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A thesis submitted for the degree of Doctor of Philosophy

by


Supervisor: Dr. Andrew Shearer

Faculty of Science
Centre For Astronomy
School of Physics
National University of Ireland, Galway
Galway, Ireland

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"Nobody knew. It turned out that it was not understood at that time. So right away I found out something about biology: it was very easy to find a question that was very interesting, and that nobody knew the answer to. In physics you had to go a little deeper before you could find any interesting question that people did not know."

R.P. Feynman
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Abstract

Optical polarization is a powerful diagnostic tool in astrophysics, allowing the investigation of asymmetries in source regions of astronomical objects, magnetic field configurations, and magnetic field strengths. Polarized sources that vary on short time scales such as Pulsars can be better understood with high-time resolution observations and, for example, may assist in finding the connection between the optical pulses and the giant radio pulses. Current astronomical optical polarimetry can at best record either the linear or circular polarization at resolutions of microseconds, but the entire stokes vector cannot be measured in a single exposure. In this thesis we demonstrate that a division of amplitude polarimeter (DOAP) can be modified and designed to measure, in a single exposure the entire Stokes vector. DOAP polarimetry is where light is initially split in two, one portion has a quarter wave of retardance added and then both portions are split in two by a polarizing beamsplitter, these four portions of the beams are measured and they are linearly related to the input stokes vector. The development of our polarimeter begins with the enhancement of a retarding beam splitter (RBS) prism, initially used by Compain and Drevillon [1], adds a quarter wave of retardance and splits the input beam in two. This RBS prism was redesigned to be achromatic by changing its geometry and choosing a more suitable glass. It was also modified for imaging polarimetry. Multiple optical designs of the polarimeter were made before arriving at the final folded layout that could record four images onto the one detector and whose polarization could then be measured. Calibration of astronomical polarimeters are difficult and we detail a method, known as the Eigenvalue Calibration Method (ECM) that can unambiguously do so. We present various laboratory and astronomical trails that test the performance and capability of the polarimeter, investigate how the errors propagate through polarimetry, conduct some Monte Carlo test for situations not possible in the laboratory and then finally present recommendations for future enhancement of this polarimeter.
Declaration

The work in this thesis is based on research carried out at the Centre for Astronomy, School of Physics, National University of Ireland, Galway. No part of this thesis has been submitted elsewhere for any other degree or qualification and it is all my own work unless referenced to the contrary in the text.

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Acronyms

APD  Avalanche Photo Diode
DOAP  Division of Amplitude Polarimetry
DOC  Degree of Circular polarization
DOL  Degree of Linear polarization
DOP  Degree of Polarization
DOTP Division of Time Polarimetry
DOWP  Division of Wavefront Polarimetry
ECM  Eigenvalue Calibration Method
FEM  FeroElectric Modulator
FOV  Field of View
FWHM  Full Width Half Maximum
GASP  Galway Astronomical Stokes Polarimeter
GRP  Giant Radio Pulse
HWP  Half Wave Plate
LCP  Left-hand Circularly Polarized
PMT  Photo Multiplier Tube
POB  Polygon OBject
PSG  Polarization State Generator
QWP  Quarter Wave Plate
RBS  Retarding Beam Splitter
RCP  Right-hand Circularly Polarized
SNR  Signal to Noise Ratio
Chapter 1

Introduction
The aim of this research project was to demonstrate the ability to measure the complete Stokes vector of astronomical polarization sources in a single exposure. In this chapter, we will look at the driving force behind the need to make such observations. We will describe where these sources of polarization are in the night sky and the possible mechanisms behind them. We will define what polarization is and how to perform polarimetry. Finally, we will finish with a look at current astronomical polarimeters and how they operate and measure polarization.

1.1 Astronomical Polarization at High-Time Resolution

Polarimetry is one of the most difficult astronomical measurements to undertake yet it has the advantage of providing much more information about a source than its intensity and/or spectral content alone. It can tell you the magnetic field strengths on stars, the orientation of dust particles in nebulae, much more of which will be explained later in section 1.3. There are many astronomical objects that emanate polarized light by differing mechanism, table 1.1, extracted from Tinbergen [2], shows a sample of these sources in each portion of the electromagnetic spectrum and the expected maximum degree of polarization that they emit.

<table>
<thead>
<tr>
<th>Source</th>
<th>DOP</th>
<th>Source</th>
<th>DOP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Radio</strong></td>
<td></td>
<td><strong>UV</strong></td>
<td></td>
</tr>
<tr>
<td>Quasars (resolved)</td>
<td>70%</td>
<td>Scattering by interstellar dust</td>
<td>4%</td>
</tr>
<tr>
<td>Pulsars</td>
<td>80%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extragalactic</td>
<td>0.5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OH Masers</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosmic Microwave background</td>
<td>0.01%</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Infrared</strong></td>
<td></td>
<td><strong>X-ray</strong></td>
<td></td>
</tr>
<tr>
<td>Dust emission</td>
<td>2%</td>
<td>Solar flares</td>
<td>5%</td>
</tr>
<tr>
<td>Scattering by interstellar dust</td>
<td>75%</td>
<td>Accreting x-ray pulsars</td>
<td>80%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Active galactic nuclei</td>
<td>20%</td>
</tr>
<tr>
<td><strong>Optical</strong></td>
<td></td>
<td><strong>γ-ray</strong></td>
<td></td>
</tr>
<tr>
<td>Planets</td>
<td>&gt; 20%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scattering by interstellar dust</td>
<td>10%</td>
<td>Pulsars (expected)</td>
<td>100%</td>
</tr>
<tr>
<td>Reflection nebulae</td>
<td>60%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crab pulsar</td>
<td>20%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1.1 Astronomical Polarization at High-Time Resolution

From this table we can see a range of degrees of polarization from 0.01% to 100% from a selection of objects. However, most of the polarizations that emanate from these are continuous or change on very long time scales and so long integrations of the signal can be taken, yielding accurate results. For some objects, the polarization can vary on the order minutes or less and it is here where a challenge lies in performing polarimetry on these targets.

1.1.1 Polars

Magnetic cataclysmic variable stars, also known as polars, are one such class of objects whose polarization can change on the order of minutes. Polars are accreting binary systems with a white dwarf primary and a late-type main sequence secondary, in which accretion onto the primary from the inner Lagrangian point, $L_1$ is dominated by a large magnetic field in the 10-200 MGauss range so that the accretion stream traces the geometry of the magnetic field [3] (see figure 1.1). A general review of the properties of polars can be found in Cropper [4], Warner [5] and Patterson [6].

Figure 1.1: Schematic of a Polar, where the accretion stream of gas from the red dwarf secondary to the white dwarf primary is redirected to the magnetic pole of the primary due to the high energy density of the magnetic field.

The accreting material, near to the white dwarf surface, is fully ionized in the presence of the strong magnetic field, producing high degrees of linear and circular polarization by cyclotron emission [7]. Since the optical flux largely comes from this region, where the accretion stream strikes the surface, the polarization signal dominates and polarimetry is a crucial diagnostic tool for studying these objects and revealing the characteristics of the magnetic field. Estimates of the magnetic field intensities as well as of some other plasma properties can be obtained through the modelling of flux and polarization variations with orbital phase [8], and on the accreting column cyclotron models [9].
The period of polars range from as low as 275 to 78 minutes [5], thus simultaneous polarimetry of linear and circular polarization at short time scales becomes difficult especially when the object can be as dim as $\sim 17^{th}$ magnitude. There are several articles on the polarimetry of polars [10], [9], while figure 1.2 below examples the very high degree of polarization observed at certain phases of the polar 1RXS J231603.6-052713.

The integration times for some of the observations for this polar was as low as 30 seconds and with changes in circular polarization of 5%-12% in less than 60 seconds, high-time resolution polarimetry becomes important. As we will see later, current instruments may struggle to obtain simultaneous linear and circular polarimetry on lower time scales due to their design. Regardless of this, polarimetry of these polars lead to an explanation of their astrophysical origins [5] and can do much more. Various authors have attempted to “inverse map” the periodic fluctuations in the polarization to generate images of the intensity and distribution of the cyclotron regions on the surface of the white dwarf star [11], [12] and faster polarimetry may prove fruitful for their efforts. Other objects that have polarization fluctuations much less than a second require faster polarimetry and such polarimetry can be useful to understand the astrophysics of these objects. We will now look at such objects.
1.1 Astronomical Polarization at High-Time Resolution

1.1.2 Pulsars

The discovery of pulsars was an accidental one and was stumbled upon by Hewish and Bell [13] while conducting interplanetary radio scintillation studies in 1968. Large periodic radio fluctuations, on the order of seconds, appeared during their observations, and were initially ignored, dismissing them as interference from terrestrial sources such as shot noise from a passing car [14]. The clue that the source was celestial came from the fact that the recurrence of this interference was sidereal. This then encouraged them to increase the temporal resolution of the radio telescope and a pulsating radio source with a period of 1.337 seconds was discovered, the first pulsar [13].

Later studies of this and other newly discovered pulsars lead to the astrophysical explanation of the phenomenon, rapidly rotating neutron stars. Neutron stars are the remnants of a supernova measuring between 10 and 15 km in diameter [15] and retains much of the angular momentum from its progenitor leading the final neutron star to rotating rapidly. The period of rotation of pulsars ranges from milliseconds to tens of seconds [16]. The emission from the pulsars originates from synchrotron radiation as charged particles spiral round the magnetic field lines of the highly magnetised, $\sim 10^{12}$G, neutron star [17]. The lighthouse model of the pulsar (figure 1.3) demonstrates the basic method of the pulsating signal, as the neutron star rotates, the beam of electromagnetic radiation sweeps past our line of sight generating a pulse every time.

![Figure 1.3: Lighthouse model of a pulsar.](image)

The pulse structure varies as a function of frequency, with varying power in various components of the pulsar light curve, namely the main pulse (MP), interpulse (IP), precursor to the main pulse (P), Low frequency component (LFC) and two high frequency components (HFC1 and HFC2) shown in figure 1.4. The pulses are strongly polarized for certain radio frequencies, a linear polarization
of almost 100% has been observed by Moffet and Hankins [18] and other high degrees of polarization have been seen coincident with the IP at 50% and HFCs at 70-80%. Moffet and Hankins also reported, with little confidence, very small amounts of circular polarization, 1-2% about the main pulse. However, there was little confidence behind this report as no circular polarization has been published for the crab pulsar.

Since 1968, it had been known that the Crab pulsar exhibits a phenomenon called a giant radio pulse (GRP) [19] in which there is a sporadic and random emission of intense pulses on average 1000 times larger than normal pulses. GRPs come in short episodes, about 5 to 20 minutes in duration, and appear extremely prominent during such phases [20]. Figure 1.5 shows the random arrival of six giant pulses where the arrival time is relative to a fixed point on the pulsar’s surface.

Hankins et al. [21] reported on observations of GRPs from the Crab pulsar made with an extremely high-time resolution, sub-microsecond, and at very high radio frequencies with large bandwidths, 5.5±1 and 8.6±1 GHz, shown in figure 1.5. They found that even over a few minutes the arrival time of GRPs jitters by several hundred microseconds in phase with either the main pulse or the interpulse. Figure 1.6 shows a magnified view of the third pulse from figure 1.5 and structure at the limit of the receiver bandwidth (∆t = 2 ns) is clearly visible. The emissions from the sub-pulses a-f were strongly circularly polarized, reaching 50-100% polarized. Evidence of the presence of circular polarization, but only resolvable at sub-µs time scales.

The population of pulsars, ~ 1800 in total, are dominated by radio pulsars and there are only there are only 5 observed optical pulsars [22]. These are
1.1 Astronomical Polarization at High-Time Resolution

Figure 1.5: Giant radio pulses from the Crab pulsar for the first 500µs of various pulsar rotations at 5.5GHz [21].

Figure 1.6: Intensity, circular polarization and nano-pulse structure of the Crab GRP [21] (LCP and RCP are left-hand and right-hand circularly polarized respectively).

PSR B0656+14, PSR B0540-69, Geminga, Vela and Crab and their respective magnitudes are 25.5, 23, 26, 24 and 16.8. The Crab pulsar stands out, because of its young age\(^1\) and is by far the brightest and therefore by far the best candidate for detailed polarimetric analysis. Polarization of the optical emission was discovered by Wampler, Scargle & Miller [24]. Even for the Crab pulsar, the photon flux is so low that phase averaging techniques over many pulsar cycles are necessary to obtain an acceptable signal to noise, and the literature also contains measurements

\(^1\)The progenitor of the Crab pulsar went supernova on 4th July 1054 and was observed by the Chinese and Japanese[23]
Introduction

Figure 1.7: Stokes parameters Q, U plotted as a vector diagram for the Crab nebula pulsar [25].

of linear polarization, but not circular.

An archetypical observation of linear polarization was made in 1985 by Smith et al.[25]. They were able to report that the detection of polarized emission throughout the whole pulsar cycle, and that the plane of linear polarization makes two complete revolutions per cycle (see figure 1.7. Subsequent observations [26], [27] all show a similar picture, extending into the UV. Such information can lead the proposal of new models of the pulsars emission. Most of the instruments used to observe the Crab pulsar can measure the polarization at high speed, but only measure linear or circular at anyone time. Some are also limited by moving parts, but we will look at these instruments in section 1.5. All of these observations were made using a phase averaging technique and report the linear polarization averaged over many, perhaps millions, of cycles further impeding the temporal resolution.

The relationship between GRP and the associated optical pulses is not at all clear, but what is clear is that polarization is extremely important to the understanding of the GRP phenomenon in the radio region, and it is also likely to be important to an understanding of the associated optical phenomenon. Shearer et al. [28] detected a correlation between GRP emission and optical emission (see figure 1.8). They found that optical pulses coincident with GRP were ~3% brighter on average.

This suggests the intriguing possibility that the extra 3% observed by Shearer et al.[28] may itself have nano-pulse structure not only intensity, but in polarization. The requirements to investigate optical counterparts to GRP and possible nano-pulse structure require time resolution of the order of microseconds for indi-
1.2 Polarization of Light, Stokes Vectors and Parameters

Figure 1.8: The Crab pulse profile showing the optical (o) and average radio light curve at 1380 MHz (r) and a single giant radio pulse at 1357.5 MHz (gr) [28].

Individual pulses and photometry with an absolute time resolution of nanoseconds for correlation with radio nano-pulse structure, if that is possible. From an optical observational perspective GRPs are significantly more difficult than normal pulsars to observe, the pulses arrive randomly albeit in phase with normal emission and at a rate significantly less than the normal pulsar rate [22]. The random nature of the GRP arrival times makes synchronised systems, such as clocked CCDs, inappropriate. Polarimetry of these types of events also restricts the type of polarimeter that can be used in the GRP studies, makes the simultaneous measurements of linear and circular polarization essential. This outlines the premise of this thesis where we wish to develop a polarimeter that can perform total instantaneous polarimetry.

We will now follow this by explaining what is polarization, how it is described and measured, how it is generated in nature and in the laboratory and a review of current astronomical polarimeters.

1.2 Polarization of Light, Stokes Vectors and Parameters

Many texts exist on the subject of polarization with Shurcliff (1964) [29] being one of the most cited and other more modern texts that include the advancement of polarization and polarimetry [30]-[31]. There are also other texts on astronomical polarimetry that concentrate more on the instrumentation and astrophysical interpretation of polarized light [2],[7]. We will not go into great detail on the
Introduction

mathematical derivations of polarization as these are covered in the aforementioned references.

The transverse wave nature of light with its instantaneous electric vector restricted to any single direction and creates linear polarization. The electric vector can be broken down into two orthogonal components, its x and y amplitudes and the resultant of these component vectors traces the polarization vector. In figure 1.9 (linear), the resultant traces a polarization vector in the xy plane of \(135^\circ\). Circular polarization occurs when the x and y components are out of phase with each other by a quarter of their wavelength \((\delta_x - \delta_y = \lambda/4\) in figure 1.9) and the resultant electric vector traces a helix in space that appears as a circle on the xy plane. When this value of relative phase \(\delta_x - \delta_y \neq \lambda/4\), the electric vector traces an ellipse and the polarization is said to be elliptical.

![Figure 1.9: Wave nature of light showing linear (left), circular (middle) and elliptical (right) polarized light.](image)

As a natural consequence of the mathematical phase and amplitude description of light, we can mathematically describe polarization in a vector form known as the Jones vector [30] shown in equation 1.1. This representation is compact and applicable to the addition of coherent beams unfortunately; it cannot deal with unpolarized light.

\[
J = \begin{bmatrix}
E_x e^{i(\omega t - kz + \delta_x)} \\
E_y e^{i(\omega t - kz + \delta_y)}
\end{bmatrix}
\]

(1.1)

where \(J\) is the Jones vector, \(E_x\) and \(E_y\) are the amplitudes of the electric vector, \(\delta_x\) and \(\delta_y\) are the phases of the electric vectors in the x and y direction.

Another mathematical way to represent polarized light is the Stokes vector. It was introduced by George Gabriel Stokes in 1852, however, this representation was lost in obscurity until the rediscovery by S. Chandrasekhar in the late 1940’s.
The Stokes vector is a $4 \times 1$ vector representation of any polarization state, including unpolarized light, as shown in equations 1.2

$$
\begin{bmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{bmatrix}
= 
\begin{bmatrix}
I \\
Q \\
U \\
V
\end{bmatrix}
= 
\begin{bmatrix}
\langle E_x^2 + E_y^2 \rangle \\
\langle E_x^2 - E_y^2 \rangle \\
\langle 2 E_x E_y \cos(\delta_y - \delta_x) \rangle \\
\langle 2 E_x E_y \sin(\delta_y - \delta_x) \rangle
\end{bmatrix} = 
\begin{bmatrix}
I \\
I_p \cos 2\theta \cos 2\phi \\
I_p \sin 2\theta \cos 2\phi \\
I_p \sin 2\phi
\end{bmatrix}
$$

where $S_0$ to $S_3$ are the four Stokes Parameters more commonly know as $I$, $Q$, $U$ and $V$, respectively. $E_x$ and $E_y$ is the amplitude of the electric field in $x$ and $y$ directions, $\delta_y - \delta_x$ is the phase difference between the $x$ and $y$ components. $I_p$ is the intensity of the polarization, $\theta$ is the angle of linear polarization and $\phi$ is the relative phase angle. In the next section, we will state how these are measured.

The Stokes parameters $Q$, $U$ and $V$ are the polarized elements of the vector, while $I$ is solely its intensity. This allows the description of unpolarized light that cannot be represented by a single electromagnetic wave. If $Q$, $U$ and $V$ equal zero, then the polarization state is said to be unpolarized. This is conventionally represented as a mix of different polarization states as in figure 1.10 [7]. A mix of linear states is used to demonstrate unpolarized light, but its is not limited this. Unpolarized light can also be a mix of circular or elliptical states although graphically this can be difficult to present. Partially polarized light features a preference towards a particular polarization state where the intensities of the dominant state are larger than orthogonal components of that state. Levels of polarization are referred to as degrees of polarization ranging from 0-100% and we will state how these are calculated from the Stokes vector in the next section.

![Figure 1.10: Conventional representations of polarized, unpolarized and partially polarized light using linear polarization only.](image-url)
while using narrowband filter may reveal that the source is polarized for different wavebands. For the spatial case, low resolution imaging polarimetry integrates the changing spatial polarization distribution thus the source appears unpolarized and when using higher image resolution the polarization structure of the image can be resolved.

Figure 1.11: Demonstration of polarized and unpolarized light for temporal, spectral and spatial cases.

Another case of the appearance of unpolarized light is the improper measurement of the Stokes vector. If we were only measuring the linear polarization ($Q$ and $U$) of an input state that was circular, it would appear to be unpolarized. The degree of linear polarization must be quoted not the degree of polarization.

1.2.1 Data Reduction and Representation

When a Stokes vector is recorded it completely describes the state of polarization for that object, but it alone is not a clear description of the information within it. Equations 1.3 to 1.7 can reduce the Stokes vectors to more meaningful values.

Equation 1.3 describes the angle of polarization$^3$, $\theta$, for some arbitrary instrument orientation, which can be predefined in the polarimeter design or calibrated out of the system later.

$$\theta = \frac{1}{2} \arctan 2(U, Q) = \frac{1}{2} \arctan(U/Q) \quad (1.3)$$

$^3\arctan 2$ as used in Matlab
1.2 Polarization of Light, Stokes Vectors and Parameters

Equation 1.4 describes the relative phase angle, $\phi$, of the polarization also know as the ellipticity. Note, this is not the absolute phase of the wave and is only the phase difference between two orthogonal polarization states.

$$\phi = \frac{1}{2} \arctan(V \sqrt{Q^2 + U^2})$$ (1.4)

Equation 1.5 describes the total degree of polarization (DOP) and can again be algebraically broken down as the square root of the sum of squares of the degree of linear and the degree of circular polarization.

$$DOP = \sqrt{Q^2 + U^2 + V^2} / I$$ (1.5)

$$DOL = \sqrt{Q^2 + U^2} / I$$ (1.6)

$$DOC = V / I$$ (1.7)

Equation 1.6 describes the degree of linear polarization (DOL), equation 1.7 describes the degree of circular polarization (DOC). DOC may be positive or negative as this will represent the handedness of the polarization state. When $DOC > 0$ it is right-handed, when $DOC < 0$ it is left-handed. Throughout this thesis there will be many references to the degrees of polarization, that is DOP, DOL and DOC, while the singular, degree of polarization refers to DOP only.

All of these reduction equations are subject to noise and errors and this topic will be covered in chapter 5 as this is a difficult topic to approach and deserves its own chapter.

A graphical representation of any polarization state can be shown by plotting the $Q$, $U$ and $V$ Stokes parameters as x, y, z coordinates in 3D space and this is

Figure 1.12: The Poincaré sphere where $\theta$ is the polarization angle, $\phi$ is the relative phase angle, $I_p$ is the intensity of polarization and $Q$, $U$, $V$ are three of the Stokes parameters.
known as the Poincaré sphere (see figure 1.12). On this sphere, the equator represents the linear states \((x, y\) or \(Q, U\) plane) and the poles represent pure circular polarization. A point anywhere else on the sphere is an elliptically polarized state. A longitudinal line, one traced perpendicular from the equator to a pole will have a constant angle of polarization, but changing relative phase angle. Points on the surface of the sphere represent 100\% polarization, while the magnitude of the radius vector is proportional to the degree of polarization [2].

1.3 Sources of Polarized Light

There are many sources of polarized light and ways to create polarization; this section looks at the materials, methods and physical, environmental and astrophysical mechanisms that can create or alter polarized light.

1.3.1 Materials

Diattenuation and retardation are the two main properties of materials that can be used to create polarized light. A diattenuator is an optical element that changes the orthogonal amplitudes unequally and can be directly used to create polarization. A retarder is an optical element that introduces a phase change between two orthogonal field components and does not directly create polarization, but modifies it. It is the anisotropy of materials as viewed by two orthogonal directions, which is the mechanism for creating / altering polarization [32].

A plate polarizer, also known as a linear dichroic filter, is a diattenuating optical element that restricts the electric vibration of light to one plane where the orthogonal plane is attenuated until it is eliminated. The ratio of the polarized to the attenuated portion of light is the extinction ratio, where ideally this would be infinity, polarizers are generally made in the range of 1000:1 to 10,000:1.

A birefringent material is one that has more than one refractive index depending upon the orientation of the material and direction of the incident light. Anisotropic optical crystals such as calcite and quartz have a crystal structure that will refract one polarization state more than the orthogonal depending on the direction of the crystal’s optic axis. These materials are used to create retarders where the crystal can be cleaved in such a way that when light is incident normally it adds a phase change to the incident beam governed by equation 1.8
1.3 Sources of Polarized Light

\[ \Delta = \frac{2\pi}{\lambda} \Delta_n \ast d \]  

(1.8)

where \( \Delta \) is the retardance, \( \lambda \) the wavelength of light, \( \Delta_n \) the birefringence of the material and \( d \) its thickness.

Stress birefringence occurs when a material is under an external or internal stress. Portions of the material can become distorted and create an anisotropy in the medium making it birefringent. It is a case to point out for polarimeters as optics may become stressed from its mountings it can induce unwanted polarization effects from stress birefringence.

The property of birefringence can also be used to make polarizers by refracting the orthogonal field components of light, known as the ordinary and extra-ordinary ray, in different directions and separating them. These are prisms constructed of uniaxial crystals where the orientation their optic axis differ for various type of polarizers such as Glan-Thomposn, Glan-Taylor, Wollaston, Forster, Rochon prisms and others. Figure 1.13 shows examples of two such prism and how the polarize light.

Figure 1.13: A Foster prism and a Glan-Taylor polarizer showing how the extra-ordinary ray (e-ray) and ordinary ray (o-ray) can be separated by using total internal reflection. The o-ray is beyond the critical angle for its birefringent refractive index while the e-ray can refract and pass through.

The Fresnel equations, which we will see later (equation 1.17 to 1.20), play a key role in polarizing light [33] [34]. The Brewster angle, shown in equation 1.9, is derived from the Fresnel equations, it shows how at a particular angle of reflection off a dielectric surface that only one polarization state can be reflected, namely the state perpendicular to the plane of reflection \( (R_s)^4 \).

\[ \theta_b = \arctan \frac{n_2}{n_1} \]  

(1.9)

\(^4s\) is used as senkrecht is the German for perpendicular.
where $\theta_b$ is the Brewster angle, $n_1$ is the refractive index of medium where the source is and $n_2$ is the refractive index of the incident media. This can be used to construct a cheap polarizer where light incident on a stack of glass plates near the Brewster angle will reflect the $R_s$ polarization with each plate, while the $R_p$ polarization (polarization parallel to the plane of reflection) will continue to pass through leavening a polarized transmitted beam with a moderate extinction ratio, $\sim 500:1$ [35].

### 1.3.2 Mueller Matrices

The Stokes vector is a mathematical description of polarization and this can be extended for materials that alter the state of polarization. The mathematical description for these polarization altering materials are known as Muller matrices [36]. A Mueller matrix is a $4 \times 4$ matrix that describes the polarimetric properties of an optical element, which can be a polarizer, a retarder, a mirror or others. Just as an optical element will alter the polarization of the light passing through it, the Mueller matrix will alter the Stokes vector by the product of the vector and the matrix and is typified by equation 1.10.

\[
S_{\text{out}} = M_3 \cdot M_2 \cdot M_1 \cdot S_{\text{in}} = M_p \cdot S_{\text{in}} \quad (1.10)
\]

where $S_{\text{in}}$ and $S_{\text{out}}$ are the input and output Stokes vectors, $M_p$ is the Muller matrix product of the 3 various Mueller matrices, $M_1$, $M_2$, $M_3$. If the order of the multiplication changes so does the product and hence a different Muller matrix is calculated, so the order matters. The order of matrix multiplication as if it was in an optical system is written down with the first to the last interacting matrix going from right to left, while the mathematical product is in the opposite direction.

\[
\begin{bmatrix}
I \\
Q \\
U \\
V_{\text{out}}
\end{bmatrix}
= \begin{bmatrix}
M_{1,1} & M_{1,2} & M_{1,3} & M_{1,4} \\
M_{2,1} & M_{2,2} & M_{2,3} & M_{2,4} \\
M_{3,1} & M_{3,2} & M_{3,3} & M_{3,4} \\
M_{4,1} & M_{4,2} & M_{4,3} & M_{4,4}
\end{bmatrix}
\begin{bmatrix}
I \\
Q \\
U \\
V_{\text{in}}
\end{bmatrix} \quad (1.11)
\]

Equation 1.11 above shows the 16 elements of these Mueller matrices, $M_{1,1}$ to $M_{4,4}$, with the input and output Stokes vectors on either side. These element can have any value, but not all combinations will lead to a physically realizable optical element. Some of the more common Mueller matrices are given in table 1.2.
1.3 Sources of Polarized Light

Table 1.2: Muller matrices for four different optical elements with their descriptions above. A quarter waveplate (QWP) is a retarder that introduces a retardance of \( \lambda/4 \) and in this case it is rotated to 45°

<table>
<thead>
<tr>
<th>MM_{air}</th>
<th>MM_{Pol@0°} (Polarizer at 0°)</th>
<th>MM_{Pol@90°} (Polarizer at 0°)</th>
<th>MM_{QWP@45°} (1/4 Waveplate at 45°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 0 0</td>
<td>0.5 0.5 0 0</td>
<td>0.5 -0.5 0 0</td>
<td>1 0 0 0</td>
</tr>
<tr>
<td>0 1 0 0</td>
<td>0.5 0.5 0 0</td>
<td>-0.5 0.5 0 0</td>
<td>0 0 0 1</td>
</tr>
<tr>
<td>0 0 1 0</td>
<td>0 0 0 0</td>
<td>0 0 0 0</td>
<td>0 0 1 0</td>
</tr>
<tr>
<td>0 0 0 1</td>
<td>0 0 0 0</td>
<td>0 0 0 0</td>
<td>0 1 0 0</td>
</tr>
</tbody>
</table>

A useful generic Muller matrix (equation 1.12) is used to generate the Mueller matrix for any dielectric or metallic surface [37]. To generate this generic Mueller matrix for a surface in transmission or reflection, the angle of incidence and the refractive index\(^5\) of the material in question must be known.

\[
M(\tau_x, \psi_x, \Delta_x) = \tau_x \begin{bmatrix}
1 & -\cos(2\psi_x) & 0 & 0 \\
-\cos(2\psi_x) & 1 & 0 & 0 \\
0 & 0 & \sin(2\psi_x) \cos(\Delta_x) & \sin(2\psi_x) \sin(\Delta_x) \\
0 & 0 & -\sin(2\psi_x) \sin(\Delta_x) & \sin(2\psi_x) \cos(\Delta_x)
\end{bmatrix}
\]

(1.12)

where \( \psi_x \) and \( \Delta_x \) are the ellipsometric angles of diattenuation(\( \psi \)) and retardance(\( \Delta \)). This Mueller matrix can be used in either transmission or reflection so the subscript \( x \) is replaced with \( r \) for the reflection and \( t \) for transmission; and \( \tau_x \) is the throughput for the material be that reflectance or transmittance. The ellipsometric angles fall into the range of \( 0° \leq \psi \leq 90° \) and \( 0° \leq \Delta \leq 360° \), \( \Delta \) is also used interchangeably in terms in waves or angle where a quarter wave is \( \lambda/4 = 90° = \pi/2 \). The ellipsometric angles are determined by equations 1.13, 1.15 and 1.14, 1.16

\[
\psi_r = \arctan(|r_p| / |r_s|) \quad (1.13) \quad \Delta_r = \arg(r_p) - \arg(r_s) \quad (1.14)
\]

\[
\psi_t = \arctan(|t_p| / |t_s|) \quad (1.15) \quad \Delta_t = \arg(t_p) - \arg(t_s) \quad (1.16)
\]

where \( r_p, r_s \) and \( t_p, t_s \), are the Fresnel coefficients for reflection and transmission, respectively, and are calculated by the equations below.

\(^5\)For a metal, the complex refractive index is used.
These Fresnel equations (see equations 1.17 to 1.20) yield the amplitude coefficients for transmission ($t$) and reflection ($r$) of light parallel ($p$) and perpendicular ($s$) to the plane of reflection where $\theta_i$ and $\theta_r$ are the angles of incidence and refraction respectively and $n_i$ and $n_r$ are the refractive indices of the respective media. Equations 1.21 and 1.22 are used to calculate the reflectance and transmittance, respectively, using the Fresnel coefficients.

When concerning ourselves with the generation of polarized light on the astronomical scale, scattering and interactions with magnetic fields are two mechanisms that induce or alter polarized light. We will now look at these mechanisms in the next section.

### 1.3.3 Scattering

Polarized light due to scattering is something that we can observe every sunny day. When we look towards the zenith of the twilight sky, it is highly polarized $\sim 70-80\%$ [38]. The further we look away from the sun the more polarized the sky and this is shown in figure 1.14 where it reaches its maximum degree of polarization (DOP) at $90^\circ$ and then decreases again. This is due to the Rayleigh scattering of light from the sun by the atmosphere. As the observer increases the viewing angle of the sun, the DOP increases as seen in figure 1.15. Light scattering off an atmospheric molecule is more efficient at scattering one polarization component compared to the orthogonal one. Thompson scattering operates much the same way as Rayleigh scattering, light scatters off a free electron instead of a molecule, which there are plenty in hot stellar atmospheres and can be seen during solar eclipse [7].

Mie scattering occurs when the particle is much larger then the wavelength of light being scattered. The DOP is a function of the viewing angle, but this function is much more complex [7] and is also a function of the size of the particle.
1.3 Sources of Polarized Light

Figure 1.14: The DOP increases as you view further away from the sun due to Rayleigh scattering.

Figure 1.15: Rayleigh scattering at a viewing angle of $\sim 90^\circ$ causes the largest amount of sky polarization.

The angle of the polarization no longer has the tangential distribution as seen in figure 1.14, but a radial one where the angles of polarization all point to the source.

Scattering plays a role in the polarization of nebulae, a dense cloud of gas and dust. A nebula may have a neighbouring star and that illuminates the cloud. This reflected light can be redirected to an observer by Rayleigh scattering of the gas and Mie scattering of the dust and thus the light can be polarized [7], [39]. As the density of the cloud is high, the probability of multiple scattering is present, which complicates the prediction of the degree of polarization [40]. Distant stars (distance from the observer $> 1000$ light years) are polarized up to 5% and one may think that these stars are polarized by scattering after passing through vast amounts of interstellar dust. However, the distribution of these polarized stars are concentrated to the galactic plane as seen in catalogues from Hiltner, Hall and Mathewson and Ford [41], [42] and [43]. It is then said that the polarization comes from dichromism. Asymmetric interstellar dust particles can become aligned to local magnetic fields and thus the particles act like low extinction polarizers [7]. Other forms of astronomical polarization come from the interaction with light and magnetic fields, which we will now look at.

1.3.4 Zeeman splitting

From a quantum perspective, the emission line from an atom is the difference between one energy level and the lower level. When an atom is subjected to a magnetic field, it will generally split its energy levels into sub-levels as seen in figure 1.16 [7] and the emission now creates three line and is called the normal
triplet.

![Energy Level Diagram](image)

Figure 1.16: Splitting of energy levels of an atom into sub-levels due to the presence of a magnetic field.

![Spectroscopic Diagram](image)

Figure 1.17: The effect of the direction magnetic the field to Zeeman splitting depends on the magnetic field direction. Parallel fields create circularly polarized doublet, while perpendicular fields create linearly polarized triplet [2].

Figure 1.17 shows that the orientation of the magnetic field (B) with respect to the observer has an effect on the structure of the triplet. A magnetic field parallel to the travelling beam (looking longitudinally) will create what appears to be a doublet and both components have opposite circular polarizations. When the magnetic field is perpendicular to the travelling beam (looking transversely), a triplet is created with a linear polarization. The polarization angle of the side lobes are perpendicular to the central peak [2],[7]. This can give us the ability to determine orientation the magnetic field with respect to the observer. Absorption spectra are more commonly seen in astronomy and the Zeeman effects also applies identically. The absorbed parts of the spectrum has a polarization as shown in figure 1.18 where by the remaining unpolarized portion can still be visible then the light can be partially polarized.

The Zeeman effect can not only tell us about the direction of the magnetic field, but also its strength. The stronger the magnetic field, the larger the change in wavelength from the nominal and this governed by equation 1.23 below.
1.3 Sources of Polarized Light

Figure 1.18: Inverse Zeeman effect showing how absorption spectra under a magnetic field (B) has the same properties as the transmission spectra [2].

\[ \delta \lambda = 4.7 \times 10^{-13} B \lambda^2 \quad (1.23) \]

where \( \lambda \) and \( \delta \lambda \) are the wavelength and spectral split expressed in Angströms and B is the magnetic field expressed in Gauss. A \( 10^5 \) Gauss magnetic field at 5000Å will split the emission line by 1.175Å, a small split, while it only takes 250 Gauss to make the same split at 10µm. However, with small spectral splits and low spectral resolution Doppler boarding, caused by a distribution of velocities of atoms or molecules, causes a spectral line to widen and this may guise as Zeeman splitting. This can be checked against a portion of a target with weak or no magnetic fields.

1.3.5 Other Polarizing Mechanism

Other mechanisms that polarize light in an astronomical situation are dependent on extremely strong magnetic fields, \( \gg 10^5 \) Gauss, such as that present in pulsars and polars. When a relativistic charged particle moving in an intense magnetic field its path becomes helical [44], this particle is subjected to a central force and emits electromagnetic wave perpendicular to the magnetic field lines that is strongly linearly or elliptically polarized and this is called synchrotron polarization. The relationship between the polarization and this effect is intensely mathematical to explain and reviews can be found in [45]. Cyclotron Polarization is a similar mechanism, where charged particles are accelerated in an intense magnetic fields, \( 10^9 \) Gauss [7], where an account of this effect is said to be found in a review by Wickramasinghe [46].

Faraday rotation is where a magnetic field parallel to the line of sight rotates the plane of polarization and is mainly visible in the radio spectrum [2]. This is proportional to the square of the wavelength of polarized light, magnetic field strength, the path length and a constant of the medium that the electromagnetic wave is travelling through, known as the Verdet constant.
Irrespective of how polarization is generated we need to know how to measure it and we will follow this section with the mathematics of polarimetry and types of polarimeters that can be made.

