Abstract

Nesting techniques are used to develop multi-scale models giving higher resolution and thus greater accuracy in nested domains. The assessment of model accuracy is therefore important for the successful implementation of nesting techniques. This paper presents details of a nested model performance assessment protocol. The basis of the protocol is the quantification of errors in the nested solution relative to a reference solution using a unique error parameter - the tidally-averaged relative error field, which simultaneously captures both spatial and temporal errors. The protocol was applied to an idealised tidal environment and was found to be an accurate measure of nested model performance. The approach is particularly useful for the development and testing of nesting techniques and the identification of suitable nested boundary locations.

Keywords: nesting, coastal modelling, performance assessment, hydrodynamics.

1 Introduction

The problem of computational cost is a perennial one for the numerical modeller. A main determinant of computational cost is spatial resolution. In the context of coastal circulation models, high spatial resolution gives enhanced representation of land-sea interactions in near-shore waters. This can lead to more accurate simulation of hydrodynamic activity and, consequently, improved simulation of sediment transport, primary production and other system processes. However, higher spatial resolution, if applied across the full model domain, can lead to a very large number of computational grid points and a substantial increase in computer processing time. The size of the computational domain also contributes to computational cost. Open boundaries must be located such that their conditions will not adversely affect model predictions in the area of interest (AOI). This often leads to a situation where the
AOI comprises only a small percentage of the computational domain. The most common tidal circulation models use a single structured computational grid. If high spatial resolution is required in an AOI it must therefore be applied across the full model domain meaning that many grid points outside the AOI are unnecessary. The most common solution to the spatial resolution problem is the use of a nesting method where a grid of smaller mesh size (the child grid, CG) is nested within a grid of larger mesh size (the parent grid, PG).

Much of the developmental work on the implementation of nesting techniques was carried out within the field of meteorological modelling [1, 2, 3, 4] and their use in that area is well-established. It was not until the very late eighties and early nineties that nesting methods were first adopted in the field of hydrodynamic modelling [5] but their use in this area has since become quite common [6, 7]. Nesting methods may be characterised as either one-way or two-way. In both cases boundary conditions for the child are obtained from the parent. If no feedback is provided from the child to the parent, i.e. interaction occurs in one direction only, the method is described as one-way. If CG values are in turn used in some way to modify PG values, i.e. interaction occurs in two directions, then the method is described as two-way. Regardless of the type of nesting method used, the aim is the same - greater accuracy in the child grid. A critical component of nested model development then is the assessment of nested model performance.

Where an analytic solution exists, a nested model solution may be compared with the analytic solution [8] to assess performance. However, for more complex modelling applications where an analytic solution does not exist, e.g. real-world applications, the nested model solution is typically compared with a reference solution computed by a high resolution single grid model [9]. Comparison of model solutions involves the analysis of spatial variations in model parameters at particular points in time (i.e. snapshot comparisons) and/or temporal variations in model parameters at discrete points in space (i.e. timeseries comparisons). Neither form of analysis can simultaneously capture the spatially- and temporally-varying nature of tidally-driven flows. The Authors have developed a performance assessment protocol (PAP) which enables simultaneous analysis of both the spatial and the temporal errors in a nested model solution. This paper describes the new assessment protocol. The mathematical basis of PAP and its method of application are explained and results from an assessment of a nested model of an idealised harbour are presented to demonstrate the effectiveness of the protocol.

2 Performance Assessment Protocol

Tidally-driven flows vary in both space and time. The accuracy of a tidal flow model will therefore also vary in both space and time and any approach to performance assessment of such models should reflect this. In a typical assessment of model performance snapshot comparisons help determine spatial accuracy at a particular instance of time while timeseries comparisons help determine temporal accuracy at
discrete locations. However, spatial accuracy at a single instance of time gives no guarantee of accuracy at other instances of time and temporal accuracy at one discrete location gives no guarantee of accuracy at other locations. PAP was designed to produce a single error field which simultaneously quantified both the spatial and temporal errors in a model solution. It was found that nested model error was best quantified by assessing the level of error in current velocities as water surface elevations are relatively insensitive to model resolution. Hereafter, reference to model errors infers that they relate to errors in current velocities.

