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'Kaldor on Debreu: The Critique of General Equilibrium Reconsidered'

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ABSTRACT

This paper revisits Kaldor's methodological critique of orthodox economics. The main target of his critique was the theory of general equilibrium as expounded in the work of Debreu and others. Kaldor deemed this theory to be seriously flawed as an empirically adequate description of real-world economies. According to Kaldor, scientific progress was not possible in economics without a major act of demolition, by which he meant the destruction of the basic conceptual framework of the theory of general equilibrium. We extend Kaldor's critique by recourse to major developments in 20th century philosophy of mathematics, and then go on to demonstrate that Debreu's work, based as it is on Bourbakist formalism and in particular Cantorian set theory, is conceptually incompatible with Kaldor's requirements for an empirical science. This aspect of Kaldor's critique has not been explored, and as a consequence a major source of substantiating his critique has remained undeveloped.

2

1. Introduction

Nicholas Kaldor's contributions to economics covered an extraordinary range of interests, including monetary theory, welfare economics and the theory of growth and distribution to name but a few. Kaldor, it has been argued, in a comparison he surely would have found pleasing, resembled Keynes more than any other 20th century economist. Among the parallels were Kaldor's 'wide-ranging contribution to theory, his insistence that theory must serve policy, his periods as an advisor to governments, his fellowship at King's and … his membership of the House of Lords' (Harcourt, 1988, p. 159).

As documented by his biographers, Kaldor developed a major critique of orthodox economics, a body of theory he found to be seriously inadequate both theoretically and empirically (see Thirlwall, 1987; Targetti, 1992 and Turner, 1993). A central target of this critique was what he termed 'equilibrium economics,' and more particularly the general equilibrium variant of this mode of theorizing, particularly as articulated in the work of Debreu. His criticism of this approach to economic theory was both fundamental and relentless. Equilibrium economics was, he argued, 'barren and irrelevant as an apparatus of thought to deal with the manner of operation of economic forces' (Kaldor, 1972, p. 1237).

He went further and argued that such was the powerful, but negative, influence exerted by 'equilibrium economics' that it 'has become a major obstacle to the development of economics as a science' (ibid., p. 1237). Kaldor's critique of equilibrium economics was derived from a number of informing principles that shaped his conception of science in general and of economics in particular. The central thrust of his methodological critique was aimed at the empirical inadequacy of orthodox equilibrium theory in representing the reality of the contemporary economic system of developed market economies. Kaldor's critique culminated with his call for 'a major act of demolition': real progress in economics would be impossible 'without destroying the basic conceptual framework' of general equilibrium theory (ibid., p. 1240).

In his own work on increasing returns, cumulative causation and other aspects of economic dynamics, Kaldor presented many perceptive and innovative lines of development for the re-orientation of economic theory. However, his methodological critique, the main interest of this paper, while radical and methodologically challenging, remained fragmented and philosophically incomplete. With the exception of Lawson (1989), who interpreted Kaldor's methodological position within a critical realist framework, little work on the philosophical evaluation of Kaldor's methodological work has been undertaken. In earlier work we disputed Lawson's critical realist reading of Kaldor and argued that Kaldor's methodological position could more plausibly be interpreted within a philosophical framework which we termed causal holism (Boylan & O'Gorman, 1991, 1995, 1997, 2001).

In this paper we do not propose to revisit these debates, but rather to engage an aspect of Kaldor's call for the 'demolition' of general equilibrium theory. The aspect of the problem that arguably troubled Kaldor most profoundly arose from the colonization of economics by mathematics in the neo-Walrasian research programme that arose after the Second World War. According to Kaldor, economists sought to create a 'mathematical crystal' (the expression is borrowed from Heisenberg) a logical system 'which cannot be further improved or perfected' (Kaldor, 1985, p. 60). Such was the fascination of economic theorists with the neo-Walrasian framework that their 'views of reality became increasingly distorted, so as to come closer to the theoretical image rather than the other way round' (ibid., pp. 60-61).

The 'mathematical crystal' they constructed was based on the Bourbakist formalism applied by Debreu to the analysis of general equilibrium. If Kaldor's programme of 'demolition' of this mode of economic theorizing is to be realized, the role of formalism in economics must be re-examined in the light of the early 20th century developments in the philosophy of mathematics. For these developments had important implications for economics arising from the different philosophical perspectives that emerged, which in turn had important potential consequences for the kind of mathematics that were most appropriate for economic analysis.

The structure of the paper is as follows. In Section 2 we provide an outline of Kaldor's theoretical critique of equilibrium economics. Section 3 summarizes his specific critique of Debreu's formalism. In Section 4 some of the demolition work that Kaldor called for

is accomplished by combining insights from debates in the philosophy of mathematics with Kaldor's criticisms of equilibrium economics. This is followed by a brief conclusion.