1.4 Polarimetry

One of the most basic ways of measuring linear polarization in the laboratory is to take a polarizer in front of a polarized source rotate that polarizer until the intensity is a minimum and this angle is the angle of polarization $+90^\circ$. Naturally, this does not measure all the components of the polarization state, but it is one of the first methods of polarimetry encountered by undergraduates. A definition of the Stokes vector not mentioned earlier is shown in equation 1.24. This shows how the Stokes parameters can be broken down into intensities of various other states.

\[
\begin{bmatrix}
I \\
Q \\
U \\
V
\end{bmatrix} = 
\begin{bmatrix}
I_{\text{lin}@0^\circ} + I_{\text{lin}@90^\circ} \\
I_{\text{lin}@0^\circ} - I_{\text{lin}@90^\circ} \\
I_{\text{lin}@45^\circ} - I_{\text{lin}@-45^\circ} \\
I_{\text{Lcirc}} - I_{\text{Rcirc}}
\end{bmatrix}
\] (1.24)

where $I$ is the intensity of the vector, $Q$, $U$ and $V$ are equal to the differences between two orthogonal states, which are linearly polarized at $0^\circ$ and $90^\circ$, $+45^\circ$ and $-45^\circ$ and left and right circularly polarized, respectively. To measure the entire Stokes vector all that is required are 6 intensities, but this is a slow and cumbersome method. This method can be reduced to four measurements by using Stokes’ famous intensity formula for measuring the four Stokes parameters [47] shown in equation 1.25.

\[
I(\theta, \phi) = \frac{1}{2} [I + Q \cos(2\theta) + U \cos(\phi) \sin(2\theta) + V \sin(\phi) \sin(2\theta)]
\] (1.25)

where $I(\theta, \phi)$ is the intensity of an output vector after passing though a polarizing element set to a polarizing angle of $\theta$ and with a relative phase angle of $\phi$. When $\phi$ is 0 it is just a polarizer, when $\phi$ is $90^\circ$ this is a quarter-wave retarder set with its fast axis rotated to $0^\circ$ followed by a polarizer at $45^\circ$ and is often called a circular polarizer. The four intensities measured are; $I_1 = I(0^\circ, 0^\circ)$, $I_2 = I(45^\circ, 0^\circ)$, $I_3 = I(90^\circ, 0^\circ)$ and $I_4 = I(45^\circ, 90^\circ)$ and a solution exists to find the Stokes parameters to be
1.4 Polarimetry

\[
\begin{bmatrix}
I \\
Q \\
U \\
V
\end{bmatrix} = \begin{bmatrix}
I(0°, 0°) + I(90°, 0°) \\
I(0°, 0°) - I(90°, 0°) \\
2I(45°, 0°) - I(0°, 0°) - I(90°, 0°) \\
2I(45°, 90°) - I(0°, 0°) - I(90°, 0°)
\end{bmatrix}
\]

Handling data and computing the Stokes vector in this way can become unruly and some polarimetrists still prefer to complete the Stokes algebra this way. A tidier way of completing the calculations is by using matrix algebra and now we will describes these basic computations and how to create a system matrix for a polarimeter.

1.4.1 Polarimetry by Using Matrix Algebra

Four linearly independent measurements are the minimum number of intensities needed to determine the entire Stokes vector. These intensities are linearly related to the Stokes vector by a matrix as is expressed in equation 1.27

\[ I = A \cdot S \]  \hspace{1cm} (1.27)

where \( I \) is a 4 × 1 vector with the 4 intensities arranged in a column, \( S \) is the Stokes vector and \( A \), a 4 × 4 matrix, is the system matrix. To determine \( S \), we pre-multiply \( I \) by the inverse of the system matrix, \( A^{-1} \). The system matrix can be experimentally measured or theoretically generated. We will look at the experimental measurement of the system matrix in chapter 3, the calibration chapter. To determine the theoretical system matrix for a polarimeter, the Muller matrices for every component/surface that the beam will pass through must be derived. Section 1.3.2 described how each of these matrices can be calculated. The product of these matrices from the input to the detector is found and the first row of the final matrix product will correspond to a row in the final matrix as expressed in equation 1.28.

\[ A_{n,i} = [MM_{n,i}]_{n} : n = 1, \ldots, 4, i = 1, \ldots, 4 \]  \hspace{1cm} (1.28)

where \( n \) is the \( n^{th} \) polarimetric analyser\(^6\), \( MM \), which is then entered at the \( n^{th} \) row of the system matrix \( A \). The first line of the Mueller matrix product is

\(^6\)An arm of a polarimeter that examines one state of polarization is often called an analyser. This can be a single polarizer or multiple polarization optics with the aim of extracting the intensity.
chosen because it relates to the intensity of the final vector to which the detector can only measure. The number of rows in the final matrix is dependent upon the number of intensities recorded, and are arranged sequentially to construct the final system matrix. Ideally, each system matrix should be square, so that it can be easily inverted, which is the case when the minimum four measurements are used. When more than four measurements are taken, then the matrix is rectangular and has to be inverted using the Moore-Penrose pseudoinverse.

Apart from the generic matrix as seen in equation 1.12, the equations for an ideal polarizer for any angle is shown in equation 1.29 and the rotation matrix shown in equation 1.30 is also used. These will go towards generating a final theoretical system matrix from a polarimeter.

\[
M_p(\theta) = \begin{bmatrix}
1 & \cos 2\theta & \sin 2\theta & 0 \\
\cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\
\sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\] (1.29)

\[
R(\theta) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta & \sin \theta & 0 \\
0 & -\sin \theta & \cos \theta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (1.30)

To rotate any Mueller matrix it must be pre-multiplied by the rotation matrix (equation 1.30) at the necessary angle and then post multiplied by the rotation matrix of the negative angle as shown.

\[
M(\theta) = R(-\theta) \cdot M \cdot R(\theta)
\] (1.31)

Taking an example of a polarimeter that would measure the Stokes vector as described in equation 1.24, six intensities would be measured, four linear and two circular. Upon inspection of the system matrix of this “polarimeter” in equation 1.32, the first four rows can be seen to be the first row of each Mueller matrix of a polarizer at 0°, 45°, 90°, 135° (-45°), respectively. The final two rows are from right and left circular polarizers. This is an idealized situation, in reality these numbers would not all equal 0.5 as some polarizers may be imperfect.
1.4 Polarimetry

\[
A = \begin{bmatrix}
0.5 & 0.5 & 0 & 0 \\
0.5 & 0 & 0.5 & 0 \\
0.5 & -0.5 & 0 & 0 \\
0.5 & 0 & -0.5 & 0 \\
0.5 & 0 & 0 & 0.5 \\
0.5 & 0 & 0 & -0.5 \\
\end{bmatrix}
\]  
(1.32)

For completeness, the system matrix for the method of measuring the Stokes vector using only 4 intensities (equation 1.24) is shown in equation 1.33 and note that the order of the rows are the same as the order of the measurements.

\[
A = \begin{bmatrix}
0.5 & 0.5 & 0 & 0 \\
0.5 & 0 & 0.5 & 0 \\
0.5 & -0.5 & 0 & 0 \\
0.5 & 0 & 0 & 0.5 \\
\end{bmatrix}
\]  
(1.33)

Once we have the system matrix for a polarimeter we can go ahead and measure the Mueller matrix for any optical element known as Muller matrix polarimetry. Some texts refer to a more complex method for measuring these [36], but Azzam [37] uses the matrix method to measure a Mueller matrix by utilizing equation 1.35, a solution of equation 1.34.

\[
I = A \cdot M \cdot W  \tag{1.34}
\]
\[
M = A^{-1} \cdot I \cdot W^{-1} \tag{1.35}
\]

where \(A\) is the system matrix of the polarimeter and \(M\) is Mueller matrix, which we are trying to find. \(W\) is known as the polarization state generator (PSG) matrix that is made up of four columns of four linearly independent Stokes vectors. Every Mueller matrix contains 16 independent elements and the product of this with the PSG matrix (\(W\)) yields an intensity matrix \(I\) that is made up of 4 columns of intensity vectors that also has 16 independent elements. The product from equation 1.35 will give us the Mueller matrix we are looking for.

Many methods exist for measuring the Stokes vector and performing polarimetry in whole or part, i.e. linear, circular or total polarimetry. The optical theory of three basic types of polarimeter currently in use throughout the optics field will be investigated here, namely, Division of Time, Division of Wavefront and Divi-
sion of Amplitude polarimetry. Other types worth mentioning, but will not be elaborated upon here are; Fourier analysis [48], null-intensity [30], interferometric [49] and holographic polarimetry [50].

1.4.2 Division of Time Polarimetry (DOTP)

Although not normally called division of time polarimetry this type of polarimeter can be realised where there is a change in the polarimeter’s analyser over a set period. This can be in the order of seconds to minutes where a modulating analyser is used. This modulation can come from a mechanically rotated polarizer or half waveplate for slower polarimeters or an electro-optic analyser such as a ferroelectric liquid crystal or piezoelectric modulator for faster polarimeters. The speed limit of the complete measurement of a Stokes vector is set by how fast this modulation can occur, but also in certain devices that use a rotating polarizer, the insertion of a quarter waveplate is needed to measure the circular component thus slowing down the polarimetry. The method of measuring a Stokes vector in the beginning of section 1.4 can be seen as a DOTP polarimeter where sequential intensities are measured.

1.4.3 Division of Wavefront Polarimetry (DOWP)

Division of wavefront polarimetry relies on analysing different parts of the wavefront with separate polarizing elements. Figure 1.19 shows such a polarimeter where different portions of the wavefront of the incident light see different polarizing elements, three linear and one circular polarizer. These portions of the beam then pass onto the detectors, where their measured intensities can yield the input Stokes vector by applying equation 1.24 or using the matrix method described earlier.

Such polarimeters can be constructed to yield instantaneous measurement of the Stokes vector whose acquisition speed is dependent on the detector and on the photon statistics of the signal. However, following the example in figure 1.19, the efficiency of the polarimeter is reduced as the entire wavefront does not strike the detectors and linear polarizers are used whereby half the light is lost. This type of application is useful for ellipsometry [52] [53], where lasers are used, but not for an imaging application. To make an imaging DOWP polarimeter, the collimated space of an optical system is split in two with a polarizing analyser in each portion. One part would have a Wollaston to measure the linear portion, the other
would have a polarization component to measure two more linearly independent polarization states and have a divergent output like the Wollastion. When the outputs are imaged, four images would be created each linearly independent and so full Stokes polarimetry could be completed, however, splitting the pupil can then have a consequence on the quality of the final images.

### 1.4.4 Division of Amplitude Polarimetry (DOAP)

In division of amplitude polarimetry, the energy of the entire wavefront is split in two by a beam splitter that partially polarizes both beams. One of these beams passes through a quarter waveplate to convert the circular polarization to linear, while the other beam continues uninterrupted. Each of these two beams are then split into two again by a polarizing beam splitter, where the intensities of the four beams are measured by the detectors as shown in figure 1.20 [54]. These four intensities are linearly related to the Stokes vector, which can be measured using the matrix method as described earlier.

This method of polarimetry, just like DOWP, has the advantage of obtaining the complete Stokes vector in a single measurement. It has the added advantage of being more efficient where polarizing beam splitters are used instead of dichroic polarizers. It is still limited in speed by its detector and the photon statistics of the detected light. It is also not exclusive to multiple detectors as seen in 1.20 and a single detector can be used if the correct optics can fold and reconjugate each image onto a common plane.
Introduction

Figure 1.20: A Division of Amplitude Polarimeter (DOAP), where PPBS is a partially polarizing beam splitter, PBS a polarizing beam splitter and QWP a Quarter waveplate, the four final intensities are measured by the detectors [30].

1.5 Astronomical Polarimeters

There are many astronomical polarimeters and each are designed for a particular use. These can be for imaging, high-speed, spectro-polarimetry or solar polarimetry. We will look at an example of some of these polarimeters and how they function. Before a detailed look at these polarimeters, some polarimeters worth mentioning: the “Pepsi” polarimeter for the large binocular telescope in Arizona, USA, which is a high resolution spectro-polarimeter [55]; the “photoelastic-modulator polarimeter” at the pine mountain observatory that uses a photoelastic-modulator at frequencies up to 50kHz [56]; “the high precision CCD imaging polarimeter,” a modified CCD on CTIO (Chile) and LNA (Brazil) that uses a rotating half waveplate then an analyser for the polarimetric observation of polars [57]. Other polarimeters include; Aries [58], PICO [59], ISIS [60] and PlanetPol [61]. The DOP accuracy and precision of polarimeters are based on how they are used, imaging polarimeters can archive on average 0.1%, photo-polarimeters, which used avalanche photo diodes, are capable of better than 0.001%[60].

1.5.1 OPTIMA and Optical Polarimetry of the Crab Pulsar

We mentioned earlier that Smith [25] made an archetypical observation of linear polarization shown in figure 1.7. The polarimeter used was the “RGO Peoples Photometer” in polarimeter mode and operated by rotating a superachromatic half waveplate (HWP) in front of a Foster prism (a type polarizing beamsplitter
1.5 Astronomical Polarimeters

see figure 1.13) and the intensities were measured with a photomultiplier tube (PMT). The period of rotation of the HWP was synchronised to multiples of the topocentric period of the pulsar and synced to the sampling rate of the PMT, which yielded 250 samples per period of the pulsar [62]. However, this was obtained by period folding of many pulsar cycles and could only perform linear polarimetry, and circular polarimetry was done separately.

![Figure 1.21: Schematic and polarizer of OPTIMA.](image)

A more recent polarimeter for measuring the polarization of the crab pulsar is a modification of the high-speed photometer, OPTIMA (Optical Pulsar Timing Analyser), built by Kanbach et al. [63]. This utilises 7 hexagonally packed tapered optical fibers at the Cassegrain focus where each fiber corresponds to 2 arcsec field. These are then fed to 7 avalanche photo diodes (APDs) as shown in figure 1.21(a). This arrangement of 7 fiber tips allows for photon counting of the sky / background intensities around the central target at very high rates, on the order of MHz. The fiber tips are mounted in a mirrored wedge to allow for finding via the CCD shown. With the optional polarizer (figure 1.21(b)) in place this high-speed photometer becomes a polarimeter. Similar to the workings of the “RGO Peoples Photometer,” OPTIMA rotates the dichroic polarizer at a set rotational period and the polarization of the pulsar from period folded intensities can be extracted. OPTIMA also has the advantage of sampling the surrounding polarization with the 6 outer fiber tips, which measures the sky / background polarization that can be subtracted from the signal.
1.5.2 IMPOL

Imaging polarimetry is much more difficult and not as polarimetrically sensitive, but one example is IMPOL, an Imaging Polarimeter for multi wavelength (400nm - 800nm) observations [64]. Figure 1.22 shows the general layout of this polarimeter, it contains a rotating half wave plate (HWP) is positioned before a Wollaston prism, the output of which is imaged CCD. To measure the polarization of a source the HWP is rotated to present angles\(^7\) of 0\(^\circ\), 22.5\(^\circ\), 45\(^\circ\) and 67.5\(^\circ\) and an image is recorded. Because a Wollaston is used, two images of the same source are seen on the detector, each of which has an orthogonal polarization state. The observation with the HWP at 0\(^\circ\) is recording the intensity of polarization at 0\(^\circ\) and 90\(^\circ\) and with the HWP at 22.5\(^\circ\) the polarisation intensities at 45\(^\circ\) and 135\(^\circ\) is recorded, the Stokes Q and U parameters are easily calculate from this. Using the HWP set at 45\(^\circ\) and 67.5\(^\circ\) will give the same results, but it can be used to cancel out any systematic instrumentation errors.

Figure 1.22: Layout of the imaging polarimeter IMPOL. Light initially passes over a grid that masks layers of the field; it then passes through a rotating HWP then through Wollaston prism (a polarizing beamsplitter), where the images in finally measured with a CCD.

In IMPOL, the divergence of the Wollaston is only 0.5\(^\circ\) so a mask must be used to stop objects overlapping on the same area of the CCD. Figure 1.23 [65] demonstrates this effect. This means that to acquire a full field the telescope pointing must be shifted by a required amount to obtain the full field. This polarimeter has been reported to obtain accuracies in the order of 0.02% in the measurement of the degree of polarization.

1.5.3 Zimpol

Zimpol, the Zurich Imaging Polarimeter is a high speed imaging polarimeter designed for solar polarimetry [66]. It utilizes an electro-optic modulator, such as a

\(^{7}\)When an HWP is rotated by \(\theta\) the plane of polarization will be rotated by \(2\theta\)
1.5 Astronomical Polarimeters

Figure 1.23: With a low divergence Wollaston prism a mask is needed to prevent any overlapping on the detector.

piezoelectric or ferroelectric modulator (FEM), to alter the retardance in the system. The modulator operates in the kHz region, (42kHz used in practice) and is synced to the CCD. To measure the polarization, the CCD has a mask over every second row of pixels and when the modulator is set to one retardance, an image is taken. The electronics then shifts the image by one pixel row, the retardance is changed and then a second image is taken. If the retardance is 0 and $\lambda/2$ then the Stokes Q value can be measured. Figure 1.24 shows a basic layout of this system.

Figure 1.24: Layout of Zimpol showing that the ferro-electric modulator and CCD are phase locked so that the detector can measure the alternate polarization intensities corresponding to the relative retardation by shifting the CCD image 1 row [67].

Zimpol II made a modification to the mask so that four images could be recoded, each with the FEM set at the relative retardance of have four linearly independent intensities, which means that the total Stokes vector can be recoded [68] in one cycle. This system completes the several cycles of pixel shifts before reading the entire CCD frame to get a high signal. The polarimeter operates on a large part of the spectrum from 0.18 to 1.1$\mu$m. Various progressive versions of the device have been made with accuracies of 0.1% to 0.0001% for solar polarimetry. The most recent version Zimpol III [69] improves the efficiency and speed ($\sim$ MHz) of the system.
1.6 Polarimeter Review

Each of the three previous polarimeters has their advantages, but the main disadvantages are efficiency and speed. Optima can polarimetrically measure a full cycle of the crab pulsar with phase folding, but half of the light is lost using a dichroic polarizer nor does it simultaneously measure circular polarization. Mechanically moving parts can fail and electronics are needed to synchronise them with the recording of the data and absolute time stamps, creating complexity.

Imaging polarimetry, as seen with IMPOL, is good for acquiring the Stokes vectors over a large field as one cannot assume that the surroundings of the target are clear for any polarization signal. IMPOL does not use dichroic polarizers, but it still loses half the light due to the mask to stop overlapping fields. It is not a fast polarimeter nor does it measure the total Stokes vector, as the author assumes the level of circular polarization is low and can be ignored.

Zimpol shows promise at being a fast imaging polarimeter, but it is quiet complex with synchronizing a ferro-electric modulator and the modulator controlling the CCD image shifting. Such modified CCD are difficult to acquire and the mask also lowers the spatial resolution, in Zimpol’s case it is reduced to \(22\mu m \times 90\mu m\). The combination of modulation and mask brings the efficiency of the system down to \(1/6^{th}\) of the CCDs sensitivity. This is not an issue for solar polarimetry, but in astronomical polarimetry, photons are scarce and cannot be wasted. More recent changes to Zimpol have negated the inefficiency by using micro-lenses and Sphere Zimpol uses two detectors to increase the efficiency [70].

In the next chapter, we shall look at the criteria for choosing the best type of polarimeter and detail the design of our chosen system learning from other polarimeters.
Chapter 2

The Polarimeter
This chapter describes the design of Galway Astronomical Stokes Polarimeter (GASP). The basis of the type of polarimeter required to fulfil the different design and tolerance requirements are discussed and under this, a Division Of Amplitude Polarimeter (DOAP) is chosen. A previous type of DOAP polarimeter is chosen and improvements, enhancements and progression from this design are elaborated upon. We then look into the optical arrangement and the outcomes to obtain the output onto a single detector. How the theoretical system matrix is calculated for this particular design of polarimeter is finally presented.

2.1 Requirements for an Ultra-High-Speed Stokes Polarimeter

Before any designs for a polarimeter are laid down, existing limitations and issues inherent to astronomical polarimetry must be acknowledged. The design and tolerance requirements discussed below form the major drivers for GASP, while still considering the optical, financial and instrumental limitation.

2.1.1 Speed

As stated in the introduction, any polarimeter that is to observe a pulsar will have to completely measure the Stokes vector at speeds in order of 0.13ms to obtain 250 sample of a pulsar rotation as done by OPTIMA in section 1.5.1. As shown in section 1.5 we saw that all astronomical polarimeters are dependant on modulating polarization elements and most designs of polarimeters do not measure the full Stokes vector only linear (Q and U) or circular (V) separately, which will further restrict the speed at which full Stokes polarimetry can be performed. If microsecond variations are to be measured an instantaneous polarimeter will be needed with a detector fast enough to record intensities at this rate.

2.1.2 Throughput

Consider the example of the polarimeter IMPOL, (see section 1.5.2), which employs a banded plate at the telescope focal plane to obscure the orthogonal polarization states from overlapping on the detector. This is a waste of 50% of the light as is with other polarimeters that use a dichroic linear polarizer as the analyser. In the case of the Crab pulsar, where the number photons per pulse are limited, a
2.1 Requirements for an Ultra-High-Speed Stokes Polarimeter

desire to record 250 samples per pulse shorten the exposure period and will have even less photons per polarimetric measurement. Therefore, the polarimeter must have no linear dichroic polarizers nor employ the grid technique as in IMPOL. Hence, one must make sure that the optical throughput for the polarimeter is as large as possible, and to try to be as efficient as possible in the design.

2.1.3 Optical Bandwidth

On a par with throughput, spectral bandwidth must also be maximized to be sure to detect as many photons as possible. A reasonable bandwidth to work with is 400nm to 1000nm. This is within transmission range of most optical materials and detection range of most silicon-based optical detectors, both avalanche photo diodes (APDs) and charge-coupled devices (CCDs). An issue arises when polarimetric precision is concerned as the larger the bandwidth the lower the precision. Such precision is affected by the spectral response characteristics of the detectors and the spectral characteristics of optics and polarization components. It is desirable to have these responses equal for all wavelengths but it is difficult to obtain. For example, the retardance of some standard achromatic quarter waveplates over 400nm to 800nm ($\delta \Delta_{400-800\text{nm}}$)\(^1\) will vary from $\sim 70^\circ$ to $100^\circ$ hence affecting the broadband polarimetric accuracy, ideally, we wish it to remain at $90^\circ$ throughout this spectral range. Figure 2.1 shows a Thorlabs achromatic waveplate, an ideal achromatic waveplate and a multi order waveplate with a $\delta \Delta_{400-800\text{nm}} = 26.97^\circ, 0^\circ, \infty^\circ$, respectively. A multi order waveplate is a birefringent plate whose selected thickness gives a quarter wave retardance for a selected wavelength. Section 2.3.4 will cover the development of an achromatic retarder for GASP.

2.1.4 Imaging

Polarimeters that achieve Degree of Polarization (DOP) accuracies of about $10^{-6}$ tend to utilize photon counting detectors such as APDs. This type of detector that would be favourable to use in the GASP, especially since acquisition rates of the order of MHz could be obtained. Unfortunately, these detectors are primarily for point source detection, while acquiring data for only one target at a time. One of the main difficulties that arises in using point source detectors is finding and alignment of the target onto the detectors. Also, in most polarimetric scenes

\(^1\)\(\delta \Delta\) is the change in retardance over the specified wavelengths
the surrounding polarization of the target can be ignored as it is non-existent or below the sensitivity of the instrument. However, where the target is embedded in a diffuse objects such as galaxies and nebulae the surrounding polarization must be accounted for. This can be done by observing the polarization a few arcseconds away from the target, interpolating the additional polarization and subtracting it from the target. For this reason, a point polarimeter would then need extra detectors to measure this, thus increasing the cost and complexity of the instrument. With this in mind, it would be better to just image the target area.

2.1.5 Sensitivity

We have seen in the introduction that the range of astronomical polarization can go from unpolarized (0%) to low polarization (0.01% - 5%) to totally polarized (100%) and thus we wish our polarimeter to be sensitive to the whole range. The accuracy of current imaging polarimeters range from $\sim 0.1\%$ to $0.01\%$ \(^2\), while photon counting polarimeters reach 0.001%, which is more desirable. If we are to construct a imaging polarimeter based on the requirements in section 2.1.4, we would hope to achieve precisions to 0.1% or better. However, the accuracy will be dependent how well we can calibrate the polarimeter and deal with the instrumental and telescope intrinsic polarization. With respect to this, we will set a reasonable accuracy target of $\pm 1\%$. Later in chapter 4 we will discuss the error analysis of the chosen polarimeter, what tolerances we can achieve and the factors influencing them.

\(^2\)with some extreme cases achieving 0.0002% from Zimpol
2.2 Polarimeter Choice: Division of Amplitude Polarimeter

Following the requirements of the polarimeter in section 2.1 it would be wise to design a type of polarimeter similar to a DOAP polarimeter (see section 1.4.4). The DOAP polarimeter is chosen for three main reasons; 1) Its ability to obtain the complete Stokes vector, 2) to do so instantaneously in a single exposure and 3) It is a highly efficiency polarimeter, where no linear polarizers or field blocks are employed. Another benefit of this choice is the lack of any moving parts thus simplifying its design and operation; however, some moving parts are needed for calibration. The development of this polarimeter begins at its prime component, the first beam splitter or herein after referred to as the Retarding Beam Splitter (RBS)\textsuperscript{3} shown as the beam splitter in figure 2.2.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{division_of_amplitude_polarimeter.png}
\caption{A Division of amplitude polarimeter where the beam splitter adds a quarter wave of retardation to the reflected path (r) and leaves the transmitted path (t) relatively untouched. A Wollaston prism then splits each beam into its orthogonal polarizations and the signal is recoded by four detectors D1 to D4.}
\end{figure}

DOAP first appeared in [54], which described how a coated beam splitter [1] could divide an incoming beam into linear and circular polarized light (see figure 2.2). These beams were then each divided by a polarizing beam splitter to leave four beams, and this process identical to that described in section 1.4.4. It becomes obvious that the main component of the polarimeter is the RBS as it converts the circularly polarized portion of light into linear so that it can be examined with a polarizing beam splitter. This beam splitter can be tuned by applying different coatings to obtain the desired retardance for a set angle of incidence and

\textsuperscript{3}Known by this acronym as it splits light in two and adds a quarter wave of retardation to the light on either transmission or reflection.
to maximize the spectral bandwidth of a polarimeter in terms of retardance and transmittance; however, this can be difficult and expensive to achieve. In Krishnan [71], the author only used their polarimeter with two narrowband sources, so the issue of a broadband behaviour of the RBS was not addressed, nor has it been addressed elsewhere. An alternative to a coated beam splitter is to modify a Fresnel rhomb to act as both a beam splitter and a quarter wave plate. Compain and Drevillon [1] achieved this and the task of this thesis is to modify and improve this prism and to use it to construct an imaging polarimeter.

A very useful effect in optics is that when a polarized beam undergoes total internal reflection it becomes retarded as if it were passing thought a wave plate. The amount of retardation is dependent on the angle of total internal reflection and the ratio of the refractive indices at the site of internal reflection. The equations that governs this retardation upon total internal reflection is found in Born and Wolf [31], and cited in equation 2.1.

\[
\Delta = 2 \arctan\left( \frac{\cos \theta_{ir} \sqrt{\sin^2 \theta_{ir} - \frac{na^2}{np^2}}}{\sin^2 \theta_{ir}} \right)
\] (2.1)

where \( \Delta \) is the retardance, \( \theta_{ir} \) is the angle of total internal reflection and \( na \) and \( np \) are the refractive indices of air and the prism, respectively. Utilizing this equation, any piece of glass with a particular geometry can make an effective quarter wave plate. In 1817, Augustin-Jean Fresnel utilized this effect and created the Fresnel rhomb [72]. This is a parallelepiped block of glass allows light in at normal incidence and due to the design of the prism angles, the total internal reflections that occur creates a retardance in the light (see figure 2.3). Two such internal reflections and hence two such retardations of \( \lambda/8 \) (\( \Delta = 45^\circ \)) will make a \( \lambda/4 \) (\( \Delta = 90^\circ \)) retardation, thus making the rhomb effectively a quarter wave plate. A rhomb whose refractive index \( \sim 1.5 \) and angle of the prism of \( \sim 54^\circ \) will make the rhomb a quarter wave plate based upon its proper size, dimensions and geometry.

![Figure 2.3: A Fresnel rhomb where linearly polarized light undergoes two total internal reflections to produce a quarter wave of retardance and thus make the beam circularly polarized.](image)
2.3 The RBS Prism

Compain and Drevillon developed a prism that split the incoming light into two, one portion passed through the prism and has 90° of retardance added while the reflected portion is relatively unaffected (see figure 2.4). This can be seen as a modification of the Fresnel rhomb where the angle of incidence (θ) is not normal and the respective prism angle (χ) has been changed such that a retardance of 90° for two total internal reflections is achieved. As with the regular DOAP, each path was followed with a polarizing beam splitter and exiting intensities were then measured to generate an intensity vector and so the mathematical description in section 1.4.1 can be applied to use this design as a polarimeter.

![Figure 2.4: An RBS used by Compain and Drevillon showing the path light takes through the prism where θ is the angle of incidence and χ is the prism angle. An unwanted back reflection from the bottom of the prism is absorbed by a tapered ground edge on the top right corner of the prism.](image)

2.3.1 RBS Design Process

The design process of the RBS prism has some interlinked factors that make the process iterative, but we first must look at how the system matrix is generated as it later becomes a criterion for the optimization of the prism.

Applying the process to generate the system matrix seen in section 1.4.1 we get the following theoretical matrix (equation 2.2), where ψ_r, Δ_r and Δ_t, ψ_t are the ellipsometric angles for the reflected and transmitted paths respectively and R and T are the reflectance and transmittance of the prism.
\[ \mathbf{A}(\psi, \Delta) = \mathbf{RT}^{1/2} \]

\[
\begin{bmatrix}
1 & -\cos(2\psi_r) & \sin(2\psi_r) \cos(\Delta_r) & \sin(2\psi_r) \sin(\Delta_r) \\
1 & -\cos(2\psi_r) & -\sin(2\psi_r) \cos(\Delta_r) & -\sin(2\psi_r) \sin(\Delta_r) \\
1 & -\cos(2\psi_t) & \sin(2\psi_t) \cos(\Delta_t) & \sin(2\psi_t) \sin(\Delta_t) \\
1 & -\cos(2\psi_t) & -\sin(2\psi_t) \cos(\Delta_t) & -\sin(2\psi_t) \sin(\Delta_t)
\end{bmatrix}
\]

where

\[
\mathbf{RT} = \begin{bmatrix}
R & 0 & 0 & 0 \\
0 & R & 0 & 0 \\
0 & 0 & T & 0 \\
0 & 0 & 0 & T
\end{bmatrix}
\]

\[
\psi_r \text{ and } \Delta_r \text{ can be calculated as given in section 1.3.2, as it is a reflection off a single surface. The } \psi_t \text{ and } \Delta_t \text{ are effectively the product of four surfaces; the first refraction, the two internal reflections and the final refraction. The } \Delta_t \text{ is twice the retardance of a single total internal reflection, the } \Delta_t \text{ for the first and last refractions are ignored as these are zero due to the surface being dielectric. The } \psi_t \text{ is a product of the first and last refractions, which are not always the same coefficients as each other and are calculated using equation } 2.4 \text{ where } t_{p1}, t_{s1} \text{ and } r_{p4}, r_{s4} \text{ are the Fresnel coefficients for the first and last surfaces. The diattenuation } (\psi) \text{ of the internal reflections need not be accounted for in this equation as their ratio is unity.}
\]

\[
\psi_t = \arctan \left( \frac{|t_{p1}|}{|t_{s1}|} \cdot \frac{|r_{p4}|}{|r_{s4}|} \right)
\]

After the system matrix has been generated, the inverse of the condition number of the matrix, a generally acknowledged metric \cite{73,1}, is used for the quality of this matrix. This is calculated by equation 2.5 where \(\lambda_1\) and \(\lambda_4\) are respectively the smallest and largest eigenvalues of \(\mathbf{A}^T \mathbf{A}\). This will consistently be referred to as the ‘inverse condition number.’

\[
s(\mathbf{A}) = \sqrt{\frac{\lambda_1}{\lambda_4}}
\]

Several criteria exist to optimize the system matrix (\(\mathbf{A}\)) when designing the polarimeter. Deviation from any of these will result in the system matrix becoming more singular, hence less invertible and leaving the polarimetry more susceptible to noise.

When calculating the angle of incidence (\(\theta_i\)) to satisfy criterion 2 in table 2.1, it is important account for loss due to the back reflections. Therefore, the transmittance becomes an effective transmittance (\(T_e\)). Another factor for the
### 2.3 The RBS Prism

#### Table 2.1: Parameters for the optimization of the system matrix

<table>
<thead>
<tr>
<th>Number</th>
<th>Criterion</th>
<th>Detail</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$S(A)$ maximum</td>
<td>Inverse condition number of the matrix should be maximized</td>
</tr>
<tr>
<td>2</td>
<td>$R = T, \ R + T$ maximum</td>
<td>Reflection and transmission should equal and be a maximum</td>
</tr>
<tr>
<td>3</td>
<td>$\Delta_r - \Delta_t = 90^\circ$</td>
<td>Difference in retardation between reflection and transmission equal 90°</td>
</tr>
</tbody>
</table>

Compliance to criterion 2 is the amount of absorption in the glass. This might not seem like much for a small prism, but for a prism whose size could lead to a path length on the order of centimetres, it is prudent to account it. The effective transmittance can be calculated by equation 2.6 where $R_1$ and $R_4$ is the reflectivity of the first and forth surface, $\alpha$ is the absorption coefficient of the glass and $L_g$ is the geometric path length inside the prism.

$$T_e = (1 - R_1).(1 - R_4).e^{-\alpha.L_g} \quad (2.6)$$

The process by which the design of the RBS prism is calculated needs four inputs, (i) the refractive index of the glass being used, (ii) its absorption coefficient, (iii) an estimate of geometric path length of the prism so that absorption can be accounted for and (iv) the entrance length ($L$) of the prism to determine its size. Figure 2.5 shows the necessary angles and dimensions for the design of the prism.

![Figure 2.5: Geometrical layout of the RBS prism where L is the entrance length, $\theta_i$ is the angle of incidence, $\theta_r$ is the angle of refraction, $\theta_{ir}$ is the angle of internal reflection, h is the height of the prism, $\chi$ is the prism angle.](image)
Figure 2.6 shows a flowchart by which the prism’s specifications are calculated and its explanation now follows.

1. Enter the initial parameters of:
   
   (a) Refractive index of the chosen glass for the wavelength it will be used  
   (b) Absorption coefficient  
   (c) Entrance length of the prism long enough to accept the footprint of the incoming beam  
   (d) Estimate of the geometric path length; this is based on the size of the prism, which is based on the entrance length.

2. Using the Fresnel coefficients (equation 1.17 to 1.20) and equation 2.6, an iterative procedure is used to find the angle of incidence to satisfy $R - T_e = 0$ (a starting value of $\theta_i = 80^\circ$ is used). The value for $R_4$ in equation 2.6 is calculated based on a parallelepiped, regardless of the prism angle. As it happens, this reflectivity value is the same as $R_1$ due to the geometry.

3. The angle needed to produce $45^\circ$ retardance for a single total internal reflection is calculated using equations 2.7a to 2.7e, which are equivalent to equation 2.1 solved for $\theta_{ir}$. Solving for $\theta_{ir}$ yields eight solutions; while all are valid answers only two angles are useful, these come about by keeping $a$ and $b$ positive and altering the sign of $x$. During this first version, we are using positive $x$ and we will discuss the use of the second angle later.
2.3 The RBS Prism

\[ x = \pm \sqrt{n p^4 - 4 n p^2 y^2 - 2 n p^2 + 1} \quad (2.7a) \]

\[ y = \tan(\Delta/2) \quad (2.7b) \]

\[ a = \frac{\sqrt{(1 + y^2)(1 + n p^2 + x)}}{2(1 + y^2) n p} \quad (2.7c) \]

\[ b = \frac{y (1 + n p^2 + x) / 2}{(1 + y^2) n p \sqrt{\frac{n p^2 - 2 y^2 - 1 + x}{1 + y^2}}} \quad (2.7d) \]

\[ \theta_{ir} = \arctan 2(\pm a, \pm b) \quad (2.7e) \]

4. Following the prism’s geometry, the prism angle (\( \chi \)) is then simply calculated by equation 2.8.

\[ \chi = \theta_{ir} + \theta_r \quad (2.8) \]

5. The geometric path length and dimensions of the prism are then calculated by using equation 2.9 and 2.10.

\[ L_g = \frac{2L \sin(\chi)}{\sin(90 - \theta_{ir})} \quad (2.9) \]

\[ h = \frac{L_g \sin(2\theta_{ir})}{2 \sin(\theta_r)} \quad (2.10) \]

This will make sure that the height of the prism is sufficient to obtain two internal reflections and that the exit point on the prism corresponds to the entrance point in terms of where on the face it was incident.

