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A critical analysis of risk and volatility
modeling in the financial markets

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Abstract

In light of the recent financial crisis, the limitations of current risk estimation techniques have become apparent. The purpose of this thesis is to see if nonlinear tools and techniques can facilitate our understanding of the financial markets, particularly during times of heightened turbulence. This is done by comparing econometric and nonlinear tools to analyse a number of asset classes, including equities, bonds and credit default swaps during periods of heightened turbulence. We analyse indices as well as individual corporate and sovereign securities. The methodology is to compare the linear stochastic framework (in particular the linear Gaussian framework) to the nonlinear time series framework. In essence the focus of the thesis is on the treatment of irregularity by these two very different frameworks. The linear stochastic framework treats irregularity as exogenous from the linear system, and expresses it in terms of probability distribution functions. The nonlinear time series framework allows the irregularity to be part of the system i.e. endogenous. The thesis is divided into five chapters, an introduction and conclusion chapter, and three chapters which represent three papers. The abstracts of the three papers i.e. chapters 2, 3 and, 4 are presented here.

The objective of chapter two is to test for nonlinear dependence in the GARCH residuals of a number of asset classes using nonlinear dynamic tools. The equity and bond market samples appear to be independent once GARCH has been applied but evidence of nonlinear dependence in the CDS

GARCH residuals is found. The sensitivity of this result is analyzed by changing the specifications of the GARCH model and the robustness of the result is verified by applying additional tests of nonlinearity; that is delay plots and the correlation dimension test. Evidence of non linear dependence in the GARCH residuals of CDS contracts has implications for the accurate modeling of the marginal distribution of the CDS market, for pricing of CDS contracts, for estimating risk neutral default probabilities in the bond market as well as for bond market hedging strategies.

Chapter 3 considers the arbitrage-free parity theory. This theory states that there will be equivalence between credit default swap (CDS) spreads and bond market spreads in equilibrium. In this chapter, we test this theory using linear and nonlinear tools. Linear stochastic modeling is reviewed, particularly the assumptions of a Gaussian distribution and of iid and stationary residuals. By applying the nonlinear dynamic tools of Cross Recurrence Plots and Cross Recurrence Plot measures, evidence of dynamically varying convergence and statistically consistent synchronization across the markets is illustrated. There is evidence that the arbitrage-free parity is conditional and equivalence is non-mean reverting. In particular there is evidence of a rising trend in equivalence in the Greek sovereign market prior to the bailout of May 2010. This trend indicates increased arbitrage activity at this time. Applying a nonlinear analysis of the markets significantly increases our understanding of the dynamically varying relationship between these two assets classes. This result has implications for supervision of arbitrage activity and for financial market policy making.

In chapter four, application of the nonlinear dynamical tool, recurrence quantification analysis (RQA) presents some evidence of periodic to chaotic transitions and bifurcation points at localized peaks of the Dow Jones Industrial Index. There is also evidence of a collapse in all the RQA measures just as the market transitions from bull to bear market. This suggests that the market becomes completely unpredictable at this time. We suggest the market is intermittently or piecewise deterministic and that the unpredictable regime allows the market dynamics to break from the past. The noisy trader theory is suggested as the economic explanation for this unpredictability. We develop a principal component series and name it the random market indicator (RMI). We suggest that this series indicates when the market is transitioning into and out of a nonstationary random regime. During this time, quantitative risk estimation techniques, such as value at risk models will produce misleading results.

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Chapter 1

Introduction

1.1 General context

The recent financial crisis has led to widespread recognition of the failure of quantitative risk estimation techniques to predict a crisis or to estimate the short or long run impact of a crisis on the economy (Danielsson 2008). The objective of this thesis is to examine risk and volatility in the financial markets during times of crisis in order to improve risk estimation techniques. Methodologically the initial analysis compares Gaussian and non-Gaussian techniques and concludes that although Gaussian techniques lead to less accurate results, non-Gaussian techniques are cumbersome and problematic. The main body of the thesis looks beyond stochastic methodologies and examines the framework of nonlinear time series to see if the conceptual framework and tools used in this field can improve our understanding of risk and volatility in the financial markets, particularly during times of heightened turbulence. This framework is applied to a number of asset classes, in a

number of structural forms and from a number of geographical regions. The assets include equity, bonds (both sovereign and corporate), and the credit derivative known as the credit default swap (both sovereign and corporate). The structural forms include indices as well as single named entities. The geographical regions include Europe, the United States, and Japan, although the majority of the securities analyzed are European. The main focus of the thesis is on credit default swaps, this is for three reasons. Firstly due to their importance for systemic stability in the financial markets. Secondly due to their unusual distributional characteristics, and thirdly due to the lack of analysis of the univariate distribution of credit default swaps in the current literature. This justification is discussed further in section 1.1.1 below and in chapter 2 and chapter 3.

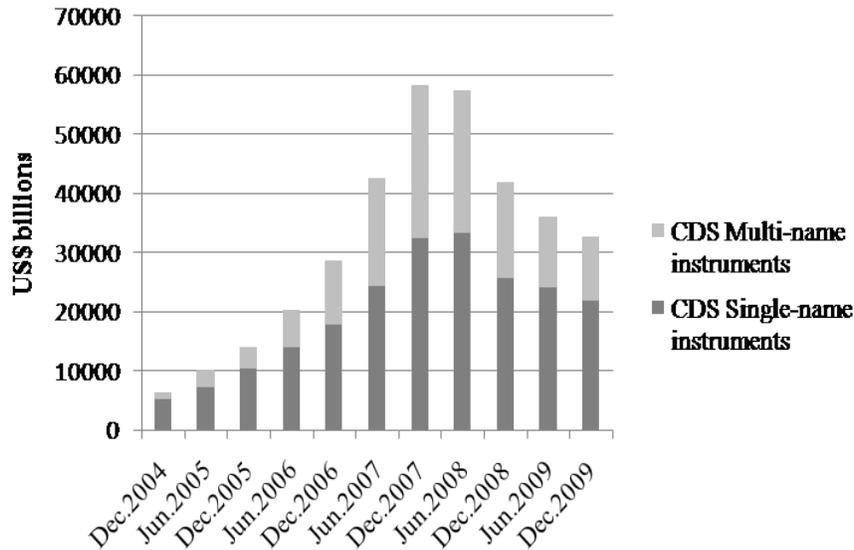
The objective of this chapter is to contextualize the aims of the thesis. The structure of this chapter is as follows; the following section discusses the systemic importance of credit default swaps. Section 1.1.2 outlines current modeling forms in modern financial econometrics. In general linear Gaussian models are applied. Section 1.1.3 outlines the stylized facts of financial markets. The implication of these facts is that Gaussian modeling may be inappropriate for financial modeling, particularly when estimating risk. In the literature the main alternative to Gaussian modeling is non-Gaussian stochastic modeling. In section 1.2 we consider this approach. The finding of this exploratory work is that non-Gaussian modeling can lead to improved estimates of risk, but that the models have a number of drawbacks. The models are cumbersome and problematic (e.g. for both multivariate portfolios and for alternative time periods). With this in mind, the aim of the

thesis is to look beyond non-Gaussian stochastics for alternative methodologies to analyze the financial markets. Section 1.3 presents the research question and reviews recent nonlinear literature. The research question asks if the application of nonlinear time series tools and concepts will facilitate our understanding of risk and volatility in the financial markets, particularly during times of heightened turbulence. Without evoking any model we use applied nonlinear tools to analyze financial data and this leads us to a number of conclusions about the nature of risk and volatility during periods of heightened turbulence. This is the contribution of the thesis. Section 1.4 outlines the structure of the thesis and section 1.5 presents the thesis outputs.

Credit Default Swaps

This thesis analyzes a number of asset classes in a number of geographical regions. Of particular interest are credit default swaps (CDS). Credit default swaps are a form of contractual agreement. The buyer is compensated by the seller in the event of a credit default by the underlying entity. This derivative contract was created by Morgan Stanley in 1995 to facilitate hedging strategies. CDS contracts are over-the counter (OTC that is non-exchange based and individually priced) tradable financial assets. They are a type of insurance policy against default of an underlying entity. Unlike most retail insurance contracts, they do not require that the buyer has an ‘insurable interest’ in the underlying entity; that is the buyer does not need to own the underlying asset. This characteristic of the CDS contract means they can be used for speculative as well as hedging purposes. As the premium on a CDS is a small percentage of the overall value of the underlying asset, CDS’s allow

Figure 1.1: CDS Notional Market Values



leveraged trading in credit based assets. Volatility and the rise in premiums in the CDS market have been suggested as important leading factors in the development of the crisis (Sharma 2008). Figure (1.1) indicates the rise and fall of the CDS market during this time.

Figure 1.1 illustrates the size of the market by examining the notional value of the underlying credit outstanding. To put the asset in context, CDS contracts were valued at US\$42 trillion as at December 2008 (BIS 2009a). In comparison, at this time, global equity markets were valued at approx. US\$32 trillion, global bond markets at approx. US\$35 trillion (Exchanges 2008); and the sum of outstanding international and domestic debt related securities was approx. US\$84 trillion (BIS 2009a). The impact of the CDS market on the financial system was illustrated when AIG was de-rated in 2009. The de-rating caused the collateral clause in many of AIG's CDS

contracts to take effect. As AIG were now seen as more likely to default, their CDS contract counterparties required collateral to maintain their positions. The resultant cash flow requirement caused financial distress for AIG, which (it has been estimated) cost the US Treasury approx. US\$30 billion (Soros 2009). AIG had totally underestimated the risk of writing and selling CDS contracts. Understanding the distributional characteristics of CDS contracts is fundamental to evaluating the risk of trading in these contracts.

In recognition of the systemic importance of these contracts, there is increasing demand from regulators to improve risk estimation techniques for the credit default swap market. The European Commission (2009) observed the increasing importance of the CDS market to the overall stability of the financial system, stating:

The crisis has highlighted that these [market] risks are particularly evident in the over-the-counter (OTC) part of the market, especially as regards credit default swaps.

(European Union 2009)

Also, ECONET, the network of economists from CESR (Committee of European Securities Regulators) reports regularly on CDS market trends including analysis of "... appropriate indicators of uncertainty and the potential for contractions (like implied volatility, value-at-risk)" (CESR 2009).

An improved understanding of the characteristics of the CDS spread will facilitate efficient supervision of the banking sector (BIS 2010). As part of Basel III, the Bank for International Settlements (BIS) has stated that CDS spreads should be included as one of a number of common 'market-related

monitoring tools' (BIS, 2010). These tools should be used by regulators to assess the liquidity risk profiles of banking organizations. From 1st January 2013, banks must apply a revised metric to better address counterparty risk and credit valuation adjustments when estimating on and off balance sheet risk. Credit default swap spreads will be used to value these risks, along with estimates of risk neutral probabilities of default. BIS (2010) suggests the review of current methods of analyzing the characteristics of CDS spreads to improve the accuracy of these valuations. In chapter 2 and 3 we critically analyze the existing models. The mathematical framework upon which these models are based is presented in section 1.1.2.

Linear stochastic models

As the thesis is a critical analysis of risk and volatility modeling in the financial markets, we present below the mathematical framework of current financial econometrics. Financial market modeling begins by assuming that financial market prices follow a random walk, for example:

$$x_t = a + x_{t-1} + \gamma_t \tag{1.1}$$

Such that x_t is the price in time t , a is a constant (known as the trend or drift) and γ_t is a stochastic white noise process (with zero mean, constant variance and zero autocorrelations). The process can be rewritten as:

$$x_t = at + x_0 + \gamma_t \tag{1.2}$$

Such that t is the number of time steps. It is assumed that x_0 is fixed.

This model is nonstationary, as neither its mean nor its variance are constant, as follows:

$$E[x_t] = at + x_0 \quad (1.3)$$

$$Var(x_t) = t\sigma^2 \quad (1.4)$$

To solve this problem, prices are converted into first differences to ensure stationarity, and the natural logs are taken to ensure additivity of returns. The equation transforms to:

$$r_t = \mu + \epsilon_t \quad (1.5)$$

Such that r_t is the logged differential of the prices, μ is a constant and ϵ_t is a stochastic white noise process.

There are a number of advantages to applying linear stochastic models to financial data. They are generally parsimonious, and they are easily comparable. They allow the application of correlation and regression analysis, which swiftly estimate relationships between variables. The assumptions of the linear stochastic models are stringent. They generally assume that the process is ergodic (based on unchanging parameters (Feller 1949)) and that the probability distribution is at least second order stationary; that is:

$$E[x_t] = \mu \quad (1.6)$$

$$\text{Var}(x_t) = \sigma^2 \tag{1.7}$$

$$\text{Cov}(x_t, x_s) = \text{Cov}(x_{t+j}, x_{t+j-s}) \tag{1.8}$$

(Patterson 2000)

As they are linear stochastic models, all irregularity is due to the random element in the model; whereas in a nonlinear model the irregularity may be part of the deterministic equation (Kantz and Schreiber 2003).

Samuelson (1968) suggested the ergodic process must be assumed, if economists hope to move economics from the realm of history into the realm of science. As neo-classical financial econometrics aims to maintain rigor and consistency; the focus has been on linear stochastic ergodic models. Danielson (2008 p. 323) criticizes current econometric risk estimation techniques. He suggests these ergodic models ignore ‘endogenous risk’; that is the risk of change in the statistical properties of a system due to changes in information. He also notes that if the endogenous risks are recognized and the system is remodeled over relevant time periods, this can lead to data quality issues (e.g., small sample size etc.). Danielson (2008) suggests that modelers “ignore what is difficult”. He points out that aggregation methods usually assume constant correlations. In fact, there is much empirical evidence of dynamically changing correlations. For example, correlations between markets rise during a crisis. This is seen as a characteristic of contagion (Baig and Goldfajn 1999).

Within the linear stochastic framework, the most commonly assumed

distribution is the Gaussian. This is due to the ease of analytical tractability. There are a number of mathematical advantages to the Gaussian distribution. We will discuss three here:

(i) Where the sample X_t ¹ is normally distributed, with a constant mean and variance, zero autocorrelations imply that the distribution is iid (independent and identically distributed) that is X_t is independent of X_s for all t and s , $t \neq s$, in the set $1, \dots, T$. (Patterson 2000).

(ii) If the Gaussian distribution is standardized, the second advantage is the 'square root of time' rule, as follows:

$$\epsilon_t \sim N(0, 1) \quad (1.9)$$

$$\sigma(\Delta r) = \sigma\sqrt{T} \quad (1.10)$$

Such that σ is the standard deviation of the logged return, Δr is the change in r , and T is the time interval. This implies that the uncertainty about the value of the variable at a certain time in the future (as measured by the standard deviation) increases as the square root of time increases (Hull 2008). This is a useful characteristic; for example for value-at-risk models (discussed below in section 1.2)

(iii) Thirdly, the estimation of the moments of a multivariate Gaussian distribution is equal to the linear summation of the univariate moments. For example:

¹Capitalized notation indicates all possible values of the variable at that time period

$$E[x, y] = E[x] + E[y] \quad (1.11)$$

$$Var(x, y) = Var(x) + Var(y) + 2Cov(x, y) \quad (1.12)$$

This third advantage gives a stability to Gaussian processes and is particularly useful when analyzing portfolios of investments. These three characteristics of Gaussian distributions facilitate the application of Gaussian models to real market data.

6 stylized facts of the financial markets

However, in 1963, Mandelbrot (1963) noted the non-Gaussian nature of cotton prices. Since that time a large body of empirical literature has led to the recognition of 6 stylized facts of financial data (see Jondeau et al. 2007 for a review). The facts can be summarized as follows:

1. Fat tails: The unconditional distribution of returns has fatter tails than that expected from a Gaussian distribution.
2. Asymmetry: The unconditional distribution is negatively skewed. The asymmetry and the fat tails persist even after adjustment for conditional heteroskedasticity, thus the conditional distribution is also non-Gaussian.
3. Aggregated normality: As the frequency of returns lengthens the return distribution gets closer to the Gaussian distribution.

4. Absence of serial correlation: Returns generally do not display significant serial correlation except at high frequencies.
5. Volatility clustering: Volatility of returns are serially correlated.
6. Time-varying cross correlation: Correlation between asset returns appear to increase across periods of high volatility.

(Jondeau et al. 2007)

In light of these stylized facts, we examine non-Gaussian modeling as an alternative. For example if equation 1.5 is altered to reflect the above stylized facts it could be rewritten as:

$$r_t = \mu_t + \varepsilon_t \quad (1.13)$$

$$\varepsilon_t = \sigma_t z_t \quad (1.14)$$

such that

$$z_t \sim g(z_t | n) \quad (1.15)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (1.16)$$

This is a non-Gaussian generalized autoregressive conditional heteroskedastic model (GARCH) In this example, we have a GARCH, 1,1 model. The 1,1 notation refers to the error squared and the variance being lagged by one time step in equation 1.16 (Bollerslev 1986). Any lag is possible in the model, although 1,1 is the most commonly applied. The constant term μ can now be conditional, the stochastic term ε_t is conditional on the lagged residual squared and the lagged variance (as mentioned above, in this case one lag is

taken but the number of lags depends on the memory of the variance). The z_t stochastic term is now non-Gaussian with n parameters. For example a four parameter stochastic process would allow for the skew and the kurtosis to be measured. The advantage of this model is that it will allow the characteristics of financial data to be measured, i.e. the skew, the kurtosis, and the autocorrelations in returns and variance. The drawbacks of this model arise as no non-Gaussian distribution has the mathematical stability of the Gaussian (as outlined above in section 1.1.2). The implications of this changeover would be very significant for financial theory. For example, the Markowitz mean-variance analysis, assumes that an asset can be assessed in terms of risk (measured as the variance) and return (measured as the mean). This assumes a two parameter Gaussian model, it ignores the higher moments, as it assumes them to be equal in all cases. Much of modern day portfolio theory is based on this framework, which would not be appropriate for non-Gaussian modeling. The Black-Scholes-Merton (BSM) option pricing model assumes that the underlying assets follow a Gaussian distribution and are time independent. Although it is widely recognized that the BSM model is flawed, it is still commonly used to price options and to develop hedging and speculative strategies. This is because the alternatives, for example the stochastic volatility model and models with jumps (Jondeau et al. 2007) are cumbersome. Copula functions were first suggested by Li (2000) to allow the joint probabilities of default of CDS contracts to be estimated. Initially the Gaussian copula was applied. It has been argued that the assumption of a Gaussian copula function leads to significant underestimation of the risk associated with CDS contracts and portfolios of CDS contracts; such

as collateralized debt obligations (CDOs) (Whitehouse 2005). Non-Gaussian copula functions become complex for more than two assets as the integration of the joint distributions cannot be done analytically (Jondeau et al. 2007). In general, the problem for financial market modelers is that they are caught between the attractions of parsimony of approach and the need for accuracy of results.

1.2 A non-Gaussian exploratory analysis

In response to the discussion above, we carried out an exploratory analysis in a non-Gaussian framework. The initial analysis is presented in this chapter as the results of these two pieces of research lead to the development of the research question of the thesis. Thus this initial exploratory analysis contextualizes the thesis. Credit default swaps were chosen as the asset under consideration due to their systemic importance to the financial markets and in recognition of the lack of analysis of their univariate distribution in the existing literature, (see chapter 2.1 for a review).

CDS tail parameter estimates: the power law, EVT, and exponential distributions.

Recognizing the systemic importance of credit default swaps and the six stylized facts of financial data; we apply a number of non-Gaussian models to estimate the (left) tail index of CDS data. We focus on the left tail as this is relevant for risk estimation techniques. Estimating one tail at a time allows for a skew in the data. Using maximum likelihood estimation

(MLE) techniques (Jondeau et al. 2007), we estimate the tail index, which can then be used for risk estimation (McNeill 1999). The normalized logged differentials are transformed into a ranked cumulative density function and the parameter value is estimated. Initially we fit the data to a power law distribution. Power laws have been found in a wide range of disciplines from linguistics to biology and to economics (Solomon and Richmond 2001). The power law cumulative density function model underlying this methodology is:

$$P(x) = \left(\frac{x}{x_{min}} \right)^{-\alpha+1} \quad (1.17)$$

(Clauset et al. 2009)

That is the cumulative probability of x relative to its minimum x_{min} is proportional to its power $(-\alpha + 1)$. The value of α has been estimated many times in existing literature and has often been found to be stable (Solomon and Richmond 2001). Finding the existence of a power law in the data has implications for the accurate modeling of the data. Power laws have been shown to be related to the logistic equation (Solomon and Richmond 2001) and have been linked to structural change in nonlinear deterministic equations (Robledo 2011). Therefore the existence of power laws in the data has far reaching implications for modeling. The parameter α is equal to the reciprocal of the tail index ξ , which measures the width of the tail of the distribution.

As a comparison, extreme value theory (EVT) is also used to estimate the tail index. The Fisher and Tippett (1928) theorem (formerly proved

by Gnedenko (1943)) states that the asymptotic distribution of the maxima will belong to one of three distributional forms, irrespective of the original distribution of the observed data. They are known as the generalized extreme value (GEV) distributions and are the Gumbel, the Weibull and the Frechet distribution. As the Frechet distribution is fat tailed, it is commonly used for financial data (Jondeau et al. 2007). The cumulative density function is formally expressed as:

$$H_{\xi}(y) = \exp\left(- (1 + \xi y)^{-\frac{1}{\xi}}\right) \quad (1.18)$$

Where y is the standardized maxima or absolute minima, $y = \frac{m-\mu}{\varphi}$, μ is the location parameter, φ is the scale parameter and ξ is the tail index such that $\xi > 0$.

The data is transformed using the Block Maxima method; that is a subsample of data is chosen by taking the minimum value (out of each block of ten) and transforming it into its absolute form. In general the data is assumed to be iid. But for correlated samples the Block Maxima method works more efficiently than the alternative Peaks Over Threshold method (Jondeau et al. 2007). The size of the block is chosen so that it is greater than the memory of the autocorrelation; in this way the subsample of data will be independent (Jondeau and Rockinger 2003). The drawback of the Block Maxima method is that it can lead to a small subsample size. MLE is used to estimate the tail index.

As the CDS data indicates linear and nonlinear dependencies (see chapter 2); a semi-parametric technique, the Hill Index, is also applied to estimate the tail index. With weakly dependent data, a semi-parametric method will

give more efficient estimates of the tail parameter (Jondeau et al. 2007). In general, under non-Gaussian conditions, the semi-parametric approach obtains better bias and mean-squared properties than its parametric counterpart (Cotter 2001). The Hill Index is applied for values of $\xi > 0$ and is expressed as:

$$\hat{\xi}_{(q,T)}^{Hill} = \frac{1}{q} \sum_{j=1}^q \ln \left(\frac{x_{T-j+1,T}}{x_{T-q,T}} \right) \quad (1.19)$$

Such that x_t refers to the logged differential at time t and q is the number of observations included in the tail. The Hill Index is estimated for a range of values of q , and the estimate with the lowest mean squared error (mse) is chosen.

For comparison purposes, the MLE parameter of the exponential model is also estimated, this is:

$$F(x; \lambda) = (1 - e^{-\lambda x}) H(x) \quad (1.20)$$

Such that $H(x)$ is a Heaviside function and λ is the parameter to be estimated.

The data used is the same data as in chapter 2; that is three samples of the iTraxx European CDS (corporate entity) index. CDS sample 1 is the iTraxx 12-22% recovery rate 5 year index, taken from 3rd August 2007 to 18th February 2009 (n=391). CDS sample 2 is the iTraxx 12-22% recovery rate 5 year index, taken from 20th March 2006 to 29th June 2007 (n=334). CDS sample 3 is the iTraxx 22-100% recovery rate 5 year index, taken from 20th March 2006 to 22nd June 2007 (n=329). The sample ‘All CDS’ adds

all three samples together, to test the sensitivity of the results to sample size (n=1054).

The results of these tests are shown in table 1.1². The first column presents the estimates of the tail index as the reciprocal of α . We compare this to the second column, which presents the estimate of the tail index based on MLE of the GEV distribution. In general, the tail index estimates are a poor fit to the data, except for CDS2. Also the tail index estimate either using MLE or the Hill Index (see column three) is $\gtrsim 0.5$. The tail index measures the fatness of the tail of the distribution. We note that in general if a random variable X is distributed with a tail index ξ , then for all integers $r < 1/\xi$ the r th moment exists (Embrechts et al. 1997). As $\hat{\xi} \gtrsim 0.5$ this implies that the variance for the unconditional CDS data is non-finite, even for the larger sample AllCDS. Looking at the fourth column of results there is some evidence that an exponential distribution may fit some of the data samples (CDS3 and AllCDS) although this result is not consistent across all the samples (similar tests were conducted for the right side tail with similar nonconclusive and poor fit results). In general the findings are inconclusive as no one distribution appears to fit all the samples. There is also some evidence of a non-finite variance. We note that the samples are assumed to be iid, therefore our second exploration allows for conditionality in the samples. This may remove the problem of the non-finite variance (Bollerslev 1986).

²The Anderson Darling test is a goodness of fit test, a high value indicates a high probability that the data set comes from this distribution

Table 1.1: Non-Gaussian left tail analysis

Left tail	MLE power law $\hat{\xi}$	MLE EVT $\hat{\xi}$ (Anderson Darling)	Non-parametric Hill $\hat{\xi}$ (mse)	Exponential $\hat{\lambda}$ (R^2)
CDS1	(R^2) 0.5076 ($R^2 = 0.9130$)	0.5971 (<0.01)	0.7458 (.0.0225)	0.4100 ($R^2 = 0.7270$)
CDS2	0.4608 ($R^2 = 0.9910$)	0.5345 (>0.25)	0.5245 (.0.0058)	0.7800 ($R^2 = 0.845$)
CDS3	0.3096 ($R^2 = 0.9150$)	0.4960 (0.1000)	0.4312 (.0.0038)	1.1900 ($R^2 = 0.9790$)
ALL CDS	0.3788 ($R^2 = 0.9270$)	0.6293 (<0.01)	0.4959 (.0.0046)	0.6700 ($R^2 = 0.9820$)

An ARGARCH value at risk model with alternative stochastic distributions

In this exploration, we wish to take account of conditionality when applying risk estimation techniques. Value at risk (VaR) is a widely used method of estimating the loss of holding an asset or group of assets. We apply VaR to estimate the minimum loss from holding an asset for one day at a 99% confidence interval (as recommended by BIS (2009)). Recognizing the autocorrelation in returns and in the variance of financial data (see 1.1.3), a conditional model (ARGARCH, see equation 1.13 to 1.16) is applied to remove the linear dependencies. This methodology follows the advice of McNeil & Frey (2000) and assumes that the residuals of the conditional model are iid. To highlight the importance of the distributional assumption, three alternative distributions will be analyzed: they are the ARGARCH – Gaussian, the ARGARCH Student t, and the ARGARCH EVT. The results will be back-tested to examine the efficacy of each model for forecasting loss.

VaR is the q th quantile of the loss distribution F (McNeil 1999); that is:

$$VaR_q = F^{-1}(q) \quad (1.21)$$

Where F^{-1} is the inverse of F and q is the quantile of the distribution (in this case, $q = 0.01$).

As the ARGARCH model is applied, both the mean and the variance are conditional. The following equations will be used to estimate VaR:

1. ARGARCH-Gaussian

$$VaR_{t+1} = \mu_{t+1} + q_{\Theta}\sigma_{t+1} \quad (1.22)$$

(Jondeau et al. 2007)

2. ARGARCH-Student t

$$VaR_{t+1} = \mu_{t+1} + q_{\Theta} \left(\sqrt{\frac{v-2}{v}} \right) \sigma_{t+1} \quad (1.23)$$

(McNeil & Frey 2000)

3. ARGARCH-EVT

$$VaR_{t+1} = \mu_{t+1} + q_{\Theta}\sigma_{t+1} \quad (1.24)$$

$$q_{\Theta} = u + \frac{\psi}{\xi} \left(\left(\frac{T}{k} (1-q) \right)^{-\xi} - 1 \right) \quad (1.25)$$

(McNeil 1999)

Such that $\Theta = 1 - q$, v is the degrees of freedom, u is the threshold, ψ is the scale parameter, ξ is the tail index, T is the number of observations in the parent sample and k is the number of observations in the subsample of absolute minima. In this case the subsample is found by applying the Peaks over Threshold method, see below for details (McNeil 1999).

When estimating VaR the value of the quantile is found based on the distributional assumption made and the estimates of certain parameter values. For the Gaussian distribution only the value of the quantile q_{Θ} for that

distribution is required. Whereas for the Student t distribution, the degrees of freedom (v) must also be estimated. For EVT, three parameter values; that is (u, ψ, ξ) must be estimated and details of the sample sizes (T, k) are required. In section 1.2.1 above the GEV formulation of EVT is used. For estimating value at risk, an extension of GEV is applied; that is the generalized Pareto distribution (GPD). This distribution is used when the Peaks Over Threshold (POT) transformation is applied to the parent sample. This was first proposed by Balkema and De Haan (1974) and it relates the limit distribution of the scaled excesses (over a chosen threshold u) to the tail index ξ . We assume that the threshold u is relatively large. GPD can be expressed in a number of generalized forms depending on the value of the tail index ξ . As the focus here is on fat tailed data, the GPD is represented as follows:

$$G_{\xi, \psi}(x) = 1 - \left(1 + \frac{\xi x}{\psi}\right)^{-\frac{1}{\xi}} \quad (1.26)$$

Such that $\xi > 0$, $\psi > 0$ and $x \geq 0$.

A semi-parametric method is used to estimate the parameters. In this example the DeDH extension (Dekkers et al. 1989) of the Hill estimator (Hill 1975) is applied to estimate the tail index. This will allow the estimation of all three parameters, (u, ψ, ξ) and will also indicate the sample sizes (T, k) .

We need a longer sample of data than in the previous exploration as we will be back-testing our results. Therefore the data to be used in this exploration is a single entity corporate credit default swap, that is the 5 years JP Morgan (JPM) CDS taken from 7th April 2004 until 6th April 2010 ($n=1565$). A subsample of data from 7th April 2004 until 5th February 2008

($n=1000$) is taken so that the estimates can be back-tested on the remaining time period, that is from 6th February 2008 until 6th April 2010 ($n=565$). Back-testing the methodologies in this way should allow the assessment of each of the value at risk models during the financial crisis.

Table 1.2 presents the estimate of the Student t degrees of freedom, based on the ARGARCH maximum likelihood estimation (MLE) of the parameter. The table also includes the z test which indicates that the value is significant.