2.1 The Tidally-Averaged Relative Error \( (RE_T) \)

The PAP approach was specifically developed for modelling applications where an analytic solution does not exist; the accuracy of the CG solution is therefore determined relative to a reference solution. The reference solution is computed by a single grid (SG) model of the full PG domain resolved at the same high resolution as the CG. The reference solution is considered the 'correct' model solution. In a typical performance assessment, where snapshot data is available for CG and SG solutions at time \( t \), the absolute error \( (AE) \) and relative error \( (RE) \) in CG data relative to SG data can be defined as:

\[
AE_{i,j}^t = |\phi_{CG}^t_{i,j} - \phi_{SG}^t_{i,j}|, \quad \text{for } i=1,2,3\ldots i_{\text{max}}; \ j=1,2,3\ldots j_{\text{max}} \tag{1}
\]

\[
RE_{i,j}^t = \frac{|\phi_{CG}^t_{i,j} - \phi_{SG}^t_{i,j}|}{\phi_{SG}^t_{i,j}} \times 100, \quad \text{for } i=1,2,3\ldots i_{\text{max}}; \ j=1,2,3\ldots j_{\text{max}} \tag{2}
\]

where \( \phi \) is a prognostic variable and \( i,j \) are the model grid coordinates (Note: the \( i,j \) coordinate system corresponds to the standard \( x,y \) cartesian coordinate system). Error data computed from equations (1) and (2) can be presented graphically as spatial distributions of error within the CG domain at a time \( t \).

To compute and plot error for snapshot data at every timestep of a model simulation to ensure accuracy in time as well as space would be prohibitively time-consuming. It was therefore decided to combine spatial and temporal error data by computing tidally-averaged errors at each grid point. The tidally-averaged absolute error field \( (AE_T) \) and the tidally-averaged relative error field \( (RE_T) \) were calculated as:

\[
AE_T^t_{i,j} = \frac{\sum_{n=1}^{N} |\phi_{CG}^n_{i,j} - \phi_{SG}^n_{i,j}|}{N}, \quad \text{for } i=1,2,\ldots, i_{\text{max}}; \ j=1,2,\ldots, j_{\text{max}} \tag{3}
\]
\[
RE_T |_{i,j} = \frac{\sum_{n=1}^{N} |\phi_{CG}^{n}_{i,j} - \phi_{SG}^{n}_{i,j}|}{\sum_{n=1}^{N} |\phi_{SG}^{n}_{i,j}|} \times 100
\quad \text{for } i=1,2,\ldots, i_{\text{max}}; j=1,2,\ldots, j_{\text{max}}
\]

where \( n \) is a snapshot identification number and \( N \) is the total number of snapshots output during the course of one complete tidal cycle. Equations (3) and (4) are merely approximations of \( AE_T \) and \( RE_T \) as they do not use snapshot data at every model timestep. For a 12.5hr tidal cycle, tests established that \( N=25 \) was the lowest number of datasets required to give accurate approximations of \( AE_T \) and \( RE_T \) at a particular grid point; thus, snapshot data was output at half-hourly intervals.

\( AE_T \) and \( RE_T \) error fields describe both the spatial and temporal errors in a CG solution and are best displayed as spatial distributions of error within the CG domain. They may also be further processed to give a single figure for model error. \( AE_T \) or \( RE_T \) can be summed across all wet grid cells in the AOI to give the total error in the AOI. Dividing the total error by the number of wet grid cells then yields the domain-averaged error, \( AE_D \) or \( RE_D \), for the CG solution; in other words, the average error per grid cell per tidal cycle. In this way, the total spatial and temporal errors in a CG simulation can be quantified by a single error parameter.

### 2.2 Error Filtering

During initial calculations of \( RE_T \) in current velocities, large \( RE_T \) values were found to occur in areas of low hydrodynamic activity. These areas were characterised by low velocities and small absolute errors of little significance to the overall system dynamics could manifest as large relative errors. Such large, but insignificant, relative errors had the effect of misrepresenting the magnitude of error in an AOI and artificially increasing the domain-averaged relative error, \( RE_D \). A method of error filtering was developed to remove insignificant errors from the error field so that \( RE_T \) plots only presented errors of hydrodynamic significance.

The most important aspect of the design of the error filter was the selection of the cut-off value at which errors were deemed insignificant. The absolute error in current velocities was chosen as the most suitable parameter for determining a cut-off. In order to select an appropriate cut-off error it was important to relate the magnitudes of the absolute errors to the corresponding magnitudes of current velocities. A maximum velocity field was computed for the AOI using SG velocities. This field recorded the maximum velocities found at each grid cell over the course of a tidal cycle and was considered to describe the level of hydrodynamic activity within the AOI. The domain-averaged maximum velocity, \( V_{\text{max,avg}} \), was then calculated and the cut-off error, \( AE_C \), was set to 3% of this value, that is:

\[
AE_C = 0.03(V_{\text{max,avg}})
\]
Once a cut-off error was determined the error filter was applied to $RE_T$ according to the following rule:

$\begin{align*}
\text{If: } & AE_T^{i,j} \leq AE_C^{i,j} \\
\text{then: } & RE_T^{i,j} = 0, \ AE_T^{i,j} = 0 \\
\end{align*}$

for $i=1,2,3...i_{\text{max}}$;

$\begin{align*}
\text{for } j=1,2,3...j_{\text{max}}
\end{align*}$

(6)

The error filter was found to successfully remove those errors of insignificant magnitudes in areas of low hydrodynamic activity whilst preserving all other significant errors. Results are presented in a later section to demonstrate the effect of the error filter.

