,6)=0

C************************************************************************** only 3 indices are required
C************************************************************************** prismatic slip systems
C************************************************************************** where angle is = 30 in slip normal
pi=2.*acos(0.)
angle=pi/3.
angle1=pi/6.

NSLIP=6
SLPDIR(1,1)=1./sqrt(1**2+tan(angle)**2)
SLPDIR(2,1)=tan(angle)/sqrt(1**2+tan(angle)**2)
C              SLPDIR(3,1)=-2./sqrt(8.)
SLPDIR(3,1)=0.

SLPDIR(1,2)=1./sqrt(1**2+tan(angle)**2)
SLPDIR(2,2)=-tan(angle)/sqrt(1**2+tan(angle)**2)
C              SLPDIR(3,2)=-2./sqrt(8.)
SLPDIR(3,2)=0.

SLPDIR(1,3)=1.
SLPDIR(2,3)=0.
C              SLPDIR(3,3)=1.
SLPDIR(3,3)=0.

SLPNOR(1,1)=1./sqrt(1**2+tan(angle1)**2)
SLPNOR(2,1)=-tan(angle1)/sqrt(1**2+tan(angle1)**2)
SLPNOR(3,1)=0.

SLPNOR(1,2)=-1./sqrt(1**2+tan(angle)**2)
SLPNOR(2,2)=-tan(angle1)/sqrt(1**2+tan(angle1)**2)
SLPNOR(3,2)=0.

SLPNOR(1,3)=0.
SLPNOR(2,3)=-1.
SLPNOR(3,3)=0.
C************************************************ only 3 indices are required
C************************************************ basal slip systems
C************************************************ where angle is equal to 60

SLPDIR(1,4)=1./sqrt(1**2+tan(angle)**2)
SLPDIR(2,4)=tan(angle)/sqrt(1**2+tan(angle)**2)

C******* SLPDIR(3,4)=-2./sqrt(8.)
SLPDIR(3,4)=0.

SLPDIR(1,5)=1./sqrt(1**2+tan(angle)**2)
SLPDIR(2,5)=-tan(angle)/sqrt(1**2+tan(angle)**2)

C******* SLPDIR(3,5)=-2./sqrt(8.)
SLPDIR(3,5)=0.

SLPDIR(1,6)=1.
SLPDIR(2,6)=0.

C******* SLPDIR(3,6)=1.
SLPDIR(3,6)=0.

******************************************************************************

SLPNOR(1,4)=0
SLPNOR(2,4)=0
C
SLPNOR(3,4)=0
SLPNOR(3,4)=1

SLPNOR(1,5)=0
SLPNOR(2,5)=0
C
SLPNOR(3,5)=0
SLPNOR(3,5)=1

SLPNOR(1,6)=0
SLPNOR(2,6)=0
C
SLPNOR(3,6)=0
SLPNOR(3,6)=1

endif
C********************************************************************** BCC lamellar phase
C********************************************************************** Initialise all variables to zero

IF ((ISP NOR(1)).EQ.4) THEN

NSLIP=0

SLPDIR(1,1)=0
SLPDIR(2,1)=0
SLPDIR(3,1)=0

SLPDIR(1,2)=0
SLPDIR(2,2)=0
SLPDIR(3,2)=0

SLPDIR(1,3)=0
SLPDIR(2,3)=0
SLPDIR(3,3)=0

SLPDIR(1,4)=0
SLPDIR(2,4)=0
SLPDIR(3,4)=0

SLPDIR(1,5)=0
SLPDIR(2,5)=0
SLPDIR(3,5)=0

SLPDIR(1,6)=0
SLPDIR(2,6)=0
SLPDIR(3,6)=0

SLPDIR(1,7)=0
SLPDIR(2,7)=0
SLPDIR(3,7)=0

SLPDIR(1,8)=0
SLPDIR(2,8)=0
SLPDIR(3,8)=0

SLPDIR(1,9)=0
SLPDIR(2,9)=0
SLPDIR(3,9)=0

SLPDIR(1,10)=0
SLPDIR(2,10)=0
SLPDIR(3,10)=0

SLPDIR(1,11)=0
SLPDIR(2,11)=0
SLPDIR(3,11)=0
SLPDIR(1,12)=0
SLPDIR(2,12)=0
SLPDIR(3,12)=0

SLPDIR(1,13)=0
SLPDIR(2,13)=0
SLPDIR(3,13)=0

SLPDIR(1,14)=0
SLPDIR(2,14)=0
SLPDIR(3,14)=0

SLPDIR(1,15)=0
SLPDIR(2,15)=0
SLPDIR(3,15)=0

SLPDIR(1,16)=0
SLPDIR(2,16)=0
SLPDIR(3,16)=0

SLPDIR(1,17)=0
SLPDIR(2,17)=0
SLPDIR(3,17)=0

SLPDIR(1,18)=0
SLPDIR(2,18)=0
SLPDIR(3,18)=0

SLPNOR(1,1)=0
SLPNOR(2,1)=0
SLPNOR(3,1)=0

SLPNOR(1,2)=0
SLPNOR(2,2)=0
SLPNOR(3,2)=0

SLPNOR(1,3)=0
SLPNOR(2,3)=0
SLPNOR(3,3)=0

SLPNOR(1,4)=0
SLPNOR(2,4)=0
SLPNOR(3,4)=0

SLPNOR(1,5)=0
SLPNOR(2,5)=0
SLPNOR(3,5)=0
\begin{verbatim}
SLPNOR(1,6)=0
SLPNOR(2,6)=0
SLPNOR(3,6)=0