6. The geometric path length (\( L_g \)) is re-entered into step 2 and the sequence is repeated. After several iterations, the change in \( L_g \) to the previous iteration will be smaller than 1mm and the final RBS dimensions are then known.

7. The final specifications of the prism are then produced and these can be used to make a virtual prism in Zemax and/or sent on to be fabricated in an optical shop.

8. The final step is to generate the Polygon Object (POB) file for Zemax so that it can be modelled and tested optically.

Taking the parameters that Compain and Drevillon used (\( n = 1.812^4 \), \( \alpha = 1.123 \ m^{-1} \), \( L_g = 10 \ cm \)) and applying them to the design process we get the

\[ ^4 \text{Glass: FBS E00-46 from Corning S.A} \]
The Polarimeter

following matrix shown in equation 2.11 where $R=T=0.1812$ for the $RT$ matrix equivalent to that shown in equation 2.3. The values of the matrix we obtain are on the right order of magnitude comparing to their theoretical matrix, but a discrepancy was noticed with a slight difference of 0.02 in some of the matrix values.

$$A = RT$$

$$\begin{bmatrix}
1 & -0.5961 & 0.8029 & 0 \\
1 & -0.5961 & -0.8029 & 0 \\
1 & 0.6079 & -0.006 & 0.7940 \\
1 & 0.6079 & 0.006 & -0.7940
\end{bmatrix}$$ (2.11)

In tracing back as to where the discrepancy might have arose, it was discovered that the theoretical matrix in this paper did not account for absorption and if we neglect it we get agreement between matrices.

Table 2.2 lists all properties and design specifications of the Compain and Drevillon RBS prism and can be used to construct the theoretical system matrix as shown in equation 2.11.

<table>
<thead>
<tr>
<th>Prism Geometry Properties</th>
<th>Glass Polarimetric Properties</th>
<th>Reflection</th>
<th>Transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_i = 78.5^{\circ}$</td>
<td>$n_p = 1.812$</td>
<td>$R = 0.375$</td>
<td>$T = 0.390$</td>
</tr>
<tr>
<td>$\theta_{ir} = 35.2^{\circ}$</td>
<td>$\alpha = 1.123 m^{-1}$</td>
<td>$\Delta_r = 0$</td>
<td>$\Delta_t = \pi/2$</td>
</tr>
<tr>
<td>$\chi = 68^{\circ}$</td>
<td>$\theta_c = 33.5^{\circ}$</td>
<td>$\psi_r = 27.44$</td>
<td>$\psi_t = 64.05$</td>
</tr>
</tbody>
</table>

2.3.2 Refractive index considerations

The Compain and Drevillon RBS had high index glass, $\sim 1.8$, necessary to provide and efficient polarimeter in tandem with a simple geometry as shown in figure 2.4. The choice of index is a consequence of the minimization of the angle of incidence, the higher the refractive index the lower the angle of incidence. To satisfy criterion 2 in table 2.1 above, a prism whose refractive index is 1.5 would need an angle of incidence of $\theta_i \sim 85^{\circ}$; this is quite steep and could be difficult to work with in an optical setup. A refractive index of 1.8 yields an angle of incidence for $R = T$ of $\theta_i \sim 78.5^{\circ}$, which reduces the sensitivity to this angle. The prism was designed for use for 488nm (also stated usable for 633nm) and it was said that ‘for a larger spectrum, a medium value of the refractive index (1.80) should be chosen’ a criterion that is not very clear. It is understood that Compain and Drevillon called this a broadband polarimeter in the sense that it operates in any narrowband over a large spectral range, rather than a large spectral range all at
Knowing the parameters of the Compain and Drevillon RBS ($\chi = 96^\circ \theta_i = 78.5^\circ$) the angle of total internal reflection for a particular $n_\lambda$ and thus the retardance is calculated by using equation 2.1. Figure 2.7 shows a plot of retardance as a function of wavelength of the glass chosen by Compain and Drevillon to make their prism. It shows that a change in retardance, for a single total internal reflection ($\Delta = 45^\circ = \lambda/8$) is $\delta \Delta_{400-800\text{nm}} = 22.88^\circ$. Comparing this to the achromatic quarter waveplate in figure 2.1, $\delta \Delta_{400-800\text{nm}} = 13.485^\circ$, this is in a similar range, hence seen as achromatic.

Using such a large refractive index glass incurs a high cost of a prism as these glasses are rarer and can cost significantly more; Schott\cite{74} quote high index glasses up to 17 times more expensive then BK7. Due to this fact, it would be prudent to investigate the use of lower index glasses.

It should be noted that Compain and Drevillon used FBS E00-46 glass from Corning S.A. ($n=1.812 @ 488\text{nm}$) for their RBS prism, but the data sheet for this glass could not be obtained. A substitute glass was used based upon the refractive indices quoted, $n_{488} = 1.812$, $n_{2000} = 1.76$ and $n_{400} = 1.83$. S-LAH52 and N-LAF36 from the Ohara Corporation and Schott AG respectively were the substitute glasses that matched FBS E00-46 in terms of dispersion and index (see Appendix A for glass specifications).

5Figure 2.1 shows a quarter wave retardance, whereas we are comparing to an eighth wave retardance this is why the range is halved.
2.3.3 Improvements to the RBS prism

Based on the elementary design of the Compain and Drevillon RBS prism as shown in figure 2.4, there is an amount of waste light dumped into the top right hand corner of the RBS and this accounts for $\sim 20\%$ of the light incident on the prism. The idea arose to improve the throughput by having the exiting face angled so that the exiting beam is incident normal to this surface and hence the loss at this face would be reduced to $\sim 4\%$. Taking equation 2.6 into account when redesigning the RBS prism, a Zemax model was created and tested (see figure 2.8). Even though the new design is more efficient, a flaw to its usage was spotted. As the input beam no longer sees a parallelepiped, but a glass wedge, angular dispersion thus occurs. The effective wedge angle of this prism would lead to $1.9^\circ$ of dispersion between 400nm and 800nm, which is not helpful when imaging, along with the fact that the small amount of light, which is back reflected, could create ghost reflections.

Figure 2.8: An attempt at a more efficient RBS with a geometry to give an exit beam normal to that face. The exit beam shows the dispersion created. Back reflected light then returns back to the entrance surface where it is reflected and exits at the lower left hand corner.

Following the geometry of the RBS prism (see figure 2.5), the prism angle $\chi = 68^\circ$ and the angle of incidence $\theta_i = 78.5^\circ$ selected from the Compain and Drevillon RBS yields an angle of total internal reflection $\theta_{ir} = 35.2^\circ$. This can be found in region A on figure 2.9 that shows that a retardance of $45^\circ$ is added to the beam for one internal reflection. Upon inspection of this graph it can be seen that there is another region, B, that would also generate the same retardance and is backed up by the results of the two usable solutions to equation 2.7.
2.3 The RBS Prism

Figure 2.9: Regions A and B show the range of internal reflection angles for the same 45° retardance for various refractive indices. A 45° retardance cannot be archived for \( n < 1.4966 \).

The other angle of total internal reflection that gives a 45° retardance for \( n = 1.812 \) is \( \theta_{ir} = 62.06° \). To achieve this angle, while keeping the same angle of incidence, the prism angle now becomes 94.8°. Replotting the retardance as a function of wavelength for the same glass (see figure 2.10) we get a retardance bandwidth of \( \delta \Delta_{400-800\text{nm}} = 0.76° \) a substantial improvement on the previous value of \( \delta \Delta_{400-800\text{nm}} = 22.88° \) seen for the original prism angle. To understand where this factor comes from, a closer inspection into the geometry of the prism and the choice of glass is needed.

Figure 2.10: Utilising retardance point B of figure 2.9 to make the RBS yields a retardance bandwidth 0.76° over 400-800nm.

Knowing the equations to create the prism angle (equation 2.8) and then plotting these against refractive index yields some interesting results (see figure 2.11). Figure 2.11(b) indicates that around a refractive index of 1.6 the gradient
The Polarimeter

of the curve tends to zero where zero occurs at 1.588. It is obvious to then select a glass of low dispersion around this refractive index to yield an even better retardance bandwidth.

Selecting at random a couple of low dispersion glasses ($V_d \approx 60, n \approx 1.6$)\(^6\), for example Ohara’s S-PHM53 ($n_d = 1.603, V_d = 65.5$)\(^7\) and S-BAL35 ($n_d = 1.596, V_d = 61.2$), an even greater improvement of the retardance bandwidth can be seen. This equates to $\delta\Delta_{400-800nm} = 0.065^\circ$ for S-PHM53 and $\delta\Delta_{400-800nm} = 0.01^\circ$ for S-BAL35, another substantial improvement over the $\delta\Delta_{400-800nm} = 22.88^\circ$ at the start.

To compare how much of an improvement this new prism angle has made to the achromaticity of a quarter waveplate, figure 2.13 plots an ideal quarter waveplate and shows how the Compain and Drevillon original RBS and Thorlabs

\(^6\) $V_d$ is the Abbe number, which describes the dispersion of the glass as a single number.

\(^7\) Subscripts on $n$ and $V$ refers to a Wavelength. Sodium ‘d’ spectral line, $\lambda = 589nm$. 

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2.3 The RBS Prism

retarders compare. The enhancement to the Compain and Drevillon original RBS design is plotted and it is barely distinguishable from the ideal. Plotting the new \( n=1.6 \) glass would be barely visible on this plots showing how achromatic the RBS now is.

![Figure 2.13: Comparison of RBS prisms to ideal and Thorlabs quarter waveplate.](image)

2.3.4 Glass Choice

Schott [74] and the Ohara Corporation [75] are two of the main producers of optical glass and they offer 213 glasses between them. This large selection of glasses must be reduced by some method so that we can give a small list to a optics manufacturer. We already have several criteria for the design of the RBS prism in table 2.1 and following figure 2.11 we can choose a glass of particular refractive index, \( n_d = 1.588 \).

Before reducing the glass list, we must take into account the design of the RBS prism as this affects the inverse condition number and absorption to which are used for a scoring system later on. In the calculation of the RBS prism dimensions and hence the inverse condition number, for reasons we will see later, we pick an entrance length of \( \sim 35 \text{mm} \) (L in figure 2.5). This will yield a geometric path length about 10-20cm, a reason to include the absorption coefficient in the glass selection. The entrance length is large because later on we will be investigating the use of the RBS in imaging polarimetry. With such an extremely low value of \( \delta \Delta \) for a randomly selected glasses, including it as a criterion for the choice of glass is not necessary. To reduce the list of glasses we limit the glass list to \( n_d = 1.588 \pm 0.025 \), which will reduce the list to 34 different glasses. Of these 34 glasses, each parameter is scored relative to the associated order of preference as
in table 2.3.

Table 2.3: Scoring system for glass choice

<table>
<thead>
<tr>
<th>Score</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dispersion</td>
<td>( \text{Score}(V_d) = \frac{V_d}{\max(V_d)} )</td>
</tr>
<tr>
<td>Absorption</td>
<td>( \text{Score}(\alpha_d) = \frac{\alpha_d}{\min(\alpha_d)} )</td>
</tr>
<tr>
<td>Inverse condition number</td>
<td>( \text{Score}(S(A)) = \frac{S(A)}{\max(S(A))} )</td>
</tr>
<tr>
<td>Cutoff wavelength(^8)</td>
<td>( \text{Score}(\lambda_{cutoff}) = \frac{\lambda_{cutoff}}{\min(\lambda_{cutoff})} )</td>
</tr>
<tr>
<td>Relative price</td>
<td>( \varepsilon_{\text{rel}} )</td>
</tr>
<tr>
<td>Total</td>
<td>( \text{Score}(V_d) + \text{Score}(\lambda_{cutoff}) + \text{Score}(S(A)\text{max}) + \text{Score}(\alpha_d) + \varepsilon_{\text{rel}} )</td>
</tr>
</tbody>
</table>

Table 2.4: Glass choice scoring results.

<table>
<thead>
<tr>
<th>Glass</th>
<th>Maker</th>
<th>n(_d)</th>
<th>Relative Price</th>
<th>(V_d) score</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-SK57</td>
<td>Schott</td>
<td>1.5869</td>
<td>2</td>
<td>59.6 1.099</td>
</tr>
<tr>
<td>N-BAK4</td>
<td>Schott</td>
<td>1.5687</td>
<td>1.5</td>
<td>55.98 1.170</td>
</tr>
<tr>
<td>N-SK5</td>
<td>Schott</td>
<td>1.5890</td>
<td>1.5</td>
<td>61.27 1.069</td>
</tr>
<tr>
<td>S-BAL11</td>
<td>Ohara</td>
<td>1.5724</td>
<td>2.5</td>
<td>57.8 1.133</td>
</tr>
<tr>
<td>LF5</td>
<td>Schott</td>
<td>1.5813</td>
<td>2</td>
<td>40.85 1.603</td>
</tr>
<tr>
<td>S-BAL35</td>
<td>Ohara</td>
<td>1.5890</td>
<td>2.5</td>
<td>61.2 1.070</td>
</tr>
<tr>
<td>S-PHM53</td>
<td>Ohara</td>
<td>1.6029</td>
<td>5</td>
<td>65.50 1.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Glass</th>
<th>(\lambda_{cutoff}) score</th>
<th>S(A) score</th>
<th>(\alpha_d) score</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-SK57</td>
<td>344.1 1.039</td>
<td>0.482 1.035</td>
<td>0.10005 1.000</td>
<td>6.173</td>
</tr>
<tr>
<td>N-BAK4</td>
<td>361.6 1.091</td>
<td>0.471 1.059</td>
<td>0.16032 1.602</td>
<td>6.423</td>
</tr>
<tr>
<td>N-SK5</td>
<td>347.3 1.048</td>
<td>0.484 1.031</td>
<td>0.20050 2.004</td>
<td>6.652</td>
</tr>
<tr>
<td>S-BAL11</td>
<td>357 1.077</td>
<td>0.474 1.053</td>
<td>0.10005 1.000</td>
<td>6.763</td>
</tr>
<tr>
<td>LF5</td>
<td>344.7 1.040</td>
<td>0.477 1.046</td>
<td>0.12018 1.201</td>
<td>6.891</td>
</tr>
<tr>
<td>S-BAL35</td>
<td>350 1.056</td>
<td>0.484 1.031</td>
<td>0.15013 1.501</td>
<td>7.158</td>
</tr>
<tr>
<td>S-PHM53</td>
<td>375.0 1.132</td>
<td>0.492 1.014</td>
<td>0.25033 2.502</td>
<td>10.648</td>
</tr>
</tbody>
</table>

The resulting table (table 2.4) shows us the scores for the glass choice and are listed in order of preference. The first 6 show the top 6 of the 34 glasses and the 7\(^{th}\) is included as it was a random glass picked earlier on which came 31\(^{st}\). It is interesting to see that S-BAL35 and S-PHM53 score so differently with the criteria of a low dispersion \(n \sim 1.6\) glass led to their initial random choice. It can be interpreted that S-PHM53 scored poorly due to its high cost. Of the top

\(^8\lambda_{cutoff} = 90\%\) transmittance at 10mm thickness
five glasses that were sent to a manufacturer, N-SK5 was obtained based upon availability and the RBS prism was made from this glass.

2.4 RBS prism for imaging use

As stated in section 2.1.4 this polarimeter needs to be an imaging polarimeter and this raises the question how to use an RBS prism in an optical imaging setup. When using a standard mounted round quarter wave plate, such a piece can be easily be implemented into an optical imaging setup. However, when a non rotationally symmetric prism is used and the angle of incidence is quite shallow, \( \sim 79^\circ \), we need to investigate how the RBS will interact with imaging optics.

If we are to pass a laser or a pencil beam of light through the prism we soon see that deviating from the optimized angle of incidence we get a different system matrix. Figure 2.14 shows how the inverse condition number of the matrix decreases either side of the optimized angle of incidence. This can be explained by the fact that criterion 2 in table 2.1 (T=R) is no longer fulfilled, as seen from the transmittance and reflectance lines. Criterion 3 in table 2.1 (\( \Delta = 90^\circ \)) is also deviated from as for a non optimized angle of incidence the total retardance is no longer 90\(^\circ\).

![Figure 2.14: Inverse condition number, transmittance and reflectance vs. angle of incidence.](image_url)

Later in section 5.5 we will find that an inverse condition number of 0.3 is a safe limit for the polarimeter to work within our specified polarimetric tolerances. This will give us a range of \( 83.2^\circ - 74.2^\circ = 9^\circ \) on the angle of incidence, which is then the angular field of view for the RBS prism. Since the optimized angle is
79°, we shorten the angular range on the lower end to 75° for reasons of symmetry thus giving us an 8° angular field of view for the RBS.

If we are to pass an imaging beam though the prism we realize that from figure 2.14 the beam cannot be diverging or converging. A converging or focusing beam will see different retardances in the cross section of the beam while propagating through the RBS prism, hence, any polarimetry performed in this case will yield an incorrect result. From this, we know that a collimated beam must pass through the RBS prism. This is true for most polarimetric optics, depending on their field of view, as it is a general rule that such optics optimally perform in collimated space.

### 2.4.1 Determining RBS prism size

Knowing that collimated light must pass through the RBS prism we can reduce the size of the prism by conjugating the pupil of the optical system to some point in the prism or on the prism. Determination of the pupil size comes from its ultimate use; as this is an astronomical polarimeter we need to know what is the best telescope that it can be used on and the field of view required.

The largest telescope in the world that will take visiting instruments is Keck in Hawaii, a 10m primary with an effective F-ratio of F/14.96. However, it is not likely that this telescope may be used during this thesis, but it leaves open the fact that the RBS prism can, at some later stage, be used in a redesigned polarimeter. Our target of interest is the Crab pulsar and it has an apparent companion star (V mag of 15.79 [76]), 5.09″ away. This can be used as a reference/check star for differential photometry and this will set the size the field should be, say ∼ 10″. Taking into account an average maximum seeing of ∼ 1″ at Keck, we should make the field of view (FOV) ∼ 12″ large. This will enable us to position the object of interest, the pulsar, in the centre of the field and then have enough room around this to encompass the companion star.

The RBS prism can accept a field of view of 8° so a magnification of 8°/12″ = 2400 is used for our sky FOV requirements. The focal length of a collimator necessary to achieve this on Keck would be 149600mm/2400 = 62.33mm and this will conjugate the pupil to a diameter of 10000mm/2400 = 4.167mm. If the RBS prism is optimized at an angle of incidence of 79° then the footprint of the collimated beam at this angle would be 4.167/\cos(79°) = 21.8mm. However, since the extent of the field is 4° either side of this the footprint should be 4.167/\cos(83°) = 34.19mm. Add ∼ 1.5mm to each side to give some play for
optical alignment and the final entrance length (L) of the RBS prism should be 37mm. Using this value of L and the N-SK5 glass, we obtain the properties of the prism as seen in table 2.5 when applying the design process as in section 2.3.1

Table 2.5: Specifications of the RBS prism.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Magnitude</th>
<th>Tolerance</th>
<th>Dimension</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>37.00mm</td>
<td>±0.25mm</td>
<td>θ_r</td>
<td>37.7932°</td>
</tr>
<tr>
<td>Thickness</td>
<td>51.00mm</td>
<td>±0.25mm</td>
<td>θ_ir</td>
<td>58.7617°</td>
</tr>
<tr>
<td>h</td>
<td>120mm</td>
<td>±0.25mm</td>
<td>L_g</td>
<td>140.4mm</td>
</tr>
<tr>
<td>Flatness</td>
<td>λ/8 at 400nm</td>
<td>α_HeNe 0.268906m⁻¹</td>
<td></td>
<td></td>
</tr>
<tr>
<td>χ</td>
<td>96.6238°</td>
<td>±4°(min ± 20″)</td>
<td>S(A)_{max}</td>
<td>0.5774</td>
</tr>
<tr>
<td>θ_i</td>
<td>78.8574°</td>
<td>±2′</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 2.4.2 RBS Prism Limitations

As with most instruments, limitations exist to its use, mostly to the size of their field of view. Restrictions placed by the RBS prism on the allowed F-number of the telescopes hinder the use of large fields as we will explain. In the determination of the prism entrance length, we said that a cross section of the collimated beam at 83° is 34.19mm. To make sure no vignetting occurs the position of the entrance surface of the RBS prism must lie along the telescope’s optical axis so that the conjugated pupil falls on the centre of this surface, as shown in figure 2.15. This means that the space along the optical axis is taken up by 18.5mm of the prism and restricts the focal length of the collimating lens (F_{limit}).

![Figure 2.15: Limiting factor for telescope F-number, obstruction by the bulk of the RBS prism. The RBS prism is positioned so that the conjugated entrance pupil falls on the centre of the entrance face to the prism.](image)

With most achromatic lenses, their back focal length can be up to 7.5mm less than the effective focal length (for a 25mm diameter lens). This then sets F_{limit} = 26mm for the collimating lens whereby the gap between the back of the lens and
The Polarimeter

the RBS prism is zero. Knowing that each time a 4.167mm exit pupil must be produced this means that an F-number no less than 26mm/4.167mm = 6.24 can be used. Extra space after the collimator will be needed for filters and some calibration optics and this should be between 5mm and 10mm, so this increases the limit to (26+7.5)mm/4.167mm=8.04. The following equations 2.12 to 2.14 are applied to the calculation of the collimator focal length of different telescopes seen in table 2.6.

\[
F_{\text{coll}} = \frac{F_t}{M}
\]

\[
M = \frac{D}{d}
\]

\[
\therefore F_{\text{coll}} = \frac{F_t \cdot d}{D}
\]

\[
d = \frac{F_{\text{coll}}}{F\#}
\]

\[
FOV(\arcsec) = \frac{F_{\text{coll}}}{F_t \cdot 8.3600}
\]

where \(F_{\text{coll}}\) and \(F_t\), are the respective focal lengths of the collimator and telescope, \(M\) is the magnification, \(D\) is the diameter of telescopes mirror, \(d\) is the conjugated pupil diameter, \(F\#\) is the telescopes F-number and \(FOV\) is the on sky field of view.

Table 2.6: Limits of Field of views for various telescopes when using the RBS.

<table>
<thead>
<tr>
<th>Telescope</th>
<th>Location</th>
<th>Primary Diameter (mm)</th>
<th>(F#)</th>
<th>(F_{\text{telescope}}) (mm)</th>
<th>(F_{\text{collimator}}) (mm)</th>
<th>FOV(\arcsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keck</td>
<td>Hawaii</td>
<td>10000</td>
<td>14.96</td>
<td>149600</td>
<td>62.33</td>
<td>12.00</td>
</tr>
<tr>
<td>VLT</td>
<td>Chile</td>
<td>8200</td>
<td>23.41</td>
<td>191960</td>
<td>97.53</td>
<td>14.63</td>
</tr>
<tr>
<td>WHT</td>
<td>La Palma</td>
<td>4200</td>
<td>10.94</td>
<td>45948</td>
<td>45.58</td>
<td>28.57</td>
</tr>
<tr>
<td>Loiano</td>
<td>Italy</td>
<td>1520</td>
<td>8</td>
<td>12160</td>
<td>33.33</td>
<td>78.94</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Telescope</th>
<th>Available lenses (mm)</th>
<th>Collimated beam diameter (mm)</th>
<th>Actual FOV(\arcsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keck</td>
<td>60</td>
<td>4.011</td>
<td>11.55</td>
</tr>
<tr>
<td>VLT</td>
<td>100</td>
<td>4.272</td>
<td>15.00</td>
</tr>
<tr>
<td>WHT</td>
<td>45</td>
<td>4.113</td>
<td>28.21</td>
</tr>
<tr>
<td>Loiano</td>
<td>35</td>
<td>4.375</td>
<td>82.89</td>
</tr>
</tbody>
</table>

It is noticed that for some telescopes the closest ‘off the shelf’ focal length lens may not match the required lens and thus generates a different sized exit pupil. However, with the extra length included for the RBS prism, a maximum allowable pupil size is \(\cos(83^\circ)(37\text{mm}) = 4.509\text{mm}\), but the placement of the RBS prism becomes sensitive to vignetting at the edges. Any telescope that has an F-number
lower that 8 can still be used; it just simply needs a focal extender to make it usable for the RBS prism.

2.4.3 Extractor Prism

As seen with the Compain and Drevillon RBS prism, an amount of light continues to internally reflect and needs to be dumped hence the tapered corner in figure 2.4 to deal with this. Altering the geometry of the RBS prism to reduce this light loss (see figure 2.8) proved unfruitful and the designed stayed as a parallelepiped. It would be possible to grind and blacken the lower half of the back of the RBS prism to absorb this reflection, but a better option would be to extract it and utilize this “wastage.” As stated in section 2.1.4 the difficulty in finding and placing a star on an APD forced the requirement of the polarimeter to be an imaging polarimeter, but if an extra imaging channel could be made available this might be a possibility later on. The wastage component due a back reflection could then be extracted and utilized for tracking or guiding and accounts for 16% of the input light.

This back reflection off the exit face of the RBS prism amounts to 20% of the incident light and to utilise this portion of light an extractor prism, made of the same glass as the RBS prism, is attached to the RBS. The final waste is absorbed by a blackened ground face C as seen in figure 2.16 and this amounts to approx 4% of the initial input. Thus, we get 16% of the initial input extracted for use with tracking, guiding or any necessary use. The design of the extractor prism, shown in figure 2.16, is as follows.

![Figure 2.16: Prism extractor dimension details.](image)

The lengths A, B, C and angle a, b, c are determined by equations 2.15 to 2.20 as depicted in figure 2.16.
\[ a = \chi - 90 + \theta_r \quad (2.15) \quad A = B \sin(a)/(b) \quad (2.18) \]
\[ b = 90 - \theta_r \quad (2.16) \quad B = L \sin(b)/\sin(a) \quad (2.19) \]
\[ c = 180 - \chi \quad (2.17) \quad C = B \sin(c)/\sin(b) \quad (2.20) \]

Face B of the extractor prism is polished to a flatness such that it wrings itself to the RBS prism and face C is ground and blackened to terminate the beam. It is also important to note that the extractor prism is designed such that the angle of refraction of the exiting beam is the same for the exiting beam in the RBS prism so no angular dispersion occurs. The Polygon object file (POB) of the extractor prism for Zemax is generated in the same manner as the POB of the RBS prism and can be used in tandem in the Non-Sequential Component Editor.

2.5 Imaging Polarimeter Development

The speed requirements in section 2.1.1 demand that we observe transient objects, such as the Crab pulsar with acquisition times of \(<0.13\)ms. This leads to needing a detector capable of delivering frame rates \(>7500\) frames per second (FPS). Any high speed detector that would eventually be used would be expensive. This was one of the main constraints of this project, as it would be extremely costly having a separate detectors for each of the four paths for a DOAP. Let alone the cost, other factors such as, calibration, gain stability, timing, data handling between each detector creates problems of its own. With this in mind, a single detector design has to be chosen and doing so is not a trivial task. This involves imaging all four beams onto one detector with folding mirrors and intermediate optics. The detector being used in our case is an *Andor\textsuperscript{TM} iXon EMCCD* with 512 x 512 pixels, 8.2\,mm square chip, 16\,\mu m square pixels, full chip frame rates of 30 FPS and the option to choose sub frames and bin the pixels to obtain frame rates about 550 FPS. Faster detectors would be suitable such as the Berkeley Visible Image Tube [77] [78], a photon counting imaging tube based on micro-channel plate array technology with temporal resolutions \(<1\mu s\). This detector was expected to be available for the project, but due to reasons beyond our control, the technology could not be acquired in the time frame of the project so the we made do with the iXon EMCCD.

The following section will focus on the different polarimeter designs for the Galway Astronomical Stokes Polarimeter (GASP) that arose from creating an imaging DOAP polarimeter using the RBS prism. The pros and cons of each
2.5 Imaging Polarimeter Development

design will be stated with the final design merits the greater attention.

2.5.1 Early Optical designs

We stated earlier that polarimetric optics work best in collimated space, however, most of these, like quarter waveplates and polarizers, are transmissive optics and are easy to work with in imaging situations. Mirrors tend to depolarize or change the state of polarization of beams reflected off them at non-normal incidence, which is especially true for high-reflectivity (HR) coated mirror. For this reason, it is recommended not to use mirrors before polarizing beam splitters in the DOAP design as this would deteriorate the system matrix. HR mirrors or plane mirrors can be used after the polarizing beam splitters as they will not alter the state of polarizing being measured only altering the intensity of the channel and this can be resolved in the calibration.

The polarizing beam splitters of choice are Wollaston prisms. These are rectangular cuboid prisms that have a large acceptance angle (>17°), a high extinction ration (∼10⁻⁶), a usable spectral range of 300-2200nm and output their orthogonally polarized beams at various divergent angles based upon the design of the prism. These Wollaston prism come off the shelf with large apertures (25mm x 25mm) and divergent angles of 5°,10° and 20°. A precaution must be noted that divergence of a Wollaston is quoted for a particular wavelength. Dispersion causes the actual divergence for a specified wavelength to change. This change from 400nm-800nm is 0.56° for a 5° Wollaston and 1.11° for a 10° Wollaston. The advantage of using mirrors after the Wollastons is that each channel could be finely steered by the mirror mount. Figure 2.17 shows one of the first designs in Zemax that uses Wollaston prisms and the beam steering technique. However, the images would have a tiny field of view as the reflected path (R-Path) traverses a large distance and when it reached the imaging lens a lot of the field would be vignette. Mechanically placing the R-path mirrors are simple enough as there is enough space, but placing mirrors for the transmitted path (T-path) is difficult as the mounts for these mirrors might get in the way and obstruct the R-path.

To solve the small field of view that would occur in the R-path, the image would need to be transported and re-conjugated to the detector. As for the mirrors after the T-path, small 45° rod-mirrors could be mounted on goniometer stages. With the difficulty of simulating the design in Zemax the output was only tested in reality, Figure 2.18 shows how this was achieved.
Figure 2.17: A Zemax model of GASP: an initial design for a imaging DOAP polarimeter using the RBS prism showing the Wollastons immediately after the beams have interacted with the RBS and then are folded down to a single detector.

Figure 2.18: Picture of the re-conjugation polarimeter design.
This design failed to be of any value as the image quality was not good enough for polarimetry, as seen in figure 2.19. This image shows that the point source was conjugated about rod mirrors and the rod mirrors themselves showed up in the output images adding to the background. The spot quality was not a symmetric Gaussian making it unmanageable to extract the intensities from them. It was difficult to align with the large amount of lenses and mirrors and unstable while using it on the telescope. Greater care needed to be involved in the design, reducing vignetting, keeping the beams on axis to the optical elements for as long a possible, which would improve the image quality.

Figure 2.19: Images of the output of this attempted design showing the poor quality of imaging.

2.5.2 System Matrix With Mirrors Before Polarizing Beam-splitters

One of the major causes of poor image quality was passing two separate beams down a single optical system. This can be seen in figure 2.18 where the R-path fold mirrors direct the two beams through single conjugating optics system where the beams are decentered from the optical axis in order to position them about the rod mirrors. To resolve this, the Wollaston would need to be immediately before the final imaging lens. This could be achieved if two detectors were used immediately after the interaction with the RBS, one for R-path one for T-path, but this was not possible, as only one detector was available. We then investigated
The Polarimeter

if the polarimeter would still work if each path had a single mirror before the Wollastons.

In section 1.4.1, the mathematics needed to derive the theoretical system matrix to perform polarimetry was described. To derive this system matrix with mirrors in it, we carry out the same procedure as for the standard system matrix except we include a general matrix, equation 1.12, for the mirrors in the proper place. Carrying out the product we get the following matrix (see equation 2.21).

\[
RT^\frac{1}{2} = \begin{bmatrix}
1 + \cos(2\psi_{m1}) \cos(2\psi_r) & -(\cos(2\psi_r) + \cos(2\psi_{m1})) \\
1 + \cos(2\psi_{m1}) \cos(2\psi_r) & -(\cos(2\psi_r) + \cos(2\psi_{m1})) \\
1 + \cos(2\psi_{m2}) \cos(2\psi_t) & -(\cos(2\psi_t) + \cos(2\psi_{m2})) \\
1 + \cos(2\psi_{m2}) \cos(2\psi_t) & -(\cos(2\psi_t) + \cos(2\psi_{m2})) \\
\cdots \\
\sin(2\psi_{m1}) \sin(2\psi_r) (\cos(\Delta_{m1}) \cos(\Delta_r) - \sin(\Delta_{m1}) \sin(\Delta_r)) & \cdots \\
-\sin(2\psi_{m1}) \sin(2\psi_r) (\cos(\Delta_{m1}) \cos(\Delta_r) - \sin(\Delta_{m1}) \sin(\Delta_r)) & \cdots \\
\sin(2\psi_{m2}) \sin(2\psi_t) (\cos(\Delta_{m2}) \cos(\Delta_t) - \sin(\Delta_{m2}) \sin(\Delta_t)) & \cdots \\
-\sin(2\psi_{m2}) \sin(2\psi_t) (\cos(\Delta_{m2}) \cos(\Delta_t) - \sin(\Delta_{m2}) \sin(\Delta_t)) & \cdots \\
\sin(2\psi_{m1}) \sin(2\psi_r) (\cos(\Delta_{m1}) \sin(\Delta_r) + \sin(\Delta_{m1}) \cos(\Delta_r)) & \cdots \\
-\sin(2\psi_{m1}) \sin(2\psi_r) (\cos(\Delta_{m1}) \sin(\Delta_r) + \sin(\Delta_{m1}) \cos(\Delta_r)) & \cdots \\
\sin(2\psi_{m2}) \sin(2\psi_t) (\cos(\Delta_{m2}) \sin(\Delta_t) + \sin(\Delta_{m2}) \cos(\Delta_t)) & \cdots \\
-\sin(2\psi_{m2}) \sin(2\psi_t) (\cos(\Delta_{m2}) \sin(\Delta_t) + \sin(\Delta_{m2}) \cos(\Delta_t)) & \cdots \\
\end{bmatrix}
\]

(2.21)

where \(\psi_{m1}, \Delta_{m1}, \psi_{m2}\) and \(\Delta_{m2}\) are the ellipsometric angles for the first and second mirror respectively, \(\psi_t, \Delta_t, \psi_r\) and \(\Delta_r\) and are the ellipsometric angles for the T-path and R-path respectively, \(RT\) is equivalent to equation 2.3. The first mirror is considered to be the mirror after the Wollaston in the R-path and the second in the T-path. Note: Equation 2.21 is an oversized \(4 \times 4\) matrix and is displayed in the following format.

\[
\begin{bmatrix}
\text{Column 1} & \text{Column 2} & \cdots \\
\cdots & \text{Column 3} & \cdots \\
\cdots & \text{Column 4} & \cdots \\
\end{bmatrix}
\]

For a given angle of incidence on the mirrors, the ellipsometric angles are calculated the same way as in section 1.3.2 except that in the calculation of the Fresnel coefficients, the refractive index is complex as it is for metallic surfaces.

With this matrix now at our disposal we tested the theoretical matrix to see if its inverse condition number would go below 0.3. The matrix was set up
with the RBS prism at the optimal angle of incidence and aluminium mirrors \(\bar{n}_d = 1.39 + i7.65\)^9 for various angles of incidence. Foreseeing that the angles of the mirrors would range from 45° to 75° in the final design of the polarimeter, an inverse condition number map was created as shown in figure 2.20.

<table>
<thead>
<tr>
<th>T-Path Mirror</th>
<th>R-Path Mirror</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
<td>0.4857, 0.4892, 0.4934, 0.4984, 0.4760, 0.4269, 0.3459</td>
</tr>
<tr>
<td>50°</td>
<td>0.4834, 0.4869, 0.4911, 0.4960, 0.4914, 0.4445, 0.3651</td>
</tr>
<tr>
<td>55°</td>
<td>0.4805, 0.4840, 0.4881, 0.4930, 0.4989, 0.4655, 0.3886</td>
</tr>
<tr>
<td>60°</td>
<td>0.4767, 0.4801, 0.4841, 0.4890, 0.4949, 0.4904, 0.4167</td>
</tr>
<tr>
<td>65°</td>
<td>0.4714, 0.4747, 0.4787, 0.4834, 0.4893, 0.4966, 0.4540</td>
</tr>
<tr>
<td>70°</td>
<td>0.4592, 0.4669, 0.4707, 0.4754, 0.4811, 0.4882, 0.4972</td>
</tr>
<tr>
<td>75°</td>
<td>0.3804, 0.3989, 0.4211, 0.4480, 0.4688, 0.4758, 0.4845</td>
</tr>
</tbody>
</table>

Figure 2.20: Inverse condition numbers for different angles of incidence upon two mirrors in a “mirror before Wollaston polarimeter.” Green areas are high inverse condition numbers, yellow areas are low.