### 2.3 Application of PAP

PAP assesses the performance of a nested model by determining the level of error in a CG solution relative to a high resolution SG solution. Application of PAP involves the following steps:

1) run nested model to produce PG and CG solutions; run high resolution reference model to produce SG solution
2) compute maximum SG velocity field for AOI
3) calculate $V_{\text{max,avg}}$ for maximum velocity field
4) calculate $AE_C$ for the error filter
5) calculate $RE_T$, in CG velocities whilst applying the error filter
6) display $RE_T$ in CG velocities as a spatial distribution plot and compute $RE_D$

PAP can also be used to assess the accuracy of a PG solution by substituting PG data for CG data. This can be useful for the selection of suitable CG boundaries as it enables identification of areas of high PG accuracy. It is also useful when developing a nested model as it allows measurement of the improvements in accuracy achieved as a result of nesting.

### 3 The Nested Model

The nested hydrodynamic model used for this research was developed by the Authors [10] and is based on the DIVAST (Depth Integrated Velocity and Solute Transport) model. It is a two-dimensional, depth-averaged, finite difference model where the hydrodynamic module computes velocity and surface elevation fields. The hydrodynamics are based on the solution of the depth integrated Navier-Stokes equations and include the effects of local and advective accelerations, the rotation of the earth, barotropic and free surface pressure gradients, wind action, bed resistance and a simple mixing length turbulence model. Full details of the hydrodynamic formulations are given in [11].
The nested model uses an overlapping grid structure and allows multiple levels of nesting so that child grids may be telescoped to any depth (i.e., a parent grid may contain one or more child grids and each child grid, in turn, may successively contain one or more child grids). Figure 1 shows an example of the grid structure. The nested model effectively consists of a series of parent and child models, dynamically linked and running simultaneously, where each model operates on the set of grids at a particular level of nesting. The open boundary conditions for each child grid are obtained from its parent using an adaptive linear interpolation scheme and are assigned to the child grid using a Dirichlet boundary condition.

![Figure 1: Nested model grid structure for multiple nesting.](image)

Model solutions are computed on a space-staggered grid system (see Figure 2) with water elevation (\( \zeta \)) discretised at the centre of the grid cell and velocity components (\( U \) and \( V \)), volumetric flux components (\( q_x \) and \( q_y \)) and water depths (\( H_x \) and \( H_y \)) discretised at the centres of the x- and y-faces of the grid cells respectively.. The model allows any integer spatial nesting ratios; however, an odd nesting ratio, such as the 3:1 ratio used in Figure 2, is preferable as it ensures that each grid value of the overlapping region of a PG coincides with a value from its CG. Although the model allows for different spatial and temporal nesting ratios, the same ratio is typically used for both. Boundary data are interpolated from the PG to the CG boundary interface which comprises both internal boundary cells and ghost cells. Only the components of velocity and flux normal to the boundary are specified at ghost cells whilst all prognostic variables are specified at internal boundary cells.

4 Model Results

PAP was applied to the idealised rectangular harbour shown in Figure 3. The harbour dimensions were 12x6 km and bed depth gradually deepened from 4 m at the back wall of the harbour to 10 m at the sea boundary. A harbour wall was included to induce a more complex, momentum-driven flow regime. The PG model covered the full extents of the harbour and was resolved at a grid spacing of 120 m
and a timestep of 120 s. To demonstrate the effectiveness of the protocol, two nested simulations were run for different CG extents and boundary locations (see Figure 3). In both simulations, the CG domain extended from the CG boundary to the back wall of the harbour. A 3:1 nesting ratio was used; the CG model was therefore resolved at a grid spacing of 40 m and a timestep of 40 s. The domain of the SG reference model covered the full PG domain but was resolved at the CG resolution. The exterior forcing function for all models was a tide of constant period (12.5 hrs) and range (3 m). Each model simulation was run for four tidal cycles (50 hours) starting at the time of high water. Model results were output and analysed for the fourth and final tidal cycle. For consistency the CG1 domain was taken as the AOI and error analyses are only presented for the AOI.