SLPNOR(1,7)=0
SLPNOR(2,7)=0
SLPNOR(3,7)=0

SLPNOR(1,8)=0
SLPNOR(2,8)=0
SLPNOR(3,8)=0

SLPNOR(1,9)=0
SLPNOR(2,9)=0
SLPNOR(3,9)=0

SLPNOR(1,10)=0
SLPNOR(2,10)=0
SLPNOR(3,10)=0

SLPNOR(1,11)=0
SLPNOR(2,11)=0
SLPNOR(3,11)=0

SLPNOR(1,12)=0
SLPNOR(2,12)=0
SLPNOR(3,12)=0

SLPNOR(1,13)=0
SLPNOR(2,13)=0
SLPNOR(3,13)=0

SLPNOR(1,14)=0
SLPNOR(2,14)=0
SLPNOR(3,14)=0

SLPNOR(1,15)=0
SLPNOR(2,15)=0
SLPNOR(3,15)=0

SLPNOR(1,16)=0
SLPNOR(2,16)=0
SLPNOR(3,16)=0

SLPNOR(1,17)=0
SLPNOR(2,17)=0
SLPNOR(3,17)=0
\end{verbatim}
SLPNOR(1,18)=0
SLPNOR(2,18)=0
SLPNOR(3,18)=0

C********************************************************************
Only 3 indices are required
C******************************************************************** Prismatic slip systems
NSLIP=18
C******************************************************************** Prismatic slip systems
pi=2.*acos(0.)
angle=pi/3.
angle1=pi/6.

SLPDIR(1,1)=1./sqrt(1**2+tan(angle)**2)
SLPDIR(2,1)=tan(angle)/sqrt(1**2+tan(angle)**2)
C SLPDIR(3,1)=-2./sqrt(8.)
SLPDIR(3,1)=0.

SLPDIR(1,2)=1./sqrt(1**2+tan(angle)**2)
SLPDIR(2,2)=-tan(angle)/sqrt(1**2+tan(angle)**2)
C SLPDIR(3,2)=-2./sqrt(8.)
SLPDIR(3,2)=0.

SLPDIR(1,3)=1.
SLPDIR(2,3)=0.
C SLPDIR(3,3)=1.
SLPDIR(3,3)=0.

SLPNOR(1,1)=1./sqrt(1**2+tan(angle1)**2)
SLPNOR(2,1)=-tan(angle1)/sqrt(1**2+tan(angle1)**2)
SLPNOR(3,1)=0.

SLPNOR(1,2)=-1./sqrt(1**2+tan(angle1)**2)
SLPNOR(2,2)=-tan(angle1)/sqrt(1**2+tan(angle1)**2)
SLPNOR(3,2)=0.

SLPNOR(1,3)=0
SLPNOR(2,3)=-1
SLPNOR(3,3)=0.

C********************************************************************
only 3 indices are required
C******************************************************************** basal slip systems

SLPDIR(1,4)=1./sqrt(1**2+tan(angle)**2)
SLPDIR(2,4)=tan(angle)/sqrt(1**2+tan(angle)**2)
C SLPDIR(3,4)=-2./sqrt(8.)
SLPDIR(3,4)=0.
SLPDIR(1,5) = 1./sqrt(1**2 + tan(angle)**2)
SLPDIR(2,5) = -tan(angle)/sqrt(1**2 + tan(angle)**2)
C    SLPDIR(3,5) = -2./sqrt(8.)
SLPDIR(3,5) = 0.

SLPDIR(1,6) = 1.
SLPDIR(2,6) = 0.
C    SLPDIR(3,6) = 1.
SLPDIR(3,6) = 0.

***************************************************

SLPNOR(1,4) = 0
SLPNOR(2,4) = 0
C    SLPNOR(3,4) = 0
SLPNOR(3,4) = 1

SLPNOR(1,5) = 0
SLPNOR(2,5) = 0
C    SLPNOR(3,5) = 0
SLPNOR(3,5) = 1

SLPNOR(1,6) = 0
SLPNOR(2,6) = 0
C    SLPNOR(3,6) = 0
SLPNOR(3,6) = 1

C***************************************************BCC DIRECTIONS

SLPDIR(1,7) = 1./sqrt(3.)
SLPDIR(2,7) = 1./sqrt(3.)
SLPDIR(3,7) = 1./sqrt(3.)

SLPDIR(1,8) = 1./sqrt(3.)
SLPDIR(2,8) = 1./sqrt(3.)
SLPDIR(3,8) = 1./sqrt(3.)