2.   Kaldor's Theoretical Critique of Equilibrium Economics[1]

Kaldor's penetrating methodological critique of neoclassical equilibrium theory is certain to 'remain one of his most important legacies' (Thirlwall, 1987, p. 316). His critical reflections on methodology first surfaced in a comment on a paper by Paul Samuelson and Franco Modigliani on the Pasinetti paradox; in his remarks Kaldor identified a number of the major themes that would preoccupy his later methodological writings (Kaldor, 1966). His critique of orthodox theory and its methodological foundations intensified during the 1970s and 1980s and culminated in the 1983 Okun Memorial Lectures and the 1984 Mattioli Lectures (Kaldor, 1984, 1985).[2]

Kaldor was unquestionably one of the pivotal post-war figures in the Cambridge critique of orthodox theory, but he pioneered an altogether broader attack on orthodoxy than many of his Cambridge colleagues. This arose from his strongly held view that there was not 'a single, overwhelming objection to orthodox economic theory: there are a number of different points that are distinct though interrelated' (Kaldor, 1975, pp. 347-48). He sometimes referred to his Cambridge colleagues as 'monists' for maintaining that exposing the logical inconsistencies of marginal productivity theory was 'alone sufficient to pull the rug from under the neoclassical value theory' (Kaldor, 1975, p. 348). Kaldor

felt strongly that this 'monist' approach was badly flawed and that marginal productivity theory was not the most significant domain of orthodoxy to contest. Other aspects of orthodox economics, he believed, are 'in some ways … even more misleading than the application of marginal productivity to the division between wages and profits, which has been the main subject of discussion' (Kaldor, 1975, p. 348).

In contrast to his Cambridge colleagues, Kaldor's non-monist critique extended to a number of key areas, all of which pointed to the emergence of his penetrating and substantive critique of equilibrium economics. The critique of these areas were elaborated in the course of Kaldor's major post-war methodological writings, but were most systematically delineated in his Okun Lectures. There Kaldor identified three major issues which he analyzed in some detail. The first referred to how markets work and why their *modus operandi* precluded a pure price system of market clearing; secondly, he addressed the issue of how prices are formed and how competition operates in the context of 'the quasi-competitiveness or quasi-monopolistic markets that embrace a very large part of a modern industrial economy' (Kaldor, 1985, p. 12); and finally Kaldor presented 'an outline of an alternative approach to orthodox equilibrium theory' (ibid., p. 12), which examined how to reincorporate the powerful influences of increasing returns into economic theory.

It was in formulating his 'alternative approach,' centred on increasing returns to scale, that Kaldor developed some of his most fundamental objections to equilibrium economics. The notion of equilibrium Kaldor had in mind was that 'of the general

economic equilibrium originally formulated by Walras,' but which had been developed 'with increasing elegance, exactness, and logical precision by the mathematical economists of our own generation,' most notably Gerard Debreu (Kaldor, 1972, p. 1237).

Thirlwall (1987) has identified three main strands to Kaldor's critique of equilibrium economics. The first was Kaldor's objection to the use made of axiomatic assumptions in equilibrium economics. For Kaldor, unlike any scientific theory, 'where the basic assumptions are chosen on the basis of direct observation of the phenomena,' the basic assumptions of economic theory 'are either of a kind that are unverifiable' - such as, consumers 'maximize' their utility or producers 'maximize' their profits - or 'are directly contradicted by observation.' The latter included the following extended list: 'perfect competition, perfect divisibility, linear-homogenous and continuously differentiable production functions, wholly impersonal market relations, exclusive role of prices in information flows and perfect knowledge of all relevant prices by all agents and perfect foresight' (Kaldor, 1972, p. 1238). The use of such assumptions, which were not just 'abstract' but 'contrary to experience' was contrary to good science and rendered economics vacuous as an empirical science.

Secondly, Kaldor argued that the primacy accorded to the principle of substitutability within the framework of the allocative function of markets, was at the expense of the principle of complementarity within the dynamic process of accumulation. For Kaldor complementarity was paramount, not just between factors of production, but between whole sectors of the economy, as his work on the relation between agriculture,

manufacturing and services demonstrated. The overarching emphasis on substitutability and trade-offs in equilibrium economics led to a neglect of the crucial role of complementarities in economic development, Kaldor argued. Allied to this concern was Kaldor's hostility to the emphasis on static allocation of a given set of resources in equilibrium economics. For him, the central economic problem was to understand the highly dynamic processes of accumulation and development (Kaldor, 1996).