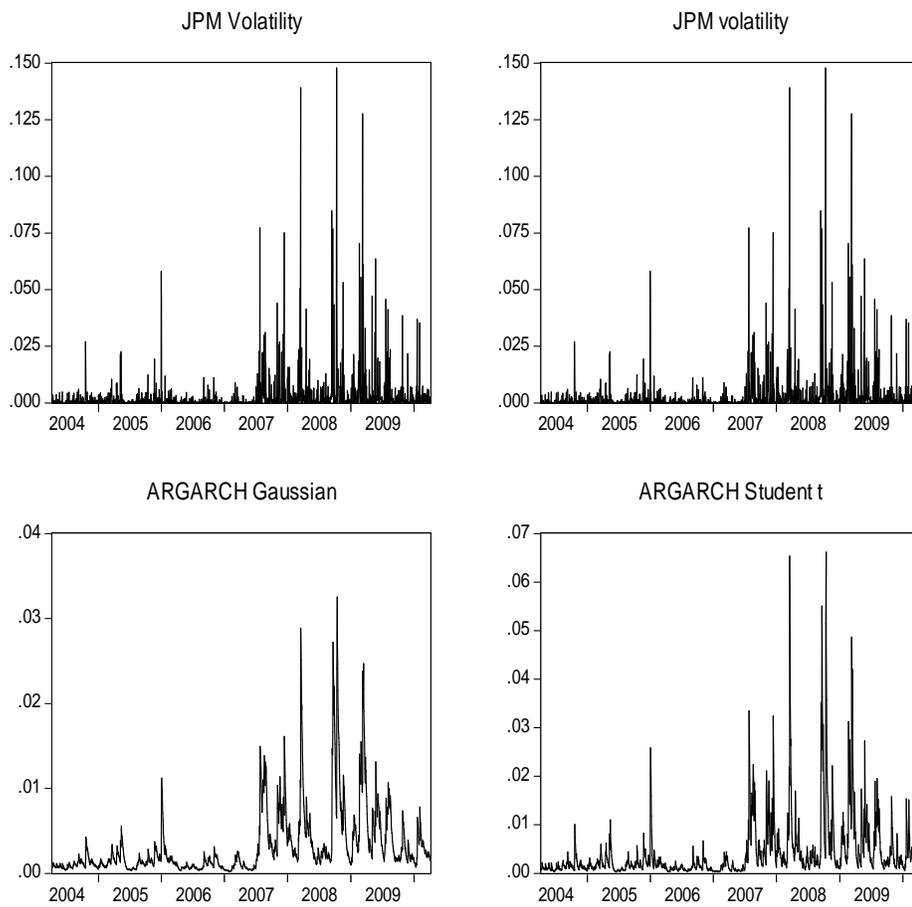
Table 1.2: ARGARCH Student t estimate of degrees of freedom (v)

	Coefficient	Std. Error	z-Statistic	Prob.
\hat{v}	3.0890	0.4044	7.6387	0.0000

Figure 1.2 illustrates the accuracy of the Gaussian and the Student t conditional models. This figure compares in turn actual JPM volatility (taking the squared logged returns as a proxy) against the forecast of variance of the ARGARCH Gaussian model and then against the forecast of variance of the ARGARCH Student t model. The top left and top right graphs are equivalent. Noting the scale of the y axis, it is clear that both models are poor forecasts of actual volatility. We note that the Student t model is an improvement on the Gaussian model. This improvement occurs as the Student t model allows for fat tails (measured by the degrees of freedom v). Similar to Curto et al (2009), the Student t is a better fit than the Gaussian distribution.

As noted by Nelson (1991), the Student t distribution is a symmetric distribution, which allows for excess kurtosis to be measured by the degrees of freedom, v (it is well known that as $v \rightarrow \infty$, the Student t distribution \rightarrow

Figure 1.2: Model comparison: Conditional Gaussian v conditional Student t



the Gaussian distribution).

We note that the result in table 1.2 gives a very low estimate for the value of v . Mood Graybill and Boes (1974) show that $v = \text{no. of moments in existence}$. Therefore the result ($v \approx 3$), indicates a nonfinite unconditional kurtosis (Jondeau et al 2007). The result meets the limiting constraint i.e $v > 2$ so the Student t distribution model can still be used. But the low value for v suggests long run instability in the parameter. In search of a better result, extreme value theory (EVT) is applied as a comparable distributional form.

As discussed above, EVT offers a generalized group of distributions in the sense that it subsumes certain other distributions under a common parametric form. For example, if $\xi > 0$ then $G_{\xi,\psi}(x)$ is a reparameterized version of the ordinary Pareto distribution. The ordinary Pareto distribution is applied to the unconditional distribution of a number of CDS samples in section 1.2.1. The Pareto or Power Law distribution has a long history in actuarial mathematics as a model for large losses (McNeil 1999). This is because it is a better fit than the Gaussian distribution as it allows for fat tails (Solomon and Levy 2000). We note that if $\xi = 0$, this implies that $G_{\xi,\psi}(x)$ corresponds to the exponential distribution as follows:

$$G_{\xi,\psi}(x) = 1 - \exp\left(-\frac{x}{\psi}\right) \quad (1.27)$$

Which is equivalent to the exponential distribution (which was applied in section 1.2.1) if $\lambda = \frac{1}{\psi}$.

If $\xi < 0$, the $G_{\xi,\psi}(x)$ has the same form as in equation 1.26 and is known as a Pareto type II distribution. This distribution would not be appropriate

for financial data. The first case is the most relevant for risk management purposes as financial data is heavy tailed and the GPD is heavy-tailed when $\xi > 0$.

To estimate the parameters of the GPD distribution, we will analyze the residuals of the ARGARCH Gaussian model (following from the Fisher Tippet theorem, see section 1.2.1). One restriction of this method is that the parent sample i.e. the ARGARCH residuals, must be iid (Jondeau et al. 2007). Engle and Gonzalez-Rivera (1991) show that if the true conditional model is non-normal then using the quasi-maximum likelihood estimation (QMLE) technique (which assumes conditional normality) will lead to reduced levels of efficiency. As the non-normal distribution becomes more normal (e.g., Student t distribution with $df > 12$) the QMLE technique is efficient. For lower levels it is found that a semi-nonparametric approach is more suitable. As $v \approx 3$, it is clear that the conditional distribution is significantly nonnormal; in fact this is evidence to suggest that the fourth moment is nonfinite. Therefore a semi-parametric approach is more suitable for the estimation of the tail index. A second advantage of this approach is that it also allows the data to be weakly dependent (see section 1.2.1). McNeil (1999) notes that the GPD is designed specifically for situations where the higher moments do not exist. As there is evidence that the unconditional second (see section 1.2.1) moment does not exist and that the conditional fourth moment (see table 1.2) does not exist; a semi-parametric GPD approach may be a more useful distributional form than the Gaussian or the Student t when analyzing credit default swaps.

As mentioned above, in order to apply the GPD model, 3 parameters,

(u, ψ, ξ) need to be estimated and also the sample sizes (T, k) . The first step is to choose a value for u the threshold. The choice of u is subject to the bias-variance trade off. Too low a threshold and the estimate will be bias (due to contamination from observations from the middle of the distribution). Too high a threshold and there are too few observations (the estimate of ξ is unbiased but inefficient due to a lack of variance in the sample). GPD is the limiting case as $u \rightarrow \infty$, therefore in general u must be large. McNeil & Frey (2000) suggest that for a sample size of $T = 1000$, the threshold should be chosen so that $k = 100$. Embrechts et al. (2005) note that the semi-parametric method the Hill Index performs well for subsamples of $20 \leq k \leq 75$. They show that the Hill estimate tends to overestimate the tail index whereas the parametric method underestimates the tail index. As the objective here is to estimate risk, a conservative risk manager would choose a semi-parametric rather than a parametric technique.³

The first step is to transform the parent sample into a subsample of absolute minima in order to estimate the parameter values. To do this we need to choose a threshold. We will then collect the subsample from all observations which exceed that threshold. The choice of threshold suffers from the bias-variance trade off (Jondeau et al. 2007). Too low a threshold and the estimate will be bias as its value will be affected by observations from

³For comparison purposes, the tail index was estimated using MLE and the DeDH non-parametric approach. The result below coincides with the findings of Embrechts et al. (2005). Looking at the value for the Anderson Darling test, we can see that the non-parametric approach gives a better fit.

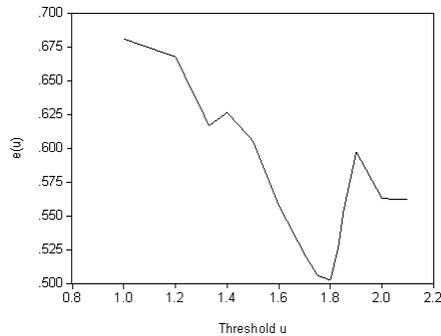
MLE EVT	Non-parametric DeDH
(Anderson Darling)	(mse)
0.2501	0.2711
< 0.0100	0.0013

the middle of the sample, rather than from the tail. Too high a threshold and there will not be sufficient variability to ensure an efficient estimate. The threshold is chosen by finding the minimum of the mean excess function. The mean excess is calculated as follows:

$$e(u) = E[X - u | X > u] \quad (1.28)$$

This is plotted against the corresponding threshold value. The optimum u is chosen as the minimum point from which the mean excess is rising in a linear fashion (Embrechts et al. 2005). The mean excess function is displayed in figure 1.3:

Figure 1.3: Mean excess plot



From this plot the threshold is chosen to be equal to 1.8, therefore $k=40$. This is a small sample size for k , which should ensure an unbiased estimate of the parameters. The other two parameters are estimated using the Hill Index (as above in section 1.2.1). From the tail index estimate, the estimate of ψ can be estimated as follows:

$$\hat{\psi} = u\hat{\xi}_{(q,T)}^{Hill} \quad (1.29)$$

The three parameter estimates are presented in table 1.3:

Table 1.3: Parameter estimates for GPD

Parameter	Cond'l GPD
$\hat{\xi}_{(q,T)}^{Hill}$	0.2711
$\hat{\psi}$	0.4879
\hat{u}	1.8000
\hat{k}	40

The GPD quantile is estimated using these parameter values. The quantile estimate for each distributional assumption are estimated and presented in table 1.4:

Table 1.4: Quantile estimates for ARGARCH models

Estimate	Gaussian	Student t	EVT
Shape parameter	n/a	3.0890	0.2711
$q_{99\%}$	2.3300	4.5410	2.6200

Comparing this result with existing literature, we note that Bystrom (2008) looks at iTraxx CDS data and using an unconditional model finds $u = 0.027$; $\xi = 0.278$, $\psi = 0.013$, for $k = 50$.

In this chapter, we use the conditional model as it removes linear dependencies in the data and allows for volatility clustering. The conditional mean and variance are found and VaR is calculated based on each quantile estimate. The results are back-tested, by comparing the forecast losses to the actual losses experienced from 6th February 2008 until 6th April 2010

($n=565$). This was a turbulent time for JPM CDS contracts due to the collapse of Lehman Brothers in September 2008 and the ongoing financial crisis. Table 1.5 presents the results of the back-test. The first column outlines the number of times the real market data exceeded the forecast loss, based on each model. The second column outlines the test of success or failure of the model.

Table 1.5: Backtesting the ARGARCH VaR approach

	No. of exceedances	p value
Expected	5.45	
ARGARCH-Gaussian	11	(0.0163)
ARGARCH-Student t	10	(0.0319)
ARGARCH-EVT	8	(0.0908)

McNeil and Frey (2000) show that the total number of violations is binomially distributed under the model, thus a binomial distribution is used as a test of the success or failure of the method. A p value of less than 0.05 implies failure of the test, according to the BIS requirements, (2010). Using this framework, only the ARGARCH-EVT method passes the test. Finally the expected shortfall is estimated for each ARGARCH-EVT exceedance. The expected shortfall (ES) is the mean of the excess distribution over VaR_q . ES is often suggested as a better estimator of loss than VaR (Jondeau et al. 2007). A simple way to estimate ES for EVT is proposed by Embrechts et al (2005), that is:

$$ES_q = VaR_q + E[X - VaR_q | X > VaR_q] = VaR_q + \frac{\psi + \xi u}{1 - \xi} \quad (1.30)$$

We apply this method and the no. of exceedances for ES is found to be 4 that is less than 5.45, which is the number you would expect if the data fit the model perfectly. Therefore ARGARCH-EVT ES appears to be the most conservative technique for risk estimation of single entity CDS data; based on an analysis of JPM CDS contracts from 7th April 2004 until 6th April 2010. This method performs well even during the financial crisis. The criticism of the ES method is that it may be too conservative on average and may be seen as overestimating the loss. The consequence of this would be to dampen investment and trading activity.

In general, a limitation of this methodology is that the residuals are assumed to follow an ergodic stochastic distribution. The distribution of the residuals must be iid and stationary. This is a questionable assumption. As shown in chapter 2 and 3, CDS data exhibits nonlinear dependencies. In table 1.6 the nonlinear dependence of JPM CDS contracts is illustrated.

Table 1.6: BDS Test of JPM ARGARCH residuals

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.0071	0.0016	4.5335	0.0000
3	0.0073	0.0014	5.1290	0.0000
4	0.0043	0.0010	4.3297	0.0000
5	0.0025	0.0006	4.0814	0.0000

The null hypothesis of the BDS test is that the data is independent (for a discussion of the BDS test please see chapter 2). Rejection of the null (see table 1.6) indicates that there are remaining nonlinear dependencies in the log squared ARGARCH Gaussian residuals of the JPM CDS contracts.

Secondly, we note the ease of using the Gaussian distribution. The Gaussian quantile for the standard normal curve is known to be 2.33 (for 99%

confidence interval). The additivity principal for the Gaussian distribution implies that $VaR^T = \sqrt{T}VaR^1$. This does not hold for the other distributions. There is no straight forward rule for time-scaling the other distributions. The Basel Committee 1996 Amendment allows the ‘square root of time’ rule (BIS 2009b) to be used for scaling VaR from 1 day to 10 day VaR, irrespective of the distributional assumptions used. This allowance can lead to poor estimation of risk for non-Gaussian data. Once again this is an example of Gaussian rules being applied to non-Gaussian data, the implications of this are discussed by a number of authors. If the square root of time rule is used the exponent, by definition is 0.5. McNeil (1999) shows that for non-Gaussian estimates, the exponent should be > 0.5 for times of low or average volatility and should be $\lesssim 0.5$ for periods of heightened volatility. Therefore, if we think of volatility as being mean reverting, this implies that applying the ‘square root of time’ rule will lead to a procyclical VaR estimate i.e. too low when markets are less volatile. As an alternative to the square root of time rule, Jondeau et al. (2007) suggest that a non-Gaussian 10 day VaR should be the sum of the VaR for each of the ten days. This would mean that the value of the quantile would have to be estimated for each of the ten days, this is a rather cumbersome approach.

In conclusion, the application of EVT to financial data allows for a more exact estimate of the distribution of the tails of the data. This is of particular importance for risk estimation techniques. But unlike the Gaussian methodology it does not allow for parsimonious estimation. Turning to non-Gaussian linear modeling, such as EVT or Student t distributions will improve accuracy of results. But it will also greatly increase the complexity of financial

modeling techniques and will necessitate the review of much of financial theory. Therefore non-Gaussian modeling brings a number of drawbacks as well as a number of solutions. For this reason, after the initial explorations, we decided not to focus on non-Gaussian linear modeling of financial data. After a review of additional literature, our focus turned to the relatively new framework of nonlinear time series analysis. The objective of the thesis is to analyze financial data using nonlinear tools and without evoking a model, to see if these tools and concepts can facilitate improved understanding of financial markets, particularly during times of heightened turbulence. The research question is further examined in the next section, 1.3.

1.3 Overview of the research question

Parsimony has a long history in scientific endeavor, from William of Occam, to Newton, to Einstein, to Mandelbrot; parsimony has been used as a justification for the adoption of many scientific principles (Zbilut 2004). As argued above, linear Gaussian modeling is parsimonious but inaccurate for risk estimation, whereas linear non-Gaussian modeling is more accurate but significantly more complex. Thus in search of parsimonious methods of risk estimation and improved understanding of financial crises, the thesis looks beyond linear stochastic modeling to the theoretical framework and the applied tools of nonlinear time series analysis. The framework and the techniques used are discussed in chapters 2, 3, and 4. We examine financial data to see if notable dynamic patterns can be discerned. The objective here is to apply the theoretical framework of nonlinear dynamics to graph and

plot the data, to search for recurring patterns, and to develop a theoretical explanation for the patterns which emerge.

The research question is: Can we improve our understanding of risk and volatility in the financial markets, particularly during times of heightened turbulence, by using applied nonlinear time series techniques?

Linear analysis interprets all regular structure in a data set through linear correlations, and all irregular behaviour of the system is assumed to be due to a random stochastic element. Whereas nonlinear analysis; such as chaos theory, has shown that irregularity may not be due to some random input but may in fact be due to the deterministic equations of motion underlying the system (Kantz and Schreiber 2003). The Polish mathematician Stanislaw Marcin Ulam (1909-1984) liked to say that “the study of nonlinear dynamics in nature is like the study of non-elephant mammals in zoology”, which is to say that most processes in nature are nonlinear (Ulam cited in Sprott 2004). The discovery of chaos theory (one subject area within nonlinear dynamics) has led to the review of financial modeling. Could it be that financial data is better modeled by nonlinear deterministic equations rather than linear stochastic equations⁴.

The application of nonlinear tools can track subtle changes in discontinuities or periodicities which cannot be picked up by linear tools. Zbilut and Webber (2008) compare the nonlinear recurrence quantification analysis

⁴Kantz and Schreiber (2003) note that nonlinear stochastic equations may also be considered, that is equations with a significant stochastic element as well as a measurable nonlinear deterministic element.

(RQA) with the linear Fourier Transform when analyzing the time series of a driver-oscillator. They show that the linear tool indicates the existence of the main oscillation, whereas the nonlinear tool indicates not only the main oscillation but also areas of laminar structures i.e. phase transitions in the data. Phase transitions occur in thermodynamics when a substance changes from one state of matter to another, for example from a solid to a liquid. In nonlinear time series analysis a phase transition occurs when there is a structural change in the deterministic equation(s) underlying the system, which causes the trajectory of the data to change (Sprott 2004). RQA analysis can indicate changes in the nonlinear deterministic equations underlying a system, which linear methods cannot distinguish. Marwan & Kurths (2002) compare the nonlinear mutual information method (Kantz and Schreiber 2003) with the linear cross correlation estimate to compare two data sets with linear and nonlinear synchronizations. They find that both tools discover the linear synchronization i.e. the cross correlation; whereas only the nonlinear tool discovers the nonlinear coupling between the two data sets. Motivated by these findings, the objective of the thesis is to apply nonlinear tools to financial data to examine the dynamical relationships.

There exists a growing body of literature which applies nonlinear tools to financial data looking for evidence of determinism. Some of this literature tests financial data to see if it can be defined as chaotic. In order for data to be chaotic it is generally accepted that the data must exhibit sensitive dependence on initial conditions, be a bounded system and have a positive Lyapunov exponent i.e. the average speed of the divergence of two initially close trajectories must be greater than zero (Kantz and Schreiber

2003). Another test applied is the BDS test which looks for dynamical non-linear dependence as this can also be an indication of attractors (which occur during chaotic or periodic regimes, see chapter 4). The results of these tests are inconclusive. An example would be the analysis of the US\$/Canadian \$ exchange rate (Hommes and Manzan 2006, Kyrtsov and Serletis 2006). These papers find a positive Lyapunov exponent for the exchange rate, although they note that the estimate of the exponent falls to a negative value as the variance of the data rises. They also find evidence of nonlinear dependence in the data. Moshiri (2004) tests crude oil futures and finds a positive Lyapunov exponent, indicating chaos. The result of a second test of chaos, the correlation dimension test finds the dimension to be rising. In order for the data to be deterministic the correlation dimension should stabilize at a relatively low number (see chapter 2). So overall Moshiri's (2004) results are inconclusive.

In chapter 2, we apply the BDS test, delay plots and the correlation dimension test to a number of financial series (CDS, equity and bond indices) and find some evidence of nonlinear dependencies. In testing for determinism the results are inconclusive. One possible reason for this inconclusivity is the requirement in the tests for deterministic data to be stationary. As noted above (see chapter 1.1.3 and chapter 1.2.1) analysis of financial data indicates that the data is nonstationary (Mandelbrot 1963) therefore application of tests which require stationarity may lead to inconclusive results. We may in fact be looking at nonstationary, intermittently-deterministic, data. In chapter 3 and 4, we apply nonlinear tools, which do not require stationarity. They are cross recurrence plots (CRP) and recurrence quantification analysis

(RQA). These tools develop a binary matrix of dynamical recurrence in the data series; analysis of this recurrence can indicate deterministic structure and relationships in the data. These tools have been applied previously to financial data by a number of authors. Gilmore (2001) looked at the conditional ARCH (autoregressive conditional heteroskedastic) model in relation to the close return test and showed that ARCH incorporates only some of the nonlinearities in the exchange rate (Sterling, German DM and Japanese Yen versus the US\$) data analyzed. Holyst et al. (2001) showed similarities in the dynamical structure of a range of financial assets (exchange rate, equity indices, Treasury Bills and individual equities) and the Lorenz system of equations (contaminated with computer generated stochastic noise). This suggests that financial data may be well described by nonlinear equations. More specifically, a number of authors have analyzed financial data during times of heightened turbulence using the RQA tool. Fabretti and Ausloo (2005) note high values in the RQA measures before a market peak; this would indicate a phase transition. While examining international equity and exchange rate markets, other authors have noted a collapse in the RQA measures just at the time when the market peaks (Basto and Caiado 2011, Guhathakurta et al. 2010, Marwan et al. 2007, Piskun and Piskun 2011, and Zbilut 2004). Although none of these authors have suggested an interpretation for this finding. The objective of this thesis is to contribute to this literature by applying nonlinear tools to a range of financial assets: CDS (indices, sovereign and corporate), equity indices, a bond index, and sovereign bonds, focusing on times of heightened turbulence, with a view to search for patterns. The aim is to interpret these patterns using the theoretical frame-

work of nonlinear dynamics and to suggest applications of this knowledge to improve quantitative risk estimation techniques in financial markets during times of heightened turbulence.

1.4 Structure of the thesis

The format of the thesis is article-based, that is it is a collection of papers which describe a coherent programme of research. This introductory chapter is followed by the three papers which are presented as three chapters. The three chapters are on various topics in nonlinear dynamics, focusing on periods of financial crises. The abstracts of the following three chapters are presented in the following section. The thesis concludes with the contributions and limitations of the research, and some suggestions for future research.

Abstract chapter 2 - Nonlinear dependence

The objective of this chapter is to test for nonlinear dependence in the GARCH residuals of a number of asset classes using nonlinear dynamic tools. The equity and bond market samples appear to be independent once GARCH has been applied but evidence of nonlinear dependence in the CDS GARCH residuals is found. The sensitivity of this result is analyzed by changing the specifications of the GARCH model and the robustness of the result is verified by applying additional tests of nonlinearity; that is delay plots and the correlation dimension test. Evidence of non linear dependence in the GARCH residuals of CDS contracts has implications for the accurate modeling of the

marginal distribution of the CDS market, for pricing of CDS contracts, for estimating risk neutral default probabilities in the bond market as well as for bond market hedging strategies.

Abstract chapter 3 - The arbitrage-free parity theory

The arbitrage-free parity theory states that there will be equivalence between credit default swap (CDS) spreads and bond market spreads in equilibrium. In this chapter, we test this theory using linear and nonlinear tools. Linear stochastic modeling is reviewed, particularly the assumptions of a Gaussian distribution and of iid and stationary residuals. By applying the nonlinear dynamic tools of Cross Recurrence Plots and Cross Recurrence Plot measures, evidence of dynamically varying convergence and statistically consistent synchronization across the markets is illustrated. There is evidence that the arbitrage-free parity is conditional and equivalence is non-mean reverting. In particular there is evidence of a rising trend in equivalence in the Greek sovereign market prior to the bailout of May 2010. This trend indicates increased arbitrage activity at this time. Applying a nonlinear analysis of the markets significantly increases our understanding of the dynamically varying relationship between these two assets classes. This result has implications for supervision of arbitrage activity and for financial market policy making.

Abstract chapter 4 - Dynamical transitions

In this chapter, application of the nonlinear dynamical tool, recurrence quantification analysis (RQA) presents some evidence of periodic to chaotic transitions and bifurcation points at localized peaks of the Dow Jones Industrial

Index. There is also evidence of a collapse in all the RQA measures just as the market transitions from bull to bear market. This suggests that the market becomes completely unpredictable at this time. We suggest the market is intermittently or piecewise deterministic and that the unpredictable regime allows the market dynamics to break from the past. The noisy trader theory is suggested as the economic explanation for this unpredictability. We develop a principal component series and name it the random market indicator (RMI). We suggest that this series indicates when the market is transitioning into and out of a nonstationary random regime. During this time, quantitative risk estimation techniques, such as value at risk models will produce misleading results.

1.5 Thesis Outputs

Listed below for information purposes are the research outputs from the thesis. These include two peer-reviewed publications and two working papers. The research has been presented at a number of conferences which are also listed below:

Papers

- ‘A linear and nonlinear review of the Arbitrage-free Parity Theory for the CDS and Bond markets’ (with S. Raghavendra), in *Topics in Numerical Methods for Finance: With Special Emphasis on Energy-Commodities-Risk Management and Credit Derivatives-Credit Risk Modeling-Liquidity*, ed. John Miller, Springer Verlag, forthcoming.

- ‘Testing for nonlinear dependence in the Credit Default Swap Market’ (with S. Raghavendra), Economics Research International, vol. 2011, article ID 708704, 12 pages, doi: 10.1155/2011/708704 2011.
- ‘Examining the dynamical transition of the Dow Jones Industrial Index from bull to bear market using recurrence quantification analysis’ (with S. Raghavendra), Working Paper 176, Working Paper Series, NUI Galway, available at http://www.economics.nuig.ie/resrch/pdf/paper_0176.pdf 2011 (Submitted.)
- ‘Quantitative risk estimation in the Credit Default Swap Market using Extreme Value Theory’ (with S. Raghavendra), Working Paper 158, Working Paper Series, NUI Galway, available at http://www.economics.nuig.ie/resrch/pdf/paper_0158.pdf 2010 (Submitted.)

Presentations

- ‘Transition dynamics of Dow Jones Index using Recurrence Quantification Analysis’, Self-Organization, Emergence and Nonlinearity in Physical, Natural and Social Systems Workshop, Complex Systems Research Centre, NUI Galway, October 2011.
- ‘Testing for non linear dependence in the Credit Default Swap Market’, presented at:
 - Numerical Methods in Finance, Conference, Kemmy Business School, University of Limerick, June 2011,

- MACSI (Mathematics Applications Consortium for Science and Industry) Seminar Series, University of Limerick April 2011,
- IEA Conference, Limerick, April 2011,
- ‘Evidence of nonlinear structure in equity, bond and credit default swap markets’, AIB-UKI Conference (selected), University of Edinburgh Business School, April 2011,
- ‘Testing for non linear dependence in the Credit Default Swap Market’, NUI Galway Seminar Series, National University of Ireland, Galway, January 2011,
- ‘Modeling Credit Default Swap Market Value at Risk using GARCH models with Gaussian, Student t and EVT distributions’, ISNE Conference. Trinity College Dublin, September 2010.
- ‘Extreme Value Theory’s contribution to measurement of Credit Default Swap Market Risk’, ISNE Conference. Kemmy Business School, University of Limerick, October 2009.

My contribution to each paper

With regard to the papers included in this thesis my supervisor, Dr. Srinivas Raghavendra, provided me with invaluable guidance in understanding the non-Gaussian and nonlinear theoretical framework. All the empirical research was conducted by me. Together, myself and Dr. Raghavendra discussed the results. All papers were written by me. I am the corresponding author in all the papers.

Chapter 2

Nonlinear dependence

2.1 Introduction

The objective of this chapter is to test for nonlinear dependence in the GARCH residuals of a number of financial time series including Credit Default Swaps. The presence of nonlinear dependence in the residuals would suggest that stochastic modeling is not appropriate and that there may be nonlinear deterministic structure remaining in the residuals. If this nonlinear structure is found, in line with the research question, we will then apply additional nonlinear techniques to further examine the characteristics of the dependence.

It has been suggested that the residuals of the GARCH model will follow a defined stochastic data generating process (Bollerslev 1986). As a result of this, GARCH has been applied to financial time series before the application of quantitative risk estimation techniques such as value at risk (Embrechts et al. 2005). The focus of this chapter is particularly on credit default swaps

(CDS) as they have been highlighted as a potential source of systemic risk (European Union 2009) and as such the analysis of the marginal distribution of the credit default swap market merits further analysis. We note the lack of univariate analysis of the CDS market and the lack of explicit testing of the GARCH residuals of financial data for non linear dependence, particularly in the CDS market. For an example of testing for non linear dependence in the exchange rate market see Kyrtsou and Serletis (2006) or Serletis et al. (2010) or in the oil futures markets see Moshiri (2004). The results are mixed with some evidence of nonlinear dependencies remaining in the data. For an example of the analysis of the CDS GARCH residuals see Li and Mizrach (2010). Li and Mizrach (2010) apply a univariate model to CDS data using the GARCH model but do not test the residuals for iid characteristics.

In general, the existing literature often places the CDS in a dependency model and analyzes inter-asset or cross asset dependencies such as correlations (Blanco et al. 2005, Cao et al. 2010, Coudert and Gex 2010, Delis and Mylonidis 2011, Jorion and Zhang 2007). The copula function is one such model, this model assumes that after a transformation, the variables under analysis follow a multivariate probability distribution, most commonly the Gaussian (Li 2000) or the student t or the Gumbel (Chen et al 2008, Frahm et al. 2005) or the exponential probability distribution (Li and Mizrach 2010). Should the marginal distribution of the CDS data be shown to be significantly nonlinear and non iid (even after the application of a GARCH model) it can no longer be assume that the data can be described by a probability distribution function, without further modeling.

It is common place in industry to use risk neutral default probabilities

from bond spreads to price CDS contracts, and in recent times to use CDS prices to imply risk neutral default probabilities in the bond market (Hull and White 2001). CDS contracts are also commonly used to hedge bond positions. This chapter analyzes the distribution of both bond and CDS contracts and shows significant differences in the dependency structures of each. This result questions the efficacy of current methods of modeling and pricing CDS contracts, of estimating implied default probabilities and of developing bond market hedging strategies. Further research in this field is required. The objective of the chapter is to test for non linear dependence in the GARCH residuals of a number of asset classes, using nonlinear dynamic tools. In section 2.2 the general principles for applying the BDS test to the GARCH residuals of the three asset classes, equities, bonds and CDS are outlined. In section 2.3, the data samples are described. In section 2.4, the BDS test is applied to the residuals of the GARCH (1,1) model. In section 2.5, the specifications of the GARCH model are varied to assess the sensitivity of the results to the model chosen. In section 2.6, some additional tests of nonlinearity are applied to ensure that the BDS result is robust. Time delay plots and the correlation dimension test are applied to verify whether the nonlinear process is stochastic or deterministic. Finally in section 2.7, the chapter is concluded with a brief discussion of the implications of the findings.