Figure 2: Schematic of the grid configuration for a 3:1 nesting ratio.

Figure 3: Idealised rectangular harbour and CG boundaries (dimensions in metres).
4.1 Application of PAP to CG1

PAP was first applied to CG1 without the error filter. Figure 4 shows $RE_T$ and $AE_T$ in CG velocities in the AOI. It can be seen that errors were present in the CG1 solution near the boundary; however, these errors resulted from inaccuracies in the boundary data rather than the nesting technique. It will be shown in the next section that the PG solution was quite inaccurate along CG1. These inaccuracies were passed to the CG solution via the boundary data. The errors decreased with distance from the boundary due to the improved resolution of the CG. In general, $RE_T$ in the CG1 solution were quite low and $RE_D$ was just 4%.

![Figure 4: (a) $RE_T$ and (b) $AE_T$ in CG1 velocities (without error filtering).](image)

A significant difference between the error distribution plots was the area of high errors in the $RE_T$ plot at the back of the harbour. While $RE_T$ values in this area were high, the corresponding $AE_T$ values were actually quite low - less than 0.001m/s. The only means by which such an order of absolute errors could represent high relative errors was if the corresponding velocities were of similarly low magnitudes, in which case the absolute errors would be relatively large. Figure 5a presents the maximum SG velocity field for the AOI. It shows that the maximum current velocities at the back of the harbour were less than 0.05m/s. This order of velocities was less than 5% of the domain maximum (approximately 1.05m/s). The hydrodynamic activity at the back of the harbour was much lower than that in the rest of the harbour and therefore much less significant in terms of general circulation within the CG domain. The high relative errors at the back of the harbour were actually insignificant as they represented errors of low absolute magnitude in an area of low hydrodynamic activity.

According to the error filtering methodology, $V_{max, avg}$ was computed for the maximum SG velocity field (0.19m/s) and used to calculate $AE_C$ (0.0019m/s). $RE_T$ in CG1 velocities was then recalculated with error filtering included; the resulting error distribution plot is shown in Figure 5b. It can be seen that the error filter removed the insignificant errors at the back of the harbour whilst preserving the significant
errors elsewhere. As a result of the error filtering, $RE_D$ was reduced from 4% to 2.7%. It should be noted that when error filtering was included in the calculation of $AE_T$ no significant change in $AE_D$ was observed as the magnitudes of those errors removed from the calculation were so low.

![Figure 5: (a) Maximum SG velocities and (b) filtered $RE_T$ in CG1 velocities.](image)

### 4.2 Application of PAP to PG

PAP was next applied to the PG solution to determine both the effect of lower resolution on model accuracy and the improvement in accuracy from PG to CG. $RE_T$ in PG velocities, shown in Figure 6, confirmed that the PG solution contained significant errors relative to the reference solution. Lower resolution meant that PG was unable to adequately resolve flow through the constricted mouth of the harbour wall. As a result, errors were highest in the areas of flow to either side of the wall. For the most part, errors ranged from 5-25% although errors greater than 50% were also found to occur in some limited areas. Also notable were the relatively large errors in the PG solution along the location of the CG1 boundary.

![Figure 6: $RE_T$ in PG current velocities (without error filtering).](image)
A comparison of $RE_T$ in PG velocities in the AOI with those in CG1 velocities is shown in Figure 7. It should be noted that the contour scale used in these plots is different to that used in the previous error plots. It is clear that the CG1 solution was more accurate than the PG solution. $RE_T$ of 5% was exceeded in 66% of the AOI for PG compared to 20% for CG while $RE_D$ was 7.1% for PG compared to just 2.7% for CG. $RE_D$ values indicated more than a 50% improvement in model accuracy as a result of nesting.

The accuracy of $RE_T$ approximations calculated using PAP was verified at the nine discrete locations shown in Figure 7b. Current velocities were output at each location every 6mins of the simulation giving 125 data values over the course of a complete tidal cycle. These data were then applied to equation (4) for $n=125$ to calculate 'true' values of $RE_T$ which were then compared to the approximated PAP values. Table 1 lists the true values with the PAP values for the nine locations. Good correlation between true and approximated errors was observed at all nine locations and it was concluded that the $RE_T$ approach to error quantification was an accurate one.