Finally, Kaldor rejected the basic assumption of constant returns which dominated theorizing in equilibrium economics. More particularly he abhorred the fact that 'the general equilibrium school (as distinct from Marshall) has always fully recognized the *absence* of increasing returns as one of the basic "axioms" of the system.' As a result, 'the existence of increasing returns and its consequences for the whole framework of economic theory have been completely neglected' (Kaldor, 1972, pp. 1241-1242). Kaldor was strongly influenced by Allyn Young, one of his teachers at the London School of Economics in the 1920s. In a now classic paper, Young (1928) drew on insights from Adam Smith to re-establish the importance of increasing returns for economic progress. Kaldor believed that Young's paper was 'many years ahead of its time,' but that economists had 'ceased to take any notice of it long before they were able to grasp its full revolutionary implications' (Kaldor, 1972, p. 1243). Kaldor was also familiar with Sraffa's (1926) contribution to this issue. Kaldor became committed to the view that, contrary to the position in equilibrium economics, increasing returns were central to understanding production processes at the level of the firm, and this in turn explained his view of the manufacturing sector as the primary 'engine of growth' in the

development of capitalist economies. In light of this theoretical critique, as summarized above, Kaldor forged a marriage of the 'Smith-Young doctrine on increasing returns with the Keynesian doctrine of effective demand' (Kaldor, 1972, p. 1250) within the framework of a theory of cumulative causation for the analysis of economic change in decentralized market economies.

However, Kaldor's critique of equilibrium economics also involved a fundamental methodological critique. His conception of 'economics as a science' was fundamental to his critique of equilibrium economics. For Kaldor, science was 'a body of theorems based on assumptions that are empirically derived, and which embody hypotheses that are capable of verification both in regard to the assumptions and predictions (Kaldor, 1972, p. 1237). Starting from this view of science, he subjected equilibrium economics to a stringent methodological critique, the prevailing theme of which was the fundamental empirical inadequacy of equilibrium theory and its incapacity to engage the complexities of advanced market economies in a meaningful way.

While this position represented a fundamental rejection of the methodological basis of equilibrium economics, Kaldor did not provide a systematically formulated, much less a complete, alternative methodology for economics. Instead, what we find scattered among his economic writings are a number of important suggestions that some of the building blocks for the construction of an alternative methodology. According to Kaldor, any attempt to construct a scientific theory must begin with a summary of the known facts in the domain under investigation. In the case of economics, since the initial summary is

normally presented in a statistical framework, the economic theorist starts with a 'stylised' compendium of the facts. These 'stylized facts' are statistical, but not universal, generalizations that describe empirical regularities. Economists then proceed to construct their economic theory on what Kaldor calls the 'as if' method. While Kaldor does not spell out the full details of this method, we can reconstruct his position as follows. Firstly, the economist 'abstracts' or develops higher-level hypotheses consistent with the stylized facts and then proceeds to construct an economic theory. Secondly, the economist attempts to express the constructed theory in a systematic way, for example in the form of an axiomatic system. Finally, the theory is inductively tested, i.e. its predictions are tested empirically by observation of the economic world. In this connection, as Lawson (1989) has pointed out, Kaldor argued that the process of inductive testing was altogether more important than the process of axiomatization.

As noted above, we have assessed Kaldor's methodological contribution in earlier work, and shall not rehearse the arguments of this methodological evaluation here. Instead we focus on a specific dimension of Kaldor's critique, which thus far has been ignored in the literature on economic methodology, namely his unequivocal rejection of general equilibrium, which in his view received its most sophisticated articulation by Debreu. In the following section we examine Kaldor's rejection of Debreu's *Theory of Value* in the context of Debreu's own commitment to mathematical formalism, as articulated by the famous German mathematician, David Hilbert. We then, in the spirit of Kaldor, show why Debreu's work is 'thoroughly misleading and pretty useless − in terms of the theory's declared objective of explaining how economic processes work in a

decentralized market economy' (Targetti and Thirlwall, 1989, p. 411). The reason for its explanatory uselessness resides in the manner in which Debreu used Cantorian set theory in his mathematization of economics.

3. A Kaldorian Reading of Debreu

As Kaldor (1972, p. 1237) pointed out, Debreu's *Theory of Value* (1959) gives us an 'elegant, exact and logically precise' account of general equilibrium. Methodologically, Debreu's work has two distinctive, though interrelated, characteristics. Firstly, Debreu ingeniously exploited the powerful mathematical resources of Cantorian set theory. His approach marks a major shift in the process of the mathematization of economics in the course of the 20th century. In the first phase, initiated by Walras and others, the mathematical resources of Cantorian set theory were not exploited. However, in the second phase, which occurred in the second half of the century and is exemplified in the work of Debreu, Cantorian set theory becomes indispensable in proving the existence of general equilibrium (see Weintraub, 2002).