This map shows that at no stage does the inverse condition number go below 0.3 where the maximum of 0.4989 occurs when mirror 1 = 65° and mirror 2 = 55°. The range of angles that can be used is large once we stay in the green areas while trying to avoid the red areas. An explanation of how these areas have a lower inverse condition number comes from the fact that the induced retardance from the reflection off the mirrors is greater in one path compared to the other thus off setting criterion 2 in table 2.1, \(\Delta_r - \Delta_t = 90°\) and this also explains why when the angle of incidence on the mirror are similar, the inverse condition number remains large. From this mathematical test we can see that theoretically, a polarimeter with the mirrors before the Wollastons should be a valid polarimeter.

### 2.5.3 Pupil Vigneting Considerations

As we found out in section 2.5.1, any optical design proposed needs to transport the two separate beams (T-Path and R-path) otherwise the field of view would be tiny. Transport of the R-path beam is not an issue, but when looking at how the T-path beam exits the RBS prism, an anomaly was spotted that would create a problem for the transport of the beam. Astigmatism is a commonly known aberration whereby a beam will create two foci, one for a tangential focus and the other for a sagittal focus [79], while astigmatism for a pupil is not commonly

---

^9\(\bar{n} = n + ik\) is the complex refractive index, the complex component, \(k\), is normally referred to as the extinction coefficient.
The Polarimeter

discussed due to the fact that the final image is usually the main concern. When the telescope pupil is conjugated to the top of the RBS prism, it was assumed that it would propagate through the RBS prism as in a normal optical system. Looking at figure 2.21 it can be seen that in the top view, figure 2.21(a), the footprint of the beam at the exit of the prism is larger than when viewing from the side, figure 2.21(b). The imaging lens can be seen to vignette the field in the top case, but not the bottom.

![Diagram of the polarimeter system](image)

(a) Top view

![Diagram of the polarimeter system](image)

(b) Side view

Figure 2.21: Zemax drawing of astigmatic pupils for both orthogonal views.

In figure 2.22, the astigmatic effect of the pupil can be seen when reconjugating the pupil after it has passed through the RBS prism. The pupil comes to a focus at differing locations for two orthogonal views. Calculation of the entrance pupil positions from the conjugated pupils puts them at -89.87mm and -8.95mm from the exit point of the prism for the top and side views, respectively. This gives a pupil separation of 80.91mm (see figure 2.22)

The geometry of the RBS prism is the cause of this issue because it is a non-rotationally symmetric system. Two solutions exist to solve this; firstly, the field of view in the x-direction of the polarimeter can be reduced until there is no vignetting, but this will yield an asymmetric field of view for the transmitted path.
2.5 Imaging Polarimeter Development

Alternately, a solution is to utilise magnification in the recollimating arms that will transport the beams. Longitudinal magnification (in the z-direction) is the square of transverse magnification [79] and this can be used to our advantage. Reconjugating the pupil and magnifying it at the same time will alter the separation of the orthogonal pupil locations as well as altering the pupil’s diameter. The choice of lenses and magnification to be used will be decided later in the final optical design.

2.5.4 Optical Design Layout

The final optical design is based upon optomechanical restriction and linked to the size of the prism. Figure 2.23 gives a brief schematic of how the design might look and equations 2.22 to 2.30 detail how these angles and distances are calculated. The solutions are based on the fact that after both beams are reflected, they must be parallel to each other before entering the Wollaston prism and then the final imaging lens.

\[
\theta_{Tmir} = \theta_i + \theta_{Rmir} \quad (2.22)
\]

\[
\theta_m = 2\theta_i - 2\theta_{Rmir} \quad (2.23)
\]

\[
R_x = \frac{h}{\sin(\theta_b)} \sin(\theta_{Rx}) \quad (2.24)
\]

\[
\theta_{Rx} = \chi + \theta_i - 90 \quad (2.25)
\]

\[
\theta_b = 180 - 2\theta_i \quad (2.26)
\]

\[
L_{R2} = \frac{R_x + L_{R1}}{\sin\theta_m} \sin(\theta_b) \quad (2.27)
\]

\[
T_x = \frac{h}{\sin(\theta_b)} \sin(\theta_{Tx}) \quad (2.28)
\]

\[
\theta_{Tx} = 180 - \theta_{Rx} - \theta_b \quad (2.29)
\]
\[ L_T = \frac{L_{R1} + R_x}{\sin(\theta_m)} \sin(\theta_{R\text{mir}}) - T_x \] (2.30)

Ideally, the angles could be chosen according to the inverse condition number mirror map in figure 2.20 to maximize the inverse condition number of the polarimeter. However, as both beams must be reflected so that they are parallel to each other this means that \(\theta_{R\text{mir}}\) is dependent on \(\theta_{T\text{mir}}\) and are offset according to equation 2.22. This is not a problem for the polarimeter once we do not end up with the mirror angle combination that gives the minimum inverse condition number on this map.

The design continues with the optomechanical limitation that \(L_{R1}\) in the R-path should be as small as possible and this ends up being \(\approx 120\text{mm}\), so that a mirror can fit at this location. We also apply the prerequisite that \(L_{R2}\) should equal \(L_T\) so that the re-collimating optics in each arm will have the same space and that they can be identical to each other. The recollimating optics serve two purposes, one, to transport the beam and two, to get around the pupil astigmatism consideration as in section 2.5.3. Solving numerically for the mirror angles necessary to give us \(L_{R2} \approx L_T\), we get \(\theta_{R\text{mir}} \approx 55^\circ\) and \(\theta_{T\text{mir}} \approx 65^\circ\) and from this \(L_{R2} \approx L_T \approx 220\text{mm}\). According to the inverse condition number mirror map in figure 2.20 this should give us an inverse condition number of \(\approx 0.48\), it may not be at the ideal position on the mirror map, but it is far from the minima. The properties of the theoretical matrix is shown in table 2.7 with the actual matrix in equation 4.1.
Table 2.7: Parameters of the single camera GASP (@ 589 nm)

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Glass Properties</th>
<th>Polarimetric Properties</th>
<th>Transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_i$ = 78.81</td>
<td>$n_p$ = 1.589</td>
<td>$R = 0.336$</td>
<td>$T = 0.336$</td>
</tr>
<tr>
<td>$\theta_r$ = 58.05</td>
<td>$\alpha = 0.219,m^{-1}$</td>
<td>$\Delta_r = 0$</td>
<td>$\Delta_t = 89.85$</td>
</tr>
<tr>
<td>$\chi = 96.624^\circ$</td>
<td>$L_g = 140.4,mm$</td>
<td>$\Delta m_1 = 196.78$</td>
<td>$\Delta m_2 = 207.53$</td>
</tr>
<tr>
<td>Mirror angles</td>
<td>$\psi_r = 30.85$</td>
<td>$\psi_{m1} = 43.52$</td>
<td>$\psi_{m2} = 42.63$</td>
</tr>
<tr>
<td>$\theta_R,mir = 55^\circ$</td>
<td>$\theta_T,mir = 65^\circ$</td>
<td>$\psi_{m1} = 43.52$</td>
<td>$\psi_{m2} = 42.63$</td>
</tr>
</tbody>
</table>

\[ A = RT \begin{bmatrix} 1 & -0.5132 & -0.8217 & -0.2477 \\ 1 & -0.5132 & 0.8217 & 0.2477 \\ 1 & 0.4388 & 0.4134 & -0.7982 \\ 1 & 0.4388 & -0.4134 & 0.7982 \end{bmatrix} \]  

(2.31)

where $R=T=0.1678$ for the $RT$ matrix equivalent to that shown in equation 2.3.

To allow for the re-collimator to fit, we give ourselves $50\,mm$ of space each side for mounts, which leads to $120\,mm$ separation for the lenses. In trying to solve the pupil astigmatism problem detailed in section 2.5.3 we arbitrarily choose a transverse magnification of 0.5, which will give us a longitudinal magnification of 0.25 thus changing the $80.91\,mm$ orthogonal pupil separation to $20.23\,mm$. To fit a re-collimator with a magnification of 0.5 into $120\,mm$, we need lenses with a focal length of $f=80\,mm$ and $f=40\,mm$. The $25\,mm$ diameter lenses that were available were $f=85\,mm$ and $f=50\,mm$ giving us a magnification of 0.588. Working with the parameters of distances, angles and optics we see the Zemax realization in figure 2.24 and 2.25.

Figure 2.24: 3D rendering of GASP in Zemax.
Figure 2.25: Final optical layout of GASP.

This gave the design a starting point so that the solution to the pupil astigmatism could be found. The recollimator in the T-path would need to be placed immediately after the beam exits the prism else the beam would vignette. The result of the solution is shown in figure 2.26 for a maximum field on an f/8 telescope.

Figure 2.26: Solution to pupil astigmatism issue.
We can see that from the top view, the field is near the limit of the optic when entering the recollimator. The final pupil conjugations from top and side views show they are closely spaced and at a position where they can be reflected through the Wollaston and on to the detector.

Having the same recollimating arm on the R-path kept the field sizes the same, but it also allowed us to carry out some optical “acrobatics.” The R-path beam would need to slip by the T-path mirror. The position of the recollimator was placed such that the exiting pupil was conjugated to the location of the T-path mirror in a way so it would not be clipped by the mirror as shown in figure 2.27. Some of the design distances and angles along the R-path would need to be adjusted to allow for this to happen, but it does not change the angles any more than 2-3° leaving the inverse condition number of the system matrix almost the same.

Figure 2.27: Magnified view of pupil conjugations to allow both beams to pass.

There are some advantages and disadvantages to this design. The first disadvantage being that the final separation of the images on the detectors are restricted by our choice of Wollaston divergence angle, which is detailed next section. However, each path is independent of each other and positioning of one with respect to the other on the detector can be controlled by changing the angle of the T-path.
mirror. An advantage with this setup is the ability to access the focal plane. Trying to access the telescope focal plane proved difficult, but with the recollimators in place we have access to intermediate focal planes where, we can place field stops (shown in figure 2.26). Field stops are necessary as allowing large fields in will results in overlapping, again described in the next section. And finally, with no field stop at the telescope focus the extractor-path is left open to see a large field where in this design there is room to place in optics to image it (see figure 2.28).

Figure 2.28: Physical realization of GASP showing the R-path, T-path and extractor arm. The RBS is clamped down and not visible and the camera is just out of view in the lower right corner.

2.5.5 Wollaston Imaging Optics

In the original design, where the folding mirrors were after the Wollaston prisms, we had control of the separation of the beams. Now that the folding mirrors are before the Wollaston and the fields are imaged immediately after it, the divergence of the Wollaston plays a key role in imaging.

When imaging a single beam immediately after a Wollaston, the focal length of the imaging lens \( F_{\text{img}} \) and prism divergence \( \theta_{\text{wol}} \) needs to be calculated so
2.5 Imaging Polarimeter Development

that the two beams do not overlap, which would effectively reduce the field of view (FOV) as demonstrated in figure 2.29. Equation 2.32 prescribes the necessary $F_{\text{img}}$ to separate the two beams dependent on the separation and divergence angle. We wish to separate the images by 4.1 mm, which is half the size of the CCD in use. Using this prescribed lens we will get field of view as calculated by equation 2.33 where $x$ is $S_{\text{chip}}$, the required separation on the CCD chip. However, this image may overlap as the image diameter might be larger than 4.1 mm. The necessary $F_{\text{img}}$ lens may not be available and this will lead to differing image separations calculated by equation 2.33 where $x$ is $S_{\text{img}}$ (equation 2.34).

$$F_{\text{img}} = \frac{S_{\text{chip}}}{\tan(\theta_{\text{wol}})} \quad (2.32)$$

$$\text{Restriction}'' = 3600. \arctan\left(\frac{x}{F_{\text{img}}}(F_{\text{col}}/F_{T})\right) \quad (2.33)$$

$$x = S_{\text{img}} = \tan(\theta_{\text{wol}})(F_{\text{img}}) \quad (2.34)$$

or

$$x = S_{\text{chip}} \quad (2.35)$$

$$\text{Ultimate FOV}'' = \frac{F_{\text{col}}}{F_{T} \cdot \text{FOV}_RBS} \cdot 3600 \quad (2.36)$$

where $F_{\text{img}}$ is the imaging lens focal length, $S_{\text{chip}}$ is the separation of images on the chip, $\theta_{\text{wol}}$ is the Wollaston divergent angle, $F_{\text{col}}$ and $F_{T}$ are the collimator and telescope focal length, respectively, and $S_{\text{img}}$ is the image separation as seen in figure 2.29. Ultimate FOV is the ultimate FOV based on the restriction posed by the field of view of the RBS ($\text{FOV}_{RBS}$).

![Figure 2.29: Illustration of field overlaps caused by large fields or incorrect Wollaston prism divergence.](image)

Taking Keck as an example telescope ($F_{T} = 149.6$ m), we use Wollastons of differing divergent angles and see what FOV’s we get as shown in table 2.8. Case 1 is perfect, that is it uses the exactly prescribed collimating lens (from equation
2.12), $F_{col} = 62.33\text{mm}$ and since the RBS prism has a limiting FOV of $8^\circ$ this will correspond to a limiting sky FOV of $12''$ as calculated by equation 2.36. Case 2 also used the exactly prescribed collimating lens, but the imaging lens is rounded off to the nearest available lens and the FOV is reduced. The rest of the cases (3a-5) use an $F_{col}$ of 60mm giving the maximum for of 11.55'.'
2.5 Imaging Polarimeter Development

CCD and this portion of the CCD is wasted. In all cases, it is the smallest of either ‘separation limited FOV’ or ‘field limited FOV’ to be used for final FOV. Irrespective of the optics used to have no overlapping images on the CCD the field angle must equal the Wollaston divergence angle.

To maximise the FOV for the polarimeter we need to obtain an $8^\circ$ Wollaston, which is not easy to find. Alternatively, we could then change the field angle before it enters the Wollaston prism and imaging lens by using a recollimator with a set magnification prior to imaging. If the magnification was set to 1.6 this would change to field angle to $8^\circ/1.6 = 5^\circ$, hence we would get the maximum FOV from the telescope without needing a specialized Wollaston. However, we did say earlier in section 2.5.4 that a solution to the pupil astigmatism was to use a recollimator with a magnification of 0.588, so, the new angle entering the Wollaston is now $8^\circ/0.588 = 13.6^\circ$, but such a Wollaston does not exist. Taking the nearest obtainable Wollaston ($10^\circ$), to have no image overlap, the field angle entering the Wollaston must also be $10^\circ$. Taking into account the magnification of the recollimator this corresponds to an RBS field of $10 \times 0.588^\circ = 5.88^\circ$, while on Keck, this would limit the field to $8.49^\circ$. So to maximise the field of view, a Wollaston prism whose divergence corresponds to the input field angle should be used.

2.5.6 Image Outputs

Figures 2.30 and 2.31 Display a calibration image of a single point source. The 2x2 configuration is used for when full frame imaging is needed for large fields. The 4x1 configuration is obtained by repositioning the detector and altering the T-path angle so that the images fall on the lower row of the CCD. By binning down the pixels and only using a portion of the chip, this allows us to obtain frame rates $>300$FPS.

Some elongation of the spot can be noticed in these images. This is due to dispersion caused by the Wollaston prism as different wavelengths will diverge (at differing angles) to the designed divergence. The change in divergence for a $5^\circ$ Wollaston is $\sim 0.28^\circ$ from 400nm to 800nm and this will elongate the image depending on the imaging lens used. As observations will be carried out through filters whose bandwidth are of the order of 100nm, this dispersion will be reduced to $.13^\circ$ for blue filters and $.02^\circ$ red filters.

We have shown that an instantaneous full stokes polarimeter can be constructed; the next chapter will examine how well this polarimeter worked.
The Polarimeter

Figure 2.30: Original 2 x 2 configuration.

Figure 2.31: Reconfigured 4 x 1 arrangement.
Chapter 3

Data processing and Calibration
In this chapter, we will look at the calibration of the polarimeter from first principals, that is, starting with calibration of the polarizing optics that will calibrate the polarimeter. We will look at issues of processing polarimetric images and that the extraction of the intensities from the images need a careful approach. We will then look at the calibration of the polarimeter when it is on the telescope.

3.1 Component Alignment Calibration

3.1.1 Polarizer Alignment

For any polarimeter to be calibrated, the polarimetric optics calibrating the polarimeters must themselves be calibrated. Only the orientation angle of transmission axis of a polarizer and the fast axis of the waveplate needs to be calibrated, the diattenuations and retardances are accounted for in another calibration used, which is detailed in section 3.4. All the same, it is best to have the extinction ratios of polarizers as high as possible (on the order of $10^{-6}$) and to use achromatic waveplates, whose properties are catalogued.

The calibration procedure for a polarizer utilizes the fact that s-polarized waves are dominant when light is reflected off a planar reflective surface at or near the Brewster angle. For this case, a reference surface is needed to determine the abscissa (x-axis) of the polarization optics and for convenience, that surface can be a glass slide placed flat on an optical bench and the procedure is as follows. This procedure will give an accuracy of alignment of $\pm 1/2$ the graduations on the polarizer.

1. As per figure 3.1, place a glass slide on an optical bench, which will be the reference plane for the polarizer being aligned.

2. Reflect a laser at 40°- 60° to the normal of the bench. This angle must be around the Brewster angle of the slide, but it does not need to be exact as a minimum of the p-polarized state can still be found.

3. Place the polarizer and then the detector in the reflected beam.

4. Rotate the polarizer until a minimum intensity is found on the detector. This is the approximate angle of the polarization axis + 90°.

5. Rotate the polarizer a small amount in one direction and note the exact angle and intensity reading and record.
6. Rotate the polarizer in the opposite direction until the intensity reading is the exact same as previously found and record that angle.

7. Repeat steps 5 and 6 several times and the mid point of these angles is the offset transmission angle $+ 90^\circ$ for the polarizer using that direction of propagation. $90^\circ$ must be added to the angle found as the calibration is looking for the crossed polarization minimum, which is perpendicular to the plane of reflection and the transmission axis.

![Figure 3.1: Starting calibration of polarizing components.](image)

### 3.1.2 Waveplate Alignment

The fact that any incident linearly polarized beam passing through the fast axis of a waveplate will remain unchanged is used to orient the waveplate to a desired angle. The following is the procedure to set the angle of a waveplate to a known angle.

1. As per figure 3.2, setup the components in the order pictured.

2. Set the first polarizer to the angle the waveplate is to be orientated to, in the figure it will be oriented to $0^\circ$

3. Set the second polarizer so that it is at $90^\circ$ to the first, i.e. the two polarizers must be crossed. If the waveplate was not aligned to the first polarizer then an intensity will show on the detector.
4. Rotate the waveplate until the extinction or minimum intensity occurs and this will be the desired angle of the waveplate for that direction of propagation.

![Angular alignment of the quarter waveplate.](image)

It must be noted that in both cases that the direction of propagation must be indicated. If the polarizer was reversed then a setting of $45^\circ$ would actually be generating a polarization state of $-45^\circ$.

### 3.1.3 Polarization State Generator Calibration

Most astronomical polarimeters solely depend on polarized stars to calibrate the device when on a telescope as the basic calibration is completed in the lab with refined alignment of the polarizing elements. As GASP is a division of amplitude polarimeter, a different method is employed. As shown in section 1.4.1, this involves entering at least four linearly independent polarization states, taking out the intensities from the polarimeter and then solving for the system matrix. These polarization states used to calibrate the polarimeter are known as the polarization state generator (PSG). The four linearly independent polarization states easiest to generate are three linear states at $0^\circ$, $45^\circ$ and $90^\circ$ and a circular polarization state.

Selection of these four polarization states are dependent on its own optimization, where by incorrectly selecting these four vectors can lead to a “noisy” calibration. To optimize these four vectors, we need to maximize their linear independence to each other. If we plot each of these vectors on the Poincaré sphere the combination that takes up the largest volume is the optimized combination, which forms a regular tetrahedron [80]. These states would then be various elliptical polarizations, which can make the PSG impractical and expensive and on this basis it leaves us to use three linear states and one circular. It is then obvious that
the PSG that will give us the largest volume on the Poincaré sphere is having the three linear polarization states $60^\circ$ apart and a total circular polarization state. Figure 3.3(a) shows the 16 output intensities, after passing through the polarimeter, as a function of rotation of the linear states of the PSG and in this we can see that at certain orientations of the PSG, low or even null intensities exist. It is better to have the output intensities as high as possible with high signal-to-noise ratios (SNR) so that it is less susceptible to noise. SNR is calculated by equation 3.1 below

$$\text{SNR} = \frac{I}{\sqrt{I + B}} \quad \text{(3.1)}$$

where $I$ is the intensity of the channel and $B$ is the background noise and when $I$ is so large compared to $B$, then SNR can just be $\sqrt{I}$. To have the output intensities with the highest signal-to-noise ratio (SNR) then we would pick the linear states for the PSG to be orientated at $0^\circ$, $60^\circ$, or $120^\circ$, when using the PSG on axis to stay away from the null intensities.

Due to the need to check the calibration of GASP from day to day, it was necessary to have a separate “calibration arm” for the polarimeter. This was achieved by having a aluminium front surface $45^\circ$ fold mirror roll into position on the polarimeter’s optical axis when a calibration was necessary. A PSG matrix with a mirror in it can be generated like that shown in section 2.5.2. Figure 3.3(b) then shows the intensity outputs for this PSG matrix and beneficially, no orientation exists where null intensities are present. Linear states of $0^\circ$, $60^\circ$ or $120^\circ$ were chosen, as this has the most intensities with the highest values.

Figure 3.3: 16 output intensities in passing the PSG through the polarimeter system matrix. Some of the 16 distinct curves may not be visible due to overlapping.

Understandably, the properties of mirror in the calibration arm may not be accurately known thus the altered polarization will have to be measured. These
Data processing and Calibration

states can be measured using the pre-existing method of simply obtaining the six intensities necessary to measure a Stokes vectors as described in section 1.4. However, this entails positioning the angles of the polarizers exactly and even at this, errors in the angle measurement can creep in. To reduce the effects of the polarizer misalignment, Malus’ law was employed. Malus’ law (equation 3.2) states that the output intensity \( I \) from a polarizer with an incident linearly polarized beam is a function of the angle between the polarizer and the linear polarization state \( \theta \) and the input intensity \( I_0 \).

\[
I = I_0 \cos^2(\theta)
\]  

(3.2)

The apparatus is set up as in figure 3.4, first without the quarter waveplate, and the intensities are measured while rotating the polarizer from 0° to 180° at set intervals. A least squares fit of the data can be made to equation 3.2 and the intensities at the desired angles (0° 45° 90°-45°) can be found. Using equation 1.24, this will reconstruct \( I, Q \) and \( U \) of the Stokes vector, while \( V \) can be found by repeating the measurements by rotating polarizer from 0° to 180° while a quarter-waveplate at 45° is placed before the polarizer. The subsequent fit to this data will allow the extraction of the intensities at 0° and 90°, which are the left and right circular polarization respectively, finally calibrating the PSG.

![Figure 3.4: Calibration of the PSG.](image-url)

Each of the PSG states yield a 4 \times 1 intensity vector, which are then arranged into a 4 \times 4 intensity matrix \( \mathbf{I} \). Likewise, the 4 Stokes vectors from the PSG are arranged into a 4 \times 4 PSG matrix \( \mathbf{W} \). They are arranged such that each column of both matrices correspond, i.e. column \( n \) of \( \mathbf{I} \) is the intensity vector from the
3.2 Image Processing

$n^{th}$ PSG state and the Stokes vector of the $n^{th}$ state matches column $n$ of $W$. The order of the intensities taken from the images to create the intensity vector must remain the same at the calibration and the polarimetry stages.

Each Stokes vector recorded for the PSG is not normalized as each vector will have a different intensity due to the attenuation of the optics of the polarizers and mirror. Normalization of each vector would create errors in the calibrated system matrix and thus errors in the final polarimetry. From this, we can note that the stability of the calibration source is critical and will be discussed later.

3.2 Image Processing

As GASP utilizes a single camera design and is an imaging polarimeter, several issues make the image processing a non-trivial task. We will be concerned with the preprocessing of the images, polarimetric field of view, and the extraction of the intensities.

3.2.1 Pre-processing

In standard astronomical preprocessing, each image must have its field flattened and the bias and dark frame subtracted. The bias and dark subtraction of the polarimetric data is very important as an offset in the intensities extracted cannot be accounted for in the calibration procedure. However, flat fielding can be omitted as this is an attempt to normalize the field to have uniform and homogeneous gain / sensitivity over the detector area. Since the first column of the system matrix is basically the gain of each channel, flat fielding can be neglected as it is taken care of by the calibration process. It is fortunate that the calibration takes care of the flat fielding issue as obtaining an evenly illuminated unpolarized field is very difficult on a telescope. Normally, astronomers use the twilight sky as a flat field, but as shown in section 1.3.3 a twilight sky is highly polarized with the angle of polarization perpendicular to the direction of the Sun. It can be conceived to rotate the instrument by $90^\circ$ while exposing for a twilight sky, but the sky is constantly dimming in the time of this exposure and an unpolarized flat field will never be satisfactorily achieved. Alternately a polarization scrambler can be introduced into the optics, but that optic itself has an effect of the flat field, so again an unpolarized flat field will never be satisfactorily achieved.

As shown in section 2.4 the polarimeter will have a polarimetric field of view
such that at every point on the imaging area of the polarimeter will have a different calibration / system matrix. This could be solved using pixel to pixel registration for each of the four channels, but the R-path and T-path do not have identical optical configurations. The image registration would not only need translation, but spatial transformation to correct for the differing distortions and field curvatures between each channel. This is resolved by using aperture photometry for the intensity extraction in each individual channel. However, with regard to the polarimetric field of view, each point must be calibrated, that is, a point that is utilized for polarimetry must be calibrated for the same location.

### 3.2.2 Aperture Selection for Intensity Extraction

Imaging polarimetry is sensitive to how the raw intensities are taken from the images as a change in intensity of one channel relative to another could result in a different measured polarization state. Aperture photometry is an image processing tool used in astronomy to measure the intensity of a star. It involves selecting an aperture of a known radius and summing all the counts in it. An annulus $\sim 3$ pixels beyond this aperture is then used to sum the background counts and the median counts per pixel from the background is then subtracted from the aperture yielding the final extracted intensity. As standard aperture photometry would have it [81] [82] [83], the selection of the size of the aperture is influenced by several factors. These include, but not limited to, rate of growth curves, factors of the FWHM (full width half max) of the star and maximization of the SNR. Figure 3.5(a) shows the rate of growth curves for the four channels and when this curve has stabilized, $\geq 6$ pixels in this case, that aperture can be used. Figure 3.5(b) displays the SNRs for the four channels and we can see that a peak SNR for each intensity occurs at a 3 pixel radius, which could possibly give us the lowest error of measurement and thus a suggestible aperture size to use.

This process of aperture selection for the intensity extraction seemed sensible, but when applied to polarimetry, some results were varying. When we then looked at the derivative of the ratios between the four points (see figure 3.6), we could see that at change in aperture size yielded a different ratio of intensities. So the peak SNR aperture selection method was abandoned.

From figure 3.6 we could then say that an aperture radius of 7 pixels could be the best choice, but the derivative of the ratios has no physical meaning. When applying a theoretical system matrix to the polarimetry of the data and looking at the change of polarization as a function of increasing aperture size we see some
meaningful results in figures 3.7 and 3.8. The system matrix need not be accurate as we are looking at the relative change in polarization measured not the absolute value. Figure 3.7(a) and 3.7(b) shows how the polarization angles and degrees of polarization change as a function of increasing aperture size. If we have a defined limit of sensitivity, we can choose an aperture size based on this. In these cases, an aperture $\geq 4$ pixels could possibly give us a precision of $\leq 0.1^\circ$ for the angles and $\leq 0.5\%$ for the degrees of polarization. We then carry on with the calibration, obtain the actual experimental matrix and repeat the aperture analysis. From this we find that the change in polarization against aperture size remains the same so the chosen aperture size is maintained.
Data processing and Calibration

Figure 3.7: Aperture size selection base upon change in polarization measurement.

![Figure 3.7](image1)

(a) Differentials of the polarization angles $\theta$ and $\phi$ for increasing aperture size

(b) Differentials of the degrees of polarization for increasing aperture size

Figure 3.8: Change in polarizations for real sky data: BD+32 3739 (unpolarized).

![Figure 3.8](image2)

(a) Differentials of the polarization angles $\theta$ and $\phi$ for increasing aperture size

(b) Differentials of the degrees of polarization for increasing aperture size

This process is acceptable for calibration data, but it becomes trickier when real sky data is used. Sometimes the size of the aperture must be limited as a larger aperture may encompass another star and change the polarization. This can be seen in figure 3.8(a) where an aperture radius > 10 pixels increases the error. Not shown, is the rate of growth for this star, which displays that the curve begins to increase in intensity again after an aperture radius > 10 pixels. Another point to note is that in the real sky data we often see that the differentials tend to be larger than the calibration data, in figure 3.8(a) and 3.8(b) a stabilization of $\sim 0.5^\circ$ and $\sim 0.5\%$ occurs. In some cases the degree of differential polarization has
only stabilised to \(\sim 2\%\) this can be due to fainter stars, poor seeing and imprecise telescope tracking, which lets noise limit the polarimetric sensitivity.

3.3 Polarimeter Calibration

In section 1.5 we mentioned how current astronomical polarimeters measure their Stokes vectors. Their calibration entails that the polarizers and waveplates are at the orientation that is required be that for a mechanical or electro-optic modulating polarization element. Further calibration continues when the polarimeters are on the telescope and calibration of the polarization angle and degrees of polarization are adjusted by observing standard calibration stars [60] [84]. GASP relies on absolute calibration before going on to the telescope, that is, the system matrix \((A)\) of the polarimeter is determined. With the PSG independently calibrated, we then pass each Stokes vector of the PSG through the polarimeter and recorded the corresponding intensity vector. The four intensity vectors are then organized into a \(4 \times 4\) matrix, known as the intensity matrix \((I)\). The system matrix can then be determined by post multiplying the inverse of the PSG matrix by the intensity matrix as shown in equation 3.3.

\[
A = I \cdot W^{-1} \tag{3.3}
\]

where \(I\) is the output intensity matrix, \(A\) is the system matrix and \(W\) is the PSG matrix.

This calibration seems very simple, but simplicity at an early stage can lead to complexity later on. GASP’s PSG uses a filter wheel to position each polarimetric element to generate a particular Stokes vector. If the gearing is loose or the stepper motor driving the filter wheel is imprecise then the Stokes vector that was previously calibrated may not match that which is generated and will produce an inaccurate and inconsistent PSG matrix. This error might be about a degree or so, but it can carry through the calibration and onto the final polarimetry.

An issue of stability of the source used for the PSG was noticed to have an influence on calibrations and becomes obvious when mathematically examined. As with DOAP polarimetry, each polarimetric measurement comes from a single instantaneous exposure. However, during calibration, there is a delay between the generation of two separate polarization states. It would then become obvious that an alteration to the calibration light source between any two polarization states would affect the measurement of relative intensities between each separate vector.
We can break down the PSG matrix \( W \) into two matrices as shown in equation 3.4

\[
W = W_{\text{nom}} \cdot W_{\text{flux}} \quad (3.4)
\]

where \( W_{\text{nom}} \) is the nominal \( W \) matrix and \( W_{\text{flux}} \) is the fluctuation matrix. The fluctuation matrix is a square \( 4 \times 4 \) diagonal matrix whose non-null values relate to the intensity of the source at each different PSG state. If the source was stable throughout the calibration then \( W_{\text{flux}} \) would be an identity matrix and thus \( W = W_{\text{nom}} \). Looking back at the calibration equation 3.3 we can rewrite this taking into account equation 3.4 such that we get

\[
A = I \cdot W_{\text{flux}}^{-1} \cdot W_{\text{nom}}^{-1} \quad (3.5)
\]

From this we can see that the change of the calibration source intensity is needed to obtain the proper system matrix. If the change in intensity is non-existent then the resultant system matrix is the properly calibrated or otherwise it may not be so. We will not go into the relationship between the change of the source and the accuracy of the calibration, we only state that it is important to note and be aware of this.

### 3.4 Eigenvalue Calibration Method (ECM)

The simple calibration method described in section 3.3 does not correct for any linear systematic or random errors introduced by the optical system, detectors, PSG calibration, source fluctuations or mechanical misalignments. Overcoming these errors would greatly enhance the accuracy of the polarimeter and leave a majority share of the polarimetric errors related to the photon statistics. A calibration method developed by Compain et al. [85], was intended to solve this issue and is called the Eigenvalue Calibration Method (ECM).

#### 3.4.1 ECM Theory

ECM is based upon measuring four chosen Mueller matrix samples and having their properties unambiguously determined from the eigenvalues of the measured matrices. The reconstructed matrices contain no errors and are then used to solve for the System and PSG matrices. The four well known matrices, are air \( (B_0) \), a polarizer at 0° \( (B_1) \), a polarizer at 90° \( (B_2) \) and a quarter waveplate orientated at 28° \( (B_3) \). An explanation of why these angles are chosen can be found in [85]
3.4 Eigenvalue Calibration Method (ECM)

and [86]. The polarization properties of the well known calibration samples can be experimentally measured from the eigenvalues of $C_i$ found in equation 3.6.

\[
C_i = aw^{-1} \cdot am_iw = B_i^{-1} \cdot B_i \quad (i = 1, 2, 3)
\]  

(3.6)

where the lower-case bold letters, $amw$, are the experimentally measured matrices as is $B_i$, which is the $i^{th}$ experimentally measured matrix of each of the samples previously stated.

Fortunately, the eigenvalues of the measured matrices are identical to perfect matrices as the errors in the measurement do not change their properties of transmittance ($\tau$), diattenuation ($\psi$) and retardance ($\Delta$). The polarizers are selected to be perfect polarizers (extinction ratios of $10^{-6}$ or greater) and do not contain any retardance. The transmittance ($\tau$) is not 0.5 as absorption and back reflections have to be taken into account and can be found by equation 3.7

\[
\tau_i = \frac{1}{2} \text{trace}(C_i) \quad (i = 1, 2)
\]  

(3.7)

where $\text{trace}(C_i)$ is the sum of the main diagonal of the matrix. The eigenvalues of the retarder will yield its transmittance ($\tau$), diattenuation ($\psi$) and retardance ($\Delta$) seen in equation 3.8

\[
\begin{align*}
\tau_3 &= \frac{\lambda_{\text{re}1} + \lambda_{\text{re}2}}{2} \\
\psi &= \arctan \sqrt{\frac{\lambda_{\text{re}1}}{\lambda_{\text{re}2}}} \\
\Delta &= \frac{1}{2} \text{arg} \left( \frac{\lambda_{\text{im}1}}{\lambda_{\text{im}2}} \right)
\end{align*}
\]  

(3.8a–3.8c)

where $\lambda_{\text{re}1}$ and $\lambda_{\text{re}2}$ are the largest and second largest real eigenvalues of $C_3$ and $\lambda_{\text{im}1}$ and $\lambda_{\text{im}2}$ are the largest and second largest complex eigenvalues of $C_3$.

Having all the necessary properties of these three matrices, assuming $\Delta = 0$ for the polarizers, they are then theoretically reconstructed to the perfect un-oriented matrices using equation 1.12. The orientation of each of the samples cannot be determined by the eigenvalue analysis yet the approximate angles are known. These matrices are generated as shown in equation 1.31 in section 1.4.1. To find the exact angles to which the samples were orientated, the application of linear mapping and non-linear optimization algorithms were used.

$H_M$ represents the linear mapping of a set of four matrices onto itself (see equation 3.9).
$$\mathbb{H}_{M_i} : X \rightarrow M_i \cdot X - X \cdot (aw)^{-1}(am_i w) = 0$$

$$\mathbb{H}_{M_i} : X \rightarrow M_i \cdot X - X \cdot C_i = 0 \quad (i = 1, 2, 3) \quad (3.9)$$

where the upper-case bold letters, $M_i$ and $X$ are the theoretical matrices and lower-case bold letters $amw$ are the experimentally measured matrices. $H_M$ has the property of containing $W$ (the PSG matrix) within its null space because without experimental errors $(aw)^{-1}(amw)$ is equal to $W^{-1} \cdot M \cdot W$. The unique solution ($X$) of this equation contains the $16 \times 1$ vector form of the matrix $W$ in its null space.

$H_M$ is the $16 \times 16$ matrix created by the calculation of equation 3.9 where the $X$ matrix contains entries $X_{1,1}$ to $X_{4,4}$. The resultant is rearranged to a $16 \times 1$ column vector and the $X$ matrix rearranged to a $1 \times 16$ row vector. The $H_M$ matrix is complied by extracting the coefficients of $X$ of each row of the column vector and placed into the corresponding positions dictated by the corresponding $X$ vector column. The process follows until all 16 rows of the $H_M$ column vector is placed in into the $16 \times 16$ matrix for all three matrices.

With all three $H_M$ matrices obtained, they are then combined in a to a K-map matrix as in the equation 3.10.

$$K = H_{M1}^T \cdot H_{M1} + H_{M2}^T \cdot H_{M2} + H_{M3}^T \cdot H_{M3} \quad (3.10)$$

where $H_{Mi}^T$ is the transpose of the $i^{th}$ $H_M$ matrix for $i = 1, 2, 3$. This K-map matrix is a positive symmetric real matrix that has 15 non-null and 1 null eigenvalue.