2.2 Test for non linearity in the GARCH residuals

The BDS test is the first test of nonlinearity that will be applied to the data. Following from Brock et al. (1995), the BDS test can be applied to any time series with n observations. The data is initially transformed into the first difference of the natural logarithm. The Null hypothesis of the BDS test is that the data is drawn from an iid process. The test uses the concept of the correlation integral, and is a non-linear test of independence. For a selected value of m , the time series is embedded into m dimensional vectors. Thus the series of scalars is converted to a series of vectors with overlapping entries. A correlation integral is calculated, as below:

$$C_{m,n}(\epsilon) = \frac{2}{n(n-1)} \sum_t \sum_{s,s<t} I_{[0,\epsilon]}(\|X_t^m - X_s^m\|) \quad (2.1)$$

Such that $I(\cdot)$ denotes the indicator function, which takes either the value of 0 or 1 according to:

$$I_{[0,\epsilon]}(s) = \begin{cases} 1 & \text{if } s \in [0, \epsilon] \\ 0 & \text{if } s \notin [0, \epsilon] \end{cases} \quad (2.2)$$

and $\|\cdot\|$ denotes the supremum norm, given by:

$$\|u\| = \sup_{i=1,\dots,m} |u_i| \quad (2.3)$$

This integral estimates the spatial correlation for a particular embedded dimension m . It estimates the probability that for the dimension m , the

vector length $|x - y|$ is less than or equal to ϵ , a predetermined distance. We note that:

$$\lim_{n \rightarrow \infty} C_{m,n}(\epsilon) = C_m(\epsilon) \quad (2.4)$$

If the data is independent then this implies that:

$$C_m(\epsilon) = [C_1(\epsilon)]^m \quad (2.5)$$

This follows from considering:

$$\begin{aligned} C_m(\epsilon) &= P[\|X - Y\| \leq \epsilon] \\ &= P[|X_1 - Y_1| \leq \epsilon, \dots, |X_m - Y_m| \leq \epsilon] \\ &= P[|X_1 - Y_1| \leq \epsilon] \times \dots \times P[|X_m - Y_m| \leq \epsilon] \\ &= [C_1(\epsilon)]^m \end{aligned}$$

The above proof from Diks (2003) assumes that the marginal probabilities are independent, if this is the case, the correlation integral should be scalable and thus $C_m(\epsilon) = [C_1(\epsilon)]^m$. The standardized BDS test statistic is:

$$T = \frac{\sqrt{n}(C_m(\epsilon) - [C_1(\epsilon)]^m)}{\sigma_m(\epsilon)} \quad (2.6)$$

Brock et al. (1995) shows that the statistic is normally distributed with $N(0, 1)$.

For a non random process, two initially close time paths are likely to remain close, thus the process will have a larger conditional probability than an iid process, and hence a larger correlation integral. In the application of

a GARCH type model to a time series, it is claimed that the dependence in the data is removed. Thus the resulting residuals are said to be iid. However this claim has not been proven conclusively for all the asset classes in the financial markets. Hence it is the objective of this chapter to verify this claim. To that end, as a first step, the BDS test is applied to the residuals of the GARCH model. If the null is accepted, this suggests that the data is iid and thus that there is no non linear dependence in the residuals. This would allow the application of a probability distribution function (pdf). If the null is rejected, the residuals cannot be said to be iid and the conditions required to fit a pdf to the data do not apply.

When applying the BDS test, general principles in the choice of the value of the embedded dimension m , the value for the length ϵ and the sample size n must be agreed. In general, for a small sample, if m is too large, there will be insufficient non-overlapping independent points (Lin 1997). If m is too small, the analysis may not pick up the higher dimensional dependencies. Thus it is customary to apply a range of values for m , usually two to five dimensions to take account for the bias – variance trade off as m increases (Brock and Sayers 1988).

In general ϵ should be kept small for time series data, as a too large ϵ will lead to a high value for the correlation integral and this will reduce the power of the test. This is because the correlation integral measures the relative frequency with which two data points are within distance ϵ of each other. If there is volatility clustering, then large shocks are likely to be followed by relatively large shocks. The probability that the distance $|X_t - X_s|$ is large and $|X_{t+1} - X_{s+1}|$ is small will be lower compared to that expected

under the null. If, in this case, ϵ is large, these low probability regions will be included in the estimates of the correlation integral, as a result of which the power of the BDS test decreases. Diks (2003) shows that when applying the BDS test to a time series sample for daily returns of the equity IBM ($n = 1013$) the most significant results are found when ϵ is set equal to one standard deviation. Brock and Sayers (1988) use Monte Carlo simulations to develop general principles, they conclude that ϵ/σ should range from 0.5 to 2, depending on the value of the embedded dimension m and that in general if $n/m > 200$, the BDS statistic will follow a standard normal distribution.

Initially the value will be set at $\epsilon/\sigma = 0.7$, the default value and allow the embedded dimension to vary from two to five. If in the samples, $n/m > 200$, it can be assumed that the BDS statistic will follow a standard normal distribution. If $n/m < 200$, then Brock and Sayers (1988) provide several tables of quantiles of the BDS statistics for these smaller samples. As the smaller data samples that is the CDS samples have < 400 observations, we apply the closest tables to this, which are the $n = 250$ tables with values of $\epsilon/\sigma = 0.5$ and $\epsilon/\sigma = 1$. In the tables, the BDS test statistic values change as m increases, and are asymmetric.

As the initial objective is to use BDS to test for non linear dependence in the GARCH residuals of financial time series, the GARCH model must first be applied to remove linear dependence and then the BDS test statistic is applied to the GARCH residuals. Thus the model applied in this chapter is similar to that in Bollerslev (1986) as below:

$$r_t = \mu_t + \varepsilon_t \tag{2.7}$$

$$\varepsilon_t = \sigma_t z_t \quad (2.8)$$

such that:

$$z_t \sim N(0, 1) \quad (2.9)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (2.10)$$

In the mean equation (2.7) r_t is the logged differential of the daily closing prices of the financial instrument. The returns are modeled to fluctuate around an expected value μ_t . The standardized residuals of the model follow a Gaussian distribution and thus must be iid. The GARCH model itself also is characterized by dynamic variance. As can be seen above in equation 2.10, the variance is an autoregressive function of the previous time lag's variance and residual squared and also a constant. Serletis et al. (2010) raise the issue of the effect of imposing the GARCH model on the sample prior to applying the BDS test. They discover evidence of episodic nonlinearity in a group of exchange rate series and question if this episodic nonlinearity is caused by inappropriate volatility modeling rather than being a characteristic of the raw data. Thus to reduce this risk, there are a number of conditions which must be met before applying the BDS test onto the GARCH residuals.

Brock et al. (1995) advise the use of the BDS test as a diagnostic tool for ARCH or GARCH as long as the mean term is small. In general when analyzing financial data this condition to the test can be accepted. Caporale et al. (2005) consider the "nuisance parameter free property", of the BDS test statistic, that is that "its asymptotic distribution does not change when it is applied to the estimates of the residuals from a model, rather than the raw material itself" (Caporale et al. 2005, p. 2). De Lima (1997) shows that the

BDS test statistic has this invariance property when applied to linear additive models or models that can be cast in this format. Looking at equation (2.7) and (2.8) above it can be seen that the residuals of the GARCH model are not additive. Caporale et al. (2005) suggest that the transformation of the residuals of the GARCH model to log squared standardized residuals, before applying the BDS test, will ensure that the errors are additive, that is:

$$v_t = \ln z_t^2 = \ln \varepsilon_t^2 - \ln \sigma_t^2 \quad (2.11)$$

Also, Caporale et al. (2005) carry out Monte Carlo simulations of the standardized residuals of a GARCH (1,1) model and show that the BDS test statistic's bias reduces as the sample size increases, and that the statistic becomes consistent for sample sizes over 200. They also show that the results are not generally affected by the moment properties of the innovations. Thus when applying BDS to GARCH residuals, The residuals need to be transformed and the sample size must be > 200 in order to assume the invariance property of the test. As the test is not sensitive to the moment properties of the innovations, it is assumed that the standardized residuals (z_t) follow a Gaussian distribution.

2.3 Data

In this chapter eight data samples from the international equity, bond and CDS markets are compared. All samples chosen represent diversified portfolios of one of the three asset classes being analyzed. This is to ensure that the results reflect the systematic characteristics of each asset class rather than

the idiosyncratic characteristics of an individual equity, bond or CDS. The four international equity samples are all market indices, the FTSE 100, the DAX, the Nikkei 225 and the S&P 500. The observations are all of a five day week frequency. They represent four geographical regions, the United Kingdom, Germany, Japan and the United States respectively. The S&P 500 Index sample of 14,821 observations from 3rd January 1950 to 26th November 2008, the FTSE 100 Index sample of 6,228 observations from 2nd April 1984 to 25th November 2008, the DAX Index sample of 4,544 observations from 26th November 1990 to 27th November 2008 and finally the Nikkei 225 Index sample of 6,127 observations from 4th January 1984 to 27th November 2008. The bond fund chosen is the Barclays Capital Pan-European Aggregate Bond Index with the sample size of 2,512 observations, running from 30th March 2000 to 16th November 2009. This bond index is chosen as it represents a broad spread of European corporate and government bonds and should give a general reflection of the characteristics of bond market returns. Considering the number of observations n and the embedded dimension m for all of these samples $n/m > 200$ and thus that the BDS statistic follows a standard normal distribution.

Credit default swaps offer the buyer protection against the credit default of an underlying entity for a fixed annual premium known as the spread (Mengle 2007). We analyze the spread in this chapter. The three samples are taken from the family of iTraxx European indices. These indices are chosen due to their diversified nature and liquidity, which should improve the robustness of the findings for the CDS market in general. The index is made up of the credit default swaps of 125 investment grade corporate

European entities. The entities are chosen for their liquidity and from a number of underlying industries, as follows;

- 30 Autos & Industrials

- 30 Consumers

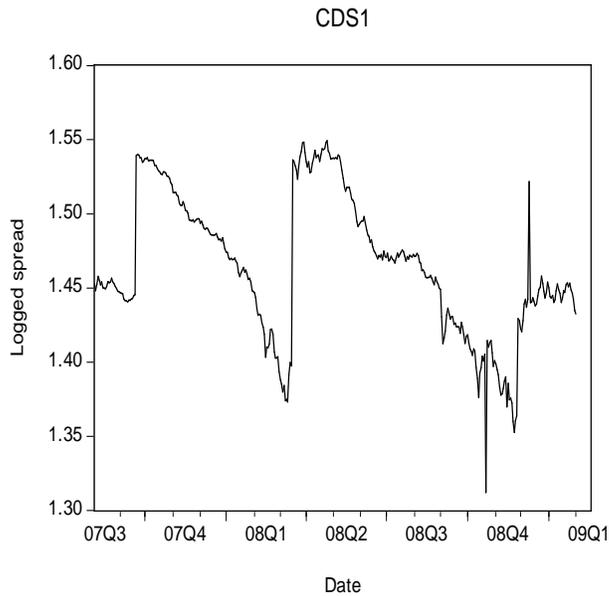
- 20 Energy

- 20 TMT

- 25 Financials

Each entity is equally weighted within the index. The index has been subdivided into a number of tranches to reflect total recovery rate, as well as maturity. In the case of a credit event, the protection seller pays 1-recovery rate of the defaulted issue to the protection buyer (Felsenheimer et al. 2004). CDS sample 1 is taken from 3rd August 2007 to 18th February 2009 and represents the iTraxx 12-22% recovery rate 5 year index. CDS sample 2 is taken from 20th March 2006 to 29th June 2007 and represents the iTraxx 12-22% recovery rate 5 year index. CDS sample 3 is taken from 20th March 2006 to 22nd June 2007 and represents the iTraxx 22-100% recovery rate 5 year index. All data samples are taken from Data Stream. For illustrative purposes we present below in figure 2.1 a time series graph of CDS 1.

Figure 2.1: CDS 1



As the CDS indices are reviewed every 18 months, the number of observations in each sample is significantly less than the equity and bond indices. There are less than 400 observations in each data sample. Thus with these samples $n/m < 200$ and the BDS statistic tables from Brock and Sayers (1988) are applied. As the sample sizes are small, bootstrapping techniques will be applied to the probability estimates.

2.4 Results

As is commonly used in the application of the GARCH model, the parsimonious GARCH (1, 1) model is applied in all cases, see table 2.1. The estimates of the parameters α_1 and β add to close to 1 for the equity and bond market samples; this implies persistence in the volatility. Referring to the Bollerslev constraints, the residual process ε_t is covariant stationary if and only if the sum of the estimates of the parameters is less than 1 (Jondeau et al. 2007). We note that the coefficients for CDS sample 1 sum to a smaller number and that the β coefficient is insignificant. For CDS sample 2 the sum of the coefficients is slightly more than 1, this would indicate a non finite i.e. non constant, variance and thus a non stationary process (Jondeau et al. 2007, Patterson 2000). CDS sample 3 behaves similarly to the equity and bond samples. Thus there is some evidence that the CDS data may be behaving differently. We note the relatively low values for the log likelihood of the CDS data samples, this implies that the model is a poor fit to the data. The ARCH LM test was applied to all the GARCH residuals as below. The hypothesis of no serial correlation in the GARCH residuals is accepted if the nR^2 statistic is less than the critical value of 6.63 (for one lag). The acceptance of the null hypothesis indicates that linear dependence has been removed through the application of a GARCH (1, 1) model. The null was accepted in all cases except for the DAX market. For the DAX, a GARCH (1, 2) model was fitted; the sum of the coefficients was 0.9866. In this case, the ARCH LM test was accepted, (0.0248(0.8749)). Thus the residuals of the GARCH (1, 2) process were used below for the DAX sample when applying the BDS test.

Table 2.1: Estimation of GARCH (1, 1) model for daily log-returns

	S&P 500	DAX	FTSE 100	NIKKEI 225	BOND FUND	CDS 1	CDS 2	CDS 3
n	14,822	4,545	6,229	6,128	2,512	391	334	329
α_0	7.35E-07	2.35E-06	1.87E-06	1.88E-06	3.39E-08	0.0002	-1.40E-07	0.0012
(s.e.)	(6.40E-08)	(3.22E-07)	(2.44E-07)	(2.57E-07)	(1.18E-08)	(9.34E-06)	(3.26E-05)	(0.0003)
α_1	0.0869	0.1020	0.0960	0.1162	0.03230	0.2187	0.0687	0.1337
(s.e.)	(0.0027)	(0.0058)	(0.0064)	(0.0063)	(0.0052)	(0.0784)	(0.0046)	(0.0145)
β	0.9100	0.8873	0.8857	0.8812	0.9599	-0.0346	0.9411	0.8595
(s.e.)	(0.0027)	(0.0060)	(0.0068)	(0.0059)	(0.0067)	(0.0529)	(0.0019)	(0.0096)
$\alpha_1 + \beta$	0.9969	0.9893	0.9817	0.9974	0.9922	0.1841	1.0098	0.9932
Log-likelihood	50950	13679	20382	18321	12042	1166	150	49
ARCH	1.8321	7.6799	0.2903	3.1443	0.4410	0.0157	1.0103	0.4108
LM test								
Prob	(0.1759)	(0.0056)	(0.5901)	(0.0762)	(0.5066)	(0.9003)	(0.3148)	(0.5216)

2.4. RESULTS

Before applying the test, the GARCH residuals are first transformed to log squared standardized residuals (Caporale et al. 2005) to ensure additivity. The test is applied to the eight data samples. Brock et al. (1995) states that the null hypothesis of iid residuals is accepted if the z statistic is less than the critical value of $|1.96|$ for a two tailed 5% significance level, for samples where $n/m > 200$. Epsilon, ϵ was chosen to be equal to 0.7σ and the embedded dimension m , was allowed to vary from 2 to 5. The null hypothesis was accepted for all the equity samples and the bond sample, see table 2.2.

Table 2.2: Empirical Size of the BDS Test, for $\epsilon/\sigma = 0.7$, for the Equity Indices and the Bond Fund

	S&P 500	DAX	FTSE 100	NIKKEI 225	BOND FUND
$m = 2$	1.2476	-0.5704	0.9501	0.2489	1.6361
$m = 3$	1.5542	-0.2726	0.2748	0.1059	1.4308
$m = 4$	0.7833	-0.4173	-0.0121	0.2425	0.7469
$m = 5$	0.4714	-0.4632	0.1807	0.3959	1.0804

For the CDS sample residuals, as $n/m < 200$, more care was needed. Caporale et al. (2005) conclude that when the sample size is within the 250 – 500 range, the estimates are still efficient. But as Brock and Sayers (1988) highlight, the BDS statistic no longer follows the standard normal distribution and the BDS tables must be used. Table 2.3 and 2.5 are taken from Lin (1997) and are for 250 observations, the closest to the data samples. The BDS test statistic has been calculated for a number of values of ϵ/σ , table 2.3 gives the critical values for $\epsilon/\sigma = 0.5$. The statistic is asymmetric, and thus when the 5% two tail test is applied, we apply the critical values for the

97.5% quantile and 2.5% quantile respectively.

Table 2.3: BDS statistic quantiles for 250 observations for $\epsilon/\sigma = 0.5$

	$m = 2$	$m = 3$	$m = 4$	$m = 5$
1%	-3.05	-3.38	-4.06	-4.86
2.5%	-2.64	-2.92	-3.37	-4.11
97.5%	2.98	3.23	3.84	4.98
99%	3.71	4.04	4.85	6.44

As the critical values are calculated for m ranging from 2 to 5, we use this range when assessing the CDS data. To double check the results and to improve the robustness of the test statistic, bootstrapped p values are also calculated, see table 2.4. For CDS 1 the null is rejected for the third and fourth embedded dimensions. For the other two CDS samples, the null hypothesis is rejected in all cases. These results are supported by the bootstrapped p values (* denotes the rejection of the null at the 5% significance level).

Table 2.4: Empirical size of the BDS Test, for $\epsilon/\sigma = 0.5$, for iTraxx CDS Index

	CDS 1	B/strp Pr	CDS 2	B/strp Pr	CDS 3	B/strp Pr
$m = 2$	2.1639	0.0568	5.7869*	0.0024	11.6932*	0.0000
$m = 3$	3.4017*	0.0064	8.6165*	0.0000	15.3103*	0.0000
$m = 4$	3.9726*	0.0080	10.3502*	0.0000	30.2382*	0.0000
$m = 5$	3.8763	0.0136	10.9620*	0.0000	62.6563*	0.0000

Due to the inconclusive results for CDS 1, the test was applied again at

$\epsilon/\sigma = 1$, see table 2.5.

Table 2.5: BDS statistic quantiles for 250 observations for $\epsilon/\sigma = 1$

	$m = 2$	$m = 3$	$m = 4$	$m = 5$
1%	-2.52	-2.47	-2.5	-2.52
2.5%	-2.15	-2.17	-2.17	-2.18
97.5%	2.27	2.37	2.39	2.56
99%	2.79	2.92	2.96	3.06

At $\epsilon/\sigma = 1$, the null hypothesis is rejected in all CDS samples, see table 2.6. These results are supported by the bootstrapped p values.¹

Table 2.6: Empirical Size of the BDS Test, for $\epsilon/\sigma = 1$, for iTraxx CDS Index

	CDS 1	B/strp Pr	CDS 2	B/strp Pr	CDS 3
$m = 2$	2.5880*	0.0192	5.5848*	0.0000	6.7298*
$m = 3$	3.2512*	0.0048	7.3115*	0.0000	5.6993*
$m = 4$	3.2693*	0.0056	8.7085*	0.0000	8.3640*
$m = 5$	3.2368*	0.0080	9.8585*	0.0000	11.1835*

2.5 Sensitivity Analysis

Considering the results of section 4, in this section the aim is to test the sensitivity of the results to the ARCH type model chosen. As there are two

¹The author carried out similar tests on single entity CDS samples with larger sample sizes > 1000 and found similar results. The results of these tests are available on request from the author.

parts to the ARCH type model, that is the mean equation and the variance equation, the mean equation specifications and the variance equation specifications are varied in order to test the robustness of the results. Four alternative ARCH type models are applied to the data before carrying out the BDS test on the respective residuals. As above, the BDS test is applied to the log squared standardized residuals of each of the ARCH type models. For the CDS samples as $n/m < 200$, bootstrapping techniques are applied to the probability estimates.

The ARCH type models selected include the AR GARCH model with mean equation:

$$r_t = \mu_t + \theta r_{t-1} + \varepsilon_t \quad (2.12)$$

This allows for autocorrelation in returns. The variance equation is as above (2.10). As evidence of autocorrelation in returns is found in some of the samples, that is the S&P 500, the Bond fund and the three CDS indices, all further models below include the AR(1) component.

Secondly, the GARCH-in-Mean or GARCH-M model is applied with a mean equation:

$$r_t = \mu_t + \theta r_{t-1} + \lambda \sigma_t + \varepsilon_t \quad (2.13)$$

As well as allowing for autoregressive dependence, the GARCH-M model also introduces the standard deviation as an independent variable in the mean equation. The estimated coefficient, λ indicates the risk-return trade off on the asset. The variance equation remains the same as (2.10).

Thirdly, the threshold GARCH or TGARCH model is applied with a mean equation as above (2.12) and with a variance equation as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} \quad (2.14)$$

where $I = 1$ if $\varepsilon_{t-1} < 0$ and 0 otherwise. This model allows good news and bad news to have an asymmetric effect on the conditional variance.

The fourth model is the exponential GARCH or EGARCH model, with a mean equation as above (2.12) and with a variance equation as follows:

$$\log \sigma_t^2 = \alpha_0 + \alpha_1 \left[\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - E \left(\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| \right) \right] + \beta \log \sigma_{t-1}^2 + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (2.15)$$

This model allows for the asymmetric effect to be exponential. Detailed discussions of all these models can be found in Jondeau et al. (2007).²

Table 2.7 gives the BDS test statistics and the probability estimates for the two GARCH models with alternative mean equations, that is AR GARCH and GARCH-M. Table 2.8 gives the BDS test statistics and the probability estimates for the two GARCH models with alternative variance equations, that is TGARCH and EGARCH. As before * indicates rejection of the null hypothesis at a 5% level.³

In general, the results are robust for the equity and bond samples and also for CDS sample 2 and 3, with both samples indicating non linear depen-

²As in section 4, the GARCH (1,2) model was used for the DAX index in all four models in this section, and GARCH (1,2) was also used for the S&P 500 in the TGARCH and EGARCH models, as these were better fits to the data.

³The estimates of the coefficients of each of the models, the log likelihoods, the ARCH LM test results etc. are available on request from the corresponding author.

Table 2.7: BDS Statistics and probability estimates for AR GARCH and GARCH-M models

AR GARCH	S&P	DAX	FTSE 100	NIKKEI 225	BOND FUND	CDS1	CDS2	CDS3
ϵ/σ	0.7	0.7	0.7	0.7	0.7	1	1	1
$m = 2$	1.2476	-0.5704	0.9501	0.2489	1.6361	1.8138	4.9689*	8.0637*
Prob	0.2122	0.5684	0.3420	0.8035	0.1018	0.0720	0.0000	0.0000
$m = 3$	1.5542	-0.2726	0.2748	0.1059	1.4308	1.6585	5.9770*	6.9778*
Prob	0.1201	0.7851	0.7835	0.9156	0.1525	0.1040	0.0000	0.0000
$m = 4$	0.7833	-0.4173	-0.0121	0.2425	0.7469	1.5263	6.6407*	6.9016*
Prob	0.4335	0.6765	0.9903	0.8084	0.4551	0.1408	0.0000	0.0000
$m = 5$	0.4714	-0.4632	0.1807	0.3959	1.0804	1.4079	7.1463*	7.2283*
Prob	0.6374	0.6432	0.8566	0.6922	0.2800	0.1736	0.0000	0.0000
GARCH-M	S&P	DAX	FTSE 100	NIKKEI 225	BOND FUND	CDS1	CDS2	CDS3
ϵ/σ	0.7	0.7	0.7	0.7	0.7	1	1	1
$m = 2$	0.5820	-0.5497	1.1761	0.2822	1.5608	0.8694	4.8089*	9.2551*
Prob	0.5606	0.5826	0.2395	0.7778	0.1186	0.3760	0.0000	0.0000
$m = 3$	1.0135	-0.1258	0.6062	0.1693	1.3082	1.4496	5.3619*	8.1994*
Prob	0.3108	0.8999	0.5444	0.8655	0.1908	0.1768	0.0000	0.0000
$m = 4$	0.4627	-0.1995	0.3017	0.3163	0.6240	1.4503	5.9207*	7.8838*
Prob	0.6436	0.8419	0.7629	0.7518	0.5326	0.1664	0.0000	0.0000
$m = 5$	0.1204	-0.3368	0.5750	0.4451	0.9062	1.1661	6.5555*	8.3414*
Prob	0.9042	0.7362	0.5653	0.6562	0.3648	0.2408	0.0000	0.0000

Table 2.8: BDS Statistics and probability estimates for TGARCH and EGARCH models

TGARCH	S&P	DAX	FTSE 100	NIKKEI 225	BOND FUND	CDS1	CDS2	CDS3
ϵ/σ	0.7	0.7	0.7	0.7	0.7	1	1	1
$m = 2$	0.5820	-0.5497	1.1761	0.2822	1.5608	0.8694	4.8089*	9.2551*
Prob	0.5606	0.5826	0.2395	0.7778	0.1186	0.3760	0.0000	0.0000
$m = 3$	1.0135	-0.1258	0.6062	0.1693	1.3082	1.4496	5.3619*	8.1994*
Prob	0.3108	0.8999	0.5444	0.8655	0.1908	0.1768	0.0000	0.0000
$m = 4$	0.4627	-0.1995	0.3017	0.3163	0.6240	1.4503	5.9207*	7.8838*
Prob	0.6436	0.8419	0.7629	0.7518	0.5326	0.1664	0.0000	0.0000
$m = 5$	0.1204	-0.3368	0.5750	0.4451	0.9062	1.1661	6.5555*	8.3414*
Prob	0.9042	0.7362	0.5653	0.6562	0.3648	0.2408	0.0000	0.0000
EGARCH	S&P	DAX	FTSE 100	NIKKEI 225	BOND FUND	CDS1	CDS2	CDS3
ϵ/σ	0.7	0.7	0.7	0.7	0.7	1	1	1
$m = 2$	0.7269	-0.2571	1.3173	0.1039	1.1104	0.5805	5.6316*	12.2350*
Prob	0.4673	0.7971	0.1877	0.9173	0.2668	0.5192	0.0000	0.0000
$m = 3$	1.1056	0.1303	0.7917	-0.0181	1.0496	0.8195	6.3627*	12.3810*
Prob	0.2689	0.8963	0.4285	0.9856	0.2939	0.3800	0.0000	0.0000
$m = 4$	0.5558	0.0422	0.4545	0.1717	0.3811	0.3267	6.1578*	13.0050*
Prob	0.5783	0.9663	0.6495	0.8637	0.7031	0.6736	0.0000	0.0000
$m = 5$	0.2490	0.0348	0.6745	0.3748	0.6792	-0.2241	6.4323*	14.6020*
Prob	0.8034	0.9722	0.5000	0.7078	0.4970	0.9136	0.0000	0.0000

dencies in the log squared standardized residuals of each of the four models. For CDS 1, the alternative GARCH models appear to be removing the non linear dependence in the data, this is a similar result as for some of the results for CDS 1 in section 2.4, for the GARCH(1,1) model at $\epsilon/\sigma = 0.5$.

Thus the sensitivity analysis using various GARCH specifications does not significantly alter the conclusions, there continues to be evidence that the GARCH model is not sufficient to incorporate the non linear dependencies in all of the CDS samples, although it is sufficient to incorporate the dependencies in the equity and bond market data. The weakness in the BDS test is that it does not infer the structure of the nonlinearity in the data, for example it does not distinguish between nonlinear stochastic and nonlinear deterministic processes. An understanding of the structure of the data would facilitate improved modeling. This is the aim of section 2.6.

2.6 Tests for a Nonlinear Deterministic Process

The objective of this section is to apply additional tests to develop an improved understanding of the structure of the data. Two additional tests for nonlinearity will be applied. Firstly we apply time delay embedding to examine the phase portraits of the three CDS samples and the S&P 500. The motivation for this analysis is that it can often reveal deterministic structure which is not evident in a plot of x_t versus t (time) (Barnett et al. 1997). Secondly the correlation dimension test (Barnett et al. 1997) is applied. The estimation of the correlation dimension of the time series can infer whether

the samples follow a stochastic or a deterministic structure.

Phase Portraits

Time delay embedding is used to plot the solution paths in phase space (x_t against x_{t-1}) for the three CDS samples and also (as a comparison) the S&P 500 sample. As this is a univariate time series analysis, a one dimensional process is assumed and thus we use a time lag of 1⁴.

For the purpose of a comparative analysis, a time series of iid random variables (that exhibit similar GARCH properties to the real data) were generated. According to Kantz and Schreiber (2003), if the sample of data is properly described by the linear process of the model, it should not be possible to find any significant differences between the phase portrait of the real data and the simulated series. The procedure is to generate random numbers and then to transform these random numbers using the dynamic GARCH variance and the estimated coefficients of the real data to create a simulated series of x_t for each sample of real data. As the simulated series have been transformed using the GARCH properties of the real data, the comparison of the two portraits (real data versus simulated series) will indicate if there are further nonlinear dependencies in the real data which are not being represented in the GARCH model⁵.

⁴Alternatively the time lag could be chosen based on the first zero crossing of the autocorrelation function (Moshiri 2004). In this case, as the CDS data and the S&P 500 have been shown to follow an AR(1) process, thus a time lag of 2 would be used. These phase portraits (x_t against x_{t-2}) were also collated and showed similar patterns to the time lag 1 phase portraits.

⁵As all four samples discussed in this section showed evidence of autoregressive dependence in the mean equation, the ARGARCH model (as in section 2.5) is used to transform the simulated series.