Secondly, Debreu's approach belongs to the axiomatic tradition, which originated with Euclid and culminated in the specific formalist view as articulated in the famous Hilbert axiomatic programme for pure mathematics. The latter characteristic, its strictly formalist character, which in turn informed Bourbakianism, is explicitly noted by Kaldor: 'In the strict sense, as Debreu says, the theory is "*logically entirely disconnected from its interpretation*"' (Kaldor, 1972, p. 1237, emphasis added). According to Debreu,

theoretical economics must attain the highest standards of logico-mathematical rigour as spelled out in the Hilbert axiomatic programme and in this programme any axiomatic system is an empty or meaningless, purely formal system. As Nagel and Newman, for instance, point out, the Hilbert programme 'involves draining the expressions occurring within the (axiomatic) system of all meaning: they are to be regarded simply as empty signs' (Nagel and Newman, 2005, p. 19). In this view 'the postulates and theorems of a completely formalized system are "strings" (or finitely long sequences) of meaningless marks constructed according to rules for combining elementary signs of the system into larger wholes' (ibid., p. 20).   Thus Hilbert is emphatic on the distinction between a formal, i.e. completely uninterpreted, system and any interpretation given to the formal system. Only when the empty symbols are given an interpretation does the issue of meaning arise. In this sense Hilbert's strictly formalist reading of a formal axiomatic system is very specific. It does not coincide with what might be called a standard understanding of a formal axiomatic system presupposed by numerous mathematicians, where an axiomatic system exposes meaningful relationships between the elements which comprise it. Such an understanding, in a Hilbertian formalist context, is but another interpretation of the more fundamental, purely formal system.[3]

Debreu was fully aware of this Hilbertian view of a purely formal system and adopted it. In his *Theory of Value* he asserts 'allegiance to rigor dictates the axiomatic form of the analysis where the theory, in the strict sense, is logically entirely disconnected from its interpretations' (Debreu, 1959, p. x). In a much later piece he is equally explicit:

According to this schema an axiomatized theory has a mathematical form that

is completely separated from its economic content … The divorce of form and

content immediately yields a new theory whenever a novel interpretation of a

primitive concept is discussed (Debreu, 1986, p. 1265).

Clearly Kaldor is correct in maintaining that Debreu's work is 'purely logical' but not

scientific in the normal sense of scientific. Any purely logico-axiomatic theory must first

be interpreted before we can decide whether it is scientific in the sense of being either a

description or explanation of events or processes in the physical, social or economic

world.

Kaldor is also correct in claiming that Debreu's theory 'is not *intended* to describe

reality*'* (Kaldor, 1972, p. 1238). If we pause, we may ask the question: if Debreu's

theory is primarily logical and not empirical and if it is not even intended to describe

reality, what makes it a piece of theoretical economics? Surely theoretical economics

ought to make claims about actual economies, either at the descriptive or explanatory

level? Again Kaldor draws the obvious conclusion. General equilibrium is neither a

description nor an explanation of actual economies, as these terms are understood by

empirical scientists. Rather it is:

a set of theorems that are *logically* deducible from precisely

formulated assumptions; and the purpose of the exercise is to find

the minimum 'basic assumptions' necessary for establishing the

existence of an 'equilibrium' set of prices (and output/input

matrixes) that is (a) unique, (b) stable, (c) satisfies the conditions of

Pareto optimality (Kaldor, 1972, p. 1237).

In other words, Debreu's *Theory of Value,* seen as a work aimed at attaining the highest

standards of logico-mathematical rigour and precision, is a purely formal uninterpreted

system having no connection whatsoever to any branch of reality in general or real

economic processes in particular. However, it is economic in that its *choice* of axioms

prior to the logical exploitation of these axioms is informed by *the desire to prove,*

when interpreted, the existence of an 'equilibrium set of prices that is (a) unique, (b)

stable, (c) satisfies the conditions of Pareto optimality' (ibid., p. 1237). In this fashion,

Debreu's *Theory of Value* became, for numerous economic theorists, 'the necessary

conceptual framework ... for any attempt at explaining how a "decentralized" system

works' (Kaldor, 1972, p. 1238).

In total opposition to this latter thesis, Kaldor maintains that general equilibrium theory

amounts to a set of:

> propositions which the *pure* mathematical economist has shown to be valid
>
> only on assumptions that are manifestly unreal – that is to say, directly
>
> contrary to experience and not just "abstract." In fact, equilibrium theory
>
> has reached the stage where the pure theorist has successfully (though
>
> perhaps inadvertently) demonstrated that the main implications of this

15

theory cannot possibly hold in reality, but has not yet managed to pass his

message down the line to the textbook writer and to the classroom (Kaldor,

1972, p. 1240).