The null eigenvector associated to the null eigenvalue of this K-map matrix yields the vector form of $W$. However, the angles entered earlier still need to be found. The ratio of the smallest to second smallest eigenvalue is the optimization value used to find the orientation angles of the 3 matrices. Minimization of this optimization value using the angles as the variables is calculated using a Nelder-Mead downhill simplex algorithm. With the optimization complete the null eigenvector of $K$ with the optimized angles is rearranged to a give the $4 \times 4$ matrix $W$. The next section is a step by step guide how to complete this ECM calibration.
3.4.2 ECM Process

1. Measure the intensity matrices of four samples, air \( (B_0) \), polarizer at 0° \( (B_1) \), polarizer at 0° \( (B_2) \) and a quarter waveplate orientated at 28° \( (B_3) \).

2. Pre-multiply the latter 3 matrices by inverse of the first to obtain \( C_1 \) to \( C_3 \).

3. Find the properties \( \tau \), \( \psi \) and \( \Delta \) of the three \( C \) matrices as described in the previous section.

4. Reconstruct the theoretical matrices for these samples using the properties obtained and rotate the matrices by the approximate angles they were set. i.e. 0°, 90° and 28°.

5. Using the \textit{fminsearch} function in MATLAB, (Nelder-Mead downhill simplex minimization algorithm), the minimization of the ratios of the smallest to the second smallest eigenvalues of the K-Map is found using the three angle as variables.

6. When the minimization is found, the unique null space eigenvector of the K-map is reshaped to a 4 \( \times \) 4 matrix, which is \( W \).

7. \( A \) is then calculated using equation 3.11, which is the product of the intensity matrix for air and the inverse of \( W \).

\[
A = (aw)W^{-1}
\]  \hspace{1cm} (3.11)

Matlab scripts for the ECM calibration can be found in appendix B

3.5 Astronomical Calibration

With GASP internally calibrated, correction and calibration off astronomical polarimetric standards may still be needed to correct for the polarization effects of the telescope. Tables 3.1 and 3.2 list some of the polarized and unpolarized standards available. A larger list of polarized and unpolarized standard stars can be found in appendix C.

The position angle (PA) is used to correct for the angle of polarization (\( \theta \)). The degree of polarization (DOP) is used to verify the correct DOP is being measured. Nearly all of these standards are linearly polarized stars and no catalogue exits
### Data processing and Calibration

**Table 3.1: Polarized Standard stars for polarimetric calibration (V band)**

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 204827</td>
<td>21 28 57.7</td>
<td>58 44 24.0</td>
<td>7.93</td>
<td>60.0</td>
<td>5.39</td>
<td>[87]</td>
</tr>
<tr>
<td>CRL 2688</td>
<td>21 02 18.8</td>
<td>+36 41 41.2</td>
<td>12</td>
<td>105</td>
<td>47.8</td>
<td>[88][89][90]</td>
</tr>
<tr>
<td>HD 236928</td>
<td>02 02 42.0</td>
<td>+60 15 26.5</td>
<td>9.07</td>
<td>98.2</td>
<td>6.69</td>
<td>[90]</td>
</tr>
<tr>
<td>BD+64° 106</td>
<td>00 57 36.7</td>
<td>+64 51 27</td>
<td>10.3</td>
<td>96.6</td>
<td>5.69 ± 0.04</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 7927</td>
<td>01 20 04.9</td>
<td>+58 13 54</td>
<td>5.0</td>
<td>92.1</td>
<td>3.32 ± 0.04</td>
<td>[92]</td>
</tr>
</tbody>
</table>

**Table 3.2: Unpolarized Standard stars for polarimetric calibration (V band)**

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>BD+323739</td>
<td>20 12 02.1</td>
<td>+32 47 43.1</td>
<td>9.31</td>
<td>35.79</td>
<td>0.025</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 212311</td>
<td>22 21 58.6</td>
<td>+56 31 52.7</td>
<td>8.12</td>
<td>50.99</td>
<td>0.034</td>
<td>[93]</td>
</tr>
<tr>
<td>βCas</td>
<td>00 09 10.7</td>
<td>+59 08 59</td>
<td>2.3</td>
<td>72.5</td>
<td>0.04 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 12021</td>
<td>01 57 56.1</td>
<td>-02 05 58</td>
<td>8.9</td>
<td>160.1</td>
<td>0.08 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>ζPeg</td>
<td>22 41 27.7</td>
<td>+10 49 53</td>
<td>3.4</td>
<td>40.0</td>
<td>0.05 ± 0.02</td>
<td>[91]</td>
</tr>
</tbody>
</table>

whereby one can calibrate for the degree of circular or the relative phase angle (φ).

Depending on the type of telescope, an amount of induced degree of polarization may have to be subtracted. This is due to the possible polarization introduced by the telescope’s optics prior to the polarimeter. An instrument placed on a telescope with a Cassegrain focus is best for polarimeters as the angles of incidence of star light on the primary and secondary mirrors are small compared to a Nasmyth or Newtonian telescope where a 45° fold mirror relays the focus to the side of the telescope rather than the back. Such a fold mirror will change the polarization just like the fold mirror in the GASP’s PSG. This is normally seen as ‘telescope polarization’ and is subtracted from the data with the telescope polarization measured from observing zero-polarization stars as in table 3.2. Like the PSG calibration, the effect of a fold mirror could be calibrated out if the properties of the mirror can be determined. It must also be said that if this fold mirror is coated, then this will seriously deteriorate the performance of a polarimeter.

In an ideal case, the telescope itself would be included in the calibration and some attempts have been made to do so. One method [94] attempts to characterize a sub-aperture of a large aperture telescope and then assume symmetry over the rest of the mirror. Another attempt [95] tries to place some calibration optics on the dome then utilize a beam expander to try to measure the Mueller matrix of
the primary mirror to account for its influence on the polarization. For our ECM calibration (section 3.4) calibrating the telescope, this would involve placing a PSG source and ECM sample before the telescope, which is impractical if not impossible for large telescopes.

3.5.1 Sky Polarization

Any observed polarized target \( S_d \) will contain a background polarization \( S_b \) that will need to be subtracted to obtain the target polarization \( S_s \) as expressed in equation 3.13. This background can come from the nebulosity around a target say in the Crab pulsar or even scattered moonlight throughout the night sky. Aperture photometry aids in this background subtraction even at the intensity extraction point. The background intensity in the annulus \( I_b \) will be subtracted from the target aperture intensity \( I_d \) yielding the intensity of the target alone \( I_s \) expressed in equation 3.13.

\[
S_s = S_d - S_b \quad (3.12)
\]

\[
I_s = I_d - I_b \quad (3.13)
\]

where \( I_s, I_d \) and \( I_b \) is the intensity vector from the source, data and background respectively and \( S_s, S_d \) and \( S_b \) are the Stokes vectors from the source, data and background, respectively. We then measure the polarization of the target and we get equation 3.14 below

\[
S_s = A^{-1}I_s \quad (3.14)
\]

Substituting equation 3.13 for \( I_s \) we then get

\[
S_s = A^{-1}(I_d - I_b) \quad (3.15)
\]

Where \( A \) is the system matrix of the polarimeter. This shows us that aperture photometry takes care of subtracting the background polarization so that the polarization only from the target is calculated.

However, an anisotropic background polarization can give a misleading background measurement in the annulus and lead to an erroneous polarization result. A solution for this is to omit the annulus subtraction in aperture photometry, survey around the target to see what the background polarization is and investigate weather the star polarization is contaminated by the background or not.
Chapter 4

Calibration, Laboratory and Astronomical Results
In this chapter, we present both laboratory and astronomical results. The laboratory results include a comparison of theoretical and actual system matrices, the effects of aperture selection on the Eigenvalue Calibration Method (ECM), stability and consistency of calibration and polarimetry, a verification test to see that the polarimeter is doing as it should and then field analysis showing the polarimetric sensitivity to image location. The astronomical results look at polarized and unpolarized sources verifying the telescope calibration, rotation tests on these sources and polarimetric stability.

4.1 Calibration Results

We compare the calibrated system matrix \( A_{\text{theo}} \) to an experimental one \( A_{\text{exp}} \) as shown in equation 4.1 and 4.2. Each matrix is normalized to the first element in the first row, while equation 4.1 comes from the theoretical derivation shown in section 2.5.4. As the order of the data taken from the calibration was set by selection of spots from the images and not the order of R-path then T-path configuration, \( A_{\text{exp}} \) was reorganized to resemble the form of \( A_{\text{theo}} \) so that we can see the signs of the matrix elements match up.

\[
A_{\text{theo}} = \begin{bmatrix}
0.1678 & 0 & 0 & 0 \\
0 & 0.1678 & 0 & 0 \\
0 & 0 & 0.1678 & 0 \\
0 & 0 & 0 & 0.1678
\end{bmatrix}
\begin{bmatrix}
1 & -0.5132 & -0.8217 & -0.2477 \\
1 & -0.5132 & 0.8217 & 0.2477 \\
1 & 0.4388 & 0.4134 & -0.7982 \\
1 & 0.4388 & -0.4134 & 0.7982
\end{bmatrix}
\] (4.1)

\[
A_{\text{exp}} = \begin{bmatrix}
0.1678 & 0 & 0 & 0 \\
0 & 0.1678 & 0 & 0 \\
0 & 0 & 0.1678 & 0 \\
0 & 0 & 0 & 0.1678
\end{bmatrix}
\begin{bmatrix}
1 & -0.5132 & -0.8217 & -0.2477 \\
1 & -0.5132 & 0.8217 & 0.2477 \\
1 & 0.4388 & 0.4134 & -0.7982 \\
1 & 0.4388 & -0.4134 & 0.7982
\end{bmatrix}
\] (4.2)

In doing so, we can see a similarity in the values between some elements of the right hand side matrices. Although not exact, we wish to verify that the format and values are of the correct order. The comparison of the elements show a large discrepancy between them, ±0.2, and this is likely due to not knowing exactly the properties of the aluminium mirrors in the polarimeter for the theoretical matrix. The main diagonal matrices on the left hand side of each equation are the gain matrices and they account for the transmittance of each channel. The top two values correspond to the R-path and the bottom two, the T-path. Even though

\(^1\)\( A_{\text{exp}} \) is a randomly selected matrix from one of the laboratory analyses.
4.1 Calibration Results

they do not all equal in equation 4.2 it does not imply that the polarimeter is far from the optimized or will fail to work. Such discrepancies are of no major concern and any concerns are allayed by looking at the inverse condition numbers. For $A_{\text{theo}}$, it is 0.4752 while for $A_{\text{exp}}$, it is 0.4469. If this value was below 0.3, the polarimeter may no give valid results. Throughout this portion of the chapter we will look at how well the experimental calibrations perform.

4.1.1 Effects of Aperture Selection on Calibration

As shown in section 3.2.2, we stated the importance of choosing the correct size aperture for the aperture photometry as it had an effect on the final polarimetry. Previously in figure 3.7, we chose a radius of $\geq 4$ pixels as we obtained a precision of $0.1^\circ$ for $\theta$ and $\phi$ and 0.5% for the degrees of polarization. However, beyond this aperture selection, the change in polarization as a function of increasing aperture did not stabilise. Here we now see how the aperture selection has an effect on the calibration parameters extracted from the ECM process.

The ECM calibration was completed for various aperture radii from 1 - 20 pixels and the following properties were extracted from each run: the transmittances ($Tr$) and orientation angle ($\theta$) for the 3 samples used, the diattenuation ($\psi$) and retardance ($\Delta$) for the quarter waveplate (QWP), the inverse condition number $S(A)$ and the minimization parameter $F_{\text{val}}$ as stated in section 3.4.2 and are shown in figure 4.1.

![Figure 4.1: Change in ECM parameters as a function of radius of aperture.](image)

The data for each parameter for increasing aperture radii stabilised as it did for the change in polarization versus aperture in figure 3.7. A stabilised mean of the parameters is created by taking the average of the parameters for aperture
radii of 15 to 20 pixels. Figure 4.1(a) shows the parameter difference from this stabilised mean versus radius of the aperture and from this figure we may conclude that each of the parameters stabilises at an aperture of radius around 8 pixels.

However, on this scale we can not be sure if the values actually stabilises so figure 4.1(b) shows the relative change as a percentage and how it stabilises. The parameters $F_{val}$ and $\theta_{Pol0}$ were omitted as they are values near zero and have percentages greater than the scale shown. From figure 4.1(b), we can see that change in the parameters settles to less than 0.25% after an aperture radius of 8 pixels similar to that seen in figure 4.1(a). The initial aperture selection method based on the change of polarizations properties against radius of aperture lead to a pixel radius selection of 4 pixels yet looking at the changes in the ECM parameters, a radius of 8 pixels is found to be the better choice. Using such aperture radius can lead to knowing the system matrix to within 0.25%, but this might be irrelevant compared to the consistency, which we will look at next.

### 4.1.2 System Matrix Calibration Consistency Tests

In terms of the stability of the system matrix, it is difficult to use a single metric to measure how consistent the calibration is from one calibration to the next. The most widely used single metric up to now has been the inverse condition number, $S(A)$ [1] [73]. In mathematical terms this is the inverse of the condition number of the system matrix, the same as that in equation 2.5, and is a measure of how non-singular or invertible the matrix is. Figure 4.2 shows the inverse condition number of an ECM calibrated system matrix of GASP every 15 minutes over 4.5 hours with each calibration taking approximately 5 minutes.

![Figure 4.2: Stability of the system matrix inverse condition number over 4.5 hours.](image)

In this figure, it appears that the inverse condition number over the 4.5 hours randomly varies with a mean and standard deviation of $0.447 \pm 0.001$ and that no systematic drift is visible. The drawback is that any two different matrices
4.1 Calibration Results

can have the same inverse condition number, and thus changes in the system matrix may not be apparent. The reverse of this is not true, as two different inverse condition numbers will definitely belong to two different matrices, so the question is, ‘how much do the matrices differ by and what is the consequence of the change?’ Quoting an inverse condition number standard deviation of 0.001 is not sufficient to say that the polarimeter is stable over this period. Alternatively, we can try to look at the change of each matrix element of the calibrated matrices. Each calibrated system matrix is normalised by the first element of the first row of that matrix and figure 4.3 shows a plot of the change of each matrix element from the first matrix in that run over the same period.

![Figure 4.3: Stability of system matrix elements over 4.5 hours.](image)

Here we see a deviation of approximately $\pm 0.05$ of the matrix elements over the 4.5 hours with some outliers at 2 and 2.5 hours; however we struggle to make an interpretation from the changes of the system matrix elements. To make sense of the changes, we can test for self consistent polarimetry. Self consistent polarimetry means we mathematically pass a Stokes vector through the first system matrix of the run, take its output intensity vector and then measure the polarization of that intensity vector with all the other system matrices. Later on in chapter 5 we will see that the error of the resulting vector is a function of the Stokes vector itself, hence larger errors can occur for different Stokes vectors. Therefore, for this self consistent polarimetry examination, we test for five different Stokes vectors, three linearly polarized at 0°, 45° and 90°, a circularly polarized and a 1% linearly polarized vector orientated at 45°. From this, we get the following results as shown in figures 4.4 and 4.5.

Looking at figure 4.4(a) we can see that the system matrices measure all the vectors with in 1% of their degree of polarization with fluctuations of some vectors less sensitive than others. It is only the measurement of the circular and 45° Stokes vector that varies greatly, the rest are with in 0.25% to 0.5%, which is important, especially for the 1% polarized Stokes vector as astronomical polarizations can be this low.
With regard to the polarization angles $\theta$ and $\phi$ in figure 4.5(a) and 4.5(b), we see an extremely consistent results for the 100% polarized vectors with deviations of only 0.2° for $\phi$ and $\theta$. When a vector is 100% circularly polarized, there is no linear component therefore there can be no polarization angle ($\theta$) to measure, so in figure 4.5(a), the degree of circular polarization is not shown for this figure. An average deviation of 4° for $\theta$ and 2° for $\phi$ is displayed for the 1% polarized vector, the error is high and in keeping with what we will see the in discussion in chapter 5. From this, we can conclude that one can use a calibrated matrix to measure polarization up to 4.5 hours later and still obtain measurements within our desired tolerance. However, the stability of the calibration for the measurement of low polarization targets may be more sensitive and more frequent calibrations may be needed. Even though this polarimetric stability test is mathematical and applied for the stability of the system matrix, later in section 4.2.1 we will look at the experimental stability of the polarimetry with real data.
4.1 Calibration Results

4.1.3 PSG Calibration Consistency Tests

Consistency of the ECM calibrated system matrix is important for the polarimetry, but the Polarization State Generator (PSG) must also be taking into consideration as the PSG itself must be stable to ensure that the calibration is stable. We first look at the PSG we hoped to generate, and compare it to what we obtained in practice. Equation 4.3 shows the theoretical PSG for linear polarization at 0°, 60°, 120° and circular polarization reflected off an aluminium mirror at 45°, as per the PSG calibration arm setup in section 3.1.3. It shows how the handedness of the polarization states has changed and that some circular polarization was induced.

\[
\text{PSG}_{\text{theo}} = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -0.52 & -0.52 & -0.03 \\
0 & -0.84 & 0.84 & -0.17 \\
0 & 0.15 & -0.15 & -0.98 \\
\end{bmatrix}
\] (4.3)

The ECM calibration then yielded the actual PSG matrix, as shown in equation 4.4, whose properties are listed in table 4.1. We see a similar amount of circular polarization added by looking at the Stokes V parameters in equation 4.4. The handedness of the circular polarization state has changed and is -98% circularly polarized as expected, but the desired orientations of the linear states were not achieved. The three linear vectors are \(\approx 72°\) apart rather than the \(60°\) separation we hoped to set. This may be due to the stepper motor position not quiet set right or some mechanical shift occurred in transport of the polarimeter. However, it is of no major concern as the ECM calibrates out these errors and corrected what we would have measured in the laboratory using the method described in section 3.1.3. While we were concerned with avoiding the presence of any null values in the intensity matrix when applying the PSG to GASP, this still did not happen for our actual PSG and only the -24° (156°) vector yielded the lowest intensity as expected from figure 3.3(b).

\[
\text{PSG}_{\text{exp}} = \begin{bmatrix}
1.0000 & 0.9807 & 1.0063 & 0.9737 \\
0.6722 & -0.4537 & -0.1636 & -0.1665 \\
-0.7505 & -0.9068 & 0.9693 & -0.1678 \\
0.0599 & 0.1657 & -0.1162 & -0.9560 \\
\end{bmatrix}
\] (4.4)

The analysis of the stability of the system matrix in the previous section was applied to the PSG and we looked at the change in the element of the normalised
Calibration, Laboratory and Astronomical Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PSG 1</th>
<th>PSG 2</th>
<th>PSG 3</th>
<th>PSG 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOP [%]</td>
<td>100.93</td>
<td>104.77</td>
<td>98.36</td>
<td>101.14</td>
</tr>
<tr>
<td>DOL [%]</td>
<td>100.75</td>
<td>103.40</td>
<td>97.68</td>
<td>24.28</td>
</tr>
<tr>
<td>DOC [%]</td>
<td>5.99</td>
<td>16.90</td>
<td>-11.55</td>
<td>-98.18</td>
</tr>
<tr>
<td>$\theta$ [°]</td>
<td>-24.07</td>
<td>121.71</td>
<td>49.79</td>
<td>-67.39</td>
</tr>
<tr>
<td>$\phi$ [°]</td>
<td>1.70</td>
<td>4.64</td>
<td>-3.37</td>
<td>-38.05</td>
</tr>
</tbody>
</table>

PSG matrices as seen in figure 4.6. Remembering that the PSG matrix is basically columns of Stokes vectors arranged in a matrix we can observe the change and interpret a result. The PSG matrix element difference of figure 4.6 shows that the change is in the order of ±0.005 meaning that the polarization is changing no more than 1% as a change of 0.01 is 1% for any of the Q, U and V Stokes parameter. This indicates a good stability of the PSG over 4.5 hours similar to the 1% stability of the system matrix (a similar conclusion came from figure 4.4 in reference to the stability of the system matrix).

![Figure 4.6: Stability of PSG matrix over 4.5 hours.](image)

Each of the Stokes vectors in each PSG was reduced to its degrees and angles of polarization and we can reaffirm that the change in polarization is not greater than 1% as shown in figure 4.7 and figure 4.8.

Figure 4.8 shows how the polarization angles were stable over this period and they varied by only 0.2° for $\theta$ and $\phi$, the same that was seen in the system matrix stability. The PSG does not have 100% circular nor a 1% polarized vector within it to be able to compare the large deviations seen there, but we can conclude that both the system matrix and PSG were stable to within 1% over 4.5 hours. This is important because it mean we can go longer on an observing run without having to check the calibration.
4.2 Laboratory Results

In this section we present several laboratory experiments conducted to test the functionality of the polarimeter. We tested to see that it can measure the correct polarization, that it remains stable over a set time frame and an analysis of the field of view was also conducted.

4.2.1 Polarimetric Stability Tests

Following on from the stability of the ECM calibrated system matrix and the PSG, we look at the stability of the polarimetry. This falls under two categories, short term (on the order of minutes) and long term (on the order of hours). As mentioned in section 3.3, short term stability of the light source is needed to ensure an effective calibration and reduce the errors within it. Long term stability
primarily concerns the polarimetry rather than the calibration, that is, “can the polarimeter measure a static polarization over time scale of hours?”

If it takes approximately 8 minutes to complete an ECM calibration it would be necessary that a calibration source should be stable to at least 0.1% over that period. For this examination, we use the source of the PSG on the internal calibration arm of GASP without any polarization element. The source, a tungsten incandescent lamp powered by a stabilised power supply that was filtered (with desired wavebands) and fibre fed to the PSG, was allowed to settle over one to two hours. We then observe the source with GASP over 15 minutes and with an uncalibrated system matrix look at the output Stokes vector. The system matrix used was not precisely calibrated as we are not interested in the absolute polarization, only the change in the polarization.

Figure 4.9(a) shows the Stokes $I$ and figure 4.9(b) the Stokes $Q$, $U$ and $V$ parameters each for a 1ms exposure every 2.5s for 15 minutes. We see that the source remains stable over the 15 minutes with a standard deviation of $I$ of 0.0017, which is a 0.17% deviation. $Q$, $U$ and $V$ have standard deviations of 0.0028, 0.0027 and 0.0025 respectively, that means that the stability of polarization of the source is within 0.25% to 0.28%. These deviations are approximately twice our desired source variation of 0.1% yet it is still satisfactory and less than 1% which was seen in the calibration stability.

We then tested for the polarization stability of a random Stokes vector over a longer period of approximately an hour (430ms exposure, every 20s for 68 minutes). The measurements took place initially after the lamp was turned on and this would mean that the source was intentionally unstable. This was then measured using a roughly calibrated matrix and its stability examined. Figure 4.10
shows the degree of polarization (DOP) and normalised Stokes $I$ parameter. The comparison between the two data sets obviously show the instability of the source over this period, while the stability of the polarization remains and is independent of the source. The standard deviation of the DOP showed a stability to within 0.05%, while the fluctuating intensity had a standard deviation of 0.41%.

In re-examining the DOP in figure 4.10(b), we see a slight drift of about $+0.1\%$ and for this reason we examine the rest of the polarimetric properties as shown in figure 4.11. Here we see a more noticeable drift in the degree of circular (DOC) in figure 4.11(a) of about $0.2\%$. The angles of polarization also show a change of $>0.1^\circ$ for $\theta$ and $\phi$ shown in figure 4.11(b). Their standard deviations over this period were $0.023^\circ$ and $0.017^\circ$ for $\theta$ and $\phi$, respectively. These drifts may be small, but this called for longer period to be investigated.

We then observed an unpolarized source to see if there was evidence of drift
over 3 hours (exposure of 0.29s, every 45s) as shown in figure 4.12. Looking at the
degrees of polarization over this period (see figure 4.12(a)) it can be shown that
DOP and the degree of linear (DOL) are drifting upwards, 3.7% to 3.9% and 4%
to 4.1% respectively, while DOC drifts downwards from 1.5% to 1.3%. A larger
change can bee seen for the angles of polarization with both having negative drifts
of $\theta = -81.1^\circ$ to $-83.4^\circ$ and $\phi = 11.3^\circ$ to $9.3^\circ$, which appears to be settling.

Figure 4.12: Examination of the stability of the degrees and angles of polarization
of an unpolarized source over 3 hours.

The changes in polarization are about 0.2% within the 1% of our desired
tolerance, but the fact this systematic drift exist is something to be aware of.
If we ignore the drifts, the standard deviations for DOP, DOL and DOC are
0.036%, 0.061% and 0.065% respectively, which shows a very high precision for
the polarimeter, more than what we require. However, this was a laboratory
experiment and photons are plentiful and does not include any other external
factors (which might occur when on a telescope) that could possibly add extra
noise to the polarimetry. The large changes in the angles of polarization suggest
some external influencing factor on the polarimeter as no polarizers were used in
the generation of the state examined. This could be a result of many possible
factors: the system matrix changing as a function of temperature, the presence of
stress birefringence in some optics or changes in the properties of the PSG mirror
that are settling down. Either way, stability on a short time scale of minutes, is
satisfactory. A change in polarization of $<1\%$ was not observed as shown in the
ECM and PSG stability test because these tests were conducted when GASP was
on a telescope and had been untouched for many hours.
4.2.2 Polarimetric Verification Tests

To confirm that GASP was measuring polarization correctly, a verification test was carried out. This involved creating a laboratory telescope positioned in front of GASP that would generate a ‘star’ whose state of polarization could be controlled, shown in figure 4.13. A “linear test” was carried out by altering the angle of linear polarization \( \theta \). This was easily done by rotating the polarizer in the collimated space of the telescope when the quarter waveplate was not present. A “circular test” would generate polarization states that would include circular polarization. With the quarter waveplate (QWP) in place, making sure the fast axis is at 45°, rotating the polarizer, while the QWP remains static, generated an oscillating elliptical polarization state. This would then effectively rotate the relative phase angle \( \phi \) and when the polarizer was at 0°, the output was 100% circular and when the polarizer was at 45°, the output was 100% linear.

These states can be visualised on the Poincaré sphere. For the linear test, a line is traced about the equator and for the circular test, a line is traced from pole to pole intersecting the equator at 45° of linear polarization. The experiment was conducted with the source intensity at its highest and an exposure long enough to ensure a very high signal-noise-ratio, \( \sim 1000 \), so high that the error bars on the measurements do not show on the scales displayed.

Figures 4.14 to 4.17 shows the output angles and degrees of polarization for both “linear” and “circular” tests with the residuals, the difference from the theoretical, plotted along side them. In figure 4.14(a), we see that the experimental matches the theoretical apart from the points at the beginning of the linear test. This is because the polarimetry data reduction yields results of \( \theta \) for -90° to 90°, while our theoretical data ignores this wrap around. The residuals, on average, have a error range of ±0.5°, which is approximately 2.5 times that of the stabil-
ity tests in section 4.1.2 and 4.1.3. We also note that the residuals appear more systematic than random, which was not expected. What is expected is the discontinuity of the circular test in figure 4.14(b). As the circular test approaches 100% circular the error on the polarization angle is high and can go off the scale, explained later in chapter 5.

Figure 4.14: Verification test for the measurement of polarization angle ($\theta$).

Figure 4.15(a) again shows a good match of the relative phase angle between the theoretical and experimental for both linear and circular tests. However, in figure 4.15(b) we again see a systematic error in the residuals. The linear test is showing what appears to be a sinusoidal systematic error with an average offset of $0.4^\circ$ from the theoretical. The standard deviation over the entire linear test is only about $\pm 0.06^\circ$, while the deviation of the circular test is $\pm 0.3^\circ$.

Figure 4.15: Verification test for the measurement of relative phase angle ($\phi$).

The measurement of the degrees of polarization for the linear test as shown in figure 4.16(a) matches up to the theory, but on this scale the error can be seen. The degree of polarization (DOP) and degree of linear (DOL) overlap and cannot be distinguished from each other as the input vectors are 100% linearly polarized.
The residuals in figure 4.16(b) show the systematic error that is more obvious than in previous tests and an average offset of $\sim 0.7\%$ for the DOL and DOP and $\sim 1.35\%$ for the degree of circular (DOC). These offsets may be explained by the presence of stress birefringence within the optical elements of the instrument and the theoretical data did not account for this. The standard deviation of the residuals are $\pm 0.67\%$ for DOL and DOP and $\pm 0.2\%$ for DOC. This again is within the range of the 1% error from the consistency tests and is less than our desired tolerance. If the offset can be accounted for, then GASP’s performance can be deemed satisfactory, but the full range of the deviation or $\sim 2.5\%$ is a concern.

![Figure 4.16: Verification test for the measurement of degrees of polarization measurement : Linear test.](image)

The measurement of the degrees of polarization for the circular test as shown in figure 4.17(a) also matches with the theory, but the residuals are greater (figure 4.17(b)) and show a larger range of $\sim 8\%$ compared to $\sim 2\%$ in the linear test. The standard deviations of the residuals of DOP, DOL and DOC are 2%, 1.8% and 2.4% respectively, while the average offsets are 0.73%, 0.23% and 1.44%, respectively. The standard deviations are much larger than our desired tolerance, but the offsets are close to it. The magnitudes of these errors suggest that GASP is more sensitive to circular errors than linear or else the calibration of this test was slightly in error.

The non-random systematic error in figures 4.14 to 4.17 stood out, even though some of the errors were within range of our desired tolerance an explanation for them is still needed. As GASP is an imaging polarimeter, we stated earlier that the calibration must be completed for the same image location as the polarimetry. If the calibration was in a different position to the polarimetry, what would the effect be? To test this, we conducted a simulation of the verification test whereby the angle of incidence for the generation of the data differed by 0.05° in $\theta_i$ on the RBS for the polarimetry.
Figure 4.17: Verification test for the measurement of degrees of polarization measurement: Circular test.

Figures 4.18 and 4.19 show the residuals of this simulation drawn on the same scales as the experimental data in figures 4.14 to 4.17. We immediately see the systematic sinusoidal errors witnessed in the experimental data. In figure 4.18(a), we can spot the match in the theoretical residuals of the circular test with the discontinuity occurring at approximately the same angle. The match of the theoretical residual to the linear test in figure 4.18(a) is not as obvious, but the magnitude of the error are quiet similar.

Figure 4.18: Theoretical residuals of angles of polarization for misaligned calibration.

Figure 4.18(b) shows the sinusoidal match of the linear test that was initially spotted in figure 4.15(b). The magnitude also matches, but no offset is visible suggesting that there was some birefringence in some of the optics. A match of the residuals of the circular test can not be discerned, some of the discontinuities in this data may match that of the experimental data, but it is not stated with confidence.
4.2 Laboratory Results

A comparison of figure 4.19(a) and 4.19(b) to figures 4.16(b) and 4.17(b) respectively does not allow us to justify a match. While the sinusoidal feature of the DOC between figure 4.19(a) and 4.16(b) match, the DOP and DOL do not match the function, but the magnitudes of these errors appear to be similar. Figure 4.19(b) does not resemble figure 4.17(b) with the magnitudes of errors a factor of 3 to 6 off. The discontinuity of the DOL may be correlated as they both occur at \( \sim 90^{\circ} \) otherwise no match can be made.

![Graphs showing residuals for degrees of polarization for linear and circular tests](image1.png)

Figure 4.19: Theoretical residuals of degrees of polarization for a misaligned calibration.

In summary, we can explain why the systematic residuals occur for the experimental data, the calibration took place at a different image location for the polarimeter. If we look carefully, we can see some random errors in comparing these experimental and theoretical residuals. For example, in figure 4.16(b) the DOC residual should be a clean sinusoid as per the theory of figure 4.19(a) and it is the presence of random errors that could distort the match. Such random errors could be a result in the positional error when rotating the polarizer of the laboratory telescope. The fact that the experimental residuals did not match that of the theoretical is due to the difference in the computational model of the system matrix compared to the experimental system matrix. The offset in RBS angle of incidence between the acquisition of the data and the polarimetry was not exactly known in the experimental. So the offset of 0.05\(^{\circ}\) used only gave an approximation of the theoretical residuals. The exact location of the object of interest in the image plane should be noted and then calibrated for later on. Thus care should be made when calibrating the polarimeter. Later in section 4.2.3 we see how the polarimetric errors are a function of the field of view (or location on image plane).
4.2.3 Field of View Analysis

In section 2.4.1 we determined that the polarimetric field of view was based on a limit of the inverse condition number of the system matrix. This limit of 0.3 set the field of view of GASP’s RBS to $\sim 8^\circ$. However, due to further optical development (section 2.5.4) and resolution of issues such as the Wollaston dispersion (section 2.5.5) and the pupil astigmatism (section 2.5.3), the RBS’s field of view was reduced to approximately $2.75^\circ$ (details of this setup is in section 4.3).

Figure 4.20(a) shows the updated theoretical plot of inverse condition number against RBS angle of incidence, which was used to determine the field of view earlier in section 2.4. A comparison of this figure with figure 2.14 shows no distinct difference apart from where the peak of the inverse condition number occurs in relation to the reflectance and transmittance lines. Figure 4.20(a) shows that the point at were reflectance equals transmittance ($\sim 78^\circ$) no longer coincides with the peak inverse condition number ($\sim 77^\circ$), which is most likely due to the attenuations caused by mirrors in the system. The condition that reflectance equals transmittance was used to optimize the prism design in section 2.3.1 and was then used to align the angle of the incidence of the RBS on GASP and set as the centre of the field of view.

Figure 4.20(b) shows the experimental measurements of the inverse condition over the full $2.75^\circ$ field as highlighted in figure 4.20(a). We can see that there is no correlation to the function only that the inverse condition numbers are within the expected range. This does not show any field dependence and to resolve this, analysis used in section 4.1.2 was applied to this data. It is only if the response for the entire detector was flat, i.e. flat fielded, that the experimental data and theoretical curves in figure 4.20(b) should match.

Figure 4.20: Inverse condition number vs Angle of incidence for GASP.
4.2 Laboratory Results

Figure 4.21(a) shows a plot of the difference in the system matrix elements to the central field position as a function of the field position and we can see a linear change of the elements when departing from the central field position. This then shows that the system matrices are field dependent as expected. Plotted on the same scale, figure 4.21(b) shows the independence of the PSG to the field position.

![Graph showing field analysis of system matrix and PSG](image)

Figure 4.21: Field analysis of the system matrix and PSG showing how the system matrix is dependent on field position, while the PSG is not.

To understand what this change in the system matrix as function of the field of view means to the polarimetry, we perform some pseudo polarimetry. This is done by mathematically passing a Stokes vector through the system matrix of the central field position, taking its output intensity vector and then measuring the polarization of that intensity vector with all the other system matrices. As said before, the sensitivity of each measurement is dependent on different stokes vectors so we tested the field dependence for 4 vectors: 100% linear at 45°, 90° and 100% and 10% left hand circularly polarized (for better visualisation of the data). At the same time, we will conduct the same field analysis for theoretical matrices and compare to the experimental data. These results are shown in figures 4.22 and 4.23.

The results from figure 4.22 and 4.23 clearly show that the polarimetry and calibration must take place within a certain area. Such limitations lead to a calibration field of view, that is, an object can wander by a certain amount to be
Figure 4.22: Pseudo polarimetry field analysis: plot of angles and degrees of polarization as a function of field position for linear polarization theoretical (lines) and experimental(points).

Figure 4.23: Pseudo polarimetry field analysis: plot of angles and degrees of polarization as a function of field position for circular polarization theoretical (lines) and experimental(points).

within a particular tolerance. We will not go into great detail on this, but solely looking at the DOP, a 1% error would come from an angle of incidence wander >0.1° or >8.5 pixels for linear polarizations and >0.4° >34 pixels for circular polarizations. In the following section, we shall talk about the astronomical results when GASP is on a telescope and the corresponding wander in the image that would have a 1% DOP error are >1" for linear polarizations and >3.7" for circular polarizations. A 1" limit would leave the linear polarization measurements susceptible to astronomical seeing.
GASP’s astronomical trials were conducted on the G.D. Cassini telescope, Italy. It is an F/8, 1.52m Ritchey-Chretien telescope that belongs to the Astronomical Observatory of Bologna located in Loiano, Italy. The setup shown in figure 2.25 in section 2.5.4 was used for our observations with the four channels arranged in a 2x2 fashion. Even though it was recommended to use a 13.6° Wollaston prism as detailed in section 2.5.5, a 5° Wollaston was used to avoid effects due to dispersion from the Wollaston. This severely limited the field of view (FOV) from a maximum available 79.9” to 28.6”. Since we are using a 5° Wollaston and an image separation of 4.1mm (half the width of the CCD chip) is needed, a 46.7mm focal length imaging lens is required. An imaging lens of $F_{img}=50$mm was used and this restricted the RBS field angle to 4.69° so that no overlap between the two Wollaston beams could occur. The recollimator then magnified this angle by 0.588, which corresponded to an input RBS field angle of 2.76°. This was then passed through the RBS, which corresponded to a sky FOV of 28.6” when a collimating lens, $F_{img}=35$mm, was used. The size of the field was still acceptable, but on a bigger telescope this would be reduced to an unusable size of 2-3”. This arrangement lead to a plate scale of 0.11 ”/pixel, which is a high spatial sampling, the ideal plate scale would be 0.5”/pixel [96]. When the four channels were adjusted to a 4x1 arrangement, a 10° Wollaston was used, $F_{img}$ of 25mm was needed and thus the field size ended up being 28.9”. For fast frame transfer, the pixels were binned down to 32x32 per channel that gave a final plate scale of 0.9”/pixel.