Four of the eight samples will be examined, that is the S&P 500 and the three CDS samples. Figures 2.2 and 2.3 below represent the S&P 500 and CDS 1. The real data and the simulated series appear to be stochastic and symmetric around the regression lines. Histograms have been included to examine if the simulated series is in general a good overall description of the real data, this appears to be the case for the S&P 500 and the CDS 1. Figures 2.4 and 2.5 represent the phase portraits for CDS 2 and 3. Certain patterns in the real data can be seen which are notably different to the patterns in the simulated series. The real data appears to have more points lying on the x and y axis. This would imply that there are nonlinear characteristics in the real data which are not being described by the GARCH model. Secondly, the points in the simulated series phase portraits are more spread out across the phase space than the real data. This is because the GARCH model assumes that the underlying dynamic variance is at all times different from zero. On average, the GARCH model simulation overestimates the variance in the real data. In the real data, there are times when the points are close together and other times when they are far apart, therefore the extreme values are less often but sometimes larger in the real data than in the simulated series. This characteristic is further illustrated by comparing the histograms of the real data in figures 2.4 and 2.5 to the simulations. The real data appears to be more leptokurtic than the simulated series. It is also clear that the real data does not follow a Gaussian distribution and that the GARCH model does not fully describe the non Gaussian characteristics of the real data. These results justify and support the results of the BDS test above, there appears to be remaining nonlinearities in the CDS real data and the real data is notably

non Gaussian.

In conclusion, if the GARCH model fully explained the dependencies in the samples, then the phase portraits of the real data would be similar to those of the simulated series. For the CDS data, particularly CDS 2 and 3, this is not the case. Certain other patterns (as discussed in the previous paragraph) can be seen which would lead to the conclusion that the traditional method of linear ARCH type modeling may not fully reveal the non linear nature of the CDS data. In response to the concerns of Serletis et al. (2010) mentioned above, this comparison of phase portraits also allows the review of the implications of imposing the GARCH model onto the sample prior to applying the BDS test. The GARCH simulations appear to be stochastic and symmetric around the regression line which would imply that the model itself may not be adding nonlinearities to the data. Finally, the phase portraits indicate that the process is attracted to the axis. The process appears to be nonlinear and stochastic rather than nonlinear and deterministic. To further test this, the correlation dimension test is applied.

Figure 2.2: Phase Portraits of samples and simulated series for S&P 500

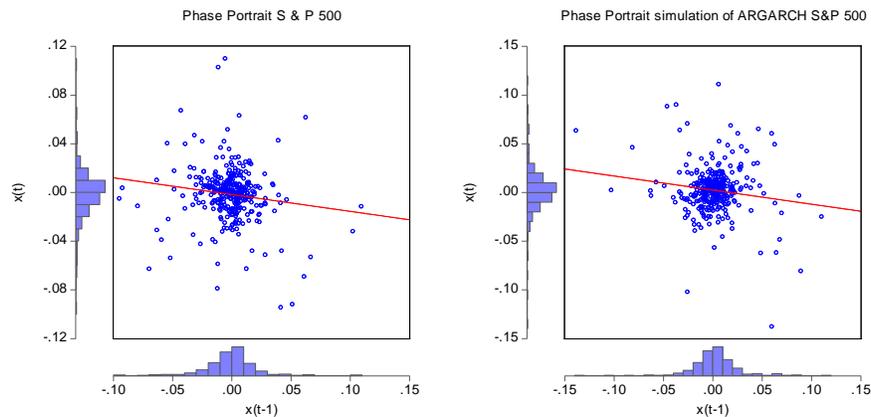


Figure 2.3: Phase Portraits of samples and simulated series for CDS 1

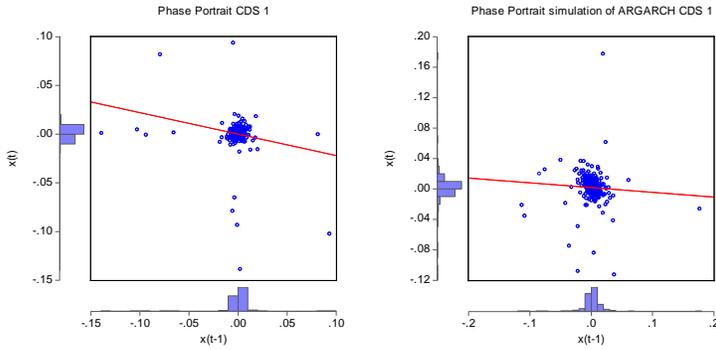


Figure 2.4: Phase Portraits of samples and simulated series for CDS 2

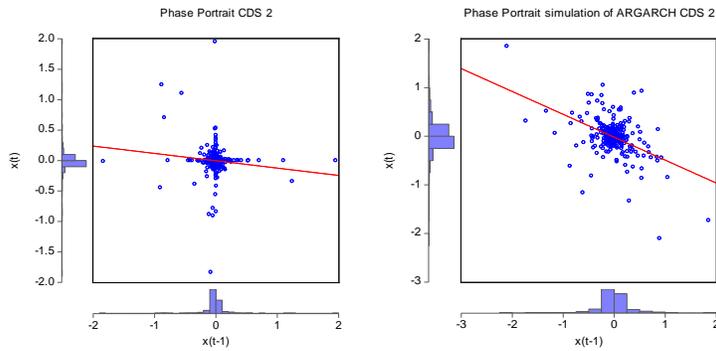
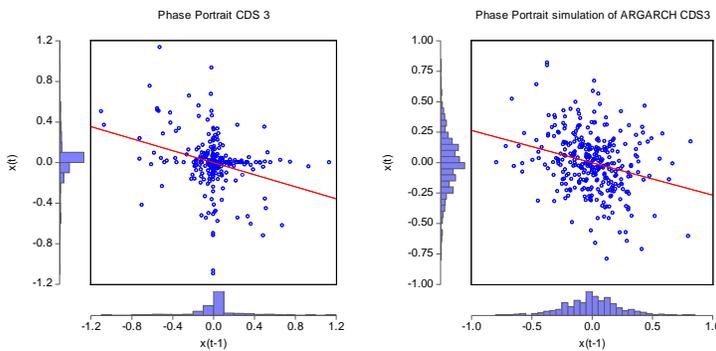


Figure 2.5: Phase Portraits of samples and simulated series for CDS 3



The Correlation Dimension Test

In section 2.4, it appears that nonlinear dependencies remains in the GARCH residuals of the CDS samples particularly for CDS sample 2 and 3. One of the weaknesses of the BDS test is that the rejection of the null does not distinguish between a non linear stochastic and a non linear deterministic process. In this section, the samples are tested further to see if they are from a stochastic or a deterministic process. To this aim, the correlation dimension test is applied. The correlation dimension was first suggested by Grassberger and Procaccia (1983) and uses the same probability estimate, that is the correlation integral (2.1) as the BDS test. Grassberger and Procaccia (1983) show that the analysis of the correlation integral $C_{m,n}(\epsilon)$, can be used to analyze the dynamics of the time series in question. As above in section 2, $C_{m,n}(\epsilon)$ (2.1) measures the probability that the distance between any two m -histories is $\leq \epsilon$. If $C_{m,n}(\epsilon)$ is large the data is said to be well correlated and if the value of the correlation integral is small, the data is said to be relatively uncorrelated (Barnett et al. 1997). As ϵ increases, one would expect $C_{m,n}(\epsilon)$ to increase, as increasing ϵ will allow for more vector lengths to be included in the sum. Grassberger and Procaccia (1983) have shown that for small values of ϵ , as ϵ increases, $C_{m,n}(\epsilon)$ grows exponentially at a rate known as the correlation dimension (D_c). Thus the definition of D_c is as follows:

$$D_c = \lim_{\epsilon \rightarrow 0} \frac{d \log C_{m,n}(\epsilon)}{d \log \epsilon} \quad (2.16)$$

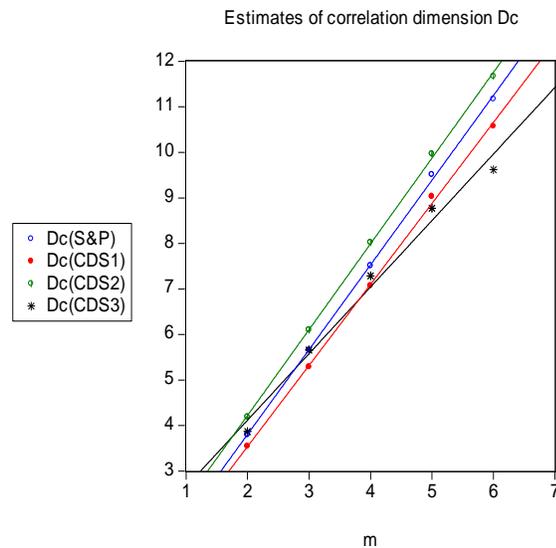
that is, the correlation dimension is equal to the slope of the regression of $\log C_{m,n}(\epsilon)$ on $\log \epsilon$ (Barnett et al. 1997). As the embedded dimension m

increases, the correlation dimension of a stochastic process will continue to increase. Whereas for a deterministic process, there will be a finite saturation point, this should be for some relatively small value of m (Barnett et al. 1997). Thus the implementation of the correlation dimension test is relatively straightforward. Firstly for a fixed embedded dimension m the correlation integral is estimated for a range of values of ϵ . Then D_c is estimated as the slope of the regression of $\log C_{m,n}(\epsilon)$ on $\log \epsilon$. This method is repeated for higher values of the embedded dimension m . One limitation of the correlation dimension test is that it can only distinguish a low dimensional deterministic series. Ruelle (1990) shows that if the correlation dimension reaches a finite saturation point, then the data generating process is believed to be deterministic only if that point is well below $2 \log_{10} n$ (where n represents the number of observations). In all cases (see table 2.9) it is shown that the estimated correlation dimension increases beyond $2 \log_{10} n$ for all the samples. Thus it appears that the samples are not from a low dimensional deterministic process, but are from a stochastic process (or possibly from a high dimensional or non stationary deterministic process). The correlation dimension test is applied to the three CDS samples and for comparison to the S&P 500 sample. Below, table 2.9 gives the estimates of the correlation dimension for varying levels of m and the probability estimates from the t tests (which were estimated from the regressions). We note that in all cases the correlation dimension continues to rise as m increases, well beyond the limit suggested by Ruelle (1990). Also we note that the estimates of the correlation dimension are significant at a 5% level in all cases. Figure 2.6 plots the correlation dimension against m for all the 4 samples.

It is clear that the samples come from a stochastic process (or possibly a high-dimensional deterministic process). It is worthwhile noting that when attempting to model a high dimensional deterministic process, it is in general difficult to distinguish from a stochastic process (Kantz and Schreiber 2003).

Table 2.9: Correlation Dimension D_c Test

	S&P		CDS1		CDS2		CDS3	
n	14822		391		334		329	
$2 \log_{10} n$	8.3418		5.1844		5.0475		5.0344	
	D_c	Prob	D_c	Prob	D_c	Prob	D_c	Prob
$m = 2$	3.7928	0.0246	3.5434	0.0265	4.1855	0.0288	3.8687	0.0297
$m = 3$	5.6521	0.0245	5.2806	0.0262	6.0988	0.0290	5.6561	0.0308
$m = 4$	7.5085	0.0245	7.0671	0.0258	8.0213	0.0281	7.2858	0.0333
$m = 5$	9.5099	0.0344	9.0264	0.0350	9.9678	0.0378	8.7685	0.0485
$m = 6$	11.1700	0.0244	10.5692	0.0262	11.6708	0.0277	9.6198	0.0464

Figure 2.6: Estimates of correlation dimension D_c 

2.7 Concluding Remarks

It is widely accepted that applying GARCH to the logged returns of a financial time series leads to iid residuals and that the application of GARCH removes linear dependence from the residuals of the time series. This claim is clearly proved in the case of the equity and bond data and it may be possible to fit this data with a probability distribution function, such as the Gaussian or the Student t . The results of the BDS test indicate that there is evidence of nonlinear dependence in the CDS data. These results are shown to be consistent for a number of ARCH type models.

We note that a weakness of the rejection of the null in the BDS test is that this result does not distinguish between nonlinear deterministic and nonlinear stochastic processes. The test does not conclusively determine the specifics of the dependence in the data. Thus further tests of nonlinearity are applied to the data. The analysis of the phase portraits and the application of the correlation dimension test indicate that the CDS data appears to be a nonlinear, non Gaussian stochastic (or possibly a nonlinear high dimensional deterministic) process.

In conclusion, as 85% of the underlying assets in CDS contracts are 5 year bonds (Li and Mizrach 2010), it is surprising to note that the GARCH residuals for the bond sample appear to be iid while the CDS sample GARCH residuals indicate evidence of nonlinear dependence. This may indicate that the source of the nonlinear dependence is particular to the CDS contract and does not relate to the assets underlying the contract. The linear GARCH model may not be sufficient to incorporate the dependencies in the CDS data. The results also question the efficacy of using bond market default probability

estimates to price CDS contracts (or vice versa) or the efficacy of using a CDS contract to hedge a bond position. Thus this chapter has a number of implications for further research. CDS contracts should be modeled to reflect their nonlinear, non-Gaussian stochastic or possibly high-dimensional deterministic structure.

Chapter 3

The arbitrage-free parity theory

3.1 Introduction

The objective of this chapter is to update recent empirical research, which tests the arbitrage-free parity theory for CDS and bond spreads. For comparison purposes, we will apply linear tools. Also and in line with the research question, we will apply nonlinear tools to examine the dynamic nature of convergence and synchronization between CDS and bond spreads, particularly during times of heightened turbulence. The focus will be on four euro zone peripheral countries, which have recently experienced increased default risk due to the financial crisis; they are Ireland, Spain, Portugal, and Greece. The time period we analyze is from November 2008 until December 2010. This will allow us to review the effect of the sovereign crisis, which gained pace during the first few months of 2010. May 2nd 2010 saw the first Greek

bailout by the EU and the IMF. May 9th 2010 saw the foundation of the European Financial Stability Facility (EFSF) to support economic stability in the euro zone area. May 14th 2010 saw the ECB intervene in the euro debt markets (through the Securities Markets Programme). This intervention was an attempt to reduce bond yields and CDS spreads. November 21st 2010 saw the bailout of Ireland by the EU and the IMF.

A credit default swap (CDS) contract gives the buyer protection against loss due to a credit event occurring in the underlying entity (Mengle 2007). The premium the buyer pays for this protection is known as the spread. A bond market spread is the premium of the bond yield above the risk free rate. Both spreads should represent the probability to default of the underlying entity (Hull 2008). The arbitrage-free parity theory states that there will be equivalence between credit default swap (CDS) spreads and bond market spreads (also known as credit spreads) in equilibrium. Deviations from equivalence will disappear almost instantaneously as traders take advantage of the opportunity for arbitrage profits. The assertion that the arbitrage-free parity theory holds in equilibrium has led to the use of CDS spreads to estimate an implied probability of default when pricing bond yields (and vice versa) (European Union 2009). This assumption also supports the strategy of hedging bond positions with CDS contracts (and vice versa).

There are a number of economic explanations for the existence of nonequivalence between the CDS and bond spread during a crisis. Fontana and Schichler (2010) highlight that as of March 2010, euro zone sovereign CDS markets represent less than 8% of the credit of the underlying sovereign bond market and therefore the CDS market is much smaller in terms of volume and

liquidity. During a crisis, they suggest that there will be a “flight to liquidity”, which will cause the inequivalence between the sovereign CDS spread and the sovereign bond spread to be maintained. Traders will be unwilling to enter into the required arbitrage trade (to remove the inequivalence) as they will be reluctant to hold the more illiquid CDS positions. Duffie (2010) has suggested that institutional impediments rise during a crisis: for example search costs for trading counterparties can rise as can the time it takes to raise capital. These rising institutional impediments can inhibit trading activities. Mitchel and Pulvino (2011) also suggest that capital moves slowly during a crisis and that this may restrict trading activity and prevent hedge fund managers from taking advantage of arbitrage opportunities.

A number of papers attempt to empirically test this theory. Blanco et al. (2005) confirm the equivalence of CDS and bond spreads for corporate entities as an equilibrium condition, using linear stochastic models. They also note strong evidence of a lack of convergence in the short run and in 10% of cases a lack of convergence in the long run. They suggest the short run deviations are due to the CDS spread leading the bond spread in the price discovery process. They suggest the long run deviations are due to imperfections in the specifications of the CDS contract (making them unsuitable for arbitrage trading) or are due to measurement error.

Prior to the financial crisis the expectation of default of sovereign bonds was extremely low. But as the crisis developed this expectation increased and higher CDS spreads and bond yields were seen. This led to further academic review of the factors influencing sovereign CDS and bond markets as well as the analysis of equivalence between the two assets. Fontana and Scheicher

(2010) and Haugh et al. (2009) apply multivariate linear regressions to CDS and bond spreads. They conclude that both markets appear to be influenced by common factors. Delis and Mylonidis (2011) replicate some of the work of Blanco et al. (2005) but for the sovereign markets. They conclude that except in times of high risk, there is evidence of CDS spreads Granger causing changes in bond spreads. In these high risk times, the causality goes both ways, indicating the flight to safety of investors from the higher risk sovereign bonds to German bunds.

The structure of the chapter is as follows, section 3.2 will outline the theoretical underpinnings of the analysis. Section 3.3 will describe the data to be analyzed. Section 3.4 will present the results of the analysis and section 3.5 will summarize the conclusions.

3.2 Theoretical Underpinnings

In the first section of the theoretical underpinnings (3.2.1) a linear stochastic test is applied; that is the Granger causality test, to see if a linear lagged relationship between sovereign CDS spreads and sovereign bond spreads can be found. We also analyze the distributional characteristics of the CDS and bond spread data, statistically analyzing the descriptive statistics and applying the Jarque-Bera test for normality. Section 3.2.2 starts with a brief introduction to the theoretical framework of nonlinear time series analysis. In section 3.2.2, 3.2.3, and 3.2.4 nonlinear time series tools are applied to test the nonlinear structure of the equivalence between CDS spreads and bond spreads.

Granger causality and descriptive statistics

The majority of the analysis of equivalence between CDS and bond spreads applies linear dynamic stochastic models (Blanco et al. 2005, Delis and Mylonidis 2011, Fontana and Scheicher 2010, Haugh et al. 2009). As an illustration and for comparison purposes, the linear Granger causality test is applied (Granger 1969). This is a bivariate linear regression test, applying lagged values of both variables as regressors. For example:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \epsilon_t \quad (3.1)$$

A Wald's f test is then applied to test the hypothesis that:

$$H_0 = \beta_1 = \beta_2 = 0 \quad (3.2)$$

Accepting the null, implies that the x_t variable does not Granger cause changes in y_t . We choose a default value of 2 lags in all cases. We initially allow the CDS spread to be the dependent variable (y_t) and allow the bond spread to be the independent variable (x_t). Acceptance of the null implies that bond spreads do not Granger cause CDS spreads. We then swap the positions of the assets, allowing the bond spread to be the dependent variable (y_t) and the CDS spreads to be the independent variable (x_t). In this case acceptance of the null implies that CDS spreads do not Granger cause bond spreads. The results are compared to those of previous literature. We note that the data are initially transformed to logged differentials prior to applying the test so that we do not need to test for cointegration (Patterson 2000).

Following this comparison the distributional characteristics of the CDS

and bond spreads in each country are examined to see if the assumption of Gaussian residuals is reasonable. We will also apply statistical t tests and f tests to compare the equality of the estimated means and variances of the two assets in each of the four countries.

The drawback of linear modeling is that all irregular behaviour has to be due to some random input (Kantz and Schreiber 2003). Also parameters are assumed to be constant over time. All dependencies in the data must be included in the linear model so that the residuals can be assumed to be iid i.e. independent and identically distributed, otherwise stochastic modeling is not appropriate (Patterson 2000). In general, a Gaussian distribution is assumed, this imposes restrictions on the forecast outcomes of the model (for example, it is assumed that approx. 99% of residuals must be within $\pm 3\sigma$ of the mean). These requirements are examined in the results section.

The BDS test

In chapter 2, the BDS test was applied to ARGARCH residuals of the iTRaxx European index of corporate CDS contracts. Nonlinear dependence was shown to remain in the residuals. This result questions the use of stochastic modeling in the analysis of CDS contracts. Here we apply this methodology to highlight the remaining nonlinearities in the ARGARCH residuals of the sovereign CDS contracts. A rejection of the null will again have implications for the application of linear stochastic modeling.

The BDS test (Brock et al. 1995) applies the correlation integral ($C_{m,n}(\epsilon)$) to test for dependence in a time series. Firstly the time series (x_t) is converted into a series of vectors (X_t^m). The embedding theorems tell us that

if only a few dominant dimensions remain in the system, we can reconstruct the motion of the system using the phase space of a single variable. The embedding dimension m is generally chosen between a value of 2 to 6 for the BDS test. The value of m determines the number of scalar points in the vector as follows:

$$X_t^m = (x_t, x_{t+1}, \dots, x_{t+m-1}) \quad (3.3)$$

The integral ($C_{m,n}(\epsilon)$) is then estimated, it measures the spatial correlation for the particular embedded dimension, m . The integral can be interpreted as the probability that (for the dimension m) the vector length $\|X_t^m - X_s^m\|$ is less than or equal to ϵ , a predetermined distance. It is estimated as follows:

$$C_{m,n}(\epsilon) = \frac{2}{n(n-1)} \sum_t \sum_{s,s < t} I_{[0,\epsilon]}(\|X_t^m - X_s^m\|) \quad (3.4)$$

Such that $I(\cdot)$ denotes the Heaviside function, which takes either the value of 0 or 1 according to:

$$I_{[0,\epsilon]}(s) = \begin{cases} 1 & \text{if } s \in [0, \epsilon] \\ 0 & \text{if } s \notin [0, \epsilon] \end{cases} \quad (3.5)$$

and $\|\cdot\|$ denotes the supremum norm, given by:

$$\|u\| = \sup_{i=1,\dots,m} |u_i| \quad (3.6)$$

If we calculate a high value for the correlation integral, this suggests that

the data is not independent. Brock et al. (1995) show that at the limit, the integral should follow a scaling principle. That is for:

$$\lim_{n \rightarrow \infty} C_{m,n}(\epsilon) = C_m(\epsilon) \quad (3.7)$$

If the data is independent then:

$$C_m(\epsilon) = [C_1(\epsilon)]^m \quad (3.8)$$

From this generalized rule, they develop the standardized BDS test statistic (T) as follows:

$$T = \frac{\sqrt{n}(C_m(\epsilon) - C_1(\epsilon)^m)}{s_m(\epsilon)} \quad (3.9)$$

Such that n is the number of observations, and $s_m(\epsilon)$ is a consistent estimator of the asymptotic standard deviation $\sigma_m(\epsilon)$ of $\sqrt{n}(C_m(\epsilon) - C_1(\epsilon)^m)$ (Brock et al. 1995). Brock et al. (1995) show that the test statistic is normally distributed with $N(0, 1)$. Dependence is found if the BDS test statistics is significantly different from the z statistics of the normal distribution.

To apply the test, the linear dependencies in the data must first be removed. We take each sample of data separately and apply the ARGARCH model, as follows:

$$r_t = \mu + \gamma r_{t-1} + \varepsilon_t \quad (3.10)$$

$$\varepsilon_t = \sigma_t z_t \quad (3.11)$$

such that:

$$z_t \sim N(0, 1) \quad (3.12)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (3.13)$$

We note that r_t is the logged differential of the series (the spread) at time t (the data is transformed to remove the nonstationary trend) $\mu, \gamma, \alpha_0, \alpha_1, \beta$ are all parameter values. ε_t is the conditional stochastic term. σ_t is the conditional standard deviation, which is dependent on a constant and ε_{t-1}^2 , the lagged residual squared and σ_{t-1}^2 the lagged variance. We assume z_t follows a standard normal distribution ¹.

Cross Recurrence Plots

As we wish to examine the equivalence between CDS spreads and bond spreads without evoking a model, the nonlinear time series tool; cross recurrence plots and cross recurrence plot measures are applied. Recurrence plots were proposed by Eckmann et al. (1987) to evaluate deterministic characteristics of systems. When considering a dynamical deterministic system, we note that the trajectory of the system will be attracted to certain points or cycles. During these times the system will revisit the same area of the phase plane. Thus by visually analyzing the recurrence in the embedded time series, we can evaluate if the system is revisiting certain areas of the plane. Continuing recurrence over time is an indication that the system is following a recurring trajectory. This would indicate the existence of attractors and suggest a deterministic system.

A recurrence plot (RP) is a visual representation of recurrence in the pro-

¹One small additional point is that in order to ensure the test has the nuisance parameter free property; that is that the test can be applied to the residuals of a model, the residuals of the ARGARCH process are first standardized and transformed into the logged squared residuals (Caporale et al. 2005, de Lima 1997).

cess. As RPs make no assumptions about the model underlying the system itself, they can be used to analyze nonstationary systems without parametric assumptions. For these reasons, recurrence plots are particularly useful in the analysis of financial time series. In this chapter, the focus is on methodological developments within the recurrence quantification analysis (RQA) literature that is the cross recurrence plot (CRP) and CRP statistical measures (Zbilut et al. 1998). This method compares two time series to see if there is equivalence between them.

As in section 3.2.2, in order for this methodology to be applied, the series of scalars need to be converted into a series of vectors. We refer to the embedding theorems (Sauer et al. 1991, Takens 1981). These theorems state that if only a few dominant dimensions remain in the system, the motion of the system can be reconstructed using the phase space of a single variable (see chapter 4.2.1 for a more in depth discussion). The time series, x_n is embedded, choosing an embedding dimension m and an additional parameter, the time delay τ (Kantz and Schreiber 2003) as follows:

$$X_i = (x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}), i = 1 \dots n \quad (3.14)$$

We will simultaneously embed another time series, as above to create the vector series Y_n . The closeness of each point of the first trajectory, X_n with each point of the second trajectory, Y_n can be tested as follows:

$$CR_{i,j} = I(\epsilon_i - \|X_i - Y_j\|), X_i, Y_j \in \mathfrak{R}^m, i, j = 1 \dots n \quad (3.15)$$

where n is the number of considered states X_i and Y_j , ϵ_i is a threshold

distance, $\|\cdot\|$ a norm and $I(\cdot)$ the Heaviside function (Marwan 2003).

In this test, the CDS spread data is converted to a series of vectors (X_i) and the bond spread data is converted to a series of vectors (Y_j) . This is a similar transformation as in the BDS test, except in this test we have an additional parameter, the time delay τ . We include the time delay here for correctness, in fact with discrete data it is preferable to allow the time delay to equal 1 (Webber and Zbilut 2005). In this case there is no difference between the transformation in equation 3.3 and equation 3.14.

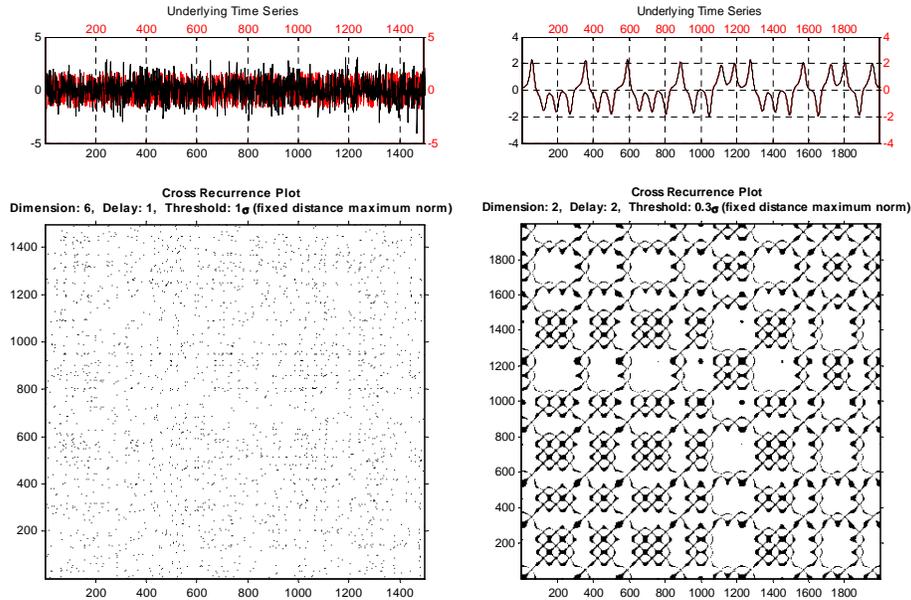
The embedding dimension m is chosen using the method of false nearest neighbours, and for continuous data, the mutual information method is used to estimate τ (Kantz and Schreiber 2003). As mentioned above, for discrete empirical data, it is best to take a value of 1 for τ , so that no data points are skipped (Webber and Zbilut 2005). With regard to the threshold distance ϵ , it has been noted by Marwan (2010, p.3) that “ a general and systematic study on the recurrence threshold selection remains an open task”. Many methods have been suggested, the key is to choose a method which maximizes the signal detection (Schinkel et al. 2008). One method is to choose the threshold equal to 5% of the maximal phase space diameter (Basto and Caiado 2011). This method did not fit well with our empirical data, as it lead to too high a level of recurrence. A threshold level is chosen, which keeps the number of recurrences low relative to the number of points in the system i.e. approximately 1%. This improves the likelihood that we are analyzing recurrence due to deterministic behaviour (Zbilut et al. 2002). The same parameters are applied across all the series to ensure consistency of results.

The application of equation 3.15 gives $CR_{i,j}$ an $n \times n$ binary matrix. The

cross recurrence plot is a visual representation of the binary matrix, with a black dot indicating a 1, and a white space indicating a 0. By examining the plot for structure, the trajectories of the equivalence through time can be interpreted. Depending on the nature of the underlying process, certain patterns will appear.

Cross recurrence plots can also be represented as distance plots by allowing the distance between the two series to be represented by a changing colour or lightening of the plot from black to white (therefore we do not choose a threshold value). We illustrate below in 3.1 (a) (on the left) a CRP for two randomly generated time series; the plot is represented by a series of random black dots. No lines are seen within the plot; this indicates that the data is randomly moving from one position to another. With dynamical systems we expect to see structure in the plot. As Eckmann et al. (1987) highlighted, small scale structures such as diagonal lines parallel to the main diagonal occur within dynamical systems. A diagonal line parallel to the main diagonal line indicates that a certain trajectory is repeated at different times. For example the plot of a sine wave will be a series of diagonal lines, as the time series repeats the same trajectory over and over again. In figure 3.1 (b) on the right, we have a plot of two non linear chaotic systems (identical except one is lagged by a time delay of 1) known as the Lorenz system of equations (Kantz and Schreiber 2003). This plot shows a checkerboard like pattern, Eckmann et al. (1987) noted that this pattern indicates that the trajectories are moving around attractors. In general, white bands represent two values or vectors which are far apart. This is an indication of extreme events or nonstationarity in the system (Marwan et al. 2007).