Unfortunately Kaldor did not spell out the methodological reasons for this unequivocal

and uncompromising claim. In the following section we address this lacuna. We show

why Kaldor was correct in maintaining his radical thesis that general equilibrium 'has

become a major obstacle to the development of economics as a *science* – meaning by

the term "science" a body of theorems based on assumptions that are *empirically*

derived (from observations) and which embody hypotheses that are capable of

verification both in regard to assumptions and the predictions' (Kaldor, 1972, p. 1237,

emphasis in original). The methodological reasons concern the notion of existence

presupposed by Debreu in his existence theorem and, secondly, the issue of the

particular mathematics used by Debreu, namely Cantorian set theory. Moreover these

two reasons are inextricably linked.

4. Existence and the Critique of General Equilibrium

In this section we argue that the Achilles' heel of Debreu's general equilibrium lies in

his use of the powerful resources of Cantorian set theory to prove the existence of

general equilibrium. To this end we focus on the distinctive characteristic of Cantorian

set theory, namely actual, as distinct from potential, infinity. In particular we focus on

different notions of mathematical existence and on acceptable methods of proving

existence claims discussed in the foundations of arithmetic. In light of these distinctions we show how some specific methods of existence proofs legitimate in the domain of Cantorian set theory fail to legitimate existence claims in either the physical or our socio-economic world. We then apply this result to Debreu's proof. To achieve this we start with the foundations of arithmetic - the Cinderella of the philosophy of economics: it is usually not even invited to the ball. For instance, Weintraub (2002) maintains it has no relevance to the correct interpretation of Debreu's work. One may feel that Weintraub is correct: *prima facie* there is no connection between issues in the foundations of arithmetic and issues in economic methodology. This *prima facie* appearance, however, is misleading, particularly when we realize that central methodological issues in the foundations of arithmetic are focused on Canatorian set theory and, as Weintraub correctly points out, it is precisely the powerful resources of this theory which are used by Debreu in his proof of the existence of general equilibrium.

In one sense, the philosophical debate, or perhaps more accurately, the philosophical battle, in the foundations of arithmetic has its origins in Cantor's highly original contribution to set theory in the 1870s. Prior to Cantor's challenging contribution, sets were either finite, e.g. the set of apostles, or potentially infinite, e.g. the set of natural numbers 1, 2, 3 … n, … and so on. The only kind of infinite set relevant to mathematics was a potentially infinite one. A potentially infinite set is one to which we can add new members *ad infinitum*. It is open-ended or never complete. The hegemony of potential infinity was challenged by Cantor: there is much more to

mathematical infinity than potential infinity. This may be seen from the following examples. The infinite set of even numbers can be put in one-to-one correspondence with the infinite set of natural numbers. There are, as it were, as many even numbers as there are natural numbers. However, the infinite set of real numbers (which includes numbers like $\sqrt{2}$) cannot be put in one-to-one correspondence with the natural numbers. The infinite set of real numbers is, as it were, 'bigger' than the infinite set of natural numbers. Hence contrary to the traditional view, there is for Cantor a variety of infinities in mathematics and these infinities, contrary to potential infinity, are complete. To mark this crucial difference, numerous philosophers of mathematics follow Hilbert in calling Cantorian infinity actual infinity, the contrast being with the traditional notion of potential infinity. As Dummett puts it, 'it is integral to Cantorian mathematics to treat infinite structures as if they could be completed and then surveyed in their totality' (Dummett, 2000, p. 41). For those committed to potential infinity this 'destroys the whole essence of infinity, which lies in the conception of a structure which is always in growth, precisely because the process of construction is never completed' (ibid., p. 41).

This ingenious Cantorian contribution to pure mathematics appears to be irrelevant to economic methodology. However, let us not jump too hastily to conclusions. In Cantorian set theory a number of theorems can be provided *which are not provable* in non-Cantorian set theory. The mathematical resources available in Cantorian set theory are, as it were, more powerful than those available in pure mathematics limited to the domain of the potentially infinite. Moreover, Velupillai has demonstrated that some of

these specific Cantorian-based theorems are indispensable to Debreu's proof of the existence of equilibrium (Velupillai, 2000).[4] In principle there is nothing *mathematically* wrong with that. On the contrary, as we already noted, a major part of Debreu's originality resided in his ability to exploit the novel and powerful resources of Cantorian set theory in formulating his mathematical proof. *The crucial methodological question is whether or not such a mathematical proof can be given an economic interpretation.* This methodological problem emerges with the discovery of skeletons in the closet of Cantorian set theory.