![Figure 7: $RE_T$ in (a) PG and (b) CG1 velocities (with error filtering).](image)

<table>
<thead>
<tr>
<th>Location</th>
<th>$RE_T$ PG [%]</th>
<th>$RE_T$ CG1 [%]</th>
</tr>
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<td>True</td>
<td>PAP</td>
<td>True</td>
</tr>
<tr>
<td>A</td>
<td>13.53</td>
<td>13.99</td>
</tr>
<tr>
<td>B</td>
<td>8.38</td>
<td>8.36</td>
</tr>
<tr>
<td>C</td>
<td>14.09</td>
<td>14.05</td>
</tr>
<tr>
<td>D</td>
<td>8.32</td>
<td>7.89</td>
</tr>
<tr>
<td>E</td>
<td>6.60</td>
<td>6.64</td>
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<tr>
<td>F</td>
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<td>7.55</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>I</td>
<td>4.88</td>
<td>4.48</td>
</tr>
</tbody>
</table>

Table 1: Comparison of true and approximated (PAP) $RE_T$ values for PG and CG1.
4.3 Selection of CG Boundaries

PAP proved extremely useful for the selection of suitable CG boundaries. It is important that CG boundaries are located in areas of high PG accuracy to reduce the errors passed from PG to CG via the CG boundary data. The PG error distribution plot of Figure 6 allowed fast and easy identification of areas of high PG accuracy suitable for location of CG boundaries. It can be seen that CG1 was placed in an area of low PG accuracy. The average $RE_T$ in PG velocities calculated along the CG1 boundary was 15%; this error was passed to the CG1 solution resulting in the inaccuracies observed in Figure 7a. Returning to Figure 6, it can be seen that PG accuracy was noticeably higher along CG2 than CG1. The average $RE_T$ in PG velocities along the CG2 boundary was just 8%. CG2 was a more suitable boundary than CG1 and was found to produce a CG solution of similar accuracy to the reference solution. Figure 8a shows $RE_T$ in CG2 velocities. $RE_T$ did not exceed 1% anywhere in the AOI; in contrast $RE_T$ of 1% was exceeded in 57% of the AOI for CG1. $RE_D$ was also less than 0.1% for CG2 compared to 2.7% for CG1.

The improved accuracy of the CG2 solution compared to CG1 can also be seen from the comparison of velocity time series at output location A in Figure 8b. SG and PG velocities are included for reference. The PG velocities can be seen to be substantially different to the SG velocities. A marked improvement in accuracy can be seen from PG to CG1 as a result of the nesting, but most striking is the high level of accuracy of CG2; most CG2 data points overly the SG data points. The high level of accuracy of the CG2 solution is directly attributable to the selection of a suitable boundary location using PAP.

Figure 8: (a) Filtered $RE_T$ in CG2 velocities and (b) velocity timeseries at location A.

5 Summary and Conclusions

The use of nesting modelling techniques for multi-scale modelling of marine waters is becoming more widespread. Determination of model accuracy in the nested
domain to establish the success, or otherwise, of the nesting process is important but it is also problematic and time-consuming due to the spatially- and temporally-varying nature of tidal flows. A performance assessment protocol (PAP) was developed which simultaneously quantifies both the spatial and temporal errors in a CG solution. To the Authors’ knowledge this is the first assessment protocol of this kind; existing protocols determine either spatial or temporal errors independently of each other and low spatial errors do not necessarily guarantee low temporal errors or vice versa.

PAP is based on the mathematical quantification of error in a CG solution relative to a reference solution using a novel error parameter – the tidally-averaged relative error field. $RE_T$ in CG velocities is approximated at each grid cell in the AOI using snapshot data output at half-hourly intervals over a tidal cycle. $RE_T$ for a particular grid cell quantifies the temporal error at that location. The spatial variation in error is simultaneously captured when the complete $RE_T$ field is graphed as an error distribution plot for the AOI. The error data can be further processed to calculate $RE_D$, a single figure quantification of the error in the CG solution. This error parameter combines both the spatial and temporal variations in model performance.

PAP was applied to an idealised harbour with tidal forcing and was found to be both efficient and effective. The protocol enables fast and accurate quantification of model error and has a number of applications. It can be used for the development and testing of nested modelling techniques as the effectiveness of a nesting technique is measured by the accuracy of the CG solution. Reduction of $RE_D$ for a CG domain indicates improvements in the effectiveness of a nesting technique. It can also be used to aid the selection of suitable CG boundaries. Application of PAP to a PG solution allows easy identification of areas of high PG accuracy where CG boundaries should be located to ensure that boundary data errors are low. Finally, the protocol is also useful for determining the level of improvement achieved by the nesting process. This may be done by comparing PG errors with CG errors. In some cases, the PG may be quite accurate for the AOI and the improvements in accuracy resulting from nesting may not be worth the additional computational cost.

References