Figure 3.1: CRPs. (a) Two random data series (L.H.S.) (b) Lorenz system with parameters, $\sigma=10$, $r=28$ and $b=8/3$ (Grassberger and Procaccia 1983) compared with itself lagged one period forward.(RHS)



Convergence and common dynamics

CRP measures are the quantification of the patterns in the cross recurrence plot through statistical values (Webber and Zbilut 1994). Analysis of these statistical values can indicate many characteristics of a process. For example, they can indicate dynamical convergence and common dynamics between two time series (Marwan 2010). Convergence between two states occurs when their respective phase space trajectories become very close. For CDS and Bond spreads, this would occur when the values of each are close as follows:

$$d(x - y) \leq \epsilon; \forall t \tag{3.16}$$

where d is the distance function and ϵ is some critical distance (Crowley 2008). In general convergence occurs when there is a recurrence. Convergence between CDS and Bond spreads would imply that the markets are similarly valuing the two spreads. Consistent convergence would be supporting evidence for the arbitrage-free parity theory. Convergence is measured by analyzing the distance between the CDS spread and the bond spread over time. If the two assets converge, we would expect the distance between them to be consistently small.

Common dynamics (or synchronization) occurs when the time evolution of two or more states are similar. This can be examined through the analysis of the phase space trajectories of the two states (Marwan and Kurths 2002). If the system is noisy and or high dimensional, it may not be possible to visually recognize the synchronization, thus the use of cross recurrence plots and CRP measures allows us to examine the potential common dynamics between two states.

As Huygens discovered in 1665, two pendulum clocks mounted on a rack of finite rigidity will synchronize over time due to the slight rocking of the rack itself. Synchronicity between states is a characteristic of a complex system (Kantz and Schreiber 2003). A complex system is a network of heterogeneous components that interact nonlinearly and give rise to emergent behaviour; complexity is a general term encompassing chaos, fractals and other nonlinear theories (Grassberger and Procaccia 1983). Evidence of non-linear synchronicity between CDS spreads and Bond spreads would support

the view that the financial markets are indeed a complex system. The key to synchronization for Huygens was the influence of the rack on the two pendulums. Synchronization between CDS and bond spreads would imply that similar common (possibly macroeconomic or trading) forces are causing changes in the two asset spreads. A lack of synchronization would imply that different forces are influencing the two asset spreads or that the extent of the influence is different. Whereas variability in the level of synchronization implies a varying coupling strength between the two states (Marwan and Kurths 2002).

Given two variables x_t and y_t , this can be thought of as:

$$\psi(x_t) - \psi(y_t) \leq \pm\omega \quad (3.17)$$

where ψ is a phase function and ω represents a critical phase shift (Crowley 2008). If we find the difference between the two functions is less than the critical phase shift, we can conclude that there are common dynamics occurring. Synchronization is measured by analyzing the nonlinear deterministic structure of the CDS spread and the bond spread over time. If the two assets are synchronized, we would expect the deterministic structure to be almost equal.

Marwan and Kurths (2002) test cross recurrence plot measures by firstly examining synchronization between noisy periodic data sets. They compare the classical cross correlation function with the CRP measures and show that both methods indicate the linear synchronization between the two data sets. Secondly, they study the nonlinear synchronization between a linear autoregressive stochastic process and the x-component of the Lorenz system.

By increasing the value of the coupling strength k , the CRP measures, recurrence rate (RR) and average diagonal line length (L) also increase. This result suggests that they are suitable measures to find the nonlinear relation between two data sets. This nonlinear relationship does not show up using cross correlation analysis. CRP measures allow us to assess the extent of linear and nonlinear convergence and synchronization between two data series.

CRP measures are calculated by analyzing the distribution of the diagonal line lengths $P_t(l)$ for each diagonal line parallel to the main diagonal. By doing so, we are focusing on the closeness of the two variables through time. It is possible to impose a time lag t in the analysis. We allow $t = 0$ thus we are comparing the two variables in concurrent time.

RR (the recurrence rate) is defined as:

$$RR(t) = \frac{1}{n-t} \sum_{l=1}^{n-t} lP_t(l) \quad (3.18)$$

(Marwan and Kurths 2002)

RR for cross recurrence analysis measures the probability of occurrence of similar states in both systems. A high density of recurrence points results in a high value for RR (Marwan and Kurths 2002). As the data here is discrete, we will be choosing a value for the threshold, ϵ , to keep RR $\lesssim 2\%$. Thus we will be analyzing the change in RR over time and comparing RR across countries. A rising RR indicates that the degree of convergence is increasing.

L is the average diagonal line length and allows us to assess the duration of the common dynamics between the two states. A high coincidence of both systems increases the lengths of these diagonals (Marwan and Kurths 2002).

L is defined as:

$$L(t) = \frac{\sum_{l=l_{min}}^{n-t} lP_t(l)}{\sum_{l=l_{min}}^{n-t} P_t(l)} \quad (3.19)$$

(Marwan and Kurths 2002)

As with RR, we will be analyzing the change in L over time and across countries to assess the nature of the synchronization i.e. changing common dynamics, between the two data series. ²

3.3 Data

The objective of the chapter is to update and review recent literature on the sovereign credit markets, testing for the arbitrage-free parity theory. We have chosen to analyze four peripheral euro zone countries that recently experienced heightened credit risk. These are Ireland, Spain, Portugal and Greece. Sovereign CDS and bonds of five year maturities are chosen as 85% of the credit default swap market relates to these maturity lengths (Li and Mizraeh 2010). In order to compare results, the number of observations in each data sample is the same and covers the same time period. The observations are all of a five day week frequency. As the expectation of default for sovereign debt has only recently increased from very low levels, the sovereign credit default swap markets are relatively new. We are restricted to the number of observations available in the shortest sovereign CDS market. The shortest data set is the Greek CDS, which is available daily from 19th November 2008 through

²Other measure of CRP can be found at www.recurrence-plot.tk/.

Data Stream. Thus each data sample was taken from 19th November 2008 until 15th December 2010, a sample size of 541 observations.

As discussed above, the arbitrage-free parity theory states that in equilibrium, there will be equivalence between the CDS spread and the relevant bond spread. The bond spread being the difference between the sovereign bond yield and the risk free rate. The risk free rate can be chosen to be the relevant benchmark bond yield (in this case the German bund yield) or the LIBOR/swap curve (Hull 2008). We note that the benchmark bond yield is used by Delis and Mylonidis (2011) and by Haugh et al. (2009). As we wish to compare our results with those of Delis and Mylonidis (2011) we use the benchmark bond yield as the risk free rate.

The data will be initially transformed into the log differentials of the spreads (as is common practice in financial econometric analysis). The justification for this transformation is to remove the nonstationarity in the data caused by the inherent trend (Patterson 2000). In order to use linear stochastic modeling, the residuals of the model must be stationary and iid i.e. independent and identically distributed. This assumption will be examined, using linear and nonlinear tools. When applying the CRP methods, the log differentials are first normalized (this allows for the comparison of our results). The method of using log differentials is also applied by Crowley (2008) when analyzing synchronization of growth cycles across EU economies, but not by Basto and Caiado (2011) when comparing developed and emerging stock markets. Analyzing the untransformed spreads allows the analysis of the nonstationary data series. Sprott (2004) argues that using log differentials may be risky as the nonstationarity may be the interesting feature of

the data. Be that as it may, we choose to analyze the log differentials as this allows us to compare results with those in the financial econometrics literature (discussed in section 3.1).

3.4 Results

The methodological objective of the chapter is to use linear and nonlinear tools to analyze the arbitrage-free parity theory for four euro zone peripheral CDS and bond markets. Initially we began with an update of the Granger causality test (table 3.1) which has been performed in many recent papers (Blanco et al. 2005, Delis and Mylonidis 2011, Fontana and Scheicher 2010). Blanco et al. (2005) analyze corporate CDS and bond markets and conclude that the CDS Granger causes changes in the bond spread in the majority of cases. Fontana and Scheicher (2010) review 10 sovereign euro denominated markets and conclude that in half of the markets, bonds Granger cause CDS spreads and in the other half, CDS contracts Granger cause bond spreads. Interestingly, they find that for Ireland, Spain, Portugal and Greece, CDS spreads Granger cause bond spreads. Delis and Mylonidis (2011) analyze sovereign CDS and bond spreads for Spain, Portugal, Greece, and Italy and conclude that (when using a 250 day rolling Granger causality test) CDS spreads Granger cause bond spreads from 2007 onwards.

The Granger causality test is applied (table 3.1).³ The results agree with the previous research on sovereign euro zone markets (Delis and Mylonidis 2011, Fontana and Scheicher 2010) that is we show that CDSs Granger cause

³As we are using log differentials we assume no cointegration between the CDS and bond spreads

bond spreads.

Linear Gaussian models assume that the residuals are stationary, iid and follow a Gaussian distribution. We will now review these assumptions by analyzing the univariate distributions of the log differentials of each of the CDS and bond spreads (table 3.1). The descriptive statistics clearly show the non-Gaussian nature of each of the distributions i.e. non-zero skew and excess kurtosis. The Jarque-Bera test (a test of normality) confirms this conclusion. By applying a t test for equality of means, comparing the mean of the CDS to the mean of the bond spread in each country, we see that the mean values appear to be statistically equivalent. Whereas by applying an f test to test the equality of the variance, we see that in all cases, the variance of the CDS spread is statistically different to that of the bond spread (table 3.1). The bond spread has a higher variance than the CDS spread. This may be due to the fact that the bond market is exchange traded whereas the CDS is OTC (Over The Counter) (Mengle 2007). Or because the volume of trading in the sovereign bond market is significantly higher than that of the relatively new sovereign CDS market (Fontana and Scheicher 2010). We conclude that using sovereign CDS spreads to price bond markets will underestimate the variance and lead to mispricing of the assets. We also conclude that assuming a Gaussian distribution will underestimate the risk, as both markets exhibit excess kurtosis.

In general, linear stochastic models require the residuals to be iid. We will run a simple linear regression around a constant to see if serial correlation in returns and variance are significant for CDS and bond spreads. If these correlations are shown to exist, they must be removed through correct

Table 3.1: Granger causality test, descriptive statistics and tests of equality

	Ireland		Spain		Portugal		Greece	
Granger causality (2 lags)	f stat	prob.						
CDS does not GC Bond spreads	18.9337	1.00E-08	16.4654	1.00E-07	28.1461	2.00E-12	3.8091	0.0228
Bond spreads does not GC CDS	0.49478	0.61	0.84987	0.428	1.61915	0.199	25.3349	3.00E-11
Descriptive statistics	Bond Spread	CDS						
Mean	0.0042	0.0025	0.0030	0.0021	0.0025	0.0028	0.0035	0.0021
Median	0.0004	0.0000	0.0041	0.0000	-1.63E-04	0.0000	0.0004	0.0000
Maximum	0.7212	0.2537	0.6336	0.2826	0.7917	0.2742	0.2259	0.2709
Minimum	-0.5989	-0.3311	-0.8044	-0.4175	-0.8769	-0.5900	-0.2083	-0.2305
Std. Dev.	0.0753	0.0521	0.0999	0.0591	0.0999	0.0623	0.0476	0.0367
Skewness	0.2980	0.1740	-0.5752	-0.4850	-0.6256	-1.4642	0.3138	0.8886
Kurtosis	31.8911	8.7830	17.7194	9.3528	24.2143	19.2119	6.4403	17.7749
Jarque-Bera	18823.46	756.5823	4913.742	930.948	10180.07	6117.882	275.67	4991.961
Probability	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	541	541	541	541	541	541	541	541
t test for equality of mean	value	prob.	value	prob.	value	prob.	value	prob.
	0.4383	0.6612	0.1648	0.8691	-0.05910	0.9529	-0.5555	0.5787
f test for equality of variance	2.0895	0.0000	2.8547	0.0000	2.5722	0.0000	1.6710	0.0000

linear modeling prior to applying a stochastic distribution to the data. The LB Q statistic is applied to the residuals and the squared residuals to test for serial correlation in returns and in variance (Patterson 2000) (table 3.2). We find evidence of serial correlation in returns in all markets except for the Portuguese bond spread and evidence of serial correlation in variance for all markets except for the Greek CDS. The linear correlation in returns is commonly removed in linear stochastic models but it is interesting to note that there is evidence of serial correlation in the variance also. A GARCH model (Bollerslev 1986) can be applied to take account of this autocorrelation. If the autocorrelation is not removed the residuals will not be iid and the estimates will be bias. This will increase the probability of a Type 1 error i.e. false rejection of the Null Hypothesis. An ARGARCH model is applied to remove the linear dependencies in the data and then test the residuals for nonlinear dependencies through the application of the BDS test (see table 3.2 and 3.3). As in chapter 2, where we analyzed the iTRaxx CDS index, we show evidence that there is remaining nonlinear dependence in the ARGARCH residuals of the sovereign CDS spreads. The bond spreads appear to be independent. This result questions the use of equivalent modeling or bivariate (linear) modeling for CDS and bond spreads as well as the use of linear stochastic modeling for CDS spreads. It would appear from this analysis that linear Gaussian/stochastic modeling may not be appropriate in the analysis of the arbitrage-free parity theory for CDS and bond spreads. Thus a new methodology is introduced that is the CRP and CRP measures.

Table 3.2: Tests for linear and nonlinear dependence, Ireland and Spain

	Ireland		Spain	
	Bond Spread	CDS	Bond Spread	CDS
LB Q stat - sc returns	8.8142	15.4070	8.4180	18.5700
prob	0.0120	0.0000	0.0150	0.0000
LBQ ² stat - sc variance	23.3990	13.7460	91.1080	15.4320
prob	0.0000	0.0010	0.0000	0.0000
BDS test - Bond Spread				
Dimension	z-Statistic	prob.	z-Statistic	prob.
2	0.6752	0.4995	-0.4464	0.6553
3	-0.0735	0.9414	-0.4171	0.6766
4	-0.3113	0.7556	-0.4058	0.6849
5	-0.2005	0.8411	-0.2065	0.8364
6	0.6365	0.5244	-0.0114	0.9909
BDS test - CDS				
Dimension	z-Statistic	prob.	z-Statistic	prob.
2	6.7746	0.0000	6.9632	0.0000
3	6.4199	0.0000	5.4492	0.0000
4	6.2409	0.0000	5.6519	0.0000
5	5.7515	0.0000	5.6022	0.0000
6	5.6200	0.0000	5.0080	0.0000

Table 3.3: Tests for linear and nonlinear dependence, Portugal and Greece

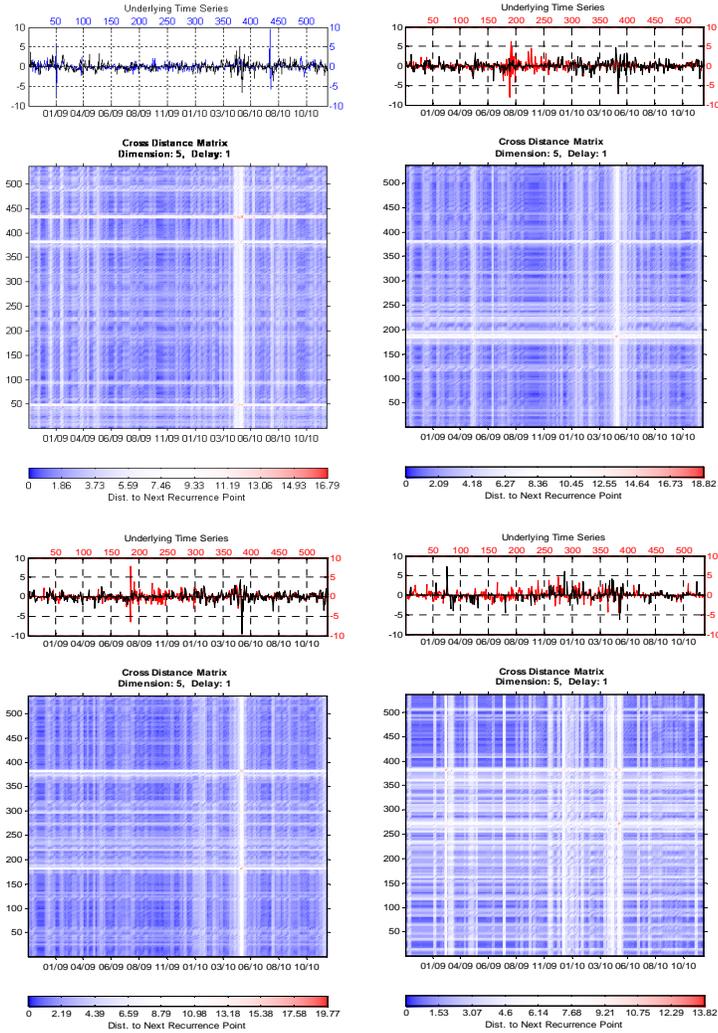
	Portugal		Greece	
	Bond Spread	CDS	Bond Spread	CDS
LB Q stat - sc returns	0.3845	29.6990	7.5565	12.1600
prob	0.8250	0.0000	2.30E-02	0.0020
LBQ ² stat - sc variance	42.3440	10.3490	23.235	4.0788
prob	0.0000	0.0060	0.0000	0.1300
BDS test - Bond Spread				
Dimension	z-Statistic	prob..	z-Statistic	prob.
2	2.3103	0.0209	-0.3810	0.7032
3	1.5787	0.1144	0.4094	0.6823
4	1.6016	0.1092	0.8678	0.3855
5	1.6926	0.0905	1.1881	0.2348
6	1.6846	0.0921	1.5124	0.1304
BDS test - CDS				
Dimension	z-Statistic	prob.	z-Statistic	prob.
2	8.7183	0.0000	38.6174	0.0000
3	8.8381	0.0000	43.3486	0.0000
4	8.5779	0.0000	49.6306	0.0000
5	8.7155	0.0000	58.3604	0.0000
6	8.4822	0.0000	74.9794	0.0000

In order to ensure consistency in our results, we applied the same embedding dimensions across all four CRP distance plots and the CRPs. The value of the embedding dimension m was chosen to be equal to 5, based on the false nearest neighbours method. The time delay τ was kept equal to 1 as the data is discrete. Initially the CRP distance plots are analyzed.

These plots do not specify the threshold value, therefore the plots indicate the closeness of the points through a colour/graying scale (figure 3.2). The CDS market is represented on the x axis, and the bond spread on the y axis. As the eye moves vertically up the y axis, we are comparing one vector of CDS spread values to each vector of bond spread values. If the maximum distance between the two vectors is small this will result in a dark spot. As the colour lightens, this indicates the vectors are further and further away from each other.

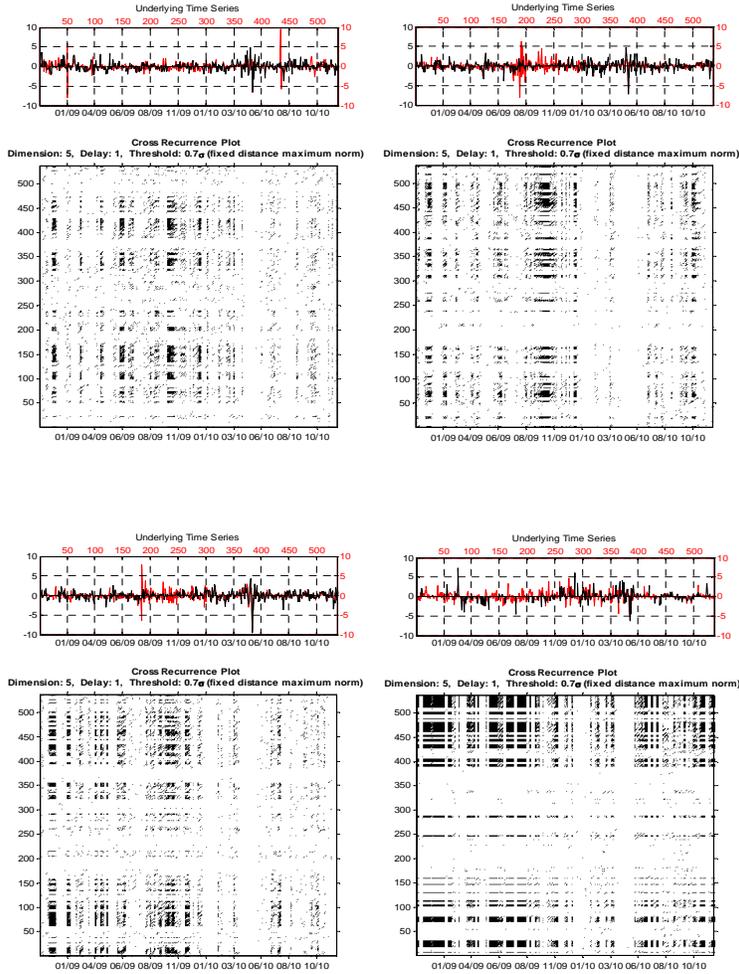
A white vertical band can be seen around May 2010, indicating a non-equivalent and possibly a nonstationary period. The CDS values are far away from any of the bond spread values during the entire observation period. We can also see clear horizontal white lines between the 350 and 400 observation point on the y axis (this is just before and after May 2010). This indicates that the bond values at this time are far away from the CDS values during the observation period. These examples of nonstationarity would imply that the equivalence is non mean reverting (Patterson 2000). In times of uncertainty and market intervention, the market values are no longer equivalent, suggesting caution when using pricing models etc. We note that the Irish bailout in November appears to have no significant effect on the Irish market, although there is a white vertical band indicated in the Greek market. When comparing countries, we note that the number of white lines increase as we move from Ireland to Spain, to Portugal and is highest for Greece. This indicates the increasing nonstationarity as we move from one market to another, with a lack of equivalence being particularly noticeable in the Greek market.

Figure 3.2: CRP distance plots (a) Ireland, top left (b) Spain, top right (c) Portugal, bottom left (d) Greece, bottom right



We will now analyze the CRP plots, a threshold of 0.7σ is chosen in all cases as this ensures consistency and comparability across markets (figure 3.3).

Figure 3.3: CRP (a) Ireland, top left (b) Spain, top right (c) Portugal, bottom left (d) Greece, bottom right.



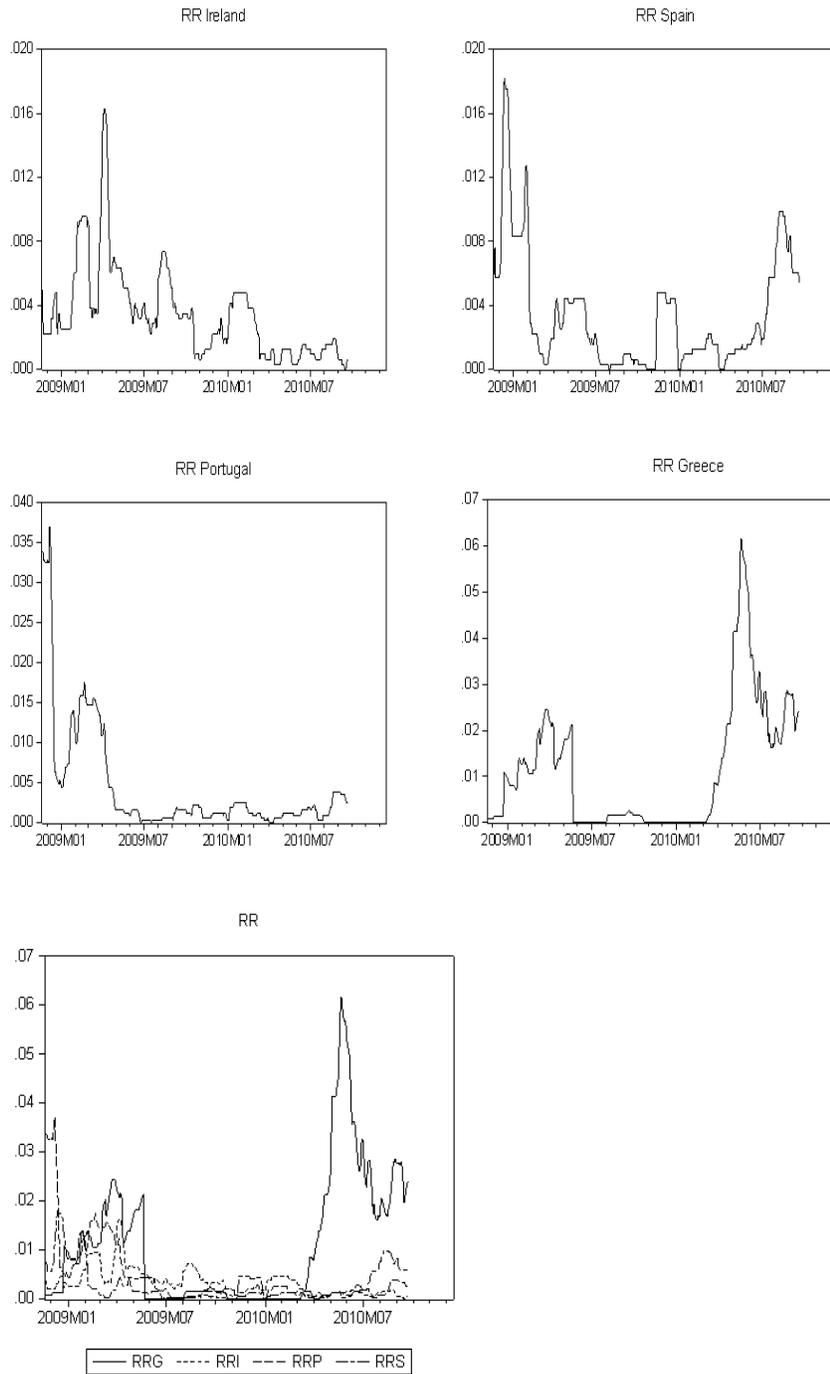
If we allowed the threshold to vary, this may cause the differences to be due to the threshold value rather than actual market differences. By comparing these plots to figure 3.1, we can see that there appears to be some kind of checkerboard structure in the data. It does not appear to be random.

This would imply that there is some deterministic relationship inherent in the two data sets and may possibly indicate attractors. We can again see white space around May 2010, but we also notice long horizontal and vertical lines. A vertical line indicates that the value in the bond spread remains trapped close to the value for the CDS spread for a period of time. A horizontal line indicates the same trapping for the CDS market. This characteristic would indicate equivalence continuing with a lag. The equivalence appears at times to be short lived. We also note that the equivalence appears to be strongest in the Greek market. The Greek market seems to fluctuate more violently than the other markets from periods of equivalence to periods of nonstationarity.

In order to analyze the convergence and synchronization in the markets the CRP measures need to be analyzed. As discussed above, we will focus on two measures, RR (recurrence rate) and L (average diagonal line length) as these have been shown to be good measures of convergence and synchronization (Marwan and Kurths 2002). RR allows us to assess the convergence between the two markets. We will analyze the change in RR over time by estimating a rolling value of the measure. We continue to use an embedding dimension m of 5 and time delay τ of 1. The threshold value is reduced to ensure that the number of recurrences is kept $\lesssim 2\%$. To do this we choose a value of 0.35σ . We keep the threshold fixed across all four countries to ensure comparability. A window size of 60 is chosen, as this relates to 60 days, or a quarter of a year. This is a common length of time to analyze when reviewing financial markets (many statistics and announcements are made quarterly). We note that Basto and Caiado (2011) chose a window size of 260 obser-

vations when analyzing equity markets. We found this window size to be too large, considering our sample size and the events we wished to analyze. Reducing the window size means the CRP measures reflect smaller scale dynamics (Marwan 2010). A window step size of 1 is chosen, this implies that we will move one day forward before estimating the measure again. Using the windowed or epoch CRP allows us to assess the change in the measure through time (Marwan 2010). As RR measures the probability of occurrence of similar states in both systems, a rising RR implies a rising probability of equivalence, thus analysis of RR through time gives us a relative expression of the validity of the arbitrage-free parity theory. We note that the absolute value of RR is not the main focus as this can be manipulated by the choice of threshold value. We are interested in the relative value across countries and through time. We present the RR values graphically for each country in figure 3.4. The first four graphs indicate the changing probability of convergence in the CDS and bond markets across time for each country. We note similarities in the markets, with higher probability of convergence at the beginning of the period, with a downward trend in the middle period and a small recovery in the latter part of 2010 in the Spanish and Portuguese markets. At this time a significant rise in the Greek market is seen (from March 2010 onwards) with no such recovery in the Irish market. We also note the extreme variability in the Greek market. The relative scale is highlighted by placing all four RR measures in one graph (figure.3.4 (e)).

Figure 3.4: Windowed RR measuring dynamic equivalence of the CDS and bond spread. (a) Ireland (b) Spain (c) Portugal (d) Greece (e) All markets



To test this further, we assess the average probability of convergence across markets using a statistical test of equality (table 3.4).

Table 3.4: RR: Probability of convergence, test of equality, estimate of skew.

RR	mean	(s.e.)	ADF test	prob.
Ireland	0.003414	0.00013	-3.057761	0.0305
Spain	0.003355	0.000163	-3.162857	0.0229
Portugal	0.004321	0.000317	-5.368402	0.0000
Greece	0.011139	0.000623	-1.860571	0.351
t test for equality of mean	Value	Prob		
	105.6099	0.0000		

The average RR for Ireland and Spain are similar, RR for Portugal is higher and the RR for Greece is far higher. A t test illustrates this, as the hypothesis of equal means is rejected. It is clear that Greece swings from periods of high equivalence to periods of low equivalence, but overall the equivalence levels are higher for Greece than the other countries analyzed. To further illustrate the point, the augmented Dicky Fuller test (for a unit root) was applied to the RR measures. Acceptance of the null hypothesis suggests that a unit root exists in the dynamic probability of convergence. We interpret this to imply that the probabilities are non-mean reverting. At a significance level of 1%, the null is rejected only for Portugal, all other markets exhibit a unit root. We note the particularly strong evidence of non mean reversion in the Greek markets, this may suggest strong variability in arbitrage-trading activity in this market.