The first hint of the existence of these skeletons arose at the beginning of the 20th century after Frege in Germany and Russell in Britain began their foundational studies in arithmetic. They used Cantorian set theory as an indispensable cornerstone in their foundational studies. In these studies, Cantorian set theory gave rise to a range of paradoxes. These paradoxes had a profound influence on future developments both in philosophy and in mathematics. Indeed it is not an exaggeration to say that the mathematical and philosophical communities divided on how best to respond to these paradoxes. For our methodological purposes we focus on the divide between Poincaré, who wished to prune pure mathematics of its Cantorian excesses, and those who cherished the growth of Cantorian actual infinity.

The French mathematician, Henri Poincaré, was regarded as the most outstanding European mathematician at the turn of the 20th century. He is probably best known among economic methodologists and historians of economic thought as the

mathematician to whom Walras turned for support for his programme of the mathematization of economics.[5] Poincaré insisted that the source of the paradoxes lay in the specifics of Cantorian set theory. To avoid the paradoxes he suggested, what we have called, the Poincaré finitist programme in the foundations of mathematics (Boylan and O'Gorman, 2008). This programme culminated in the 1930s in the birth of what is today called constructive mathematics – which basically is mathematics without actual Cantorian infinities.[6] In the terminology of Benacerraf and Putnam, Poincaré's approach to the practice of pure mathematics is informed by his own 'epistemology of mathematics' (Benacerraf and Putnam, 1983, p. 2). A basic cornerstone of Poincaré's epistemology is that pure mathematics is the outcome of mathematical activity on the part of mathematicians. The pure mathematician is more a constructor than a discoverer. Mathematicians construct their logico-mathematical edifices and what is crucially important is that the rigorous conceptual resources used in these constructions are both linguistically based and finite. Rigorous mathematics is the output of finitely bounded, rational, linguistic agents. Thus Poincaré objects to Cantorian mathematics because he refuses 'to argue on the hypothesis of some infinitely talkative divinity capable of thinking an infinite number of words in a finite length of time' (Poincaré, 1963: 67). In Carnap's terminology, Poincaré wants to replace 'theological mathematics' with 'anthropological' mathematics (Carnap 1983: 50).[7]

Contrary to Cantorian mathematics, constructive mathematics in the Poincaré programme is limited to the domain of the finite and potentially infinite, thereby excluding Cantor's actual infinity. Moreover, in this programme any genuine

mathematical proof must in principle be capable of being carried out in a finite number of steps.[8]   No such epistemological restriction is imposed on proofs in Cantorian set theory.   In particular, and crucially for economic methodology, in the Poincaré programme of constructive mathematics, some of the theorems of Cantor's set theory used by Debreu in proving the existence of equilibrium are not theorems at all.   Their method of proof violates the Poincaré principle that any legitimate mathematical proof must be capable of being carried out in a finite number of steps.   We will return to this point later.

Other philosophers and mathematicians argued that a Poincaré-type solution was too draconian.   These wished to retain Cantorian set theory.   Among these were Platonists and Hilbertian formalists.   For Platonists, Cantorian actual infinity subsists in a real, Platonistic world.   This Platonistic world consists of real objects, which, unlike objects in the empirical world, neither initiate nor undergo change.   Thus for Platonists, mathematical existence transcends empirical existence in general and socio-historical existence in particular.   Existence in this Platonistic world is independent of spatio-temporal existence.

As we already noted, Hilbert proposed a strict formalist reading of pure mathematics. In this strictly formalist setting Hilbert proposed an ingenious, non-Platonistic way of retaining Cantorian set theory. He divided pure mathematics into a finitist part (à la Poincaré) and an idealized, infinite part (à la Cantor). The idealized infinite part is not open to interpretation; only the finite part may be interpreted. However, the idealized

infinite part is heuristically indispensable as an instrument for deriving finitist results otherwise unobtainable. In this reading of Hilbert's ingenious solution, Cantorian actual infinity is a non-empirical, non-finite, heuristic fiction, justified by its enormous mathematical power and utility. Crucially for Hilbert such idealized fictions cannot be arbitrarily introduced into mathematics: the extended system of Cantorian infinity combined with the finite must be proven to be consistent. In this way one could say that Hilbertian formalism equates Cantorian mathematical existence with freedom from contradiction.[9]