To reduce the effect of background sky polarization from a lunar source, a requirement was made to have the observations planned around a new moon, which occurred on the 30th August 2008. The results presented came from observations between the 27th August and 1st September 2008 and we will be largely focusing on the measurements of the degrees of polarization.

The availability of astronomical objects that have polarization states changing in the order of seconds or less are rare. Those that are available are faint, > 16th magnitude, making them difficult to acquire using a 1.5m telescope. An attempt was made to observe the Crab pulsar at high-time resolution. Unknown to the user at the time of acquisition, operating the Andor Ixon camera at high gain and clocking frequencies in the Mhz range brought on the effect known as clock induced charge (CIC). This adds extra photoelectrons to the imaging and reduced the sensitivity of the detector in photon counting mode [97]. Thus, observing a
faint source at 250 frames per second with full gain and cooled to $-90^\circ$C, CIC noise dominated and the pulsar could not be seen at high-time resolution. However, we could still perform tests on other targets and this included rotation and stability tests on various polarized and unpolarized objects.

After each of these observations, calibration data was taken for the location on the image for which the object was recorded. This attempted to ensure that the resulting polarimetry was as accurate as possible. However, in the interest of stability of the polarimeter, the calibration may not have occurred directly after the observation, but no more than three hours from the start of the observation.

Exposures of each observation used a Sloan R, G or I filter or no filter. Objects for polarimetric analysis had a typical exposure time of 100ms, higher gains were used to increase the SNR and failing that longer exposures were used to ensure a desirable SNR of $\sim 50$. These short exposures were recorded sequentially then stacked into a single FITS $^2$ data cube and this type of acquisition is referred to as a kinetic exposure. Kinetic exposures ensured that any interruption while recording a data set could be excluded from the analysis. Such interruptions could involve poor tracking, loss of signal due to clouds or abrupt cessation of recording data. Because of the poor seeing and telescope jitter, a stacked image would give a poor image to extract intensities. Each frame would then have its intensity extracted and yielded the polarization result, which could later be summed.

### 4.3.1 Calibration

As stated previously in section 3.5, further calibrations were needed when GASP was on a telescope, which would correct for the induced telescope polarization and possible polarization effects from the instrument/telescope combination. This was done by observing a standard zero polarization star, recording the induced polarization, then observing a standard polarized star and verifying that this subtraction gave the expected result. Table 4.2 [91] shows five standard polarization stars used with their degrees of polarization for different filters. These filters are Cousins R and I and Johnson V whose full width half max (FWHM) and effective wavelength ($\lambda_{\text{eff}}$) are shown in table 4.3, which also lists the properties of the Sloan G, R and I filters used on GASP. A visual comparison of these filters are shown in figure 4.24.

Table 4.2 does not show the DOPs for the zero polarizing standards for R

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$^2$Flexible Image Transport System, a standard astronomical file format.
4.3 Astronomical Results: Loiano

<table>
<thead>
<tr>
<th>Object</th>
<th>V Filter</th>
<th>R Filter</th>
<th>I Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD +32 3739</td>
<td>0.025 ± 0.02</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HD 212311</td>
<td>0.034 ± 0.02</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HD 204827</td>
<td>5.32 ± 0.01</td>
<td>4.89 ± 0.03</td>
<td>4.19 ± 0.03</td>
</tr>
<tr>
<td>HD 236928</td>
<td>6.70 ± 0.02</td>
<td>6.43 ± 0.02</td>
<td>5.80 ± 0.02</td>
</tr>
<tr>
<td>CRL 2688[88]</td>
<td>44.9 ± 0.3</td>
<td>49.9 ± 0.4</td>
<td>49.0 ± 0.5</td>
</tr>
</tbody>
</table>

Table 4.2: Expected DOPs for five polarization calibration stars for each filter.

<table>
<thead>
<tr>
<th>Filter</th>
<th>(\lambda_{eff}) (nm)</th>
<th>FWHM (nm)</th>
<th>Filter</th>
<th>(\lambda_{eff}) (nm)</th>
<th>FWHM (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Johnson V</td>
<td>546.5</td>
<td>87</td>
<td>Sloan G</td>
<td>480</td>
<td>150</td>
</tr>
<tr>
<td>Cousins R</td>
<td>760</td>
<td>250</td>
<td>Sloan R</td>
<td>620</td>
<td>129</td>
</tr>
<tr>
<td>Cousins I</td>
<td>800</td>
<td>150</td>
<td>Sloan I</td>
<td>767</td>
<td>143</td>
</tr>
</tbody>
</table>

Table 4.3: Effective wavelengths (\(\lambda_{eff}\)) and bandwidths (FWHM) of selected astronomical filters.

and I, but one would expect for these filters that there may be a slight change of about 0.02%. This change is expected to be so small that they remain a zero polarization standard for other filters. Tables 4.4 and 4.5 present the results of calibration observations on nights of 27\(^{th}\) and 30\(^{th}\) of August 2008.

In table 4.4, we can see that the instrumental polarization is about 8%. We expect this to be in the order of 1% as is reported for other polarimeters [98][59]. The reason for such a high instrumental polarization is not clear, but an investigation later with a rotation test may yield some answers. The measured polarization from HD 204827 was initially questioned, but the resulting corrected polarization puts it \(\sim 1\)% from the expected values. This 1% error from the expected result could be due to the calibration accuracy, but it is near the accuracy that we wish to achieve for GASP leaving it some what tolerable.

<table>
<thead>
<tr>
<th>Object</th>
<th>%</th>
<th>NO filter</th>
<th>G filter</th>
<th>R filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD +32 3739</td>
<td>DOP</td>
<td>7.83 ± 0.01</td>
<td>8.03 ± 0.03</td>
<td>9.22 ± 0.02</td>
</tr>
<tr>
<td>(unpolarized)</td>
<td>DOC</td>
<td>-1.38 ± 0.01</td>
<td>-2.14 ± 0.03</td>
<td>-1.38 ± 0.02</td>
</tr>
<tr>
<td>(standard)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HD 204827</td>
<td>DOP</td>
<td>12.745 ± 0.006</td>
<td>12.25 ± 0.02</td>
<td>15.07 ± 0.01</td>
</tr>
<tr>
<td>(polarized)</td>
<td>DOL</td>
<td>12.456 ± 0.006</td>
<td>11.89 ± 0.02</td>
<td>15.03 ± 0.01</td>
</tr>
<tr>
<td>(standard)</td>
<td>DOC</td>
<td>-2.699 ± 0.006</td>
<td>-2.96 ± 0.02</td>
<td>-1.06 ± 0.01</td>
</tr>
<tr>
<td>HD 204827</td>
<td>DOP</td>
<td>4.92 ± 0.01</td>
<td>4.22 ± 0.04</td>
<td>5.85 ± 0.02</td>
</tr>
<tr>
<td>(corrected)</td>
<td>DOL</td>
<td>4.75 ± 0.01</td>
<td>4.14 ± 0.04</td>
<td>5.92 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>DOC</td>
<td>-1.32 ± 0.01</td>
<td>-0.82 ± 0.04</td>
<td>0.32 ± 0.02</td>
</tr>
</tbody>
</table>

Table 4.4: Experimental results from calibration stars. Date: 27/08/2008

The results of observations on 29\(^{th}\), shown in table 4.5, again show a high instrumental polarization and at a similar level to the observations on the 27\(^{th}\),
but higher by about 0.5 - 2%. The corrected observations for HD 236928 then show a larger deviation from the expected value, with a difference of 0.58% in the G band and 3% in the I band. Such large errors could show an issue with the chromatic calibration, calibration at different filters, either due to GASP or the telescope. Since the largest error is in the G filter, we can also suggest that the mismatch between the Sloan G and Johnson V also adds to this polarization error.

Table 4.5: Experimental results from calibration stars. Date: 29/08/2008

The change in telescope polarization from night to night was investigated and table 4.6 shows the instrumental polarization between the 28th and 29th using HD 212311 as our zero polarization standard. The instrumental polarization may be different from filter to filter, but the change from night to night remained at ∼ 4%, at least a consistency for filters, but the change from one night to the other raises questions. When a polarized calibration star (HD 236928) was observed with a G filter on the night of the 28th, the results remain close to expected with DOP=5.436 ± 0.029, DOL=5.817 ± 0.038 and DOC=−0.264 ± 0.039, independent of the large change in instrumental polarization. The possible reason for the high instrumental polarization in the first place is not likely due to the telescope
and more so the calibration/acquisition location of the data. The reasons for the change in the instrument polarization from one night to another could be due to many factors. Environmental changes of the telescope and instrument might have a role to play, but such a change would not be in the order of 4%. Even though the observations were planned for a moonless sky, the site is not a dark site and scattering of distant urban lighting and nearby rural lighting might add to the polarization and a change in the lighting conditions from night to night may correlate. However, it is most likely again due to the calibration/acquisition location of the data as they would change from night to night.

<table>
<thead>
<tr>
<th>Night</th>
<th>Filter</th>
<th>DOP [%]</th>
<th>DOL [%]</th>
<th>DOC [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>28th</td>
<td>G</td>
<td>6.672 ± 0.013</td>
<td>6.103 ± 0.013</td>
<td>-2.700 ± 0.012</td>
</tr>
<tr>
<td>29th</td>
<td>G</td>
<td>10.225 ± 0.006</td>
<td>10.134 ± 0.006</td>
<td>-1.366 ± 0.006</td>
</tr>
<tr>
<td>Change</td>
<td></td>
<td>3.553</td>
<td>4.031</td>
<td>1.331</td>
</tr>
<tr>
<td>28th</td>
<td>R</td>
<td>5.123 ± 0.013</td>
<td>4.756 ± 0.013</td>
<td>-1.904 ± 0.012</td>
</tr>
<tr>
<td>29th</td>
<td>R</td>
<td>8.965 ± 0.006</td>
<td>8.749 ± 0.006</td>
<td>-1.954 ± 0.006</td>
</tr>
<tr>
<td>Change</td>
<td></td>
<td>3.841</td>
<td>3.993</td>
<td>-0.050</td>
</tr>
<tr>
<td>28th</td>
<td>I</td>
<td>4.202 ± 0.023</td>
<td>4.104 ± 0.023</td>
<td>-0.901 ± 0.020</td>
</tr>
<tr>
<td>29th</td>
<td>I</td>
<td>8.52 ± 0.01</td>
<td>8.49 ± 0.01</td>
<td>-0.64 ± 0.01</td>
</tr>
<tr>
<td>Change</td>
<td></td>
<td>4.318</td>
<td>4.386</td>
<td>0.261</td>
</tr>
<tr>
<td>28th</td>
<td>None</td>
<td>6.459 ± 0.008</td>
<td>5.759 ± 0.008</td>
<td>-2.924 ± 0.007</td>
</tr>
<tr>
<td>29th</td>
<td>None</td>
<td>10.621 ± 0.004</td>
<td>10.416 ± 0.004</td>
<td>-2.080 ± 0.003</td>
</tr>
<tr>
<td>Change</td>
<td></td>
<td>4.162</td>
<td>4.656</td>
<td>0.844</td>
</tr>
</tbody>
</table>

Table 4.6: comparison of instrumental polarization between nights of the 28th and 29th of August when observing HD 212311

It is difficult to comment on and compare the circular polarizations from our calibration stars as only the linear polarizations of the standards are quoted. The corrected circular polarization of our polarized targets are on the order of -1 to 1%, while the instrumental circular polarizations are in the order of -2 to -.05%. The amount of instrumental circular polarization is substantially less than the linear polarization, but a reasonable explanation may come from an inaccurate calibration as we saw some similar issues with circular polarization in the laboratory test in section 4.2.2. Later in section 4.3.3 we conduct rotation tests on these targets to get a better understanding of the instrumental and intrinsic stellar circular polarisation.

One final contemplation of our results, looking at table 4.5 we notice some wavelength dependence for our targets. The DOP for HD 212311 decreased with increasing wavelength, while the DOP for HD 236928 increased. For the quoted polarization standards in table 4.2, we see that the DOP decreases with increasing wavelength and this is found to be in line with "Serkowskis’s law" [99]. This
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generalises that due to interstellar dust the DOP rises from the ultraviolet to peak in the visible and then fall to the near infrared. We would expect that the unpolarized stars have no wavelength dependence as they are close to us and would not be affected by the interstellar dust, yet from our measurements they do. Such an occurrence leads to re-evaluate the filters and source we used in the calibration and the observation. It is difficult to make a truly achromatic polarimeter where a flat spectral response for all components and sources is needed. Our calibration source was tungsten incandescent lamp (emits more in the red), the camera used was more sensitive in the blue and other components could have different spectral characteristics. The combination of these spectral responses in tandem with the filters could give us the differing polarization measurements of a target at different filters as we have seen.

4.3.2 Stability Tests

As performed in the laboratory, we looked at the polarimetric stability of GASP while observing astronomical targets. We looked first at low and unpolarized targets and then on highly polarized targets. We investigated how the stability of astronomical targets compares over time. Unfortunately, it was not practical to conduct such a test of hours, but the following tests looked at the stability in the order of 5 minutes with 100ms frames or else otherwise stated.

4.3.2.1 Low and unpolarized targets

In the previous section, we saw GASP perform measurements of DOP with accuracies around 1% and getting as bad as 3% either due to the filters, induced polarization or just poor calibration. We now ask the question of how much does the intra-measurement stability of these measurements have a role to play.

Figures 4.25(a) and 4.25(b) show the stability of a low polarized and an unpolarized source, respectively. These are 300s kinetic exposures with 100ms per acquisition acquired using Sloan G filter, summed up to 1s exposures. Both data sets appear to be stable within the 300s and fluctuations are slightly larger than the error bars for BD +32 3739. HD 204827 has fluctuations significantly larger than the error bars, around the 50-100s mark, showing evidence of a possible variability. On looking into this, HD 2048728’s polarization was found to vary on a scale of months [100] [101], so the change may be due to astronomical seeing and the extraction of intensities during the aperture photometry or even intrinsic to
the star itself.

![Image](a) HD 204827: 5.32% polarized (b) +BD 32 3739: Unpolarized

Figure 4.25: Stability of degrees of polarization for low/unpolarized standard stars.

<table>
<thead>
<tr>
<th>Target</th>
<th>DOP [%]</th>
<th>DOL [%]</th>
<th>DOC [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 204827</td>
<td>12.37 ± 1.73%</td>
<td>12.01 ± 1.6%,</td>
<td>−2.96 ± 1.03%</td>
</tr>
<tr>
<td>BD +32 3739</td>
<td>8.78 ± 2.67%</td>
<td>8.28 ± 2.55%</td>
<td>−2.14 ± 2.13%</td>
</tr>
</tbody>
</table>

Table 4.7: The mean and standard deviations of the datasets found in figure 4.25

The mean and standard deviations of the datasets are shown in table 4.7. We will notice that the results differ slightly for the DOP and DOL compared to that in table 4.4. This is because the result is taken from the average of all the DOPs in the dataset, while the results from table 4.4 are taken from a summation of the Stokes vector. To explain why this difference occurs, imagine two 100% linear stokes vectors at 0° and 45°. The average of the two DOPs would be 100%, but in reality these vectors must be summed then the DOP calculated is 70.7%.

![Image](a) HD 204827: 5.32% polarized (b) +BD 32 3739: Unpolarized

Figure 4.26: Stability of angles of polarization for low/unpolarized standard stars.
Figure 4.26 shows how the angles of polarization fluctuate for both targets and that in both the variation of the angle of polarization ($\theta$) is greater than the relative phase angle ($\phi$). Table 4.8 shows the means and standard deviations for both targets from both the dataset and summed data. The greater variation of $\theta$ signifies that the resulting mean of the dataset will give a DOP less than that of the summed results as is always the case. The amount of variation of $\phi$ in the dataset is not large enough to result in a lower DOC than the summed result, the precision is not high enough to show this.

<table>
<thead>
<tr>
<th>Target</th>
<th>$\theta$ [°]</th>
<th>$\phi$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 204827 dataset</td>
<td>$-56.34 \pm 4.37$</td>
<td>$-6.27 \pm 1.95$</td>
</tr>
<tr>
<td>HD 204827 summed</td>
<td>$-56.66 \pm 0.04$</td>
<td>$-6.99 \pm 0.04$</td>
</tr>
<tr>
<td>BD +32 3739 dataset</td>
<td>$102.22 \pm 12.10$</td>
<td>$-6.55 \pm 7.47$</td>
</tr>
<tr>
<td>BD +32 3739 summed</td>
<td>$102.4 \pm 0.1$</td>
<td>$-7.7 \pm 0.1$</td>
</tr>
</tbody>
</table>

Table 4.8: Results for the angle of polarization ($\theta$) and relative phase angle ($\phi$) for both summed and averaged data of HD204827 and BD +32 3739

Our laboratory tests showed a stable measurement of polarization with no abrupt changes, the changes where slow and drifted no more than 0.5% over hours. Seeing changes within HD 204827 of $\sim 5\%$ over 10s can be due to many possible reasons such as, the aperture photometry, seeing or even intrinsic variability in the star itself. Comparing the changes of polarization between the two targets over 5 minutes, BD +32 3739 do not show such a large change yet HD 204827 does. This may lead us to conclude that HD 204827 may have intrinsic variations on these short timescales. However, we will not rule out the fact that the seeing and shifting of the image has an effect later on down in the data processing that may make us think the target is changing. As we saw earlier in section 4.2.3, a change in the position of acquisition and calibration can lead to an inaccurate result.

4.3.2.2 Highly polarized targets

Highly polarized stars (DOP $> 10\%$) are few and far between. CRL 2688 has a degree of polarization of $\sim 50\%$ and is found as an HST polarimetric calibration target [90]. CRL 2688 was misread as a star, but after re-examination due to the large PSF of the object it was clarified that it was actually a reflection nebula, the Egg nebula, which explains the high DOP. Different portions of the nebula have differing degrees of polarization [88], so an aperture equivalent to $\sim 15''$ (14 pixel aperture) was used in the aperture photometry to make sure the entire nebulae was measured. The data presented for CRL 2688 was measured with
no filter with kinetic exposure of 5 minutes with 100ms acquisitions (Figures 4.27 to 4.29), then with the data binned for 1s acquisitions (figure 4.30) and 5s acquisitions (figure 4.31). The results for the summed data of CRL2688 are as follows, \( \theta = -0.967 \pm 0.024^\circ, \phi = -0.488 \pm 0.015^\circ, DOP = 59.102 \pm 0.037\%, \)
\( DOL = 59.093 \pm 0.037\% \) and \( DOC = -1.006 \pm 0.032\% \). When corrected for instrumental polarization we get \( DOP = 48.48 \pm 0.04\%, DOL = 48.68 \pm 0.04\% \) and \( DOC = -3.08 \pm 0.03\% \). This is \( \sim 1\% \) off the expected results, but a direct comparison cannot be made as this data was recoded with no filter.

![Figure 4.27: Stokes I of CRL2688 with 100ms exposures over 5 minutes.](image)

We have already seen in figure 4.10 that the laboratory polarization measurements are independent of the intensity fluctuations. Figure 4.27 shows how the intensity of CRL 2688 varies over the 5 minutes and comparing this to the polarimetric measurements in figure 4.28, we again see that while on the telescope, GASP measures polarization independently of changing intensity.

![Figure 4.28: Polarization results for CRL2688: 100ms exposures over 5 minutes.](image)

Figure 4.28 shows the degrees and angles of polarization without the error bars, but with the value of errors shown in figure 4.29. We can initially see that
the polarization measurements are quiet stable, but this can not be conclusively stated as the noise level is too high. What we also notice is how the error of the polarization measurement in figure 4.29 decreases by a factor of \( \sim 2 \) as the intensity increases. This is in keeping with the error measurement for which we will comment on later in chapter 5.

![Figure 4.29: Polarimetric errors for CRL2688 corresponding to figure 4.28.](image)

To investigate how stable the measurements are, we sum the data to 1s bins as shown in figure 4.30. This is the same binning as the low/unpolarized stability test above in figure 4.25 and figure 4.26 so we can compare like with like. CRL2688 shows deviations over this data set of 1.6% for DOP and DOL and 1.7% for DOC, which is on the order or better than the deviations seen for HD 204827 and BD +32 3739. However, CRL2688 appears to be more stable over this period in relation to the polarization angles with standard deviations of 1.18° for \( \theta \) and 0.85° for \( \phi \) compared to that seen in table 4.8 for the low/unpolarized targets. These lower standard deviations are in keeping with the fact that the error or the angles of polarization are inversely proportional to the degree of polarization, which we will see in chapter 5.

Finally, in figure 4.31, the data is binned to 5s, we see a standard deviation over the dataset of 0.89% for DOP and DOL and 0.71% for DOC, which would seem quiet stable. However, upon further inspection we can see some drift in figure 4.32 of about 3% from start to finish of the acquisition. This could be possibly due to a drift in the instrument itself as we have seen evidence of slow drift in the laboratory tests (section 4.2.1). However, this was a drift of 0.1% over 70 minutes, which means that there could be polarimetric drift elsewhere, possibly due to a change in the environmental conditions around the instrument.
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Figure 4.30: 1s data binning for the degrees and angles of polarization of CRL2688.

Figure 4.31: 5s data binning for the degrees and angles of polarization of CRL2688.

Figure 4.32: 5s data binning for the degrees of polarization of CRL2688 to show the change in polarization at the start of recording as in figure 4.31.

4.3.3 Rotation tests

In section 4.2.2 we conducted verification tests where we alter an input polarization state and then compare it to the measured results afterwards. Naturally, we can
not modify the polarization of the star, but on the Loiano telescope there was the ability to rotate the instrument about the optical axis of the telescope, known as a derotator. This meant that we could rotate the instrument with respect to the sky and the angle of polarization should also appear to rotate. We expect to see similar results as in figures 4.14 to 4.16 where we can only change the angle of polarization.

Using CRL 2688 as the polarized object we rotated the derotator in steps of what ended up being 8.451°. This angle came about as there was no scale was on the derotator and 213 turns of the wormwheel controlling the derotator equalled one full rotation. Hence the experiments were done in steps of 5 turns, which equalled 8.451°. One of the greatest difficulties in doing this experiment was that GASP’s optical axis was not co-axial to the optical axis of the telescope. This meant that for every change in the rotation position, the location of the target on the detector had to be repositioned to the same point after each change. This then entailed turning off the tracking, repositioning the telescope then re-engaging the tracking. This made acquisition of fainter targets much more difficult to obtain. Each acquisition of CRL 2688 was a 150 x 0.1s kinetic exposure with maximum gain yielding an SNR of 700.

![Graphs](image)

Figure 4.33: Polarization angles θ and φ for the rotation test of CRL2688. Error bars not visible, but are θ=±0.44°, φ=±0.06°.

Figure 4.33 shows the polarization angles θ and φ as a function of the rotation of GASP. We immediately see the sinusoidal nature of the residuals, clearly shown in figure 4.33(b). This error is synonymous to the error in residuals seen in section 4.2.2 and its reasons were explained therein. However, the level is error is much greater and reduces our precision to ±5°. It is most likely larger because the acquisition and calibration point were a greater distance away on the image.
Figure 4.34 shows the output degrees of polarization for the rotation tests and a very large amount of error is visible. The DOP and DOL goes from 59% at 0° to 46% at 90°, while the DOC continues to have sinusoidal nature with an average of 0.05% and range of ±4% over the entire rotation. Both DOP and DOC should be unchanged as a function of rotation and the error can be due to calibration as stated in section 4.2.2. Earlier in figure 4.19(a) we saw a similar effect of this misaligned calibration. The DOP residuals were 3 times greater than the DOC residuals and now in figure 4.34 a similar ratio is seen in relative range in errors, 13% for DOP 4% for DOC. This yields some agreement to the cause of the errors where it is due to the misalignment of the target and calibration location.

It was a coincidence that the measurement of CRL 2688 was at the rotational alignment of 0°, which lead to an agreement with GASPs DOP measurement to that which was expected. For such result to be valid at any other instrument orientation, the instrumental polarization must also change as a function of rotation and a rotation test of an unpolarized star might show this. Figures 4.35(a) and 4.35(b) show the degrees and angles of polarization respectively for the rotation test of an unpolarized target, BD +32 3739. The angles of polarization follow an expected trend up to 120° where the data becomes erroneous. This is most likely due to a truncation of one of the targets’ channels when the data was being recorded. From figure 4.34, we would expect the instrumental DOP to decrease by 13% from 0° to 90°. The degree of polarization of figure 4.35(b) does show the same downward trend at the same rotation angles, but reaches a minimum around 60° then rises again after 90°. The DOP after 60° may have gone negative, but this trend can not go negative as DOP is never negative due to how it is calculated. What is interesting is the minimum instrumental polarization is ∼ 1%, that which would have been expected earlier on and raises a question about the
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telescope/instrument combination.

![Graph of polarization angles](image1)

(a) Angles of polarization

![Graph of polarization degrees](image2)

(b) Degrees of polarization

Figure 4.35: Rotation test for unpolarized target BD +32 3739.

Unfortunately, the rotation test of BD +32 3739 was completed on a different night to that of the rotation test of CRL2688. As we found earlier, the instrumental polarization changed from night to night so we cannot adjust the CRL 2688 rotation data with the data from BD +32 3739. However, figure 4.36(a) shows the unfiltered rotation test of HD 204827 (∼5% polarized), which was conducted on the same night as the rotation test of BD +32 3739. It too shows how the DOP drops as function of rotation angle and reaching a minimum at ∼60°, but immediately rises again. Figure 4.36(b) then shows the corrected degrees of polarization (noting that the range is from 0° to 120°) and initially we can see that the function of DOP against the rotation angle is gone, but the DOP is not as static as expected. At several points on figure 4.36(b) between 0° and 50° the corrected results is ∼5% as expected, where it is not near 5% is most likely due to the mismatch calibration/acquisition location, which was a major problem for these rotation tests.

In the rotation tests, the DOC for CRL2688 (figure 4.34) had an average of 0.05% where the sinusoidal nature is explained. The DOC for BD +32 3739 (unpolarized target) remained relatively unchanged at about 0.5%, which is expected, but the DOC for HD 204827 (polarized target) decreased linearly up to about 90° then a smaller rise then ensued. The fact that there is no visible sinusoidal nature due to the attempt to match the calibration/acquisition location, but obviously not good enough. As to the fact of trying to see if the star had intrinsic circular polarization, the average of the DOC over a 180° rotation would cancel out the instrumental DOC and show only the stars circular polarization and from this we could say that CRL 2688 has a possible DOC of 0.05%. However, we can not show that the instrument was accurate to this level thus this result can not be trusted.
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Figure 4.36: Degrees of polarization for rotation test of HD 204827 and corrected for instrumental polarization using BD +32 3739.

One final point must be made about these rotation tests. The rotation of the instrument about the telescopes’ axis can add other polarimetric effects as the telescope/instrument combination is always changing, similar to the rotation of a polarizer and a quarter waveplate. A true rotation test would involve rotation of the telescope and instrument relative to the sky. This is not possible on an equatorially mounted telescope, but if one was to use an altazimuth mounted telescope the target would rotate relative to the telescope and instrument as it was observed over an entire night. Such a test would show constant degrees of polarization against rotation that would be expected.

Overall, one of the major points to take from these results is the sensitivity to the field position. This was evident not only on the telescope, but in the laboratory where conditions are controlled. Such sensitivity to the field position can be said to invalidate the astronomical results even though we got some reasonable results. We will comment on the possible solutions in the final chapter. The stability of the polarimeter within short time scale of minutes appeared satisfactory in both the laboratory and astronomical results especially its independence with the changing intensity of the source. However, there was the presence of some polarimetric drifts in the results, which we attribute to environmental changes and will discuss the more in the final chapter. The error bars of these measurements were calculate for each exposure not from the statistics of the dataset and the next chapter details how these errors were calculated.
Chapter 5

Error Analysis
In this chapter we will look at how the errors of the laboratory and astronomical results in chapter 4 were calculated. We will describe the propagation of errors for the division of amplitude polarimeters (DOAP), namely the Compain and Drevillon DOAP and the Galway Astronomical Stokes Polarimeter (GASP). We will see how the errors in the measurement of the intensities propagate through to the data reduction of the Stokes vectors, which will lead us to see how individual errors are measured. We will then compare errors from a theoretical system matrix to that of laboratory measurements. Finally, for measurements that are not possible in the laboratory we will simulate them with a Monte Carlo analysis.

5.1 Polarimetry Error Metrics and Error Propagation

Earlier in section 2.4 we defined the inverse condition number \(1/S(A)\) as a metric to determine the field of view of the polarimeter. This was then measured in the laboratory, as shown in section 4.2.3, but it was at this point that the metric became difficult to relate to the polarimetric errors.

There is a relationship between the inverse condition number of the system matrix and its sensitivity, but this is dependent on how well conditioned it is. If we have a system with a small perturbation of the intensity vector, \(\delta I\), then a unique solution exists (equation 5.1) to find the perturbation of the Stokes vector, \(\delta S\), which we hope is also small in comparison to \(\delta I\).

\[
A^{-1}(I + \delta I) = S + \delta S
\]  
(5.1)

where \(A\) is the system matrix \(S\) and \(I\) are the Stokes and intensity vectors respectively and \(\delta S\) and \(\delta I\) are the respective perturbations.

It has been stated [102] that the relationship between condition number and sensitivity is as follows

\[
\frac{|\delta I|}{|I|} \leq S(A) \frac{|\delta S|}{|S|}
\]  
(5.2)

where \(| \cdot |\) is the vector norm and \(S(A)\) is the condition number of the system matrix \(A\). Rephrasing this for our convention of the inverse condition number we get

\[
\frac{1}{S(A)} \frac{|\delta I|}{|I|} \leq \frac{|\delta S|}{|S|}
\]  
(5.3)

and if a system matrix is well conditioned then small values of \(|\delta I|/|I|\) imply small
5.1 Polarimetry Error Metrics and Error Propagation

values of $|\delta S|/|S|$. This makes the inverse condition number a useful metric for sensitivity. However, if a system matrix is ill-conditioned, it cannot be guaranteed that a small values of $|\delta I|/|I|$ can lead to small values of $|\delta S|/|S|$. From this, we do not know what value of an inverse condition number will lead to our limiting tolerance and as shown later in section 5.5 the same condition number can have two different sensitivities. This leads us to evaluate the sensitivity of the polarimeter based on the data reduction of Stokes vectors not the inverse condition number.

To assess the errors of any polarization state using DOAP polarimetry we could use the standard deviation from a dataset, as we did in section 4.3.2.1, but this itself can give an improper estimate of the error as exemplified in section 4.3.2.1. We need to know what the polarization error of a single acquisition is. Section 3.2 has already described how to measure the intensities from each of the four channels using aperture photometry and their associated errors, the background signal. However, this is difficult to theoretically model due to not knowing what other external factors can contribute to the intensity errors. To simplify the model, an assumption is made that all read-noises are much lower than the signal, and that the intensities are thus Poisson limited, that is to say that their error is the square root of the mean photon count of each channel.

The matrix product of $A^{-1}I$ may give the resultant Stokes vector, but the error of that Stokes vector ($\sigma_S$) is calculated by equation 5.4 or 5.5 [80]

\[ \sigma^2_S = \sum_{i=1}^{4} \sigma^2_{I_i} (A^{-1}_{ki})^2 \]  

or

\[ \sigma^2_S = (A^{-1})^{*2} \sigma^2_I \]  

where $\sigma_{S_k}$ and $\sigma_{I}$ is variance of the $k^{th}$ elements of the Stokes and intensity vectors respectively and $X^{*2}$ is the square of each element not the square of the matrix or vector.

Equations 1.3 to 1.7 described how to reduce the Stokes vector to the angles and degrees of polarization. Utilising equation 5.6, a general function of several variables,

\[ z = f(x, y, \ldots) \]  

the variances of each of variable is derived from the error equation 5.7.

\[ \sigma^2_z = \left( \frac{\partial z}{\partial x} \right)^2 \sigma^2_x + \left( \frac{\partial z}{\partial y} \right)^2 \sigma^2_y + \ldots \]  

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where $\partial z/\partial x$, $\partial z/\partial y$ are the partial derivatives of $z$ with respect to $x$ and $y$ and $\sigma^2_x$ and $\sigma^2_y$ are the variances of $x$ and $y$. Applying this to equations 1.3 to 1.7 we get the following for the variances of the angles and degrees of polarization:

$$\sigma^2_\theta = \frac{\sigma^2_Q U^2 + \sigma^2_U Q^2}{4 (Q^2 + U^2)^2}$$ (5.8)

$$\sigma^2_\phi = \frac{\sigma^2_V (Q^2 + U^2)^2 + \sigma^2_Q V^2 Q^2 + \sigma^2_U V^2 U^2}{4 (Q^2 + U^2) (Q^2 + U^2 + V^2)^2}$$ (5.9)

$$\sigma^2_{DOP} = \frac{\sigma^2_Q I^2 Q^2 + \sigma^2_U I^2 U^2 + \sigma^2_V I^2 V^2 + \sigma^2_I (Q^2 + U^2 + V^2)^2}{I^4 (Q^2 + U^2 + V^2)^2}$$ (5.10)

$$\sigma^2_{DOL} = \frac{\sigma^2_Q I^2 Q^2 + \sigma^2_U I^2 U^2 + \sigma^2_I (Q^2 + U^2)^2}{I^4 (Q^2 + U^2)^2}$$ (5.11)

$$\sigma^2_{DOC} = \frac{\sigma^2_I I^2 + \sigma^2_V V^2}{I^4}$$ (5.12)

where $\sigma^2_\theta$, $\sigma^2_\phi$, $\sigma^2_{DOP}$, $\sigma^2_{DOL}$, $\sigma^2_{DOC}$ are the variances for polarization angle $\theta$, relative phase angle $\phi$, degree of polarization (DOP), degree of linear (DOL) and degree of circular (DOC) respectively and where $\sigma^2_I$, $\sigma^2_Q$, $\sigma^2_U$, $\sigma^2_V$ are the variances for each Stokes parameter. From these equations we can see that the polarization errors are dependent on the Stokes vectors themselves, some more so than others. In the next section we investigate this in more detail.

### 5.2 Representation of Polarimetric Errors

The fact that the polarization errors are function of the Stokes vectors makes it difficult to visualise them. It was found that the best way to resolve this was to use the Poincaré sphere, as previously described in section 1.2.1, to display the errors. We can map the calculated error on to the surface of the sphere showing an error for each polarization state. We start with a theoretical DOAP system matrix ($A_{std}$) shown in equation 5.13 as generated from section 2.3.1, which is approximately the same as the Compain and Drevillon DOAP.

For this system matrix we look at the errors in DOP on the Poincaré sphere as shown in figure 5.1. This shows two views of the Poincaré sphere where ‘View 1’ is
5.2 Representation of Polarimetric Errors

looking down over the right hand circularly polarized hemisphere of the Poincaré sphere where the “equator” is drawn which represents the plane of linearly polarized light. A longitudinal line goes from the right hand circularly polarized “pole” and intersects the equator where the polarization state is 0° linearly polarized. ‘View 2’ looks over the antipodal point of the previous view where the intersection of the longitudinal line and the equator is visible as is left hand circularly polarized pole. This is the convention for all of the error projections on the Poincaré sphere for the other figures.

\[
A_{std} = \begin{bmatrix}
0.1812 & -0.1080 & 0.1455 & 0 \\
0.1812 & -0.1080 & -0.1455 & 0 \\
0.1812 & 0.1102 & -0.0001 & 0.1438 \\
0.1812 & 0.1102 & 0.0001 & -0.1438
\end{bmatrix}
\]

(5.13)

\[
\delta \sigma_x = \frac{R}{\sqrt{I}}
\]

(5.14)

Figure 5.1: Error in DOP resulting from a theoretical standard system matrix (equation 5.13) where the input polarization is 100% and intensity is 20,000 counts.

In this figure we see, as expected, that errors are a function of the Stokes vectors and quiet noticeably so in a graphical sense. These DOP errors range from 1.82% to 1.50%, a range of 0.32%, over the entire sphere for a source corresponding to a mean photon count of 20,000. This range may not seem large, but fainter and brighter objects will give differing error ranges and we can numerically calculate this relationship. When using 5,000 counts we get the exact same distribution of errors on the Poincaré sphere as seen in figure 5.1 whose DOP errors range from 3.64% to 3.00% and errors of 0.81% to 0.67% for 100,000 counts. This gives us the relationship between the errors shown in equation 5.14.
Error Analysis

where $\delta \sigma_x$ is the error range of any polarimetric results, $I$ is the mean photon count of the source and $R$ is a range constant related to the system matrix and type of polarimetric error being analysed. One might then say that an error change over the Poincaré sphere of 0.05% would translate to saying that the polarimetric error is independent of the Stokes vector. In the case above, we can then calculate the range constant $R$ to be $\sim 45$ and with this, to achieve an error range of 0.05% would require 810,000 counts for this current system matrix. We accept that such error analysis can be redundant, but mean photon counts in astronomy are low and this allows us to continue the error analysis.