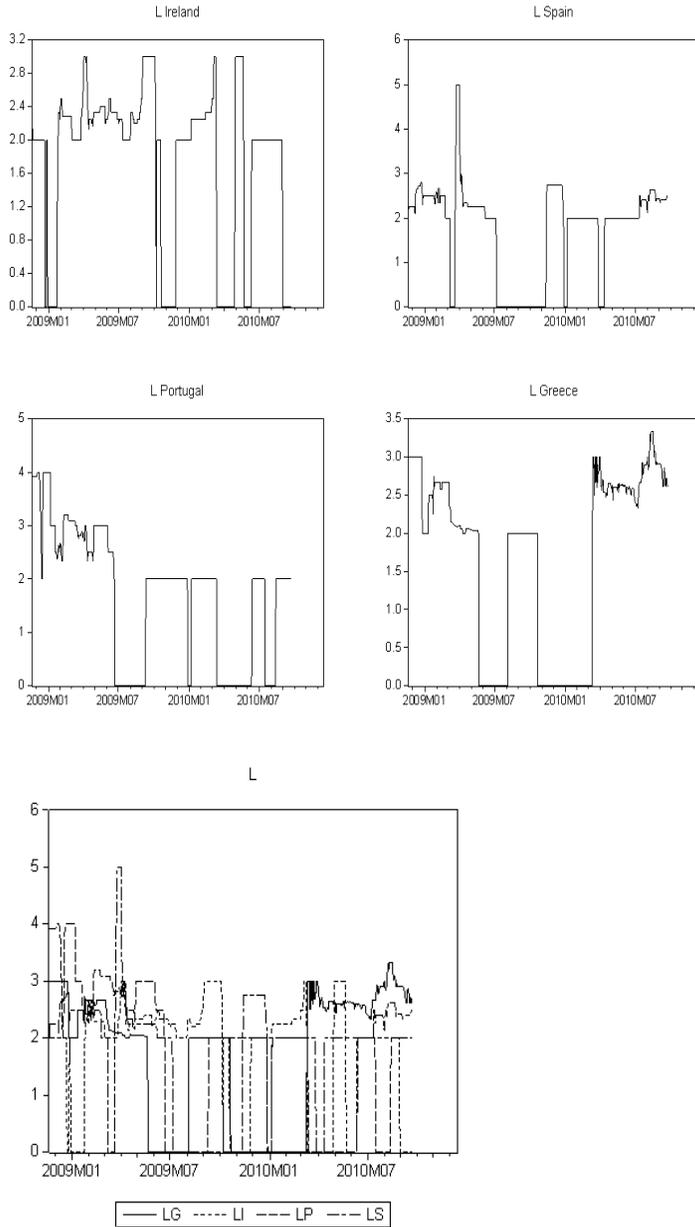
The second CRP measure we will analyze is L (the average diagonal line length). This will indicate the synchronization, or common dynamics across the CDS and bond spreads. Long diagonal lines indicate long periods of

common dynamics. Thus a rising L implies rising synchronization between the two spreads and vice versa with a falling L . We are interested in the relative value across countries and through time. We present the L values graphically for each country in figure 3.5. The first four graphs indicate the characteristics of the changing common dynamics between the CDS and bond spread in each of the countries. We note that each country appears to have a different pattern, with periods of high common dynamics and also periods where the dynamics or synchronization collapse to zero. Greece again is notable from March 2010 i.e. prior to the bailout as a significant rise in the common dynamics is seen. It is curious to note this recovery is prior to the bailout. Overall, if we place the four markets onto one graph (figure 3.5 (e)), it appears as if the level of common dynamics is similar across the markets. We estimate the average value for L in each country and test the equality of the means in table 3.5. The t test clearly shows equality of mean estimates for common dynamics across the countries, suggesting that on average the level of synchronization is equivalent.

Table 3.5: L : Synchronization/common dynamics, test of equality

L	Mean	(s.e.)
Ireland	1.677955	0.054751
Spain	1.728222	0.046088
Portugal	1.715994	0.057406
Greece	1.750707	0.049023
t test for equality of mean	Value	Prob
	0.342761	0.7944

Figure 3.5: Windowed L measuring common dynamics for the CDS and bond spread. (a) Ireland (b) Spain (c) Portugal (d) Greece (e) All countries



3.5 Conclusions

We have applied linear and nonlinear tools to test the arbitrage-free parity theory. The results of the linear Granger causality test concur with the results of previous literature (Blanco et al. 2005, Delis and Mylonidis 2011, Fontana and Scheicher 2010). In general sovereign CDS spreads are shown to Granger cause changes in sovereign bond spreads for the four countries under examination, that is Ireland, Spain, Portugal, and Greece. This linear tool assumes that the assets follow a Gaussian distribution. The analysis of the descriptive statistics of the CDS and bond spreads, in each of the four sovereign markets indicate that both assets are non-Gaussian. Using a Gaussian model will lead to unreliable results. We also illustrate evidence of a statistically higher variance in the bond spread than in the CDS. This implies that using equivalent linear Gaussian models for both assets will lead to poor fits and unreliable results.

We apply an ARGARCH model to each asset to remove the linear dependencies (in returns and in the variance) and note that nonlinear dependencies remain in the CDS data series. We conclude that linear stochastic models are not appropriate in the analysis of the CDS market. We also conclude that the application of linear Gaussian bivariate models for CDS and bond spread data will lead to unreliable results. We suggest nonlinear nonparametric tool should be applied instead.

Therefore we apply the nonlinear CRP and the CRP measures. The advantage of these nonlinear tools is that they can be applied to nonstationary, high-dimensional data series. This methodology is unique in that it allows the examination of the deterministic structure of the series without evoking

a model. The CRP distance plot indicates that there are periods of extreme events or nonstationarity in the equivalence between the two assets; particularly around the time of May 2010. As outlined in the introduction, this was a time of significant government intervention in the market. The existence of nonstationarities questions the mean reversion assumption of the arbitrage-free parity theory. The CRPs illustrate evidence of deterministic structures in the data and evidence of market values being trapped at certain levels. This indicates that there may be an intermittent nonlinear relationship inherent in the two series. We note in both the CRP distance plots and the CRPs; that Greece appears to present stronger evidence of a deterministic structure as well as more nonstationarity. This suggests that country by country the markets behave differently over time.

The CRP measures allow us to examine convergence and synchronization across the two data series, allowing a dynamical review of the arbitrage free parity theory. The probability of convergence (measured as RR) fluctuates widely across time and is not statistically equivalent across countries. In general we conclude that convergence trends down initially (prior to the intervention) in all markets and trends upwards (after the intervention) in all but the Irish market. This trend in convergence indicates varying levels of arbitrage over time. We also note that Greece behaves differently, with a significant rise in the convergence prior to the May 2010 intervention. By applying the ADF test, we conclude that the probability to converge is non-mean reverting (except in the Portuguese market). This result questions the assumption of a stable equilibrium, which is central to the arbitrage free parity theory.

In the analysis of synchronization (measured as L) we note relatively stable dynamics across countries and through time (although there are periods when the common dynamics fall to zero). This indicates that there are common fundamental factors influencing the dynamics of the two markets. As evidence of synchronicity is found between the two markets, we suggest further analysis of the markets as a complex system.

There are a number of policy implications of this chapter. Firstly, policy makers should be wary of results obtained from linear Gaussian models (for either of these two assets, be they univariate or bivariate). Secondly, they should be wary of results from linear stochastic models for CDS spreads. Thirdly, as the markets behave in an extreme and nonstationary manner around the time of government intervention; policy makers should be wary at this time of predictions made from stationary models. Fourthly, policy makers should be wary of the arbitrage free parity theory, we show significant evidence to refute this theory. We note in particular the non-mean reversion of the equivalence between the asset classes. Policy makers cannot assume that market participants will take advantage of any arbitrage opportunity. In fact we show evidence of trends in the probability to converge varying across countries. This indicates that specific markets go in and out of focus for market participants; that at times arbitrage increases and at times it falls. We show in particular that it rose for Greece prior to the ECB intervention. This suggests that rising expectation of government intervention in a market leads to rising levels of arbitrage. By applying cross recurrence measures policy makers will be able to calibrate and to supervise the probability of convergence between asset classes and the level of synchronization across

countries. This knowledge and awareness of their influence on the markets could be particularly useful when drawing up new market interventions or bailout plans.

Chapter 4

Dynamical transitions

4.1 Introduction

The objective of this chapter is to examine the transition from bull to bear market in the Dow Jones Industrial Index. In line with the research question, the nonlinear dynamic tools, recurrence plots (RP) and recurrence quantification analysis (RQA) are applied to examine the period of transition in the market. It is common practice in financial econometrics to use linear stochastic models to analyze variance and risk in the markets (Jondeau et al. 2007). The weaknesses of these models are that they assume fixed parameter values, stationary probability distributions and that all irregularity must be due to the random element in the model (Kantz and Schreiber 2003, Patterson 2000). Even a nonlinear dynamic model requires stationarity. There is significant evidence to show that during times of heightened risk, financial data becomes nonstationary even when first differences are used (Danielsson 2008, Jondeau et al. 2007). The question here is how should nonstationary

financial data be examined so that reliable quantitative risk estimation techniques can be estimated? Out of recent developments in nonlinear dynamics and chaos theory has come the methodology of recurrence plot and recurrence quantification analysis (Zbilut and Webber 1992, Webber and Zbilut 1994). This methodology allows us to apply dynamical nonparametric tools, which do not require stationarity, to find quantifiable indicators of transitions in the market. We will examine these quantifiable indicators to see if they can be used as indicators of a transition in the equity markets from bull to bear market. If so, they could be used as indicators of heightened risk of price collapse.

In this chapter we will be analyzing samples of localized peaks in the Dow Jones Industrial Index and comparing them to localized control samples. The control samples are chosen to highlight the findings for the peak samples. As with all the social sciences, comparison to control samples is not exact. But even with this limitation, some identifiable indicators of the transition to collapse are found. The results show evidence of periodic to chaotic and chaotic to chaotic phase transitions in the data during the peak samples of 1929 and 2007 and also evidence of a collapse in the RQA measures at the time of transition in all of the samples. The collapse in the RQA measures indicates that the data transitions to a random process. Zbilut (2004) suggested the existence of non Lipschitz stochastic repellors as an explanation for this characteristic. Using Zbilut's (2004) terminology, the stock market appears to be "piecewise deterministic", that is it presents intermittent deterministic behaviour. We suggest that the collapse in determinism is supporting evidence for the noisy trader theory. This theory states that ra-

tional but uninformed traders chase noise. We suggest this as an explanation for the rising variance and the collapse in the RQA measures. A principal component series, named the random market indicator (RMI) is developed. RMI indicates when the market loses its predictability. At these times, linear or nonlinear stochastic models should not be used for quantitative risk estimation as their results will be misleading. The evidence presented in this chapter may facilitate improved regulation and supervision of financial markets and financial institutions.

Section 4.2 discusses the conceptual framework underlying the nonlinear tools, which will be applied in this chapter. As an illustration, we apply these tools to a number of known models: that is a periodic, chaotic, and random process, as well as a simulation of the ARGARCH (autoregressive generalized autoregressive conditional heteroscedasticity) model. This model is commonly used as part of financial risk estimation techniques. Section 4.3 outlines the data to be used in the empirical analysis. Section 4.4 displays our results, examining four localized peaks of the Dow Jones Industrial Index using recurrence plots and RQA measures and comparing them to the four control samples. Section 4.5 concludes the chapter, summarizing the findings.

4.2 Nonlinear dynamic tools

Nonlinear dynamic tools such as phase space reconstruction, recurrence plots and recurrence quantification analysis can be utilized to reconstruct and assess determinism in a system.

Phase space reconstruction

When considering a purely deterministic system, a phase space is the vector space of the system; such that specifying a point in this space specifies the state of the system (Kantz and Schreiber 2003). As dynamical equations are defined in phase space, the optimal way of studying dynamical systems is through phase space reconstruction. The phase space is assumed to be a finite dimensional vector space \mathfrak{R}^m . A state is specified by a vector x , such that:

$$x_{n+1} = F(x_n), n \in Z \quad (4.1)$$

for discrete time or:

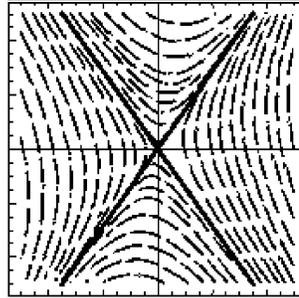
$$\frac{d}{dt}x(t) = f(x(t)), t \in R \quad (4.2)$$

for continuous time, known as the 'flow'.

It is common in physics and mathematics to analyze the flow of a nonlinear system in the vicinity of its equilibria points. As the flow is usually smooth at these points, the nonlinear system is often linearized. This gives a qualitative picture of how the system will behave in the entire state space. Using differential equations, the eigenvalues (λ) and eigenvectors can be computed and used to interpret the Jacobian matrix. For example, for a one dimensional linear system, if $\lambda < 0$, nearby initial conditions will converge on the equilibrium and they are said to be stable (Sprott 2004). If $\lambda > 0$, the nearby initial conditions will diverge away from the equilibrium point and the system is unstable. If $\lambda = 0$, the nearby initial conditions will rotate

around the equilibrium point. In a two dimensional continuous linear system there will be two eigenvalues and it is possible for one to be positive and the other negative. Some trajectories will be attracted to the equilibrium point and other trajectories will be repelled, this is known as a 'saddle point', see figure 4.1.

Figure 4.1: saddle point



This type of stretching and folding can produce chaos. In higher dimensional systems this is the most common type of equilibrium (Sprott 2004). A chaotic system is generally agreed to be a bounded system such that the distance between two trajectories diverges over time i.e. a positive Lyapunov exponent and also one where the trajectory is sensitive to initial conditions (Sprott 2004). By solving the eigenvectors, we can discover any point in the plane, as the eigenvector gives us the ratio of the dimensions (for example in a two dimensional system the ratio of x and y). Computation of eigenvalues and eigenvectors can be applied for three and higher dimensional nonlinear systems. In three dimensions, there are more types of equilibrium points: there are also limit cycles, tori, and strange attractors. As the number of dimensions increases so does the number of eigenvalues, thus it becomes more and more likely to have stable and unstable solutions and thus to have sad-

dle points. In order to analyze a deterministic system, the trajectory of the system in phase space should be analyzed. But as we have a time series and not a phase space object, we use the embedding theorems and the method of delays to reconstruct the object in phase space.

Time Delay Embedding

A central tool in nonlinear time series analysis is time delay reconstruction. This is where a vector space (which is equivalent to the original state space of a system) is constructed from a scalar time series. In order to plot the phase space reconstruction of the variable, the data need to be transformed from a series of scalars to a series of vectors. Two parameters τ the time delay and m the embedding dimension are used.

The time delay τ

Efficiency with a limited amount of data is enhanced by particular choices of the time delay τ . Assuming the time series is continuous, the nonlinear mutual information function (Fraser and Swinney 1986) can be used to assess the value of τ . If the process is discontinuous or discrete, Webber and Zbilut (2005) advise that the delay is best set equal to 1 as no points in the time series will be skipped. Basto & Caiado (2011) and Karagianni & Kyrtsov (2011) also advise the use of τ equal to 1 for financial time series as the data is discrete.

The embedding dimension m

With empirical data it is generally advised that the method of false nearest neighbours is used to find the embedding dimension m (Kantz and Schreiber 2003). Kantz & Schreiber (2003) advise parsimony when choosing m as an unnecessarily large m will degrade the performance of many algorithms. If the system is stochastic, too high an embedding dimension will lead to the display of an artificial pattern of recurrence (Marwan 2010). The embedding dimension must be large enough so that the attractor has been completely unfolded, Sauer et al. (1991) show that this is guaranteed for $m > 2D^F$ (see equation 4.4 below). If the embedding dimension chosen is too low there will be a lot of false neighbours as the attractor intersects itself. Webber and Zbilut (2005) note that the method of false nearest neighbours works well on stable and low noise systems such as the Lorenz attractor but it has been shown that noise inflates the dimension (Parker and Chua 1989) and non-stationarities modulate the critical m . It is important to be conscious of these drawbacks when using the false nearest neighbours method for empirical noisy data.

Once the embedding parameters have been chosen, the empirical data can be embedded. If s_n is the sample of scalars then this transforms into:

$$x_i = (s_i, s_{i+\tau}, \dots, s_{i+(m-1)\tau}) \quad (4.3)$$

There are a number of embedding theorems, one of the most important being the Fractal Delay Embedding Prevalence Theorem (Sauer et al. 1991). This states that when considering fractal A with box counting dimension $D^F < D$

(the dimensional smooth manifold) almost every Cartesian C^1 map from A to the \mathfrak{R}^m with $m > 2D^F$ forms an embedding. Note the box counting dimension D^F is calculated as follows:

$$D^F = \lim_{n \rightarrow \infty} \frac{\ln N}{\ln \epsilon} \quad (4.4)$$

N = no. of elements forming a finite cover of the relevant metric space.

ϵ = is a bound on the diameter of the sets involved.

The theorem states that if successive observations are deterministically interrelated, the mapping F (from the state vector at a certain time to its position one sampling interval later) is unique. Thus time delay embedding is a time independent map from A to \mathfrak{R}^m . Knowing $x(s_{ni})$ at successive times of ni is the same as knowing a set of different coordinates at a single moment i.e. if the map F couples the different degrees of freedom. Thus if only a few dominant dimensions remain in the system, we can reconstruct the motion using the phase space of a single variable. In conclusion if a D^F dimensional dissipative system is being considered, time delay embedding of one scalar variable from the system will reconstruct the system state space. From this we can evaluate the trajectories and the attractors of the system; this will indicate the underlying structure of the dynamical system and facilitate modeling.

Recurrence Plots (RP)

As shown in chapter 2 financial data exhibits the characteristics of a high-dimensional deterministic system. Thus a 3 dimensional phase space reconstruction of the embedded time series may not reflect the motion of the system. For higher dimensional systems, such as financial data, an alternative approach must be used. One approach was recently suggested by Eckmann et al. (1987). When considering a dynamical deterministic system, we note that the trajectory of the system will be attracted to certain points or cycles. During these times the system will revisit the same area of the phase plane. Thus by analyzing the recurrence in the embedded time series we can evaluate if the system is revisiting certain areas of the plane. Continuing recurrence over time is an indication that the system is following a recurring trajectory. This indicates the existence of attractors. A recurrence plot is a visual representation of the recurrence in the system.

The recurrence plot can be represented as follows:

$$R_{i,j} = I(\epsilon_i - \|x_i - x_j\|), x_i \in \mathfrak{R}^m, i, j = 1 \dots n \quad (4.5)$$

where n is the number of considered states x_i , ϵ_i is a threshold distance, $\|\cdot\|$ a norm and $I(\cdot)$ the Heaviside function (Marwan 2003). In the recurrence plot, two close vectors are represented by a black dot, whereas all other vectors are represented by a white space.

The threshold, ϵ

The correct choice of threshold (also known as the radius) can be highly influential on our results. Too small a threshold will exclude trajectories which are in fact close and will under-present the attractors. Too large a threshold will include noise and suggest spurious results. In this chapter we are analyzing one data set, the Dow Jones Index, at different points in time, looking for similarities in the dynamical behaviour of the system. Thus it is important to use the same threshold for all the empirical time series. Many methods have been suggested for the correct selection of the threshold value. One such method is to choose the threshold equal to 5% of the maximal phase space diameter (Basto and Caiado 2011). We found this approach lead to too large a threshold. Instead we followed the advice of Webber and Zbilut (2005) and applied the following guidelines;

(i) Threshold must fall within the linear scaling region of the double logarithmic plot of recurrences versus threshold.

(ii) Threshold must be such that the number of recurrences is kept low ($\lesssim 2\%$)

Guideline (i) above has strong theoretical underpinnings as we chose a threshold such that the box counting dimension can be estimated (see equation 4.4). Thus this threshold should illustrate the deterministic structure of the system. The second guideline ensures that we can be confident that we are focusing on the deterministic structure rather than the noise. Thus the choice of threshold was made by applying both guidelines to all data sets and averaging the resultant estimated threshold.

Interpreting recurrence plots

RPs can be interpreted by following the pattern in the plot as the eye moves from the bottom left corner up to the top right corner following the central diagonal line known as the line of identity (LOI). By familiarizing ourselves with the recurrence plots of known deterministic systems (e.g., the Lorenz system) we can search for similar patterns in empirical data (Kantz and Schreiber 2003). Below the phase space reconstruction of the Lorenz system of equations and the corresponding recurrence plot are illustrated.

The Lorenz system of equations are:

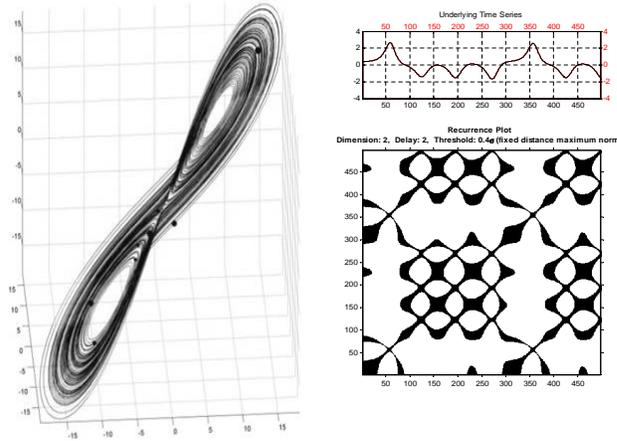
$$\frac{dx}{dt} = \sigma(y - x) \quad (4.6)$$

$$\frac{dy}{dt} = -xz + rx - y \quad (4.7)$$

$$\frac{dz}{dt} = xy - bz \quad (4.8)$$

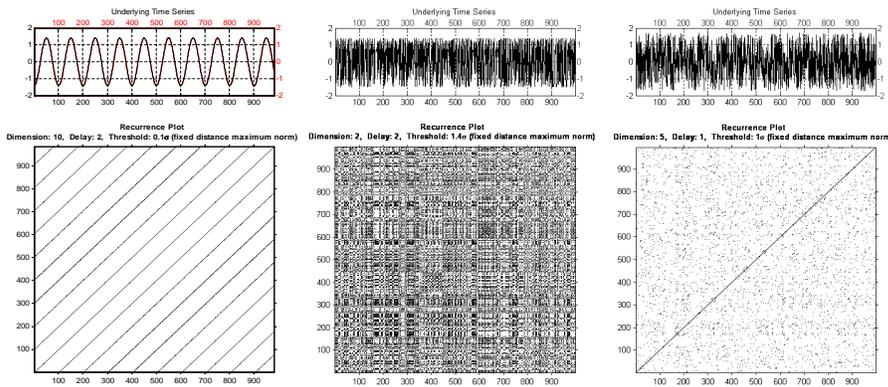
As noted by Eckmann et al. (1987) the recurrence plot “checkerboard type” structure indicates that the trajectories of the system are spinning around attractors.

Figure 4.2: (a) Three dimensional phase space reconstruction of a Lorenz curve with parameters $\sigma = 10, r = 28, b = \frac{8}{3}$ (b) its corresponding recurrence plot.



The analysis of recurrence plots can indicate if we are indeed analyzing a periodic, chaotic or random process. In figure 4.3, we present three such recurrence plots.

Figure 4.3: Recurrence plots of (a) periodic (b) chaotic and (c) random processes



The periodic process is iterated from the following equation:

$$y = \sin \left[\frac{\pi}{50} (t) \right] \quad (4.9)$$

The chaotic process is iterated from the logistic map with $r = 3.999$:

$$x_{n+1} = rx_n(1 - x_n) \quad (4.10)$$

The random process is generated from (computer generated) random numbers. The periodic process is represented by long diagonal lines indicating the recurrence of the trajectories in the same areas of the phase plane. The chaotic process can be represented by a number of different structures; such as the checkerboard structure above for the Lorenz curve, or by a much more subtle structure; such as for the logistic map with $r=3.999$. At this parameter value the logistic map is extremely chaotic, yet small square-like structures can be seen in the recurrence plot. This indicates that the trajectories are spinning around, moving close to or being repelled from attractors. A completely random process will show no structure and will be represented by a series of random black dots. As RPs make no assumptions about the process, they can be used to analyze nonstationary systems and the phase transition (Marwan 2003) of a process into differing states. For this reason, recurrence plots are particularly useful in the analysis of bull and bear markets in financial time series. With this in mind, we simulated an ARGARCH (1,1) process, from random Gaussian numbers and the S&P 500. Details of the simulation are contained in the data section. The ARGARCH (1,1) process represents the following equations:

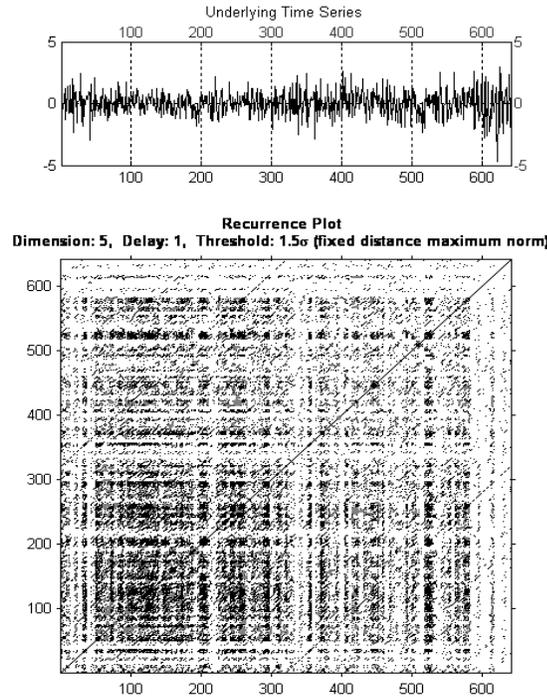
$$r_t = \mu_t + \theta r_{t-1} + \varepsilon_t \quad (4.11)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (4.12)$$

$$\varepsilon_t \sim N(0, 1) \quad (4.13)$$

This model was first suggested by Bollerslev (1986) and is commonly used in financial econometrics as it recognizes the autocorrelation in returns and in the variance of the financial time series (see section 1.1.3, or Jondeau et al 2007 for a review). It has been found to be most useful in quantitative risk estimation, although there is significant evidence to show that financial data residuals are non-Gaussian (chapter 3, table 3.1 or Jondeau et al 2007 for a review). Even if a non-Gaussian distribution such as a generalized extreme value distribution (GEV) is applied, the ARGARCH model still underpredicts risk in the financial markets (chapter 1, table 1.5 or Jondeau et al 2007 for a review). Applying the recurrence plot methodology, we present the recurrence plot in figure 4.4.

Figure 4.4: ARGARCH process



The structure of the recurrence plot of an ARGARCH model shows square structures. Zbilut (2004) noted similar squares in an recurrence plot of the S&P 500 and suggested they indicate the autocorrelations in the system. The ARGARCH model allows for periodic heteroscedastic variance, which is a stylized fact of the financial markets (section 1.1.3). In this model the variance is increasing as we move towards observation 600. This is taken to be a stationary model based on the Bollerslev constraints i.e. $\alpha_1 + \beta \leq 1$ (Bollerslev 1986). For example, in this simulation that is $0.083 + 0.916 = 0.999$. The recurrence plot fades as we move from left to right up the centre

line (LOI). This indicates a trend in the system (Eckmann et al. 1987). To further analyze these dynamical systems we need to introduce recurrence quantification analysis.

Recurrence Quantification Analysis

The quantification of the patterns in recurrence plots came from the field of physiology (Zbilut and Webber 1992, Webber and Zbilut 1994) where statistical values known as recurrence quantification analysis (RQA) were first proposed. These statistical values can indicate many surprising characteristics in a system, and can facilitate distinction between random, periodic, and chaotic processes. More than fifteen interrelated statistical values have been proposed. We will focus on three of these values: determinism (DET), maximum diagonal line length (Lmax), and maximum vertical line length (Vmax). We will also consider the proportion of recurrences (RR). These measures can be used to extract meaningful information from dynamical systems which have no supporting mathematical theory or conceptualization (Webber et al. 2009). Physiological signals are notoriously nonlinear, non-stationary and noisy (Webber et al. 2009). The same can be said of financial data, (Danielsson, 2008). Hence the RQA approach allows us to analyze this data without evoking a model.

The first of the statistical values is the recurrence rate (RR) which is the proportion of recurrence points in the matrix relative to the total number of points. This can be expressed mathematically as:

$$RR_{(\epsilon)} = \frac{1}{n^2} \sum_{i,j=1}^n R_{i,j(\epsilon)} \quad (4.14)$$

This measures the density of the recurrence points in the recurrence plot. It corresponds to the definition of the correlation integral (see chapter 2) except that the LOI is not included. At the limit $n \rightarrow \infty$, it gives the probability that a state recurs in the ϵ -neighbourhood of the phase space (Marwan et al. 2007).

This density measure is then used as a benchmark for examining the dynamics of the recurrences. For example, determinism (DET) is the proportion of diagonal lines relative to the recurrence rate. To measure this we develop a histogram $P(l)$ of diagonal lines of length l . As we see above in figure 4.3 (a), periodic processes have long diagonal lines parallel to the main diagonal (LOI) and no isolated recurrence points. Whereas random processes (figure 4.3 (c)) have no or very few diagonal lines and many isolated recurrence points. Thus analysis of the proportion of diagonal lines (of at least length l_{min}) relative to the total number of recurrences can indicate if a deterministic process is present. The measure DET can be expressed mathematically as:

$$DET = \frac{\sum_{l=l_{min}}^n lP(l)}{\sum_{l=1}^n lP(l)} \quad (4.15)$$

Marwan et al. (2007) describe this as a measure of the predictability of the system. The higher DET, the higher the number of recurrences that are in diagonal lines, indicating that through time the trajectory of the system is recurring. A recurring dynamical trajectory suggests the existence of at-

tractors and that the system is deterministic. We note in the chaotic system (figure 4.3(b)) that there are many short diagonal lines. Therefore additional measures, such as the maximum diagonal line length, can also be useful in system diagnosis. The maximum diagonal line length can be expressed as:

$$Lmax = max(\{l_i\}_{i=1}^{n_l}) \quad (4.16)$$

Lmax has been associated with the speed of the divergence of the phase space trajectory. The faster the divergence, the smaller Lmax will be. For example, the chaotic system in figure 4.3(b) has short diagonal lines. Eckmann et al. (1987) suggest that the length of the diagonal lines is related to the dynamical invariant, the Lyapunov exponent. Following from this, Trulla et al. (1996) studied the bifurcation behaviour of the logistic equation (equation 4.10). They noted a strong linear correlation ($\rho = 0.912$) between the inverse of Lmax and the Lyapunov exponent during the chaotic regions of the logistic map i.e. $r > 3.5688$. Marwan et al. (2007) develop this analysis further and show that in fact the inverse of Lmax (known as DIV) is related to the K_2 entropy i.e. the lower limit of the sum of the positive Lyapunov exponents. By analyzing Lmax, we can examine the speed of expansion of nearby trajectories into new areas of the state space. The lower Lmax is, the more chaotic the system and the less predictable.