Clearly in these Cantorian settings Debreu's proof is a genuine one. In short in the context of the Poincaré programme Debreu's proof is invalid as a piece of mathematics, whereas in a pro Cantorian framework it is a valid proof! The moral is clear: the process of the mathematization of economics via Cantorian set theory requires closer methodological scrutiny. In particular, we are now in a position to address the crucial methodological question noted above, namely whether or not Debreu's proof can be given an economic interpretation? Debreu's *Theory of Value* is said to prove the existence of a set of signals, market prices, in a Walrasian exchange economy, leading economic agents to make decisions which are mutually compatible. This is the economic interpretation of Debreu's mathematical proof. Our thesis, which we call the P-K thesis (Poincaré-Kaldor), is that there is no justification for this *economic interpretation* of Debreu's ingenious piece of Cantorian pure mathematics. Debreu's proof does *not* support this economic interpretation. Debreu's so-called economic equilibrium only exists in the domain of Cantorian actual infinity which transcends any

process limited to socio-historical time. More precisely, since the method of the proof of existence is inherently non-constructive, i.e. cannot be carried out in a finite number of steps taken one at a time, Debreu's equilibrium cannot be given either a finite or a potentially infinite interpretation.[10] Debreu's equilibrium point is merely shown to exist in a non-temporal, actual infinite Platonic domain, which cannot in any finite effective way be realized in the socio-historical world in which economic agents operate. Alternatively, in the language of the Hilbertian formalist, there is no evidence to support the assumption that the logical possibility, established by Debreu's proof of existence, could be realized in any socio-economic system where real historical time operates.

There are a number of aspects to the P-K thesis which should be noted. Firstly, there is no obligation on economic methodologists to take the Poincaré side in the philosophico-mathematical debate in the foundations of mathematics. The P-K thesis assumes that, even though Platonists and the Hilbert 'school' fail in different ways to defend Cantor's paradise of actual infinity, Cantorian set theory is an authentic part of pure mathematics. In other words, in the spirit of Debreu's own distinction between a rigorous proof in pure mathematics and its economic interpretation discussed in the previous section, the P-K thesis accepts that as a piece of pure mathematics, without an economic interpretation, Debreu's proof is valid. What is crucial to the defense of the P-K thesis hinges on what is proven to exist by Debreu's proof. Mathematically Debreu's proof establishes existence in the Cantorian domain of actual infinity which transcends the domains of the strictly finite and potentially infinite. This claim is

justified by the fact that, as Velupillai for instance demonstrated, the method of proof is endemically non-constructive, i.e. cannot be carried out, in a potentially infinite setting, by a finite number of steps, however large, taken one step at a time (Velupillai, 2002).[11] In short Debreu's mathematical proof establishes existence in an idealized world, either Platonic or Hilbertian, which in principle is not accessible to any mathematician operating with a finite number of steps, however large, when each step is taken one at a time.

By virtue of this inaccessibility, Debreu's equilibrium solution, though valid in pure mathematics, cannot in any empirically meaningful way be interpreted as obtaining in our historically situated, socio-economic world where real time matters. In particular, any interpretation in terms of price signals would necessarily imply that these signals would take more than a potentially infinite period of time to be transmitted! Similarly economic agents, to arrive at an equilibrium decision, would require more than a potential infinite period of time. In short, the legitimacy of Debreu's proof is in the realm of Cantor's paradise which in principle is not realizable in our socio-economic world where decisions have to be arrived at in finite time settings and signals must be transmitted under similar real time constraints. In Poincaré's terminology, any economic interpretation of such a proof assumes that the economist is 'some infinitely talkative divinity capable of thinking an infinite number of words in a finite length of time' (Poincaré, 1963: 67). Theological economics of this nature is beyond the reach of us humanly bounded rational agents. Thus Kaldor's uncompromising claim is fully vindicated: general equilibrium, as articulated by Debreu, 'is shown to be valid only on

assumptions that are manifestly unreal' (Kaldor, 1972: 1240). The P-K thesis shows how this economic unreality is endemic to Debreu's mathematical proof. The so-called equilibrium point, by virtue of the Cantorian non-constructive manner in which it is demonstrated to exist by Debreu, exists only in a non-temporal, idealized realm, which is completely cut off from the economic world of finitely bounded economic agents, with limited capacities, where real time impinges on decisions taken and signals given.

Conclusion

The P-K thesis is very specific: it is concerned with the Debreu articulation of general equilibrium and, in particular, its existence proof. It is not concerned with either uniqueness or stability conditions. It presupposes, á la Weintraub, an appreciation of the second phase in the mathematization of economics, namely the recourse to Cantorian set theory in proving the existence of general equilibrium. Moreover it also presupposes, á la Debreu, the distinction between a rigorous piece of pure mathematics and its subsequent economic interpretation.