We now want to look at the errors for our theoretical system matrix for GASP ($A_{\text{theo GASP}}$). Equation 5.15 shows the matrix used for this examination, while Figure 5.2 shows the errors in DOP for this system matrix.

\[
A_{\text{theo GASP}} = \begin{bmatrix}
0.1678 & -0.0861 & -0.1379 & -0.0416 \\
0.1678 & -0.0861 & 0.1379 & 0.0416 \\
0.1678 & 0.0736 & 0.0694 & -0.1339 \\
0.1678 & 0.0736 & -0.0694 & 0.1339
\end{bmatrix}
\] (5.15)

Figure 5.2: Error in DOP resulting from a theoretical GASP system matrix (equation 5.15) where the input polarization is 100% and intensity is 20,000 counts.

Figure 5.2 shows a marked difference between the DOP errors when using the standard system matrix and the GASP system matrix. The errors are higher, 1.57% - 2.03%, the range is larger, 0.46% suggesting that the standard matrix is more optimized than the GASP system matrix. The distribution of the errors for the GASP system matrix may appear to be on average lower than the standard system matrix, but this is not the case. Taking the mean error over the entire sphere from figure 5.2 we get 1.76%, while the mean from standard system matrix in figure 5.1 is 1.67%. This implies that an error for any random Stokes vector can
be slightly larger for the GASP system matrix compared to the standard system matrix. One advantage of the distribution of DOP errors for the GASP system matrix is that it can be tuned. If we were to orientate GASP so that the input Stokes vector appears to be $\sim 0^\circ$ or $\sim 90^\circ$ then the polarimeter becomes more sensitive for these input vectors whose polarization angle can be corrected later.

5.2.1 Experimental Matrix Error Analysis: High Input Polarization

We now want to look at the errors for our experimental system matrix for GASP ($A_{\text{exp \ GASP}}$). Equation 5.16 shows the matrix used for this examination. We will then complete an analysis of the errors in DOP, DOL, DOC, $\theta$ and $\phi$ shown in figures 5.3 to 5.8.

\[
A_{\text{exp \ GASP}} = \begin{bmatrix}
1.2748 & -0.5275 & -0.9492 & -0.5800 \\
1.2152 & -0.5726 & 0.9589 & 0.5783 \\
1.0577 & 0.5959 & 0.6691 & -0.5436 \\
1 & 0.4775 & -0.6726 & 0.522
\end{bmatrix}
\]  

(5.16)

![Figure 5.3](image)

(a) View 1

(b) View 2

Figure 5.3: Error in DOP resulting from an experimental GASP system matrix (equation 5.16) where the input polarization is 100% and intensity is 20,000 counts.

Comparing the error distribution from the experimental and theoretical GASP matrices, figures 5.3 and 5.2 respectively, we notice a similar error distribution, but with the regions of higher errors extending towards the poles, giving higher relative errors in the experimental case for circular polarized light. However, the value of the error has reduced to 0.79% - 0.57%, a range of 0.22% and a mean over the entire sphere of 0.71% all of which are half that of the theoretical GASP system matrix.
matrix. This can possibly be due to the fact that the GASP theoretical model does not match the experimental model. The differences in each are most likely attributed to not knowing exactly the angles and properties of the aluminium mirrors in the generation of the Mueller matrices for the theoretical model.

Another observation that we did not mention for figure 5.2 is that in figure 5.3 we can see a difference between portions of the sphere where the angle of polarization is +45° and -45°. The sensitivity is lower for -45° region (figure 5.3(b)) compared to the +45° region (figure 5.3(a)), while the minima of the sphere are the same at 0° and 90°. This is created due to the asymmetry of each path in the polarimeter where the transmittances of each channel, after passing through the retarding beam splitter, do not match. We can simulate this by generating two theoretical GASP system matrices for an angle of incidence of 74° and 82°. In figure 5.4 (both images are showing the same viewpoint), we can see how the error distribution has shifted to one side of the Poincaré sphere for each angle of incidence. We also notice the amount of error has increased, but we will discuss this later in section 5.4.3.

![Figure 5.4: DOP errors from two theoretical GASP system matrices (input polarization 100%, intensity is 20,000 counts) comparing for two different angles of incidence.](image)

Figures 5.5 and 5.6 below show the errors in the degree of linear and degree of circular polarization, respectively. We continue to see the asymmetric distribution of these errors and these are more prominent in figure 5.6. The maximum errors for these spheres are 0.79% for the DOL and 0.78% for the DOC, quiet similar to each other, but the mean error over the entire sphere is 0.63% for DOL and 0.72% for DOC. This may not appear to be much, but keep in mind that these errors scale as a function of the signal.

The results from the error in the angle of polarization from figure 5.7 is intuitive and expected. When the polarization state is highly circular, then the errors
5.2 Representation of Polarimetric Errors

Figure 5.5: Error in DOL resulting from an experimental GASP system matrix (equation 5.16) where the input polarization is 100% and intensity is 20,000 counts.

Figure 5.6: Error in the DOC resulting from an experimental GASP system matrix (equation 5.16) where the input polarization is 100% and intensity is 20,000 counts.

asymptotically approach infinity (not shown on graph for better visualisation). In this case, the presented errors have a minimum of error of 0.13° along the equator, while the highly circular polarized regions have errors of 6.34°. The equator has an asymmetric error distribution, but this is minimal compared to the errors at the poles.

Finally, the error in the relative phase angle is shown in figure 5.8. This has a likeness to the DOC error (figure 5.6), with both the maximum and minimum errors on the equator and occurring at similar angles of polarization. However, the distribution of errors are not the same and are more complex than that of the DOC errors. Unlike the error in angle of polarization, the error in the relative phase angle does not go to infinity with a maximum error of 0.21° and a mean over the entire sphere of 0.18°.
Error Analysis

Figure 5.7: Error in $\theta$ resulting from an experimental GASP system matrix (equation 5.16) where the input polarization is 100% and intensity is 20,000 counts. The colour bar scale is in log units for better visualisation.

Figure 5.8: Error in $\phi$ resulting from an experimental GASP system matrix (equation 5.16) where the input polarization is 100% and intensity is 20,000 counts.

Up to this point we have been talking about the distribution of errors over the Poincaré sphere. We mentioned earlier that knowing the distribution of errors that we could tune the polarimeter to measure Stokes vectors with a certain polarization angle by orientating the instrument to that angle to get better sensitivity. This can help in the planning of observations as the instrument axis can be rotated on most telescopes and the instrument can be oriented to receive a polarization to which the polarimeter is more sensitive. However, not all measurements of a particular Stokes vector will have a higher sensitivity for all polarimetric results. For example, if the polarization angle of Stokes vector to be measured is at approximately $0^\circ$ then the polarimeter will give a more sensitive result for DOP, DOL, DOC and $\phi$, but not the actual angle of polarization itself. Therefore, rotating the instrument to get a more sensitive angle of polarization result, we compromise all the other measurement. If the Stokes vector is varying by $> 10^\circ$ in either $\theta$ or $\phi$ it becomes more difficult to find the optimum alignment of the polarimeter by
5.2 Representation of Polarimetric Errors

which the analysis above has shown us this can be expected.

5.2.2 Experimental Matrix Error Analysis: Low Input Polarization

The examination of errors for 100% polarized Stokes vectors is useful for the laboratory test, but we need to see the effect of errors on examining states with lower degrees of polarization, something that is more commonly seen in astronomy. The following figures (figures 5.9 to 5.13) show the errors for DOP, DOL, DOC, $\theta$ and $\phi$, which are calculated for the same intensity as previous Stokes error test (20,000 counts), but the degree of polarization is set to 10%.

![Figure 5.9](image1)

(a) View 1
(b) View 2

Figure 5.9: Error in DOP resulting from an experimental GASP system matrix (equation 5.16) where the input polarization is 10% and intensity is 20,000 counts.

![Figure 5.10](image2)

(a) View 1
(b) View 2

Figure 5.10: Error in DOL resulting from an experimental GASP system matrix (equation 5.16) where the input polarization is 10% and intensity is 20,000 counts.

Comparing the 100% polarized error test to that of the 10% polarized error test, we see that the distribution of errors are quite similar for DOP, DOC, $\theta$
Figure 5.11: Error in DOC resulting from an experimental GASP system matrix (equation 5.16) where the input polarization is 10% and intensity is 20,000 counts.

Figure 5.12: Error in $\theta$ resulting from an experimental GASP system matrix (equation 5.16) where the input polarization is 10% and intensity is 20,000 counts. Colourbar scale is in log for better visualisation of errors with the range of errors in angles is $1.3^\circ$ to $63.1^\circ$.

Figure 5.13: Error in $\phi$ resulting from a theoretical GASP system matrix (equation 5.16) where the input polarization is 10% and intensity is 20,000 counts.
5.3 Comparison of Experimental and Mathematical Polarimetric Errors

and $\phi$. The error distribution for the DOL (Figure 5.10) is markedly different to the 100% polarized error test and this begins to show the relationship between the errors and the input degree of polarization. Unexpectedly, the maximum and mean of the errors for DOP, DOL and DOC are all lower than the 100% polarized test. The maxima are reduced by $\sim 0.1\%$ and the means are reduced by $\sim 0.05\%$. This can be attributed to the fact that as we approach unpolarized Stokes vectors all the intensities are equal in each intensity channel and from this, the signal-to-noise ratio is higher in each and this less noise for certain Stokes vectors. However, the errors for $\theta$ and $\phi$ are both 10 times that of the 100% polarized test, again showing the relationship of the polarimetric errors to the degree of input polarization. Later in section 5.4 we examine this relationship of the polarimetric errors as a function of degrees of polarization. In summary, we see that the errors are function of the input stokes vector even though at certain intensities the range of errors is low. However, the polarimeter can be orientated to accept input where the polarimeter is more sensitive. Next, we will look at how the mathematical error calculations compare to errors from experimental data.

### 5.3 Comparison of Experimental and Mathematical Polarimetric Errors

In the previous section we were able to display the theoretical errors for every polarization state on the Poincaré sphere arising from a single experimental system matrix. Earlier in section 4.2.2 we verified the ability of the polarimeter to measure polarization by inputting known linear polarization states. We now show the experimental errors from this laboratory test. Figures 5.14(a) and 5.14(b) compare the theoretical and actual errors from this test.

Figure 5.14(a) shows the errors in $\theta$ and $\phi$ as a function of input linear polarizations states $0^\circ$–$180^\circ$. We can see that the experimental data matches the theoretical curve, but with some slight offset. In the measurement of the degrees of polarization shown in figure 5.14(b), we again see functional matches, but with offsets at certain points. The cause of this offset is due to the actual intensity. Looking at the equations that calculate the errors in the degrees of polarization (equations 5.10 to 5.12) we see that they are dependant on the Stokes intensity parameter, $I$. In the calculation of the data for the mathematical error estimate, we took the intensity from one data point ($10^\circ$) and calculated it for the rest of the set with that same intensity. An explanation of the offset in figure 5.14(b)
can then be attributed to a change in intensity for later measurements. This then might explain the offset for the angles of polarization in figure 5.14(a), but looking at the equations to calculate these errors (equations 5.8 to 5.9) they are not dependant on the intensity $I$. However, we are not dealing with normalised or averaged Stokes vectors, but Stokes vectors that have the intensity of each Stokes parameters within them, thus, a change in the intensity will change the estimation of the error. Practically, the intensity vector that then gives the Stokes vector must be converted from the cameras analogue-to-digital unit’s (ADU) to the correct number of photoelectrons. This conversion is dependent on the cameras setting’s and was described in the thesis by Sheehan [97]. This conversion tends to fluctuate, but it is more stable than the change of intensity of the source that occurred in the lab tests.

We now look at how the polarization errors are a function of intensity. Both the mathematically calculated and experimentally measured errors are shown in figure 5.15. It is more convenient to talk about the polarimetric error as a function of the signal to noise ratio (SNR) as calculated by equation 3.1. However, we realise that we recorded 4 intensities and thus we will have 4 SNRs, which would be difficult to correlate to the polarimetric error. It is then easier to talk about the polarimetric errors a function of the source intensity.

Figures 5.15(a) to 5.15(e) show the inverse power law relation between the relative errors and intensity where both the mathematical and experimental follow this law. Each graph shows that the experimental error for the measurement of a linearly polarized source at $\sim 45^\circ$ is within the bounds of the theoretical error. The
5.3 Comparison of Experimental and Mathematical Polarimetric Errors

Figure 5.15: Experimental and the theoretical errors against signal intensity showing the theoretical maximum and minimum possible error over the entire Poincaré sphere with the mean theoretical error also shown.

Agreement of these graphs is satisfactory, but the residuals in figure 5.15(f) show an interesting development. These residuals also follow an inverse power law and can be explained by the fact that the theoretical error does not account for other noises other than Poissonian, so, detector noise can dominate in the experimental case, which show up as the residuals. The difference in these theoretical and experimental errors are of no concern when at high intensities, but one must be
Error Analysis

aware that an underestimation of the errors should be quoted when the source is of low intensity.

From these two demonstrations, we can see that the error estimates for various Stokes vectors matches the mathematical and that the inverse power law of the intensity against polarization errors provides a useful tool for planning observations.

5.4 Monte Carlo Simulation of Error Propagation

We previously mentioned that it is difficult to experimentally test the polarimetric errors as a function of the input degree of polarization and because of this, a Monte Carlo simulation was used. This Monte Carlo simulation could not only test for the propagation of errors with simulated Poissonian noise, but also test GASP at different field positions thus showing us the sensitivity over the field of view.

We begin this test by the generation of Stokes vectors that are equally distributed about the Poincaré sphere. The simplest way to achieve this was to adapt a method for the generation of evenly distributed random points on a unit sphere [103], [104], [105] and apply it to the generation of random Stokes vectors by using equation 5.17.

\[
S = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{x^2 + y^2 + z^2}} \begin{bmatrix} 0 \\ x \\ y \\ z \end{bmatrix}
\]

(5.17)

where \( S \) is the resulting Stokes vector and \( x, y \) and \( z \) are independent Gaussian random variables. To set the intensity, we multiply this vector by the necessary counts. To generate a set degree of input polarization we multiply the \( Q, U \) and \( V \) parameters by the fractional DOP. In our simulation, we generated 10,000+ Stokes vectors for any one trial.

A theoretical system matrix was generated, depending on the field position we wish to test. For every Stokes vector, the intensity vectors were generated from this system matrix and Poissonian distributed noise was added, where the standard deviation of the noise was the square root of the input counts of each channel.
5.4 Monte Carlo Simulation of Error Propagation

Polarimetry and data reduction was performed on the noisy data using the same system matrix. The error of each individual Stokes vector came from the difference between the input and output reduced data. Just as we took the mean error over the entire sphere in the previous section we now take the root-mean-square (RMS) error of the reduced data to represent the error over the entire sphere. This process was repeated 250 times for each intensity, field position and degree of polarization with the noise and Stokes vector sample changing every time, effectively measuring the error for 2,500,000 Stokes vectors. The intensity, field position and degree of polarization were changed for whatever range we wish to test. The results are presented showing the errors for all of the 250 trials.

5.4.1 Comparison of Monte Carlo Simulation to Mathematical Polarimetric Errors Estimates

Before we examine the results from the Monte Carlo simulations we wish to see if the mathematical error calculation compares well to the errors that come from the Monte Carlo simulation. We have already seen that the laboratory error calculation compares well to the mathematical in figure 5.15, so we do the same for the mathematical and simulated data.

Figure 5.16: Polarization errors as a function of intensity for both the mean mathematical estimates (dashed lines) and Monte Carlo simulation (points) using the same system matrix as for the experimental comparison in section 5.3.

The mathematical error estimates (lines) in both figures 5.16 and 5.17 compare favourably for DOP, DOL, DOC and \( \phi \), but not \( \theta \). The DOP, DOL and DOC errors are not exactly the same compared to the Monte Carlo data as we have seen in 5.15 there is a range of errors coming from the mathematical errors over the Poincaré sphere. In these figures we present the mean error over the Poincaré
sphere, while the Monte Carlo error estimates are within the bounds of the mathematical error as demonstrated figure 5.15. The reason for the poor match for $\theta$ in figure 5.16 and 5.17 is that in the mathematical calculation of errors, the presence of circular polarization alters the average error over the Poincaré sphere. Because of this, we get a large spread of mean values for $\theta$ as some simulations may have a larger spread of highly circular polarization than others. We can see that the lower bounds of the simulation data is close to the mathematical error estimate and if we also compare this large spread of errors in $\theta$ with figure 5.15(d) we see that the errors are within the expected range and below the upper bounds of the errors.

In figure 5.16(b) the mathematical error estimate of $\theta$ may not match that of the experimental as previously stated, but it is close to the lower bounds of the Monte Carlo simulation. As we increase in intensity, there is a spread of the error estimate of the simulated data and this is because the errors are low when no purely circularly polarized vector is present and large when one is present. What is strange is that the range in the error estimate gets larger with increasing intensity. However, the maximum possible error settles at approximately $1^\circ$ and can possibly be much lower, which is tolerable.

Figure 5.17: Errors in Polarization as a function of intensity for both the mean mathematical estimates (dashed lines) and Monte Carlo simulation (points) using the theoretical GASP system matrix.

As a frame of reference for the following sections, figure 5.17 is the presentation of results from a theoretical GASP system matrix and this matrix is used to test the sensitivity related to changing input DOP and the dependence of polarimetric errors on the field of view (FOV). Comparison of the Monte Carlo simulation to the mathematical error estimates does leave us to question the mathematical assessment of the errors, however, the DOP, DOL, DOC, and $\phi$ are still within
range of the errors and will suffice for the next assessment.

### 5.4.2 Error in Varying Input Degree of Polarization

At the end of section 5.2 we said that we could not conduct an experimental test to see how the polarization errors change as a function of the degree of polarization. Seeing that the Monte Carlo simulation compared favourably to the mathematical error estimate, we can conduct such a test numerically.

![Graph showing error in angles of polarization versus input degree of polarization for two different intensity scenarios](image)

(a) Intensity at 10,000 counts  
(b) Intensity at 500,000 counts

Figure 5.18: Angle errors versus input degree of polarization for examination of mathematical (dashed lines) and Monte Carlo simulation (dotted lines) errors for two different intensity scenarios.

The Monte Carlo simulation from figure 5.18 was generated from a theoretical GASP system matrix whose angle of incidence could be changed, whereby the analysis of such changes are described later in section 5.4.3. The errors in this figure were calculated for a retarding beams splitter (RBS) angle of incidence of $78^\circ$ and shows the errors in the polarization angles when changing the degree of polarization. Figure 5.18(a) shows the errors when the intensity is at 10,000 counts and the values corresponding to 100% polarized are as expected matching figure 5.17(b). What is surprising is when the DOP is lower than 3% the different error estimates separate. The mathematical estimates of both angles continues with the inverses power law, while the simulation errors levels out to $\sim 30^\circ$ for $\phi$ and $\sim 50^\circ$ for $\theta$. It is not clear why the simulated and mathematical error estimates differ so much at the low end of DOP, but it can be attributed to the effect of not including the correct signal errors and noise factors for the mathematical estimates. When we increase the intensity to 500,000 counts (figure 5.18(b)) we see that the point of separation between the simulated and mathematical error estimates occurs at a much lower DOP, $\sim 0.6\%$. This again demonstrates the effect of the difference
between simulation and the mathematical error estimate. The intensity is now higher and the error is negligible in the simulation and thus not differentiable from the mathematical at errors beyond 1% DOP.

![Graph showing degrees of polarization errors versus input degree of polarization (1 trial shown) for examination of mathematical (dashed lines) and Monte Carlo simulation (dotted lines) errors for two different intensity scenarios.](image)

Figure 5.19: Degrees of polarization errors versus input degree of polarization (1 trial shown) for examination of mathematical (dashed lines) and Monte Carlo simulation (dotted lines) errors for two different intensity scenarios.

The relationship between altering the input DOP and the measurement of the output degrees of polarization for the simulated and mathematical error estimates are shown in Figure 5.19. When the input polarization is 100%, the values of the different errors match to that shown in figure 5.17(a) and the offset more visible. What is apparent is that the functions do not match in any way. The values of the errors may be of the same order, but a valid explanation as to why they differ can not be easily found. We can only use the same conclusion that arose from the test of angles of polarization seen in figure 5.18 and that the two differ due to exclusion of the correct signal noise factors. However, this statement is contravened by the fact that the mathematical functions do not approach a similarity to the simulated data when the intensity is at 500,000 counts as was shown previously. The only solace is that the range of errors is the same in the cases of differing intensities.

It is difficult to validate that the Monte Carlo simulation matches that of reality when we see the difference to the mathematical estimate occurring at low DOPs. Even though difficult to achieve, a laboratory simulation of error measurement as a function of decreasing DOP is recommended.

### 5.4.3 Field of View Sensitivity

We saw earlier in section 4.2.3 that if the acquisition and calibration field position differed we would get an error proportional to the amount of misalignment.
However, this restricts us from testing a well calibrated field and seeing how the sensitivity is a function of the field of view. Using the Monte Carlo simulation we can test this and figures 5.20 and 5.21 show us the errors in degrees and angles of polarization, respectively, as a function of the angle of incidence ($\theta_i$) on the retarding beamsplitter (RBS).

Figure 5.20: Monte Carlo simulation of errors for degrees of polarization for two intensities, 10,000 (dark colours) and 100,000 (light colours), against RBS angle of incidence.

Figure 5.20 shows the errors for the degrees of polarization for an input polarization of both 100% in figure 5.20(a) and 1% in figure 5.20(b). Within each figure we set two different intensities of 10,000 and 100,000. From these figures, we see that the sensitivity is a function of the RBS angle of incidence. The maximum ensemble sensitivity occurs at $\sim 78^\circ$, which was the optimized angle of incidence for which the prism was designed. We say maximum ensemble sensitivity because we can see that DOP and DOL do indeed have a minimum error within the presented angular range, but the DOC error continues to minimise with decreasing angle of incidence. This fact is echoed in the error in angles of polarization in figure 5.21 where $\theta$ has a minimum error at $\sim 78^\circ$ and $\phi$ has a minimum error $< 71^\circ$.

We can see in all these figures that there is a shift in the ensemble error to one side of the prism, that is, where the angles of incidence is $< 78^\circ$. This is useful to show that one side of the field of view is more sensitive than the other and this can be used in the planning of observations. The cause of this effect is most likely due to the fact that the reflectivity of the RBS is a function of the angle of incidence. We defined earlier in table 2.1 that when the transmittance ($T$) through the RBS equalled the reflectance ($R$) that the polarimeter was optimized. The absolute difference between $R$ to $T$ at $78^\circ$ is 0 and when calculating this difference, $6^\circ$ either side of $78^\circ$, it is greater when $\theta_i = 84^\circ$. This means that when at this angle...
Figure 5.21: Monte Carlo simulation of errors for angles of polarization for two intensities, 10,000 (dark colours) and 100,000 (light colours), against RBS angle of incidence.

of incidence one path has a lower signal than the other and hence a larger induced error.

The explanation as to why the sensitivity for DOC and $\phi$ is more favourable at the $\theta_i < 78^\circ$ side of the prism is because the signal for the T-path is much higher therefore the induced retardance on the signal will have less errors when examining the circular polarization of the input beam. The fact of having a higher sensitivity of DOC and $\phi$ can lead to planning observations based on this knowledge, with targets positioned at one part of the field for more sensitive DOC measurements and positioned at the centre for more sensitive DOP and DOL measurements.

5.5 Sensitivity Relationship of the Inverse Condition Number

At the start of this chapter we stated that there were issues in using the inverse condition number to quantify the sensitivity of the polarimeter. However, it was used to restrict the design and optimisation of the RBS and so we will examine its relationship to the sensitivity of the polarimeter using the Monte Carlo simulation. The inverse condition number varied as function of the RBS angle of incidence and we examine this solely for the DOP as shown in figure 5.22.

This plot is for the initial standard system matrix as developed by Compain and Drevillon. This matrix was used as it was used for the initial design of the RBS. Figure 5.22(a) shows how the error in DOP changes with the inverse condition number for two differing intensities. We can see how the sensitivity
5.5 Sensitivity Relationship of the Inverse Condition Number

Figure 5.22: DOP errors for changing condition numbers and angles of incidence calculated using the standard system matrix for two different intensities.

*increases with an increasing inverse condition number as expected. What is also expected are the two branches of these curves. We mentioned earlier that two differing system matrices can have the same inverse condition number, but with two different system matrices, we can have two differing sensitivities for the same inverse condition number. Just as we commented in the previous section on how one portion of the field of view was more sensitive than the other, figure 5.22(b) shows that the same effect carries on to the inverse condition number.*

Figure 5.23: GASP theoretical matrix Error for DOP as a function of inverse condition number for two different intensities.

*In defining the field of view in section 2.4.1 we used an inverse condition number \((1/S(A))\) of 0.3, in actual fact this number was passed down by word of mouth as it is a rule of thumb for the limit of sensitivity for a polarimeter. However, there is no scientific validation for choosing this number, but by looking at figure 5.22(a) we can derive such validation from it. Looking at the line where the intensity*
10,000 counts we get an error of \( \sim 2\% \) for one side of the prism and \( \sim 1.8\% \) for the other side, an average of \( \sim 1.9\% \) when the \( 1/S(A) = 0.3 \). Granted that the choice of intensity is arbitrary, but looking at the second line where the intensity is 40,000 counts, the DOP error for \( 1/S(A) = 0.3 \) is \( \sim 1.1\% \) and \( \sim 0.95\% \), an average error of \( \sim 1\% \), which is the DOP threshold we set starting out. Figure 5.23 above is the same as figure 5.22 except that we are now analysing the theoretical system matrix for GASP. We again see that same that inverse condition number can have two differing errors. Looking at the line where the intensity is 40,000 counts, the limit of \( 1/S(A) = 0.3 \) has an average error in DOP of \( \sim 1.125\% \), which means that the limit of the field of view (FOV) for the RBS is near our DOP threshold of 1%. We can also argue that we can increase the FOV of the RBS and still get a result within our tolerance for sources with higher intensities. However, observing beyond the limiting FOV will mean that the image will be vignettet by the prism unless the optics are changed or a larger prism is used.

In summary, the estimation of the polarimetric error is important for defining the theoretical sensitivity of the polarimeter before it is used, but our analysis has shown that it is difficult to define a metric that can describe the limits of the polarimeter\(^1\). The evaluation of the error propagation lead us to use the average error over the entire Poincaré sphere as a single metric. However, in reality, we may know the expected polarization that is to be measured. Reassessing the sensitivity around the region of the expected polarization can lead to a better error estimator thus predicting if the sensitivity is adequate for the planned observations. Comparing the polarization errors from laboratory tests and Monte Carlo analysis to the mathematical error estimator we saw some promising results when the input DOP was high. However, at a low DOP, a difference in the Monte Carlo and mathematical estimator showed us that a reassessment of the error propagation is necessary. Such a result was useful to reveal as most astronomical targets are low polarized ones and the sensitivities need assessing at this regime. We must also remember that the range of the errors are a function of the intensity of the source and when the intensity is high, \( > 100,000 \) counts, this range may seem small and it can be argued that the error is the same for any stokes vector or for any field position in the image. When dealing in high-time resolution astronomy, photons are rare this range will increase making it an important matter to know how to best tune the polarimeter for the observation.

\(^1\) Viva correction: It was noted that a single metric does exist to evaluate the performance of the polarimeter and it is called the polarimetric efficiency\( (\varepsilon)\)\(^\text{[106]}\) given by the following equation

\[
\varepsilon = \frac{1}{\sqrt{n} \sum_{i=1}^{n} (A_{ij}^2)}
\]

where \( n \) is the number of rows in the system matrix (4 in our case).
Conclusion
The aim of this research project was to design, build and test an instantaneous full Stokes astronomical polarimeter and in this we succeeded. We found good agreement in the laboratory measurements to known polarization values and also for the astronomical measurements, even though with some errors. We can critique its performance both polarimetrically and optically, but we will acknowledge that this is one of the first astronomical polarimeters to measure the complete Stokes vector of a source in a single exposure. In this section, we summarise the entire thesis and then comment on future recommendations within each topic.

6.1 Summary and Conclusions

6.1.1 Polarimeter Design Summary

Choice of Type of Polarimeter
Based on the design requirements for our polarimeter, only division of wavefront polarimetry (DOWP) and division of amplitude polarimetry (DOAP) could, in a single exposure, measure the complete Stokes vector. Due to the inefficiency of division of wavefront polarimetry, division of amplitude polarimetry was chosen and developed upon. No such polarimeter has existed in astronomy and the development of the Compain and Drevillon [1] polarimeter used in the field of applied optics was modified for astronomical use.

Development of the RBS Prism and Glass Choice
The main component of the polarimeter was the retarding beam splitter (RBS), a prism that acted as a beam splitter and a quarter waveplate. It was the development of this component that was crucial to the workings of the polarimeter. With the successful enhancement of the pre-existing RBS, we were able to design an RBS that would have compelling achromicity, however we ran into an issue as to what glass to use for the prism. In section 2.3.4, we attempted a scoring system that would aid in the choosing the best glass to make our RBS prism.

The factors for the choice of glass were, proximity to the desired refractive index, dispersion, cut-off wavelengths, absorption, the inverse condition number used to score the resulting polarimeter; and cost. However, some of these could be excluded, like cost, and more parameters could be included to improve the selection of glass for the prism. Such parameters could be, thermal expansion coefficient for mounting and stress birefringence considerations, change of refractive index with temperature for polarimetric stability in changing environmental
circumstances; and how easy it is for the desired glass to be cut and polished in a optical workshop. The final score was dependant on an amalgamation of these, but more preferably, a top down approach should have been taken by using the final sensitivity of the polarimeter as a function of the glasses for the final metric.

**RBS Size and Imaging Polarimetry Development**

The decision to implement an imaging polarimeter was based on the need to capture an entire field of view (FOV). This made it necessary to develop the RBS for use in an imaging polarimeter, which was not a trivial task. The relationship between the size of the astronomical FOV and the RBS size are inexorably linked. Since the FOV of the RBS cannot go beyond 8°, due to the inverse condition number limit, then to increase the size on the astronomical FOV, the prism must be enlarged. The need for much larger fields would lead to an unacceptably large RBS prism. However, as we later found, the metric of inverse condition number for restricting the FOV of the polarimeter was somewhat arbitrary and could be relaxed on the basis of sensitivity. If we relaxed this restriction then we would find an optimum RBS FOV that would allow a larger sky FOV, but by only a few arcseconds. For example, in section 2.4.1 we derived the entrance length of the RBS prism to be 37mm when designed for the Keck telescope. Back calculating and using the same logic, a 37mm prism using an 8° RBS FOV would actually give a FOV of 13” assuming we had the exact focal length lenses and the edge of the off-axis collimated beam was touching the edge of the RBS prism. Calculating the FOV for various RBS field sizes we find that the optimum RBS FOV is \( \sim 11° \) and this will give us a sky FOV of only 14”. While applying this to the Loiano telescope, we find the same optimum RBS FOV and get a final sky FOV of 1’33.6” and increase of 7”, which is not much of an improvement. From this, we reiterate that to increase the sky FOV for the polarimeter we must increase size of the RBS prism.

One of the more convincing arguments that GASP was an instantaneous full Stokes polarimeter was that the final images were recoded on a single detector. The only argument that can counter this is to look at the time of flight of the photons entering the polarimeter. An effort was made when designing the polarimeter to make the R-path and T-path of equal path length, but this was not achieved nor was it thought to be a concern as the difference in path length would equate to 0.1ns, significantly smaller than any time resolution we wished to achieve. Unfortunately, we were not able to acquire images at time-scales >>300 frames per sec due to limiting technology, but using APDs could be the solution to do so and will discus this in section 6.2.1.
The final image quality lead to stars whose FWHM\(^1\) were \(\sim 12\) pixels. This was because the plate scale was \(\sim 0.1''/\text{pixel}\), where the optimum plate scale is \(0.5''/\text{pixel}\). To obtain this plate scale we would need to increase the linear extent the field of view by a factor of 5, which is difficult to achieve as we stated previously. We could increase the plate scale by changing the final imaging lens. By reducing the focal length we will reduce the number of pixels used for the entire field and obtain the desired plate scale, however, since the Wollaston has a set divergence then separation of the images is also reduced and thus the images overlap reducing the effective field of view. We said in section 2.5.5 that we could circumvent the Wollaston restriction by altering the field angle before the beam enters the Wollaston however we then run into problems with vignetting due to the pupil astigmatism caused by the RBS prism. The final imaging issue was the dispersion cause by the Wollaston prism. This restricted us to using a low divergence Wollaston so when the input was filtered the divergence would not affect the image quality too much. Such arguments do disparage the design of GASP, but in all, the final design was satisfactory in giving the desired polarimetric results. In section 6.2.1 we will talk about what final design recommendation that could improve the image quality, but with certain sacrifices.

6.1.2 Calibration Summary

Alignment of Polarizers and Waveplates
The accuracy of the alignment of polarizers and waveplates used for later calibrations were not that important for the workings of the polarimeters as further calibrations methods, such as the ECM, were able to take account for any errors in the basic alignment. This basic alignment did not need high quality polarizers, but it did give the frame of reference to which the rest of the polarimeter was aligned, i.e. it sets the coordinate system for the measurement of the angels of polarization.

Choice of PSG States
In choosing the states of polarization necessary for the polarization state generator (PSG), which was then used for later calibrations, we found it necessary to make sure that the 4 polarization states be as linearly independent from each other as possible. The resulting 16 intensities after passing the PSG through the polarimeter needed to be as large as possible and with the design of GASP, the 16 intensities never reach a null value, which then lead to better noise-reduced

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\(^1\)Full Width at Half Maximum
6.1 Summary and Conclusions

calibrations.

**Image Processing**
The image processing of the outputs from GASP and taking their intensity values presented one of largest obstacles for calibration and polarimetry. We initially saw that the method of aperture photometry presented problems in that the final polarization results were dependent on the selection of aperture size. We were able to show how this could finally be resolved by conducting the aperture analysis and seeing what aperture would yield the most sensitive polarization result. However, we could see that the sensitivity was no better than 0.5% in the degrees of polarization. This was within our tolerance, but for future higher precision measurements we require a more radical solution (suggestions follow in section 6.2.2).

**Basic and ECM Calibration**
The basic calibration described in section 3.3 was sufficient for the initial calibration and gave approximate polarimetric results, however, flaws in the alignment of the PSG states would eventually give a poor calibration. It was with the eigenvalue calibration method (ECM) that resolved these problems and could unambiguously calibrate not only the system matrix of the polarimeter, but also the PSG matrix to which we could see how much the alignment of the PSG was out by. All that was needed to obtain a high accuracy calibration was a PSG, some high quality polarization samples orientated at the approximate angles and a stable source. This went a long way to increasing the confidence in the functionality and accuracy of the polarimeter.

**Astronomical calibration**
We were satisfied with the calibration of the polarimeter in the laboratory, but the expected issues with the astronomical calibration was a great hindrance. The fact that one cannot calibrate the polarimeter and telescope as one unit, lead to the necessity to calibrate off know polarized and unpolarized standard stars. We could find the amount of polarization introduced by the telescope by looking at unpolarized stars and then we could find the polarization angle from polarized stars. However, we admit that a mistake was made in using interference filters in the polarimeter, which invalidated some results. In an ideal case we would calibrate the telescope and polarimeter as we would the polarimeter on its own, which would imply placing a PSG and ECM samples before the telescope. Such an implementation is not possible for large telescopes, but one could possibly use
the basic calibration method on 4 linearly independent polarized stars. The only issue with this availability of highly polarized circular standard stars, we need not know what these stars are, but they must create a linearly independent PSG and the ECM will take care of the rest.

6.1.3 Results Summary

Laboratory Calibration and Polarimetric Consistency
We started to compare the theoretical and experimental system matrices from GASP and initially saw that they did not match exactly, however we were satisfied that this difference is due to not knowing every part of the model and there was no concern in the difference of these system matrices as the ECM calibration took care of this. The aperture photometry analysis was able to show that choosing an optimum aperture size lead to the parameters from the ECM staying within 0.25% as the aperture sized increased, easing any concerns that the aperture choice had an effect on the calibration. The stability of the system matrix showed satisfactory consistency as changes no greater than 1% in the degrees of polarization for up to 4.5 hours were present. The same can also be said about the consistency of the PSG, which reduced how often GASP needed to be calibrated in order to have a measurement within our tolerance.

The initial polarimetric stability measurements showed that the polarimetry was stable to within 0.25% for a 15 minute trial, where the source was also stable to less than 0.25%. This was followed by polarimetry of a deliberately unstable source over 70 minutes where it was shown that the polarimetric measurements were completely independent of the source. The degrees of polarization were stable to within 0.05%, but there were signs of a small amount of drift during this period of 70 minutes. This was then re-examined over a longer period of 3 hours and we found a change of about 0.2% in the measurement of the degrees of polarization. This can probably be explained by a change in the system matrix due to environmental changes over this time. However, such a drift was not seen in the stability of the system matrix nor of the PSG, which would mean that the polarization of the source itself might be changing over this period. Anyhow, this change was much less than our tolerance and in the observations over a few minutes it would not be expected to cause any problems.