SLPDIR(1,9) = 1./sqrt(3.)
SLPDIR(2,9) = 1./sqrt(3.)
SLPDIR(3,9) = 1./sqrt(3.)

SLPDIR(1,10) = -1./sqrt(3.)
SLPDIR(2,10) = 1./sqrt(3.)
SLPDIR(3,10)=1./sqrt(3.)
SLPDIR(1,11)=-1./sqrt(3.)
SLPDIR(2,11)=1./sqrt(3.)
SLPDIR(3,11)=1./sqrt(3.)
SLPDIR(1,12)=-1./sqrt(3.)
SLPDIR(2,12)=1./sqrt(3.)
SLPDIR(3,12)=1./sqrt(3.)
SLPDIR(1,13)=1./sqrt(3.)
SLPDIR(2,13)=-1./sqrt(3.)
SLPDIR(3,13)=1./sqrt(3.)
SLPDIR(1,14)=1./sqrt(3.)
SLPDIR(2,14)=-1./sqrt(3.)
SLPDIR(3,14)=1./sqrt(3.)
SLPDIR(1,15)=1./sqrt(3.)
SLPDIR(2,15)=-1./sqrt(3.)
SLPDIR(3,15)=1./sqrt(3.)
SLPDIR(1,16)=1./sqrt(3.)
SLPDIR(2,16)=1./sqrt(3.)
SLPDIR(3,16)=-1./sqrt(3.)
SLPDIR(1,17)=1./sqrt(3.)
SLPDIR(2,17)=1./sqrt(3.)
SLPDIR(3,17)=-1./sqrt(3.)
SLPDIR(1,18)=1./sqrt(3.)
SLPDIR(2,18)=1./sqrt(3.)
SLPDIR(3,18)=-1./sqrt(3.)

C******************************************************************BCC_NORMALS

SLPNOR(1,7)=0.
SLPNOR(2,7)=-1./sqrt(2.)
SLPNOR(3,7)=1./sqrt(2.)
SLPNOR(1,8)=1./sqrt(2.)
SLPNOR(2,8)=0.
SLPNOR(3,8)=-1./sqrt(2.)
SLPNOR(1,9)=-1./sqrt(2.)
SLPNOR(2,9)=1./sqrt(2.)
SLPNOR(3,9)=0.
SLPNOR(1,10) = 1./sqrt(2.)
SLPNOR(2,10) = 0.
SLPNOR(3,10) = 1./sqrt(2.)

SLPNOR(1,11) = 1./sqrt(2.)
SLPNOR(2,11) = 1./sqrt(2.)
SLPNOR(3,11) = 0.

SLPNOR(1,12) = 0.
SLPNOR(2,12) = -1./sqrt(2.)
SLPNOR(3,12) = 1./sqrt(2.)

SLPNOR(1,13) = 0.
SLPNOR(2,13) = 1./sqrt(2.)
SLPNOR(3,13) = 1./sqrt(2.)

SLPNOR(1,14) = 1./sqrt(2.)
SLPNOR(2,14) = 1./sqrt(2.)
SLPNOR(3,14) = 0.

SLPNOR(1,15) = 1./sqrt(2.)
SLPNOR(2,15) = 0.
SLPNOR(3,15) = -1./sqrt(2.)

SLPNOR(1,16) = 0.
SLPNOR(2,16) = 1./sqrt(2.)
SLPNOR(3,16) = 1./sqrt(2.)

SLPNOR(1,17) = 1./sqrt(2.)
SLPNOR(2,17) = 0.
SLPNOR(3,17) = 1./sqrt(2.)

SLPNOR(1,18) = -1./sqrt(2.)
SLPNOR(2,18) = 1./sqrt(2.)
SLPNOR(3,18) = 0.

Endif
Appendix 1.5

The following appendix contains stress versus depth for both CP and $f_2$ plasticity models for 316L SS material described in Chapter 6.

Figure 8-14. Tensile and shear stress ranges into the depth of the specimen for the CP plasticity model for a $\sigma_{amp} = 225$ MPa.

Figure 8-15. Tensile and shear stress ranges into the depth of the specimen for the CP plasticity model for a $\sigma_{amp} = 213.75$ MPa.
Figure 8-16. Tensile and shear stress ranges into the depth of the specimen for the CP plasticity model for a $\sigma_{amp} = 202.5$ MPa.

Figure 8-17. Tensile and shear stress ranges into the depth of the specimen for the CP plasticity model for a $\sigma_{amp} = 191.25$ MPa.
Figure 8-18. Tensile and shear stress ranges into the depth of the specimen for the $J_2$ plasticity model for a $\sigma_{amp} = 225$ MPa.

Figure 8-19. Tensile and shear stress ranges into the depth of the specimen for the $J_2$ plasticity model for a $\sigma_{amp} = 213.75$ MPa.
Figure 8-20. Tensile and shear stress ranges into the depth of the specimen for the $J_2$ plasticity model for a $\sigma_{amp} = 202.5$ MPa.

Figure 8-21. Tensile and shear stress ranges into the depth of the specimen for the $J_2$ plasticity model for a $\sigma_{amp} = 191.25$ MPa.