The P-K thesis is based on the fact that there are real time constraints on existence claims in economic theory which do not apply in pure Cantorian mathematics. A non-constructive, existence proof in the domain of Cantor's actual infinity places what is proven to exist outside the real time constraints of existence in the economic sphere. Mathematical existence in Cantor's actual infinite paradise cannot be given an empirical interpretation in economic theory where real historical time constraints are

operational. Cantor's paradise contains mathematical truths which are empirical fictions and among these mathematically true, empirical fictions is Debreu's equilibrium solution. In this fashion Kaldor is fully justified in claiming that 'in fact equilibrium theory has reached the stage where the pure theorist has successfully (though perhaps inadvertently) demonstrated that the main implications of the theory *cannot possibly* hold in reality …' (Kaldor, 1972, p. 1240, emphasis added). Debreu's non-constructive proof 'inadvertently,' but endemically, necessitates that which is demonstrated to exist cannot be an equilibrium set of prices which could subsist in real historical time. In short Kaldor is, in Carnapian terminology, calling for an 'anthropological' economics in place of Debreu's 'theological' economics.

Footnotes

[1] This section draws in part on material in section 1 of Boylan and O'Gorman (1997).

[2]   While Kaldor's concern was to dismantle the whole edifice of general equilibrium theory as contained in his critical writings, particularly from the 1970s, it is interesting to note that at this time there emerged a series of papers that are conventionally interpreted as representing a major 'internalist' technical critique of the failure of general equilibrium theory to provide proofs of the uniqueness and stability based on general characterizations of preferences and technologies.   These were the papers by Sonnerscheim (1973), Mantel (1974) and Debreu (1974) – generally referred to as the SMD theorem – which inflicted what appeared to be a fatal wound to stability analysis in general equilibrium theory, certainly to any version of that theory which employed the Walrasian tâtonnement process and aggregate excess demand functions. These authors demonstrated that the only general properties possessed by the aggregate excess demand function (which is used to characterize the competitive equilibria) were those of continuity, homogeneity of degree zero, and the validity of Walras's law. Beyond that, as contained in the memorable phrase of Mas-Collell, Whinston and Green (1995, p. 548), 'anything goes.' The SMD results showed, as Tohmé succinctly summarized them, 'that for every given system of equilibrium prices and its associated excess demands, an arbitrary economy can be defined, exhibiting the same aggregate behaviour and the same equilibria. That is, *prices do not convey all the relevant information about the economy*,

27

since a "mock" one is able to generate the same aggregate demand' (Tohmé, 2006, p. 214, emphasis in original). Kirman has recently noted, 'The full force of the Sonnenscheim, Mantel and Debreu (SMD) result is often not appreciated. Without stability or uniqueness, the intrinsic interest of economic analysis based on the general equilibrium model is extremely limited' (Kirman, 2006, p. 257). While this sentiment would surely have found favour with Kaldor, it arguably falls far short of his more fundamental call for the 'demolition' of general equilibrium theory as a major inhibition to the development of economics as a science, and certainly as an empirical science. We do not pursue the implications of the SMD results here as this would constitute a different exercise and would take us too far away from our central concerns in this paper.

[3] For a more detailed account of this Hilbertian, purely formalist reading of an axiomatic system see Boylan and O'Gorman (2008).

[4] Debreu presupposed what is technically known as Brouwer's fixed point theorem in his proof of the existence of equilibrium. However, this theorem cannot be proved in non-Cantorian, computable mathematics. Moreover efforts by Scarf and others to render equilibrium constructable also fail. For more on this see Boylan and O'Gorman (2008).

[5] We are currently completing an extended analysis of the Poincaré-Walras correspondence, with particular reference to the mathematization of economics as developed by Walras and its later development in the Neo-Walrasian programme.

[6] These developments in non-Cantorian mathematics and their relevance to economic methodology are discussed in more detail in Boylan and O'Gorman (forthcoming 2009).

[7] The phrase 'theological economics' was suggested to us by Carnap's characterization of Ramsey's work as 'theological mathematics' (Carnap (1983) in Benacerraf and Putnam (1983), p. 50).

[8] For Poincaré, the notion of what is in principle attainable in a finite number of steps, requires the notion of potential infinity. Suppose we reject the relevance of the notion of potential infinity to the correct explication of the notion of what is in principle possible in a finite number of steps. On this supposition, there is some finite upper limit to the number of steps which are in principle possible. Call this upper limit ℓ. As Poincaré notes, in pure arithmetic we are not limited: there is nothing in principle wrong with the number $\ell + 1$ etc. Thus the Poincaré programme for pure mathematics is attempting to hold a middle ground between the too restrictive nature of a strict finitist approach to mathematics and the excesses of Cantorian actual infinity. This middle ground is the potentially infinite.

[9] Brown (2002) provides an interesting and readable introduction to these topics.

[10] As Poincaré notes, the number of steps 'is greater than aleph zero' (Poincaré, 1963, p. 67), the first of Cantor's transfinite numbers.

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[11] Velupillai is well aware of the efforts of Scarf and others to develop constructive general equilibrium. Velupillai shows how the non-constructive, Brouwer's fixed point theorem is used in these efforts.

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