Verification Tests and Field of View Analysis
The plots of known polarizations (figure 4.14 to 4.17) showed excellent agreement
6.1 Summary and Conclusions

to that, which was measured, but when we looked at the residuals we could see error greater than our 1% tolerance, but more so these were systematic errors. The search for the cause of these errors lead to the discovery and conclusion that the calibration and acquisition positions were not identical and would cause such a problem as shown in figures 4.18 and 4.19.

Further field of view analysis showed how the measurement of the inverse condition number for various matrices did not show the expected dependency on the field position. It has not been clearly understood as to why this is. One such reason might be that as the field position changes, other factors, such as the gain variations across the CCD, are being corrected for by the ECM, thus we get a larger change in the inverse condition number than expected. We proceeded with analysis similar to that conducted for the calibration consistency and indeed saw, as expected, that the system matrix was dependent on the field position, while the PSG was not. Knowing the field positions and the corresponding angles of incidence on the RBS, we were able to simulate some off-axis polarimetry, showing how polarimetry completed with a matrix that was used for a different field position gave erroneous results. Figures 4.22 and 4.23 were able to show such agreement to this when analysing the field of view analysis data. However, this points out one of the greatest weaknesses of GASP, its strong field dependency. Depending on the amount of astronomical seeing and on the state of polarization being measured, a 1” seeing disc could lead to an error of greater than 1% in the degrees of polarization. Section 6.2.2 discusses some recommendations to rectify this.

Astronomical Results

There was some moderate success in GASP’s astronomical observations where the corrected polarization standards of HD 236928 and HD 204827 were off by 1-2% in DOP. The observation of the reflection nebulae CRL 2688 was within 1.5% of the expected DOP. However, much greater problems were discovered especially in the observation of zero polarization standards. The telescope polarization was larger than expected, \(~7\%-10\%\) in DOP and also changed from night to night. We presumed that such are large telescope polarization was actually due to the types of filters used in GASP. Such interference filters were detrimental to the polarization as they were placed just after the collimator of GASP. In retrospect, these should have been placed after the Wollaston. Even though they would affect the intensity of each orthogonal beam after the Wollaston it would be calibrated out by the ECM calibration. Apart from this, GASP was able to perform some astronomical polarimetry and validate this prototype.
Polarimetric Stability

The absolute polarimetry of GASP yielded results with an error greater than 1% for targets under a long exposure, but the stability of the polarimetry for short exposure observations were of interest especially for an instantaneous full Stokes polarimeter. We saw that the polarimetry for polarized and unpolarized targets with exposure times of $10 \times 100$ms had fluctuations of 1%-2.5% in the measurement of the uncorrected degrees of polarization (see table 4.7.) These measurements only displayed random errors with no systematic drift over 3000 acquisitions or 5 minutes. Such random errors can be attributed to the astronomical seeing at the time and large jolts in the data were possible due to the telescope’s poor sky tracking. These effects would then appear as jitter over the field and thus the same field dependency issue would cause such larger errors. Had the seeing be less and the telescope more stable we could predict precisions to less than 1%. The recommendations in 6.2.2 regarding the image processing would also go along way to reducing this value of precision and increasing the sensitivity.

In the polarimetric stability tests on a highly polarized target (CRL 2688), we again saw the independence of the polarization measurements to that of the intensity fluctuations (figure 4.28) with changes in the intensity of $\pm 50\%$ about the mean (figure 4.27). In comparison to the low-polarized and unpolarized targets, the polarimetry of CRL 2688 was stable to 1.6%, within the range of 1%-2.5% stated in the previous paragraph. No systematic drift was visible for the data bins of 0.1s and 1s, but upon integrating to 5s bins, we saw some drift at the start of the measurement of DOP and DOL, but none for the DOC. This showed a possible correlation to the change in transparency else it was due to a small drift in the telescopes position and hence field dependency issues.

Rotation Tests

A test of the measurement of the angle of polarization was the rotation test, whereby we could rotate the instrument about the telescopes axis and in a sense rotate the sky. This test was conducted on CRL 2688 and we first saw a similar, but larger ($\pm 5^\circ$) sinusoidal residual in the angel of polarization as we did in the laboratory; we can say this was due to the acquisition/calibration misalignment. The greater concern came upon the inspection of the DOP against rotation, whereby the range shifted by $\sim 15\%$, a range larger than the instrumental polarization. It was then noted that the measurement of targets and the telescope polarization that would correct these measurements were all at the same instrumental rotation angle seeing that the measurements were a function of the instrument rotation position. However, a rotation test on an unpolarized standard on the same night
as that of CRL 2688 was not conducted. A rotation test of HD 204827 (~5% polarized) and BD +32 3739 (unpolarized) was conducted on the same night and it showed how the polarized target changed by 12.5% over the rotation test and had a smaller variation when corrected (figure 4.36).

In all, the laboratory and astronomical observations showed that GASP could measure polarization, but its performance was hindered by the errors introduced by the sensitivity to the field position. On many occasions we could achieve better than 0.1% precision in our measurement, but we understood that the accuracy may not reach this level due to telescopes intrinsic polarization, instrumental polarization and other errors.

### 6.1.4 Error Analysis Summary

For situations of high polarization with high intensity (> 20,000 counts), the error calculations matched well to the experimental, even if the value of the errors were quiet small. The presentation of the errors on the Poincaré sphere gave an intuitive description of the errors as a function of the polarization states and could only be improved upon by using spherical projections such as Mercator or Mollweide. The error plots of intensity against the measurements of polarization all fell within the range of expected errors even if the residual to the minimum error showed a power law.

The Monte Carlo simulations were useful for testing situations that we could not conduct in the lab, such as errors as a function of the degrees of polarization. This was intuitive at high polarizations, but for lower polarizations the deviation from the theoretical was unexpected. The same Monte Carlo simulations were also used to test the dependency of the errors for field of view showing how portions of the field are more sensitive than others. We were able to show how the same inverse condition number gave two different sensitivities and that the inverse condition number should be excluded as an error metric and a task based error metric should be used, i.e. error in the final measurement.

### 6.2 Recommendations

Apart from the recommendations we have already mentioned, there are other improvements that could be made to GASP not only in its design, but its use, calibration and sensitivity analysis. Here we present all of the recommendations
Conclusion

to better the design and use of the polarimeter.

6.2.1 Design Recommendations

In respect to the use of GASP, the field dependency of the polarimeter was one of the largest issues that stood out. This is a consequence of the shallow angle of incidence on GASP’s RBS prism. The only resolution to this is to use DOAP polarimeter design like that depicted in figure 1.20 and 2.2, where there is a normal angle of incidence on the QWP. However, this leads to a problem of trying to develop a QWP that is highly achromatic and has no field dependency.

The complex design of GASP lead to a restricted field of view not only due to the RBS, but also due to the Wollaston and the imaging optics. One of the best solutions would be to develop a 4 camera design while using Forster prisms as the polarizing beam splitters. This would mean that there would be room for optics and a camera for all four channels. A Forster prism will also improve the image quality as it will not create any angular dispersion of the light. However, there will be a problem in synchronising all four images especially when high speed detection is needed.

Unfortunately, we were not able to obtain the technology to image at rates faster than 7500 frames per second, especially since one of GASP’s best features is that it only needs one camera. However, the option of using APDs is still available. We did say initially that we would develop an imaging polarimeter because it was difficult to place a target on an APD. Utilizing GASP’s extractor arm, using APDs could be possible as the extractor arm can still see the full image field and knowing relative position of the APDs on the extractor camera, it is easier to place a target on the APDs.

In the design of the RBS, the optimisation process could have included more parameters. This could include, but not limited to; final field of view, the choice of mirror material and their angles, attenuations to compensate the difference in throughput of each path, error metrics of final possible polarization state to be measured, tolerance to astronomical seeing and the multiple mechanical and optical parameters of the glass choice. We will also suggest here that it may have been possible to place an HR coating on the RBS prism to reduce the angle of incidence and thus the size of the prism and cost, but the main premise of the development of the RBS was that it had no coating and only depended on the glass used.
6.2 Recommendations

6.2.2 Calibration Recommendations

Knowing that it is difficult to correct for the field dependency of the polarimeter in the design stage it is possible that a different type of calibration can enhance its performance. While still using the aperture photometry to obtain the intensities, we could use an array of field points and try to interpolate how the system matrix changes as a function of the calibrated field. However, if there are gain fluctuations in the CCD then such interpolations may not work. An alternate idea is to use a pixel by pixel calibration. This will initially be difficult as we have to correlate the locations of respective pixels in each of the four portions of the CCD. As the optics are different for the T-path and R-path in GASP, there is most likely to be different amounts of aberration in each and thus a translation of 1 pixel in one channel may amount to 2 pixels in another channel. When the pixel to pixel correlation is found, it is then a case of having a uniform illuminated field and completing a full field calibration. This could be computationally intensive depending on the amount of pixels used, for this system there would be 65,536 \((256 \times 256)\) calibrations.

Stability Recommendations
The speed and repeatability of each calibration can be improved by using a PSG that uses electro-optic modulators to generate any polarization state and does not rely on any moving components. The samples for the ECM calibration may have to remain as static components as these are of high quality and using an electro-optic modulator may not be able to match this quality requirement.

The stability of the polarimeter appeared to alter over the space of hours and this was possibly due to environmental reasons. A test could be conducted to see how some environmental variables, most likely temperature, have an effect of the performance of GASP. A final test on the stability of the polarimeter is motion. Unless the polarimeter remains static and the image from the telescope is fed to the polarimeter like that on a telescope with Coudé focus, a test on how the instrument performs for different orientations on a telescope should be completed to test its performance.

6.2.3 Error Analysis Recommendations

A future test of GASP’s sensitivity that should be completed is a laboratory examination of how the polarimetric errors behave as a function of the degree of
polarization. Such a test needs a specialised input that can generate any degree of polarization. However, this test needs to consider how to generate low degrees of polarization. Do we mix differing linear or circular states? do we do so spatially, temporally or spectrally as we described in figure 1.11 and do different forms of a low degree of polarization have different error responses?

With regard to the Monte Carlo tests, they could include many other types of tests, but at first it should also include the error introduced by the aperture photometry, which may show significant error at low intensities. The other test that can be competed to see how it affects the polarimetric errors are: error in the position of the Wollaston, effects of changes in intensity of the source during calibration, changes of the PSG after it itself has been calibrated and the effects of changing gain/flat field with respect to other channels.
Appendices
Appendix A

Refractive Indices of listed glasses
### Table of listed glasses

Table A.1: Table of refractive indices of glasses used in thesis

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\lambda [nm]$</th>
<th>S-LAH52</th>
<th>N-LAF36</th>
<th>N-SK5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{2325}$</td>
<td>2325</td>
<td>1.7549465</td>
<td>1.75555</td>
<td>1.55966</td>
</tr>
<tr>
<td>$n_{1970}$</td>
<td>1970</td>
<td>1.7620222</td>
<td>1.76246</td>
<td>1.56539</td>
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<tr>
<td>$n_{1530}$</td>
<td>1530</td>
<td>1.7697586</td>
<td>1.77001</td>
<td>1.57140</td>
</tr>
<tr>
<td>$n_{1060}$</td>
<td>1060</td>
<td>1.7785210</td>
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<td>312.6</td>
<td>-</td>
<td>-</td>
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Appendix B

Matlab scripts for ECM calibration
Matlab scripts

The following appendix contain the matlab scripts to perform the ECM calibration and functions to generate a theoretical system matrix. Annotations are listed within each file to help in the use of the file. The following files are, *ECM_Point_2x2.m* the root file for running the ECM, *angopt.m* and *HMgen.m* necessary sub-scripts needed for the ECM, *A_gen_mir.m* for generating a theoretical system matrix for GASP and *Stokes_analysis.m* a simple code for reducing the Stokes vector the polarimetric properties.

The following ECM calibrations require the input data in the following form

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<th>Star2</th>
<th>Star3</th>
<th>Star4</th>
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<td>57501</td>
<td>35580</td>
<td>82655</td>
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</tbody>
</table>

The file output above, automatically compiled from a matlab code during the intensity extraction, has the 16 combinations of the 4 PSG states and 4 ECM samples. The format of the filename is *p’PSG state’\’s’sample’* where the names of the state and sample can be change but so must the search string in the ECM code.
% ECM Eigenvalue Calibration Method
% calibrate the system matrix and PSG for imaging polarimetry
clear, clc;
% load data following a prescribed format.
[filename, pathname] = uigetfile(’*.txt’, ’Select data file containing intensity info’);
   cd(pathname);
   [name] = textread(filename, ’%s’

   res=importdata(filename);

% list psg and sample search string used in the filename of the data
pa=’p15’;
pb=’p75’;
pd=’p135’;
pe=’none’;
sa=’sair’;
sb=’s0’;
sc=’s90’;
sd=’sr28’;
% Alternate psg search strings
% pa=’p23’;
% pb=’p60’;
% pc=’p130’;

% nested loops for searching input file for relevant data and inserting it into relevant matlab variables
for m=1:length(name)
   a=char(name(m));
   for n=1:length(a)-4
      if a(n:n+length(pa)-1)==pa
         for k=1:length(a)-4
            if a(k:k+length(sa)-1)==sa
               pa_sair=res.data(m-1,1:4)’;
            elseif a(k:k+length(sb)-1)==sb
               pa_s0=res.data(m-1,1:4)’;
            elseif a(k:k+length(sc)-1)==sc
               pa_s90=res.data(m-1,1:4)’;
            elseif a(k:k+length(sd)-1)==sd
               pa_sr28=res.data(m-1,1:4)’;
            end
         end
      elseif a(n:n+length(pb)-1)==pb
         for k=1:length(a)-4
            if a(k:k+length(sa)-1)==sa
               pb_sair=res.data(m-1,1:4)’;
            end
      end
   end
elseif a(k:k+length(sb)−1)==sb
    pb_s0=res.data(m−1,1:4) ‘;
elseif a(k:k+length(sc)−1)==sc
    pb_s90=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sd)−1)==sd
    pb_sr28=res.data(m−1,1:4) ‘;
end
end

eelseif a(n:n+length(pc)−1)==pc
for k=1:length(a)−4
    if a(k:k+length(sa)−1)==sa
        pc_sair=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sb)−1)==sb
        pc_s0=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sc)−1)==sc
        pc_s90=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sd)−1)==sd
        pc_sr28=res.data(m−1,1:4) ‘;
end
end

eelseif a(n:n+length(pd)−1)==pd
for k=1:length(a)−4
    if a(k:k+length(sa)−1)==sa
        pcircsair=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sb)−1)==sb
        pcircs0=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sc)−1)==sc
        pcircs90=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sd)−1)==sd
        pcircsr28=res.data(m−1,1:4) ‘;
end
end

eelseif a(n:n+length(pe)−1)==pe
for k=1:length(a)−4
    if a(k:k+length(sa)−1)==sa
        pnonesair=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sb)−1)==sb
        pnones0=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sc)−1)==sc
        pnones90=res.data(m−1,1:4) ‘;
ellseif a(k:k+length(sd)−1)==sd
        pnonesr28=res.data(m−1,1:4) ‘;
end
end
end
end
Appendix B

\%form B matrices
B0=[p_sair p_bair p_cair p_circsair];
B1=[p_s0 p_b0 p_c0 p_circ0];
B2=[p_s90 p_b90 p_c90 p_circ90];
B3=[p_sr28 p_bsr28 p_csr28 p_circsr28];

\%Calculate C matrices
B_inv=inv(B0);
C(:,:,1)=B_inv*B1;
C(:,:,2)=B_inv*B2;
C(:,:,3)=B_inv*B3;

\%Calculate properties of samples
Tpol0=trace(C(:,:,1));
Tpol90=trace(C(:,:,2));
x=eig(C(:,:,3));
eigreal=sort(real(eig(C(:,:,3))));
eig1real=eigreal(4);
eig2real=eigreal(3);
Ma = find(imag(x)==max(imag(x))); % Finds the maximum imaginary
eigenvalue
eig1complex= x(Ma);
Ma = find(imag(x)==min(imag(x)))); % Finds the minimum imag value i.e. the last
eig2complex= x(Ma); % complex value

Tret=(eig1real+eig2real)/2;
psi_ret=atan(sqrt(eig1real/eig2real));

\%arg is angle of the complex vector
del_ret=0.5*abs(angle(eig1complex/eig2complex));

\%Reconstruct Mueller Matrices after determining the characteristics from the values above. They are the unrotated form as the will be rotated in HMgen which can be used in fminsearch
MM(:,:,1)=Tpol0/2*[1 1 0 0
1 1 0 0
0 0 0 0
0 0 0 0];
MM(:,:,2)=Tpol90/2*[1 1 0 0
1 1 0 0
0 0 0 0
0 0 0 0];
MM(:,:,3)=Tret*[1 -cos(2*psi_ret) 0 0
0 0 0 0
0 0 0 0
0 0 0 0]
\[
-\cos(2 \cdot \psi_{ret}) \quad 1 \quad 0 \quad 0 \\
0 \quad 0 \quad \sin(2 \cdot \psi_{ret}) \cdot \cos(\delta_{ret}) \quad \sin(2 \cdot \psi_{ret}) \cdot \sin(\delta_{ret}) \\
0 \quad 0 \quad -\sin(2 \cdot \psi_{ret}) \cdot \sin(\delta_{ret}) \quad \sin(2 \cdot \psi_{ret}) \cdot \cos(\delta_{ret})
\]

% set approximate angles for the samples as starting points for the optimization
angles(1) = 0;
angles(2) = 90 \cdot \pi / 180;
angles(3) = 28 \cdot \pi / 180;

% use Nelder–Mead minimisation to search for samples angles. ANGOPT is a custom function for calculation minimisation parameters for matrices at set angle also needs HMGEN.m
[angles, fval] = fminsearch(@angopt, [0, 1.5708, 0.4887], [], MM, C);
Angles = angles;

% function to generate the HM matrices for all 3 samples when minimized angles are found
HM = HMgen(angles, MM, C);

K = HM(:, :, 1)' \cdot HM(:, :, 1) + HM(:, :, 2)' \cdot HM(:, :, 2) + HM(:, :, 3)' \cdot HM(:, :, 3);

% use SVD to calculate W (PSG matrix)
[U, S, V] = svd(K);
Wdiag = zeros(16, 1);
Wdiag(16) = 1;
W = (V \times Wdiag);
W = reshape(W, [4, 4])';

m = eig(K);

% calculate A (System matrix)
Acalc = B0 \times \text{inv}(W)

% calculate Mueller matrices of samples using calibrated matrices
Bp0 = \text{inv}(Acalc) \times B1 \times \text{inv}(W)
Bp90 = \text{inv}(Acalc) \times B2 \times \text{inv}(W)
Br28 = \text{inv}(Acalc) \times B3 \times \text{inv}(W)

% show condition number of system matrix
condition = 1 / \text{cond}(Acalc)

% display Mueller matrices of the samples
imagesc(Bp0)
title(’POL at 0’)
figure
imagesc(Bp90)
title(’POL at 90’)
figure
imagesc(Br28)
title(’RET at 28’)

% write A and W to file
xlswrite(strcat(filename(1:16), ’_A.GASP.xls’), Acalc)
Appendix B

```matlab
xlswrite(strcat(filename(1:16), 'WW.xls'), W)
% write all properties of ECM to file
time = clock;
dataname = strcat(filename(1:16), '_Sample_properties.txt');
fid = fopen(dataname, 'w');
fprintf(fid, '{
    Tr/l=100\%
    tAngle/deg\nPol0/t%8.4f\nPol90/t%8.4f\nRet/t%8.4f\nPsiRet/t%8.4f\nDelRet/t%8.4f
    nFval/t%10.8f\n1/cond/t%8.6f\n
    Tpol0, 180/pi*angles(1),
    Tpol90, 180/pi*angles(2), Tret, 180/pi*angles(3), psi_ret*180/pi,
    del_ret*180/pi, fval, 1/cond(Acalc)
};
fprintf(fid, 'Meuller Matrices for the samples\nP0\n');
fprintf(fid, '%8.4f\nBp0\n');
fprintf(fid, '%8.4f\nBp90\n');
fprintf(fid, '%8.4f\nBr28\n');
fprintf(fid, 'Acalc - calibration matrix\n');
fprintf(fid, '%8.4f\nWcalc - PSG matrix\n');
fclose(fid);
```

matlab_files/ECM_Point_2x2.m

```matlab
function mi=angopt(angles, MM, C)
% acquire HM matrix (custom function)
HM = HMgen(angles, MM, C);
% form the K matrix
K = HM(:, :, 1)'*HM(:, :, 2)*HM(:, :, 3)';
eigK = eig(K);
% calculate minimization parameter
mi = eigK(1)/eigK(2);
```

matlab_files/angopt.m
\begin{equation}
\begin{bmatrix}
0 & -\sin(2(-\text{angles}(n))) & \cos(2(-\text{angles}(n))) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix};
\end{equation}
\end{quote}

% rotate each matrix to the relevant angle
\begin{verbatim}
for n=1:3
    MM(:,:,n)=arot(:,:,n)*MM(:,:,n)*rot(:,:,n);
end
end
end
\end{verbatim}

% nested loop for formation of HM matrices
\begin{verbatim}
for n=1:3
    for i=1:4
        for j=1:4
            % for k=1:4
            HM(i+0,((i-1)*4)+j,n)=HM(i+0,((i-1)*4)+j,n)-C(j,1,n);
            HM(i+4,((i-1)*4)+j,n)=HM(i+4,((i-1)*4)+j,n)-C(j,2,n);
            HM(i+8,((i-1)*4)+j,n)=HM(i+8,((i-1)*4)+j,n)-C(j,3,n);
            HM(i+12,((i-1)*4)+j,n)=HM(i+12,((i-1)*4)+j,n)-C(j,4,n);
            \% HM(i+4*(k-1),((i-1)*4)+j,n)=HM(i+4*(k-1),((i-1)*4)+j+n)+MM(i,j,n);
        \end{verbatim}

end
\end{verbatim}

matlab_files/HMgen.m
Appendix B

Generate a system matrix of GASP with mirrors in the polarimeter

%inputs are angles, n, Pang,
%output A matrix

function [A] = A_gen_mir(np, na, th_i, pang)

function [A] = A_gen_mir(np, na, th_i, pang)

%inputs required are np: prism refractive index, na: refractive index of air, Th._i: angle of incidence on the prism, pang: prism angle.

th_i = th_i * pi / 180;

th_ideal = 78.81 * pi / 180;

pang = pang * pi / 180;

th_r = asin(na .* sin(th_i) ./ np);

th_ir = pang - th_r;

npa = np ./ na;

nap = na ./ np;

%calculate the Fresnel coefficients of the prism

rs = (cos(th_i) - ((npa.^2) - (sin(th_i).^2)) .* 0.5) ./ (cos(th_i) + ((npa.^2) - (sin(th_i).^2)) .* 0.5);

rp = (((npa.^2) .* (cos(th_i)) - ((npa.^2) - (sin(th_i).^2)) .* 0.5)) ./ (((npa.^2) - (sin(th_i).^2)) .* 0.5);

t_s = (2 .* na .* cos(th_i)) ./ (na .* cos(th_i) + np .* cos(th_r));

t_p = (2 .* na .* cos(th_i)) ./ (na .* cos(th_r) + np .* cos(th_i));

r_s1 = (((1 ./ npa.^2) - (sin(th_r).^2)) .* 0.5) ./ (cos(th_r) + (((1 ./ npa.^2) - (sin(th_r).^2)) .* 0.5));

r_p1 = (((1 ./ npa.^2) .* (cos(th_r)) - (((1 ./ npa.^2) - (sin(th_r).^2)) .* 0.5)) ./ (((1 ./ npa.^2) .* (cos(th_r)) + (((1 ./ npa.^2) - (sin(th_r).^2)) .* 0.5)));

%calculate the reflection and transmission values

R = ((rs.^2) + (rp.^2)) / 2;

R2 = ((rs1.^2) + (rp1.^2)) / 2;

Tabs = exp(-0.2186 * 0.1404); %absorption of prism using beer lambert law

T = (1 - R) * (1 - R2) * Tabs;

%calculate the ellipsometric angles

psi_r = atan(rp / rs);

psi_t = atan(tp / ts * tp1 / ts1);

del_t = atan(((cos(th_ir) .* (((sin(th_ir).^2) - nap.^2) .* 0.5)) ./ ((sin(th_ir).^2)) .* 4);

del_r = 0;

%complex refractive index of the mirror (aluminium)
n_m=1.39+7.65i;

% set the mirror angles
M1_ang=90-th_ideal+55;
M2_ang=65;

% calculate the angles of incidence for the mirrors
th_im1=180/pi*(th_i)+M1_ang-90*pi/180;
th_im2=(M2_ang+1.7*(180/pi*(th_i-th_ideal)))*pi/180;

th_r1=asin(na.*sin(th_im1)./n_m);
th_r2=asin(na.*sin(th_im2)./n_m);

nma=n_m./na;
nam=na./n_m;

% calculate the Fresnel coefficients for mirror 1 and mirror 2
rm1s=(cos(th_im1)-((nma.^2)-(sin(th_im1).^2)).^0.5))./(cos(th_im1)
+(nma.^2)-(sin(th_im1).^2)).^0.5));
rm1p=((nma.^2).*(cos(th_im1)))-(nma.^2)-(sin(th_im1).^2)).^0.5)
 ./(((nma.^2).*(cos(th_im1)))+((nma.^2)-(sin(th_im1).^2)).^0.5));
tm1s=(2.*na.*cos(th_im1))./(na.*cos(th_im1)+n_m.*cos(th_r1));
tm1p=(2.*na.*cos(th_im1))./(na.*cos(th_r1)+n_m.*cos(th_im1));

% calculate the ellipsometric angles of the mirrors
rm2s=(cos(th_im2)-((nma.^2)-(sin(th_im2).^2)).^0.5))./(cos(th_im2)
+(nma.^2)-(sin(th_im2).^2)).^0.5));
rm2p=((nma.^2).*(cos(th_im2)))-(nma.^2)-(sin(th_im2).^2)).^0.5)
 ./(((nma.^2).*(cos(th_im2)))+((nma.^2)-(sin(th_im2).^2)).^0.5));
tm2s=(2.*na.*cos(th_im2))./(na.*cos(th_im2)+n_m.*cos(th_r2));
tm2p=(2.*na.*cos(th_im2))./(na.*cos(th_r2)+n_m.*cos(th_im2));

Rm1=((abs(rm1s).^2)+(abs(rm1p)).^2))./2;
Tm1=(1-Rm1);
Rm2=((abs(rm2s).^2)+(abs(rm2p)).^2))./2;
Tm2=(1-Rm2);

del_m1=abs(log(rm1p/abs(rm1p)))/i-log(rm1s/abs(rm1s))/i;
del_m2=abs(log(rm2p/abs(rm2p)))/i-log(rm2s/abs(rm2s))/i;

psi_r1=atan(abs(rm1p)/abs(rm1s));
psi_r2=atan(abs(rm2p)/abs(rm2s));

Reff=R*Rm1;
Teff=T*Rm2;

RT=[Reff 0 0 0
     0 Reff 0 0
     0 0 0 0
     0 0 0 0]
%generate the system matrix
A = 0.5*RT*[1+(cos(2*psi_rm1)*cos(2*psi_r)) - (cos(2*psi_r) + cos(2*psi_rm1))
  sin(2*psi_rm1)*sin(2*psi_r)*(cos(del_m1)*cos(del_r) - sin(del_m1)*sin(del_r))
  sin(2*psi_rm1)*sin(2*psi_r)*(cos(del_m1)*sin(del_r) + sin(del_m1)*cos(del_r))
  1+(cos(2*psi_r)*cos(2*psi_t)) - (cos(2*psi_t) + cos(2*psi_r))
  sin(2*psi_r)*sin(2*psi_t)*(cos(del_m2)*cos(del_t) - sin(del_m2)*sin(del_t))
  sin(2*psi_r)*sin(2*psi_t)*(cos(del_m2)*sin(del_t) + sin(del_m2)*cos(del_t))
  1+(cos(2*psi_t)*cos(2*psi_r)) - (cos(2*psi_r) + cos(2*psi_t))
  sin(2*psi_t)*sin(2*psi_r)*(cos(del_m2)*cos(del_t) - sin(del_m2)*sin(del_t))
  sin(2*psi_t)*sin(2*psi_r)*(cos(del_m2)*sin(del_t) + sin(del_m2)*cos(del_t))];

matlab_files/A_gen_mir.m

%% Stokes Analysis
%function to take in a stokes vector and reduce the
%data to its 5 component results. Theta, Phi, DOP, DOL, DOC
%[Theta, Phi, DOP, DOL, DOC]=Stokes_analysis(stokes)

function [Theta, Phi, DOP, DOL, DOC]=Stokes_analysis(stokes)
if size(stokes)==[4 1] size(stokes)==[1 4]
  Theta=180/pi*(0.5*atan2(stokes(3),stokes(2)));
  %Theta=180/pi*(0.5*atan(stokes(3)/stokes(2)));
  Phi=180/pi*(0.5*atan(stokes(4)/sqrt((stokes(2)^2)+(stokes(3)^2))));
  DOP=100*sqrt((stokes(2)^2)+(stokes(3)^2)+(stokes(4)^2))/stokes(1);
  DOL=100*sqrt((stokes(2)^2)+(stokes(3)^2))/stokes(1);
  DOC=100*stokes(4)/stokes(1);
else
end

matlab_files/Stokes_analysis.m
Appendix C

List of polarimetric calibration standard stars
## List of polarimetric calibration standard stars

Table C.1: Polarized standard stars for polarimetric calibration (V band)

<table>
<thead>
<tr>
<th>Name</th>
<th>RA(2000)</th>
<th>DEC(2000)</th>
<th>Mag</th>
<th>PA</th>
<th>DOP</th>
<th>Citation</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 204827</td>
<td>21 28 57.7</td>
<td>58 44 24.0</td>
<td>7.93</td>
<td>60.0</td>
<td>5.39</td>
<td>[87]</td>
</tr>
<tr>
<td>CRL 2688</td>
<td>21 02 18.8</td>
<td>+36 41 41.2</td>
<td>12</td>
<td>105</td>
<td>47.8</td>
<td>[88][89][90]</td>
</tr>
<tr>
<td>HD 236928</td>
<td>02 02 42.0</td>
<td>+60 15 26.5</td>
<td>9.07</td>
<td>98.2</td>
<td>6.69</td>
<td>[90]</td>
</tr>
<tr>
<td>BD+64d106</td>
<td>00 57 36.7</td>
<td>+64 51 27</td>
<td>10.3</td>
<td>96.6</td>
<td>5.69 ± 0.04</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 7927</td>
<td>01 20 04.9</td>
<td>+58 13 54</td>
<td>5.0</td>
<td>92.1</td>
<td>3.32 ± 0.04</td>
<td>[92]</td>
</tr>
<tr>
<td>BD+59d389</td>
<td>02 02 42.1</td>
<td>+60 15 27</td>
<td>9.1</td>
<td>98.1</td>
<td>6.70 ± 0.2</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 19820</td>
<td>03 14 05.4</td>
<td>+59 33 48</td>
<td>7.1</td>
<td>115.4</td>
<td>4.82 ± 0.03</td>
<td>[92]</td>
</tr>
<tr>
<td>HD 25443</td>
<td>04 06 08.1</td>
<td>+62 06 07</td>
<td>6.8</td>
<td>135.1</td>
<td>5.15 ± 0.03</td>
<td>[92]</td>
</tr>
<tr>
<td>HD 251204</td>
<td>06 05 05.7</td>
<td>+23 23 39</td>
<td>10.3</td>
<td>147</td>
<td>4.04 ± 0.07</td>
<td>[90]</td>
</tr>
<tr>
<td>HD 43384</td>
<td>06 16 58.7</td>
<td>+23 44 27</td>
<td>6.3</td>
<td>169.8</td>
<td>2.94 ± 0.04</td>
<td>[107]</td>
</tr>
<tr>
<td>HD 154445</td>
<td>17 05 32.2</td>
<td>-00 53 32</td>
<td>5.6</td>
<td>88.6</td>
<td>3.67 ± 0.05</td>
<td>[92]</td>
</tr>
<tr>
<td>HD 155197</td>
<td>17 10 15.6</td>
<td>-04 50 03</td>
<td>9.2</td>
<td>103.2</td>
<td>4.38 ± 0.03</td>
<td>[90]</td>
</tr>
<tr>
<td>HD 161056</td>
<td>17 43 47.0</td>
<td>-07 04 46</td>
<td>6.3</td>
<td>66.3</td>
<td>4.00 ± 0.01</td>
<td>[92]</td>
</tr>
<tr>
<td>Hiltner 960</td>
<td>20 23 28.4</td>
<td>+39 20 56</td>
<td>10.6</td>
<td>54.8</td>
<td>5.66 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>VI Cyg12</td>
<td>20 32 40.9</td>
<td>+41 14 26</td>
<td>11.5</td>
<td>115.0</td>
<td>8.95 ± 0.09</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 204827</td>
<td>21 28 57.7</td>
<td>+58 44 24</td>
<td>7.9</td>
<td>58.7</td>
<td>5.34 ± 0.02</td>
<td>[92]</td>
</tr>
</tbody>
</table>
## Appendix C

### Table C.2: Unpolarized Standard stars for polarimetric calibration (V band)

<table>
<thead>
<tr>
<th>Name</th>
<th>RA(2000) [H M S]</th>
<th>DEC(2000) [°'&quot;]</th>
<th>Mag</th>
<th>PA [°]</th>
<th>DOP [%]</th>
<th>Citation</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD+323739</td>
<td>20 12 02.1</td>
<td>+32 47 43.1</td>
<td>9.31</td>
<td>35.79</td>
<td>0.025</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 212311</td>
<td>22 21 58.6</td>
<td>+56 31 52.7</td>
<td>8.12</td>
<td>50.99</td>
<td>0.034</td>
<td>[93]</td>
</tr>
<tr>
<td>βCas</td>
<td>00 09 10.7</td>
<td>+59 08 59</td>
<td>2.3</td>
<td>72.5</td>
<td>0.04 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 12021</td>
<td>01 57 56.1</td>
<td>-02 05 58</td>
<td>8.9</td>
<td>160.1</td>
<td>0.08 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 14069</td>
<td>02 16 45.2</td>
<td>+07 41 11</td>
<td>9.0</td>
<td>156.6</td>
<td>0.02 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 21447</td>
<td>03 30 00.2</td>
<td>+55 27 07</td>
<td>5.1</td>
<td>171.5</td>
<td>0.05 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>G191B2B</td>
<td>05 05 20.6</td>
<td>+52 49 54</td>
<td>11.8</td>
<td>147.7</td>
<td>0.06 ± 0.04</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 94851</td>
<td>10 56 44.2</td>
<td>-20 39 52</td>
<td>9.2</td>
<td>—</td>
<td>0.06 ± 0.02</td>
<td>[90]</td>
</tr>
<tr>
<td>GD 319</td>
<td>12 50 04.5</td>
<td>+55 06 03</td>
<td>12.3</td>
<td>140.2</td>
<td>0.09 ± 0.09</td>
<td>[91]</td>
</tr>
<tr>
<td>γBoo</td>
<td>14 32 04.7</td>
<td>+38 18 30</td>
<td>3.0</td>
<td>21.3</td>
<td>0.07 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 154892</td>
<td>17 07 41.4</td>
<td>+15 12 38</td>
<td>8.0</td>
<td>—</td>
<td>0.05 ± 0.03</td>
<td>[90]</td>
</tr>
<tr>
<td>BD+32d3739</td>
<td>20 12 02.1</td>
<td>+32 47 44</td>
<td>9.3</td>
<td>35.8</td>
<td>0.03 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>BD+28d4211</td>
<td>21 51 11.1</td>
<td>+28 51 52</td>
<td>10.5</td>
<td>54.2</td>
<td>0.05 ± 0.03</td>
<td>[91]</td>
</tr>
<tr>
<td>HD 212311</td>
<td>22 21 58.6</td>
<td>+56 31 53</td>
<td>8.1</td>
<td>51.0</td>
<td>0.03 ± 0.02</td>
<td>[91]</td>
</tr>
<tr>
<td>ζPeg</td>
<td>22 41 27.7</td>
<td>+10 49 53</td>
<td>3.4</td>
<td>40.0</td>
<td>0.05 ± 0.02</td>
<td>[91]</td>
</tr>
</tbody>
</table>
Bibliography


[72] A. Fresnel, “Mémoire sur les modifications que la réflexion imprime la lumière polarisée” (memoir on the alterations that reflection imparts to polarized light). Mmoires de l’Académie des sciences de l’Institute de France, vol. 11, pages 373-434, 1832. (Note: This manuscript was submitted to the Institute on 10 November 1817 and was read to the Institute on 7 January 1823.).