The final measure we will be using is the maximum vertical line length (Vmax). This value is similar to Lmax except it measures the maximum of all the vertical line lengths (v). This can be expressed as:

$$Vmax = max(\{v_l\}_{l=1}^{n_v}) \quad (4.17)$$

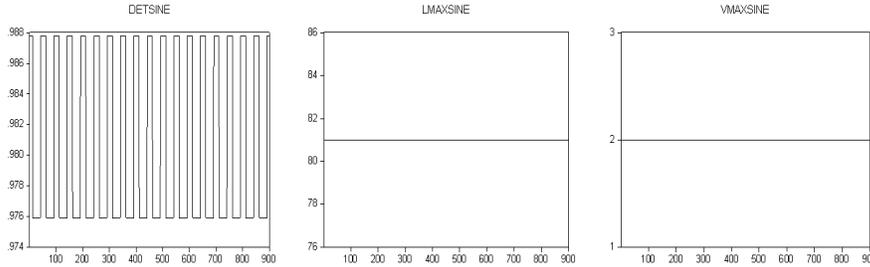
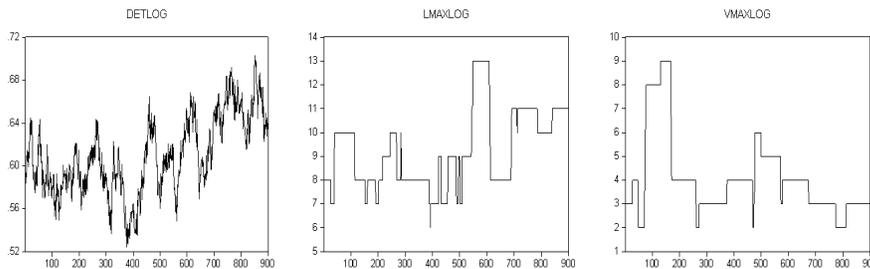
Marwan et al. (2007) describe a vertical line as a time when the state of the system is “trapped” and suggest that this is typical of laminar states i.e. intermittency. Intermittencies occur when the system alternates between chaotic and periodic phases (Sprott 2004). Thus analysis of V_{\max} can facilitate the discovery of phase transitions and laminar states.

Interpreting RQA

Although some information can be gathered by measuring each statistical value for the whole data set, analysis of moving window (epoch) RQA measures can reveal far more about the dynamical nature of the system. We begin by presenting moving window RQA results for the periodic and chaotic processes presented above in figure 4.3. The epoch window size is 100 observations and the step size is 1. The embedding dimensions (m, τ, ϵ) are $(10, 2, 0.1)$ for the periodic process and $(2, 2, 0.5)$ for the chaotic process. Firstly we will present the periodic RQA measures, in figure 4.5. DET is ≈ 1 , L_{\max} is fixed at 81 and V_{\max} is fixed at 2, indicating the periodic nature of the process. Marwan et al. (2002) note that periodic states are associated with vanishing V_{\max} . In general we would expect V_{\max} to be low or zero for a periodic process. The RQA measures for the chaotic process are presented in figure 4.6.¹

¹As the random numbers used above in figure 4.3 (c) are computer generated (from an algorithm) the RQA measures for the ‘random’ process are similar to that of a chaotic process. A truly random process would have very low or zero values for DET, L_{\max} and V_{\max} .

Figure 4.5: RQA for sine wave

Figure 4.6: RQA for logistic map with $r=3.999$ 

The level of determinism (DET) is much lower for the chaotic process and the value varies over time. This indicates the chaotic nature of the determinism. Lmax also varies and is much lower than for the periodic sine wave. This indicates the lower predictability of the chaotic process. A low value for Lmax indicates a high value for DIV i.e. $1/L_{\max}$. This correlates with a high positive Lyapunov exponent, suggesting a chaotic regime. Vmax is higher and variable for the chaotic process.

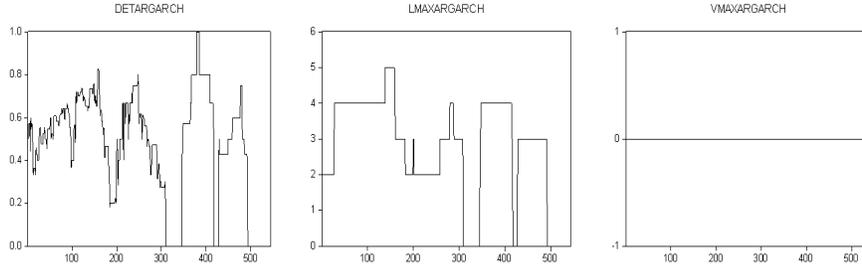
Moving window RQA measures can be used to indicate transitions in the system: such as periodic to chaotic transitions or chaotic to chaotic transitions. Marwan et al. (2002) use moving window RQA measures to indicate bifurcation points i.e. period doubling. Bifurcation points occur as

the system changes; the structurally stable attractors become unstable and new attractors are born (Sprott 2004). The main RQA characteristics that indicate transitions and bifurcation points are summarized in table 4.1 below.

We will use table 4.1 to interpret the empirical findings presented in section 4.4.

The RQA measures of an ARGARCH simulation are presented in figure 4.7 below. The epoch window size is 100 observations and the step size is 1. The embedding dimensions (m, τ, ϵ) are $(5, 1, 0.45)$.

Figure 4.7: RQA for ARGARCH simulation



We note that just after observation 300 DET and Lmax collapse to zero, and that Vmax is zero throughout. We present in figure 4.8 a scaled moving window variance (VAR) for the ARGARCH process, calculated with a window size of 100 and a step size of 1. The moving window variance is scaled by dividing it by the average variance for the entire series such that:

$$VAR = \frac{\sigma_{i \dots i+99}}{\sigma}, i = 1 \dots (n - 100) \quad (4.18)$$

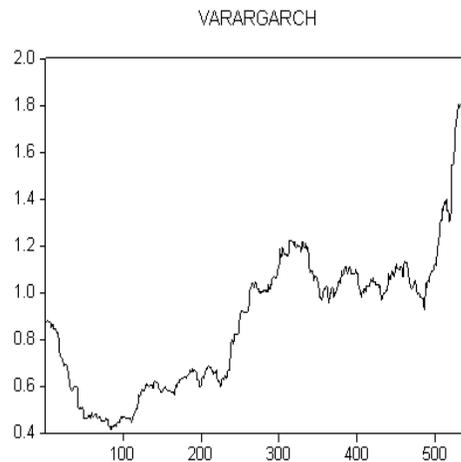
Looking at the time series graph (above the recurrence plot in figure 4.4) and VAR (figure 4.8), we can see that the RQA measures collapse at around

Table 4.1: Using RQA measures to identify phase transitions.

Author	Year	RQA	Characteristic of transition
Trulla et al.	1996	DET	Sharp rise indicates chaotic-periodic transitions
		Lmax	Should collapse prior to bifurcation points
		1/Lmax	Is related to the Lyapunov exponent during chaotic windows
Marwan et al.	2002	Lmax	Maxima at periodic-chaotic/chaotic-periodic transitions
		Ymax	Minima at periodic-chaotic transitions and maxima at chaotic-chaotic transitions, vanishes during periodic windows.
Fabretti and Ausloo	2005	DET	Collapses to lowest level during the ‘critical regime’ of the simulated log periodic model
		Lmax	Lmax collapses prior to the ‘critical regime’ of the simulated log periodic model
Marwan et al.	2007	Lmax,	Peaks at periodic-chaos transitions. Lmax finds all periodic-chaos transitions. Size of Lmax is related to the predictability of the underlying system.
		DET	Peaks at periodic-chaos transitions, although DET only finds some of the transitions.
		Ymax	Peaks at chaos-chaos transitions (e.g. laminar states/intermittencies, band merging and inner crisis). Transitions can still be identified for small noise levels.
Zbilut & Webber	2008	Lmax	Should decrease precipitously prior to the inception of a laminar state and be at a minimum just prior to the laminar areas (unstable singularities, i.e. repellors)
Marwan	2010	DET	DET rising to 1 implies a chaotic-periodic transition.

the same time as the volatility rises and clusters. It appears as if the rise in the dynamic variance is canceling out the autoregressive signal. The system appears to be completely random for some time. As VAR falls again, the signal returns.

Figure 4.8: Moving window scaled variance of the ARGARCH model



Collapses in RQA measures have been noted around the time of stock and currency market collapse (Basto and Caiado 2011, Guhathakurta et al. 2010, Marwan et al. 2007, Piskun and Piskun 2011, and Zbilut 2004) but this has not been explained or modeled. It appears from the representation of the ARGARCH simulation in figure 4.7 and 4.8, that the collapse in the RQA measures is connected to the volatility clustering in the data. We will further test this assertion by analyzing empirical time series of the Dow Jones Industrial Index.

4.3 Data

The data to be used in this analysis is taken from the Dow Jones Index, which represents 30 top ranking industrial equities trading on the US Stock Exchange. This index is chosen as it is one of the longest data samples available for the equity markets. The objective of the research is to analyze the transition from bubble to collapse; therefore we focus on the time just before and just after the market has reached a localized peak. The four peaks and subsequent crashes chosen are 1929, 1973, 2000, and 2007. These samples are chosen due to the economic significance of each crash and also due to the large overall fall in the Dow Jones Index during each crash. The crash of 1973 and 2000 are generally agreed to be attributable to the oil and technology sectors respectively; the crashes of 1929 and 2007/2008 have been attributed to banking and financial market collapse. The details of each crash are outlined in table 4.2 below:

Table 4.2: Dow Jones Industrial Average peaks and troughs

Date of Peak	Date of Trough	Peak value	Trough value	Decline %
03/09/1929	8/7/1932	381.17	41.22	-89%
08/01/1973	6/12/1974	1047.86	577.60	-45%
14/01/2000	9/10/2002	11722.98	7286.21	-38%
10/09/2007	9/3/2009	14164.53	6547.05	-54%

According to Johansen and Sornette (2000) there are distinguishable periodic oscillations in the log of the time-to-crash. They present a Log Periodic Power Law model (LPPL) and suggest this can be used to predict a market crash. The LPPL model has been tested and questioned by Bree et al (2011) who conclude that the model is not reliable, but others disagree. We also

wish to review the LPPL model. To do so we will simply test to see if there is evidence of a periodic regime prior to a market peak. Johansen and Sornette (2000) note the oscillations can be exhibited for up to 100 days prior to the market peak. Thus a sample size of 12 months (250 trading days) prior to and 12 months (250 trading days) after the market peak is taken in order to analyze the lead up to the peak and the turbulence or crash following on from the peak. The focus is on the dynamics of the transition from bull into bear market. If for example, we note a rise in DET up to 1, this will indicate the existence of a periodic phase prior to the peak. The sample size in each case is 502 observations. One observation is lost as the data is converted to log differentials, giving us 501 observations with the market peak in the middle at observation 501. By placing the peak in the middle of the sample, the changing dynamics prior to and after the event can be analyzed (Marwan 2010). We will focus on the bubble prior to the peak with the objective of assessing the possibility of early warning signals of the oncoming crash.

This sample neither take into account the entire period of the market crash which can take as long as 3 ¼ years to occur (as in the case of 1929); nor the entire period of market recovery after the crash. Specifically we are focusing on the lead up to the market peak. The details of the four samples taken are outlined in table 4.3. The observations are all of a five day week frequency.

In experimental sciences it is customary to compare the relevant sample to a control sample. In Marwan et al. (2002) samples of data are taken just prior to the onset of a ventricular tachyarrhythmia (VT) and at a control time i.e. when the patient is at rest thus without a life threatening arrhythmia.

Table 4.3: Dow Jones Industrial Average peak sample details

Sample Name	Start Date	Finish Date	Start value	Finish value
1929	31/08/1928	4/09/1930	240.41	236.04
1973	7/01/1972	07/01/1974	910.37	876.85
2000	29/01/1999	04/01/2001	9358.80	10912.40
2007	09/10/2006	07/10/2008	11857.80	9955.50

In this chapter we are interested in developing indicators of an upcoming transition from bubble to collapse. As economic data is collected not from a controlled laboratory environment but from actual market data; a reliable control is difficult to obtain. We propose taking a sample just prior to the peak sample under question. This 'control' sample will be similar to the peak sample in terms of the number of observations and will occur in the same general epoch. However it differs because a significant localized peak of historical economic importance does not occur at the middle point of the control sample. This methodology has some success although we note that market volatilities can occur in the control period. As there is a 20% collapse in the market from April to May 1970, we remove this period from the 1971 sample to maintain its control characteristics. Thus the 1971 sample has 90 observations less than the others. We use the full sample for all other samples including 1998. This sample may not be fully appropriate as a control as 1998 saw significant turbulence in the markets due to the Russian currency crisis and the collapse of LTCM (Long Term Capital Management). The details of the control samples are outlined in table 4.4.

The prices are transformed into log differentials. Log differentials are used to remove the nonstationary trend in the data and are a customary

Table 4.4: Dow Jones Industrial Average control sample details

Sample Name	Start Date	Finish Date	Start value	Finish value
1927	03/09/1926	30/08/1928	163.75	238.85
1971	27/05/1970	6/01/1972	663.2	908.49
1998	12/02/1997	28/01/1999	6961.63	9281.32
2005	12/10/2004	06/10/2006	10002.32	11866.69

transformation in financial econometrics (Patterson 2000). We note that the transformation of the prices to log differentials may have its drawbacks. As Sprott (2004) argues the nonstationarity may be the interesting feature of the data. The transformation is applied to ensure our results are comparable to existing financial econometric results and to ensure that any evidence of determinism is not merely as a result of a linear trend in the data. Table 4.5 presents a summary of the chosen parameters to be found in recent financial literature. We note the significant heterogeneity in the parameters chosen, this can lead to difficulties in comparison of results².

As discussed in section 4.2.2.2, the false nearest neighbours method is used to estimate the embedding dimension m and it is found to be equal to 5. The time delay τ is kept equal to 1 because the data is discrete. The threshold is found to be 0.73 (using the methodology outlined above in section 2.3.1). As suggested by Zbilut (2004) when he analyzed the S&P 500, in all cases a window size of 90 observations and a step size of 1 is taken. This gives 411 estimates of each RQA. We note that Sornette and Johansen (2000) suggest that on average it takes 30 days for the log periodic model to lead to the 'critical' time, that is market collapse. Thus 90 days should give

²note that ws is window size, ss is step size, ld is logged differentials, td is transformed differences, n is normalized i.e. $\frac{x_i - x_{min}}{x_{max} - x_{min}}$, and ns is not specified.

Table 4.5: Embedding dimensions for financial data

Author	Year	Data	m	tau	ϵ/σ	ws/ss	ld/td/n	No. of obs
Fabretti and Ausloo	2005	Nasdaq and DAX	5	10	ns	100/10	ld	1,482
Marwan et al.	2007	Exchange rates	20	1	0.048-4.350/ σ	-	ld	168
Strozzi et al.	2007	High frequency exchange rates	11	260	0.043	336/48	td	268,554
Crowley	2008	EU and US GDP	4	1	0.009/ σ	ns	ld	147
Gubhathakurta et al.	2010	Dow, Nifty Hong Kong Index	3	10	0.1/ σ	200-100/100	n	766-921
Karagianni & Kýrtson	2011	CPI and Dow Jones	1	1	10	100/10	ld	561
Basto & Caiado	2011	46 MSCI Indices	11	1	0.16/ σ	260/ns	n	3,914
Aparicio et al.	2011	Simulations and equity data	1-6	1	ns	-	ld	2,211
Moloney & Raghavendra	2011	EU CDS & Bonds	5	1	0.35/ σ	60/1	ld	541

enough time to allow the dynamics to unfold, without being so large as to average out any small scale dynamical changes.

The ARGARCH simulation applied in section 4.2 is generated from random normally distributed numbers that have been transformed using the saved ARGARCH variance series and the estimated coefficients of a sample of the S&P 500 from 8th May 2006 until 26th November 2008 (x_o is set equal to the value of the log differential on 8th May 2006).³

For illustrative purposes we will also analyze a series of data from 9th October 2006 until 5th October 2010 named the 2008 sample. This data set extends the 2007 data set by a further 502 observations.

4.4 Empirical Findings

Initially we will present the RPs of the four peak samples in figure 4.9 and the four control samples in figure 4.10 to compare and contrast the dynamic structures.

We can see evidence of extreme or nonstationary events indicated by the white bands or lines (Marwan et al. 2002). During the peak samples there appears to be more white bands both in number and width, particularly after the peak (observation 251). In the recurrence plot figure 4.9 (a) 1929 and figure 4.9(d) 2007 we see large black squares, followed by thick white lines indicating that the system swings from having many close trajectories i.e. low turbulence, to extreme turbulence. The RPs of the control samples are more uniform and have small squares indicating small scale correlations

³This simulation is also used in chapter 2 to compare phase portraits of Gaussian and non Gaussian series

Figure 4.9: RPs of the Dow Jones Index peak samples (a) 1929 (b) 1973 (c) 2000 (d) 2007

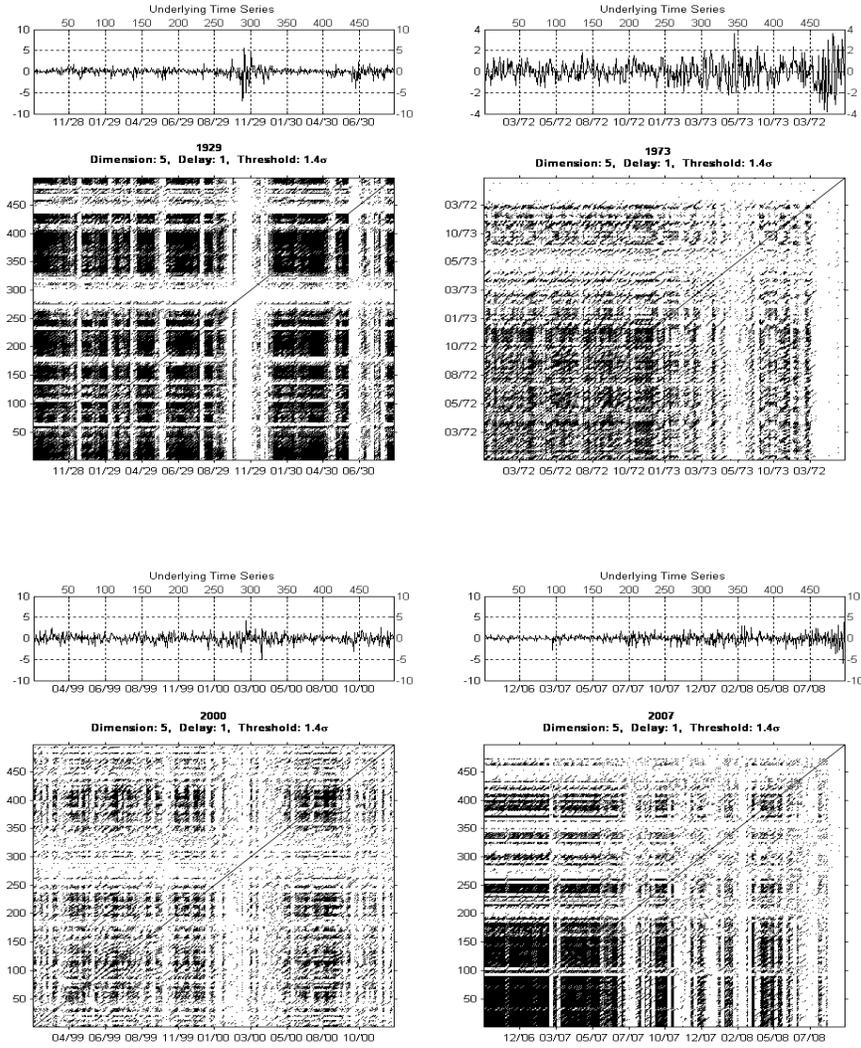
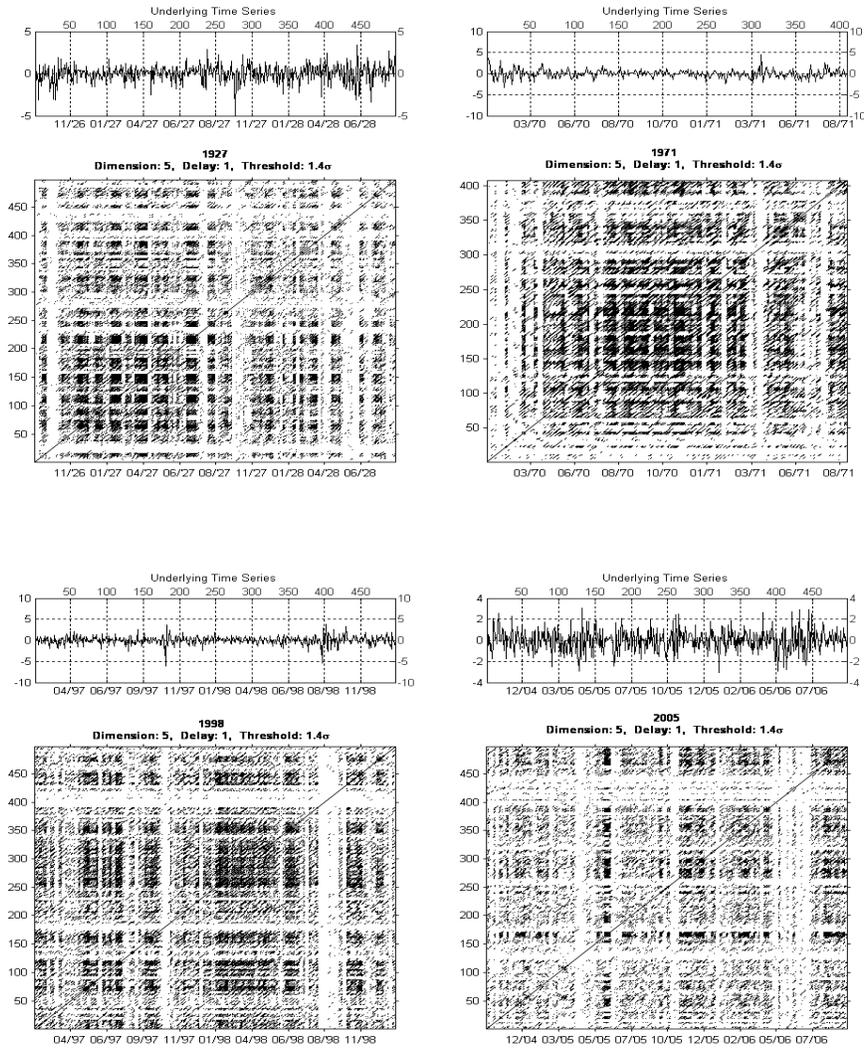


Figure 4.10: RPs of the Dow Jones Index control samples (a) 1927 (b) 1971 (c) 1998 (d) 2005



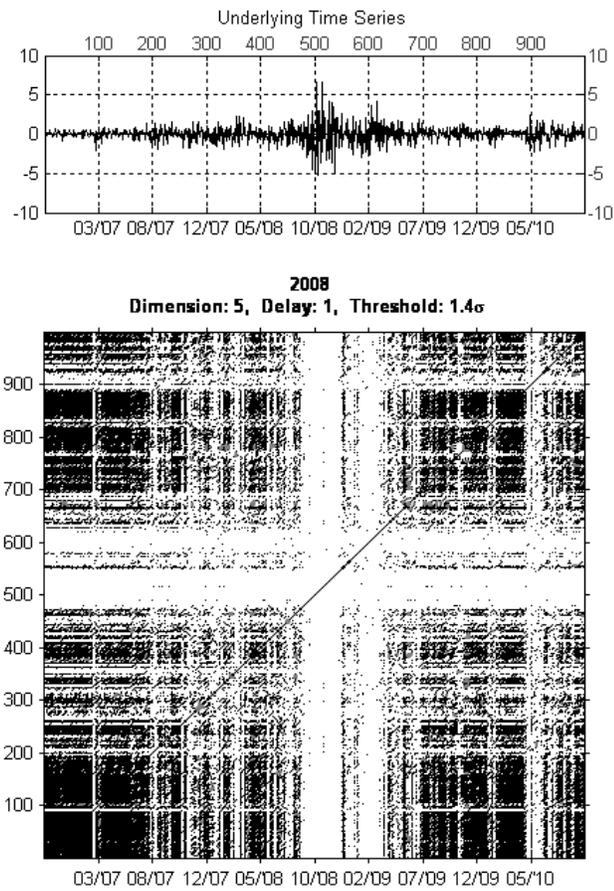
throughout the period. We note that figure 4.10 (c) 1998 is somewhat similar to the peak samples as there are large black squares followed by a large white band around August 1998. This may be explained as the summer of 1998 saw the Russian currency crisis and the LTCM crisis, which was noted earlier. There is a fading of the recurrence plot figure 4.9 (d) 2007 as the eye moves up the LOI. Similarly to the ARGARCH simulation, this indicates an underlying trend in the system. Analysis of the time series (shown above the recurrence plot) shows a rising variance. With this in mind, the sample is extended as outlined in the data section and illustrate the recurrence plot for sample 2008 (figure 4.11). The overall pattern in this recurrence plot is very similar to that of figure 4.9 (a) 1929. A large white band can be seen around September 2008, which was when Lehman Brothers collapsed. This initial analysis suggests similarities in the dynamics of the market during the peak samples, with particular similarities in the RPs of 1929 and 2008.

In both cases the time series and the RPs indicate that a rising variance in the logged differentials is coupled with extreme events or nonstationarities i.e. white bands in the RPs.

Phase transitions

The moving window RQA measures for all eight samples are calculated. The objective of the chapter is to examine the transition in the samples from one phase of the market to another, i.e. from bull to bear market. The terminology, phase transition, is borrowed from physics where it is used to define the transition of a substance from one form to another, e.g. from liquid to a gas. In nonlinear dynamics, phase transitions occur as structure of the

Figure 4.11: RP of the Dow Jones Index 2008 sample



trajectory changes due to changes in the parameter values of the underlying system. The trajectory can change from periodic to chaotic, or from chaotic to chaotic., for example. Bifurcations points occur at a period doubling. RQA measures have been shown to indicate phase transitions and bifurcation points. Table 4.1 above indicates that L_{max} has been shown to peak during a periodic to chaotic transition and to collapse prior to bifurcation points. V_{max} has been shown to peak during a chaotic to chaotic transition. Evidence of phase transitions around the time of a market peak and collapse could be a useful indicator of rising risk and a changing market. In figure 4.12 and 4.13 below, we present graphs of the windowed L_{max} and V_{max} for all eight samples of the Dow Jones Index. Looking at the scales of the graphs it appears as if L_{max} peaks at values above 20 in 1929 and 2007. We also note a smaller peak in L_{max} , (above 15) in January 1998.

Following the methodology of Marwan et al. (2002) (when examining heart-rate variability), we use the Mann-Whitney U test to compare the medians of the peak samples with the control samples. The null hypothesis of the test is that the two samples have the same distribution. As part of the test the two samples are ranked and the medians are calculated and compared. Marwan et al. (2002) interpret a rejection of the null as evidence of a phase transition in the data and concluded that RQA measures can be used to indicate an oncoming life-threatening arrhythmia. The results are illustrated in table 4.6 and 4.7 below. In all cases the null hypothesis is rejected. If we compare the estimates of the median for L_{max} (table 4.6) we note that only for peak 1929 and 2007 is $L_{max}(\text{peak})$ greater than $L_{max}(\text{control})$ i.e. $L_p > L_c$. In the other two samples, although the null is

Figure 4.12: Lmax & Vmax (a) 1929 (b) 1973 (c) 2000 (d) 2007

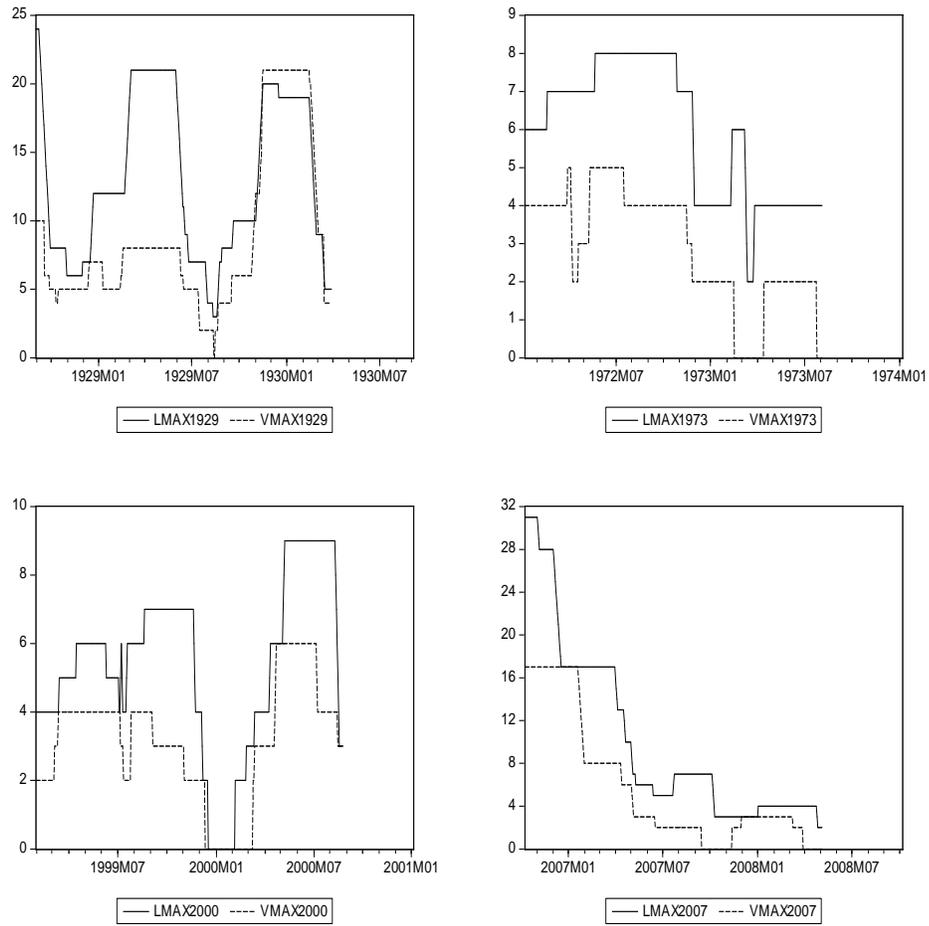
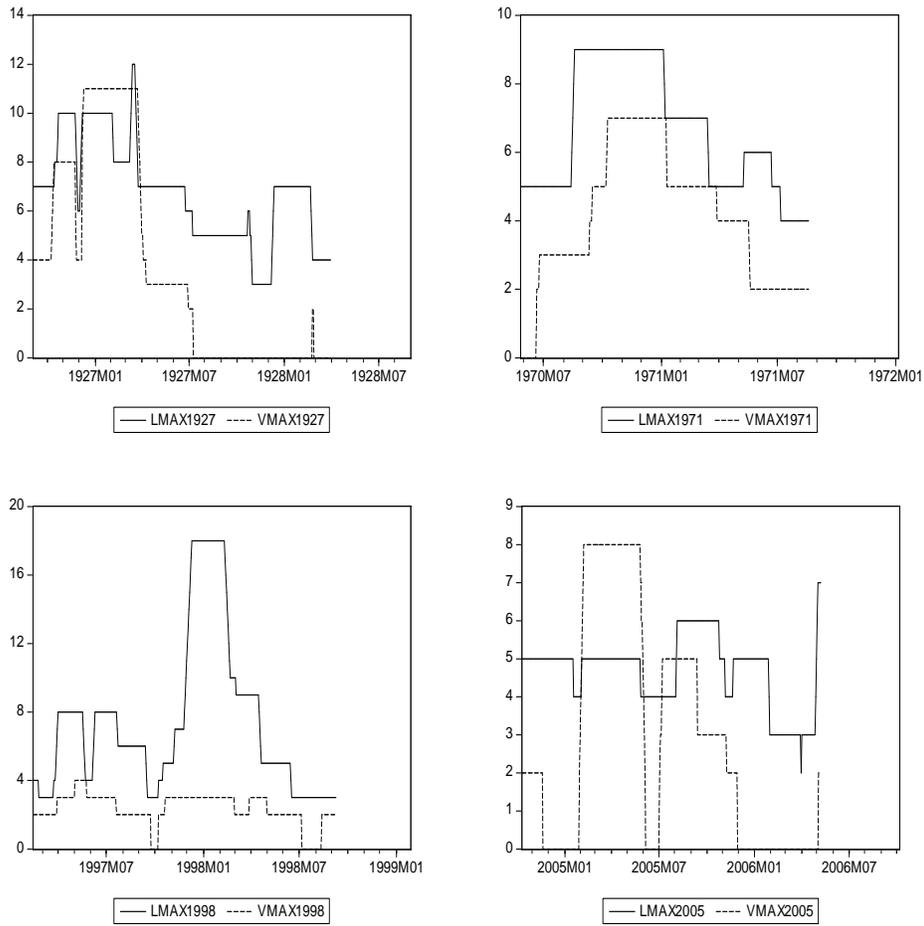
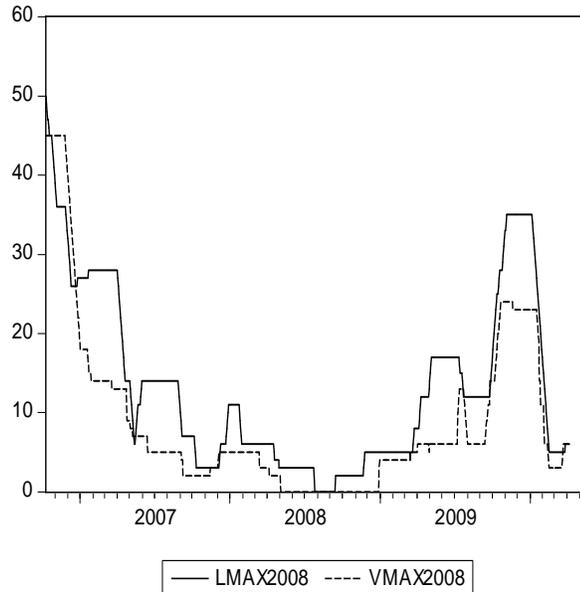


Figure 4.13: Lmax & Vmax (a) 1927 (b) 1971 (c) 1998 (d) 2005



rejected, the median L_{\max} is the same. Thus we can only be confident of suggesting evidence of periodic to chaotic transitions in samples 1929 and 2007. Looking at the graphs themselves, we see L_{\max} collapses around the middle of the peak samples i.e. around observation 251, the market peak. A collapse on L_{\max} suggests a rise in the Lyapunov exponent and this can indicate bifurcation points around the market peak (see table 4.1).

Figure 4.14: L_{\max} & V_{\max} (a) 2008



Similarly to the L_{\max} test, the null hypothesis is rejected in all cases and the median value for $V_{\max}(\text{peak})$ is great than the median value for $V_{\max}(\text{control})$ i.e. $V_p > V_c$ for the samples 1929 and 2007. Therefore it appears as if there is evidence of chaotic to chaotic transitions in these peak samples, as well as evidence of periodic to chaotic transitions.

Table 4.6: Mann-Whitney U test for Lmax

Sample	median	mean rank	Mean	(s.e.)	Mann U	p value	
1927	7	279.3333	6.7664	0.1033			
1929	12	543.6667	13.0827	0.2932	15.9591	0.0000	$L_p > L_c$
1971	6	409.4783	6.57683	0.1026			
1973	6	333.7202	5.9440	0.0874	4.8073	0.0000	$L_p \lesssim L_c$
1998	6	443.3224	7.2871	0.2247			
2000	6	379.6776	5.3577	0.1253	3.8424	0.0001	$L_p \lesssim L_c$
2005	5	345.8796	4.6813	0.0467			
2007	7	477.1204	10.1800	0.4127	7.9236	0.0000	$L_p > L_c$

Table 4.7: Mann-Whitney U test for Vmax

Sample	median	mean rank	Mean	(s.e.)	Mann U	p value	
1927	3	299.3345	3.6521	0.2092			
1929	7	523.6655	8.8127	0.2936	13.5440	0	$V_p > V_c$
1971	4	429.7019	4.0155	0.1118			
1973	3	317.8759	2.9273	0.0744	7.0961	0	$V_p \lesssim V_c$
1998	2	337.4647	2.3406	0.0453			
2000	3	485.5353	3.1192	0.0868	8.9397	0	$V_p \gtrsim V_c$
2005	2	352.6156	2.8029	0.1515			
2007	3	470.3844	5.7202	0.2919	7.1102	0	$V_p > V_c$

This suggests that there is significant and measurable change in the deterministic structure of the trajectory of the market around the time of these economically and historically important market peaks. Without evoking a model, we suggest that these indicators could be used as warning signals of transition in the market. For illustrative purposes we include the 2008 sample in figure 4.14. We note that for all of the peak samples (including 2008) there is a collapse in V_{\max} as well as L_{\max} at or close to the observation 251 i.e. the market peak. This pattern is repeated for DET below. We can see a clear repeated pattern in all the peak samples of a collapse in the RQA measures around the market peak. This collapse may be another indicator of the transition from bull to bear market. The following section further analyzes this pattern and suggests an interpretation.

Collapse in RQA

To summarize the collapse in the RQA measures, we examine the windowed DET and compare it to the scaled variance, VAR (see equation 4.18). We note that the scaled variance VAR rises around the same time as DET collapses. We also note that VAR falls as DET recovers. This can be illustrated in all four peak samples (figure 4.15).

In the peak samples (including 2008) DET and VAR vary much more widely than in the control samples; at times DET rises to close to 1 and collapses to zero. VAR rises as high as 9.

This negative relationship between DET and VAR is not as striking in the control samples. In the control samples DET varies between 0.5 and 0.9.

Figure 4.15: DET & VAR (a) 1929 (b) 1973 (c) 2000 (d) 2007

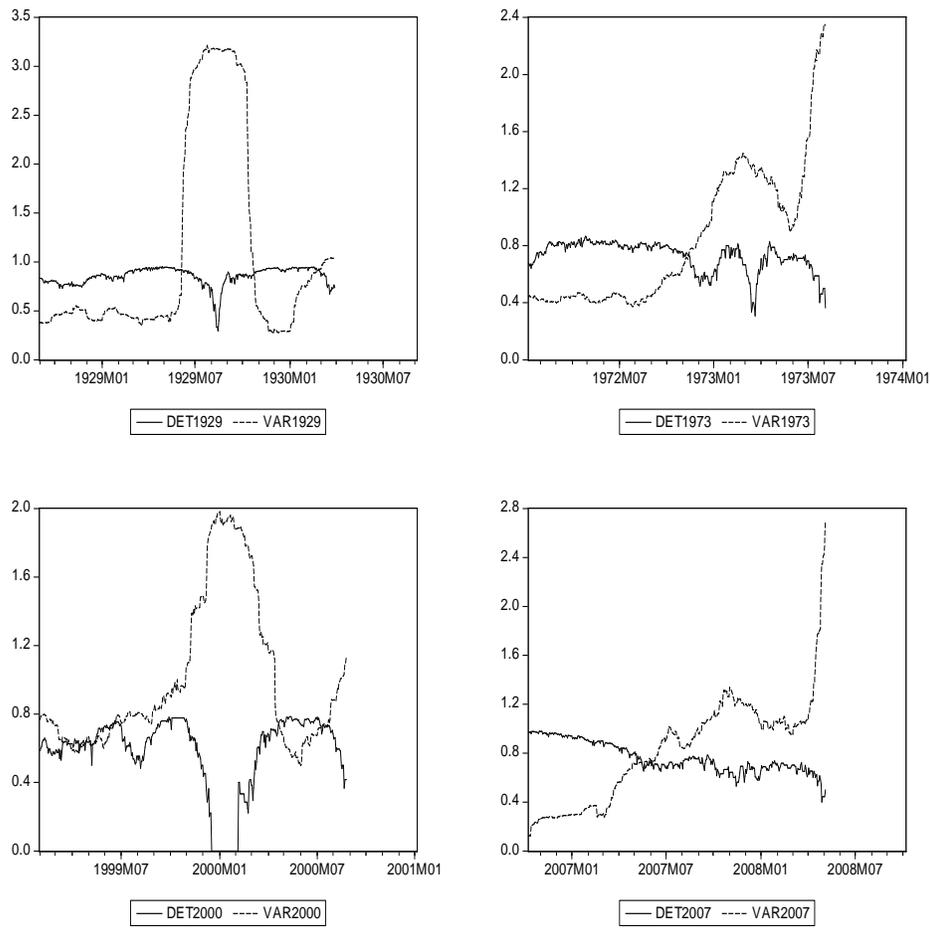


Figure 4.16: DET & VAR (a) 1927 (b) 1971 (c) 1998 (d) 2005

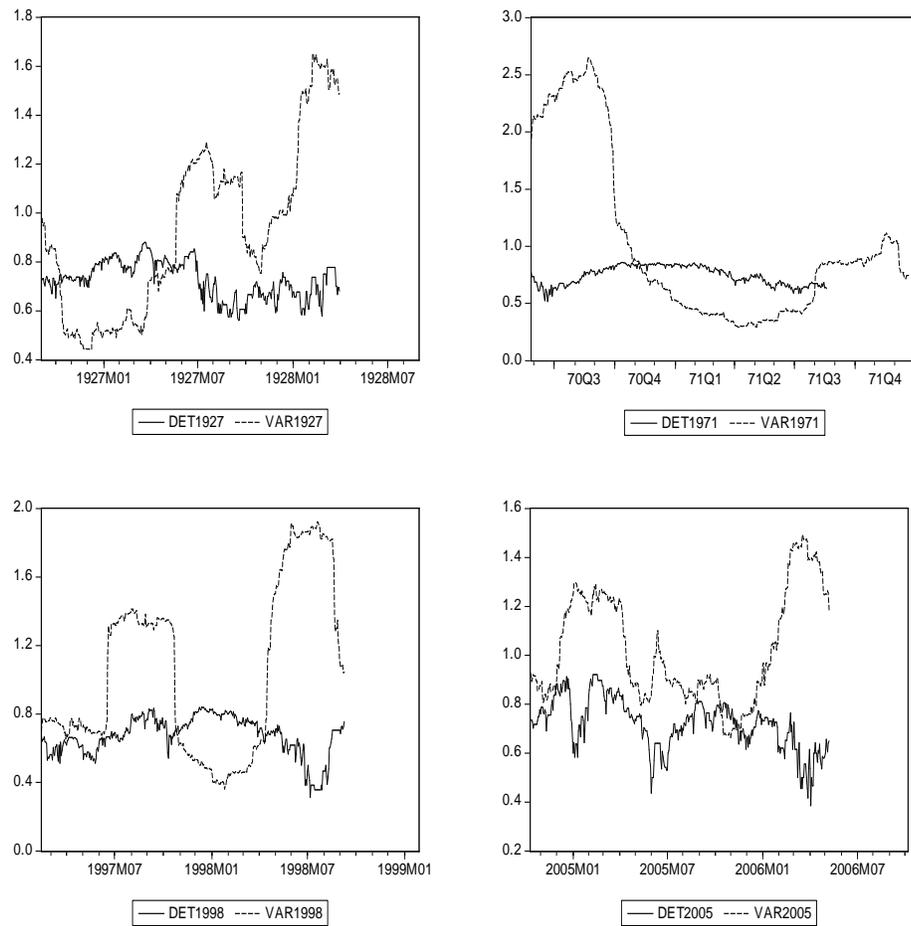
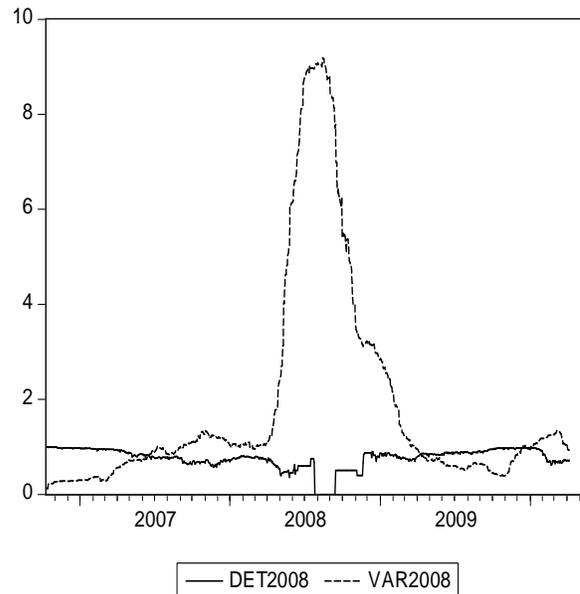


Figure 4.17: DET & VAR 2008



We conclude that around the time of the collapse in DET, the scaled variance VAR peaks. Why is this happening? An additional force could be entering the system, causing the relationship between signal and noise to alter. We examine this proposition in more detail in the next section.

Random Market Indicator (RMI)

Let us visualize the system as follows. Let us assume that the signal is deterministic with a map f which is not known to us. All that we do have knowledge of are noisy measurements (s_n) of this signal (x_n). Following from the work of Kantz and Schreiber (2003) we present the system using time delay embedding such that:

$$s_n = f(x_{n-1}, \dots, x_{n-m}) + \epsilon_t \quad (4.19)$$

$$\epsilon_t \sim g(\bar{x}, \sigma) \quad (4.20)$$

We assume that the noise variable (ϵ_t) is random and has no correlation with the signal and also that the system is stationary. This is a stochastic model (it could be linear or nonlinear). It is clear from the analysis of VAR that during times of transition from bull to bear market, variance rises considerably. If we take VAR as a proxy of the variance of the noise ϵ_t , we can see that during peak times variance is correlated with f (measured by DET) and cannot be said to be stationary (figure 4.15 and 4.17). We suggest that during these times the system is no longer conservative but is experiencing an exogenous force which is causing ϵ_t to rise. So much so, that the deterministic signal f can no longer be measured. All the RQA measures collapse to zero and the system appears completely random.

Classical Newtonian dynamics assume the existence of derivatives at all times i.e. Lipschitz conditions. This allows for the use of differential equations as a mathematical framework. Zbilut (2004) and Zbilut and Webber (2008) suggest that under 'non-Lipschitz dynamics' if a system experiences an exogenous force that this will cause unstable singularities and a 'stochastic' repeller to be born. The trajectory of the system becomes probabilistic and the dynamics of the system are irreversible. Zbilut asserts that non-Lipschitz dynamics allow for a 'dynamic pause' as the system adapts to its environment. In physiological or social systems, non-Lipschitz dynamics would allow

the system to 'learn'. By definition, chaotic systems require that the dynamical structure of the system is dependent on initial conditions (Sprott 2004). Zbilut (2004) argues that for physiological and social systems this is unlikely. For example, it is unlikely that the behaviour of Coca Cola's share price today is determined by the IPO (Initial Public Offering) price back in 1919. Although it is also true that social systems do not appear to be completely random. They appear to have deterministic structure. It is argued by Trulla et al. (1996), Zbilut (2004), and Zbilut and Webber (2008) that these systems should be modeled so that they can adapt or learn. Zbilut's approach allows a dynamical pause in the system which allows the system to forget and adapt. Zbilut (2004) describes this as "piecewise determinism". We assert that the collapse in the RQA measures indicated in the above graphs illustrates a transition in the stock markets from a deterministic to a random system. After the market peak, the RQA measures recover as the variance falls, and the deterministic relationships within the system reappear. This collapse in the RQA measures when VAR increases has also been illustrated by the analysis of the ARGARCH model (figure 4.7 and 4.8).

With this in mind, we use principal component analysis (PCA) to linearize the relationship between DET and VAR and to create a principal component series. We suggest that this series could be useful in indicating a market peak and collapse. PCA allows us to compose the principal components of a set of variables by computing the eigenvalue decomposition of the observed variance matrix. We will use the first principal component, which is the unit length linear combination of the original variables with maximum variance.

In table 4.8, we present the eigenvalues for the eight samples, the 2008 sample, and the ARGARCH sample. The proportion is the eigenvalue divided by the number of data series (in this case two). It can be interpreted as the percentage of variance between the variables i.e. DET and VAR which is accounted for by the first principal component.

Table 4.8: Principal Component Analysis of DET & VAR

Sample	Eigenvalue	Proportion	Correlation
1927	1.4326	0.7163	-0.4326
1929	1.4409	0.7205	-0.4409
1971	1.1859	0.5929	-0.1859
1973	1.6729	0.8364	-0.6729
1998	1.5213	0.7607	-0.5213
2000	1.8356	0.9178	-0.8356
2005	1.1990	0.5995	-0.1990
2007	1.9257	0.9629	-0.9257
2008	1.7699	0.8849	-0.7699
ARGARCH	1.5535	0.7767	-0.5535

In general the proportion is higher in the peak samples, indicating the appropriateness of the principal component series as a linear summation of the relationship between DET and VAR. This point is also highlighted by comparing the correlations, which in all cases are negative, but which are higher in the peak samples than the control. The correlations are particularly strong in the later samples of 2000 2007, and 2008. This suggests that the linear relationship between DET and VAR is strengthening through the twentieth century and into the twenty first century. We note that the correlation for the ARGARCH model is not as strong as for the actual market samples. This could be due to the fact that the ARGARCH model is by design stationary

and thus will not allow for the extreme changes we see in the actual market. This point is highlighted in figure 4.18 which presents the squared return series for the S&P 500 (as a proxy for realized volatility) with the forecast variance from the ARGARCH Gaussian model. The ARGARCH Gaussian model reflects the volatility of the actual data but consistently underestimates it. Therefore a principal component series based on the ARGARCH model would underestimate the variance of the market. Figures 4.19, 4.20, and 4.21 illustrate graphs of the principal component series (which we have named the random market indicator (RMI)) for the eight samples and for 2008. Comparing the samples, we can see that RMI is higher during the peak samples and peaks around the time of the market peak. This is particularly noticeable for figure 4.19 (a) 1929, figure 4.19 (c) 2000 and figure 4.21 2008. Comparing the absolute value of the RMI, it appears as if a value well above 3 indicates a transition in the market; variance is rising high and DET is collapsing to close to or to zero. During these times the signal f can no longer be read and the variance VAR is high. To use a stationary stochastic model, such as that in equation 4.19, would lead to poor estimates of risk. The system appears nonstationary and the covariance between f and ϵ_t is nonzero.

Figure 4.18: Comparison of ARGARCH model with S&P 500

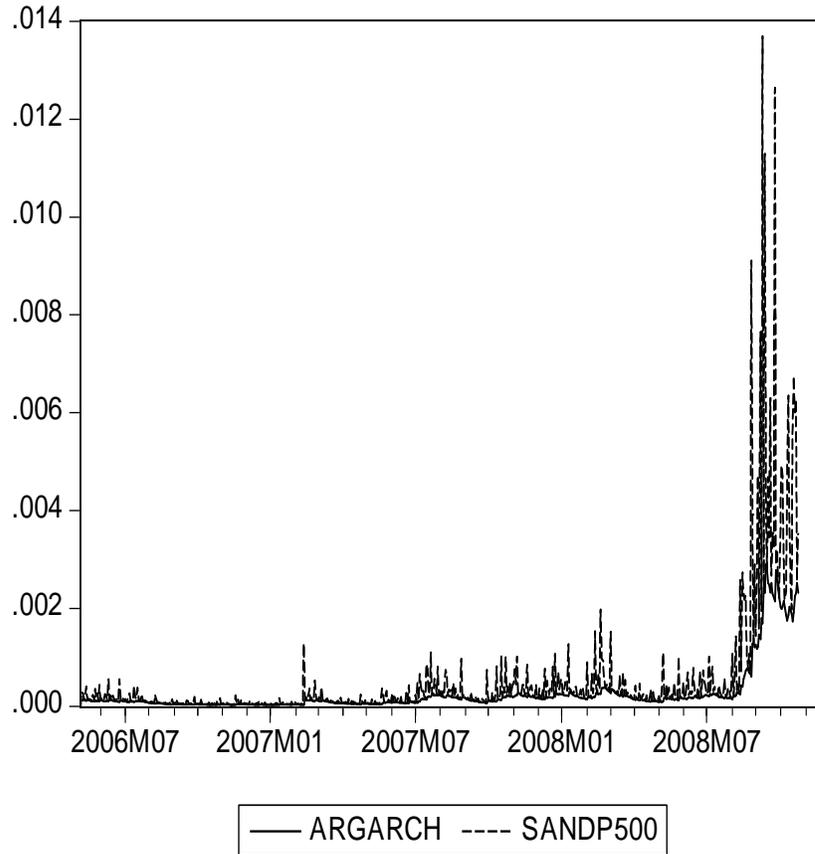


Figure 4.19: RMI (a) 1929 (b) 1973 (c) 2000 (d) 2007

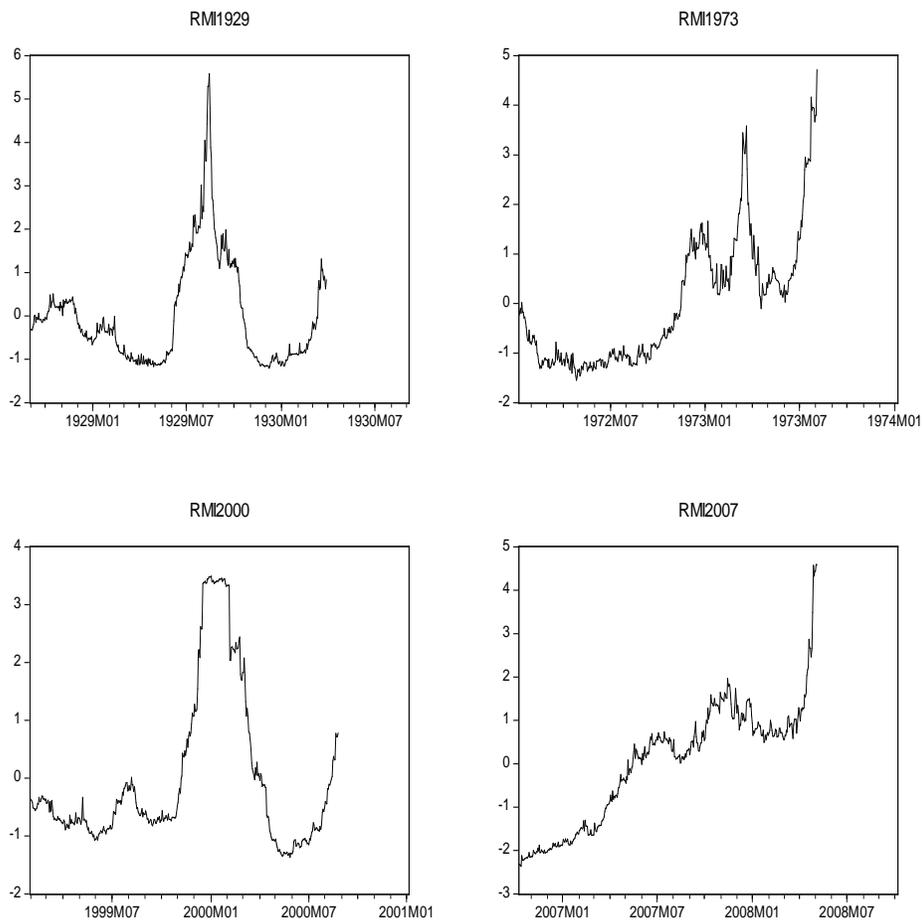
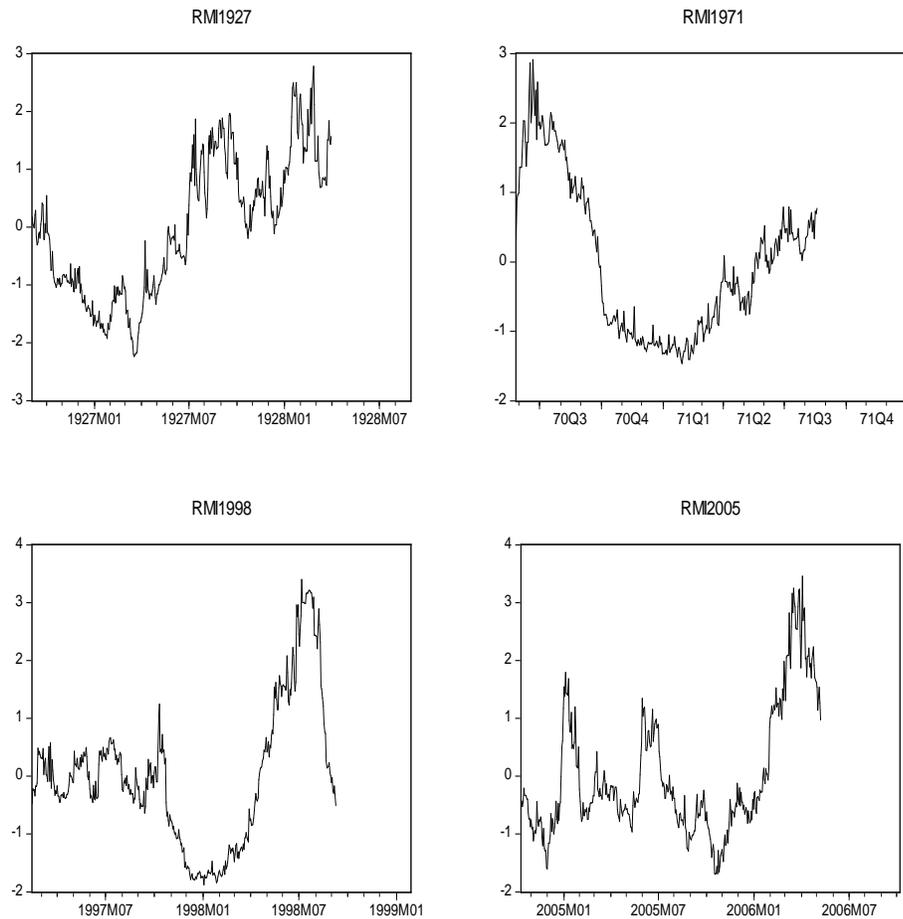


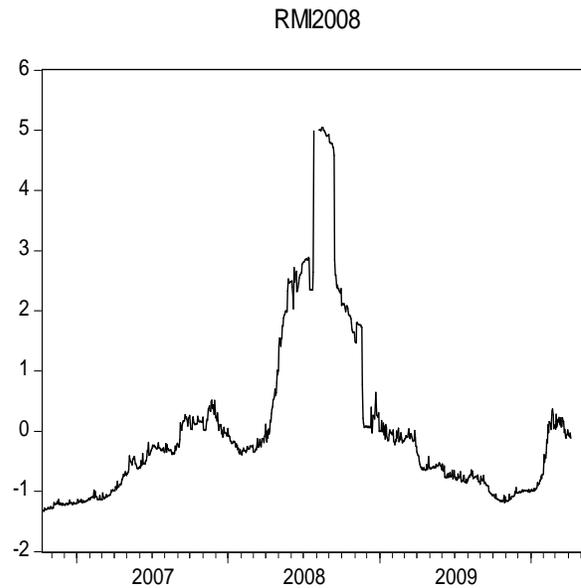
Figure 4.20: RMI (a) 1927 (b) 1971 (c) 1998 (d) 2005



The regulatory implications of this paper are that when the RMI rises to high levels, (say above 3) stationary quantitative risk models should not be used or allowed. They should be switched off as the market is behaving in a random and unpredictable manner. Once the RMI falls back to acceptable levels the models can be used again. One central question remains: what is this force? Much work has been done in the economic literature on the

relationship between sentiment and noise (Brown and Cliff 2004, Malcolm and Wurgler 2006, Mendel and Shleifer, forthcoming).

Figure 4.21: RMI 2008



The 'noisy trader theory' suggests that 'noise traders' are rational but uninformed and as a result chase noise. By chasing noise, these traders cause the market to fall into and remain in disequilibrium for a period of time. If we consider this force as sentiment or ignorance or fear, we can explain the collapse in the RQA measures as a result of noise traders. When sentiment recovers, trading becomes more predictable and the market calms down; variance falls, and the RQA measures recover.

4.5 Conclusions

The comparison of recurrence plots and recurrence quantification analysis measures for peak and control samples of the Dow Jones Industrial Index have illustrated some similarities across the time periods, particularly the 1929 and 2007 samples. We find some evidence of periodic to chaotic transitions and chaotic to chaotic transitions prior to the market peak in the 1929 and 2007 time series. Secondly we find that the RQA measures collapse just prior to or around the time of the market peak. This would indicate that the trajectory of the market loses its deterministic structure at this time. As the markets do not appear to be periodic, our results do not support Johansen and Sornette's LPPL model (2000).

There is also evidence of a collapsing L_{max} and thus a rising Lyapunov exponent prior to each of the peaks, which indicates a chaos to chaos phase transition. This suggests that the dynamics of the system are changing prior to the market collapse and that the system becomes structurally unstable. In the middle of the peak samples, there is evidence of collapsing RQA measures, which implies that the market is losing all its deterministic structure and is behaving in a completely random manner.

Following from Zbilut (2004) and Zbilut and Webber (2008) the system can be thought of as piecewise deterministic. We argue that as noise increases (due to an exogenous force) the determinism in the model is lost and the system transitions from chaotic to random. We suggest that the cause of the exogenous force is due to rational but uninformed noise traders. The relationship between the RQA measures and the variance can be summarized through a principal component series named the random market indi-

cator (RMI). If RMI increases above 3 this is an indication of a transition in the data to a nonstationary random process. Quantitative risk estimation techniques should be turned off during these times until the market has transitioned back to a more predictable deterministic process.

One criticism of recurrence quantification analysis is that it is presented without a model. In fact applying recurrence quantification analysis allows us to understand the dynamics of the system without evoking a model. This chapter suggests that the system is in fact piecewise deterministic: applying any stationary, fixed parameter model onto the entire data series will lead to misleading forecasts. Recurrence quantification analysis allows us to reconsider the methodological steps taken during the modeling procedure as well as allowing us to question the assumptions made as part of the mathematical framework imposed.

This methodology has been used by many disciplines to better understand their dynamics: including biology, physiology, psychology, and meteorology. For example, Trulla et al. (1996) suggest that:

biological processes are high-dimensional entities, living on transients amidst a field of relatively weak attractors

(Trulla et al 1999)

This chapter suggests that social systems such as the stock market are similar such entities. Allowing for the market to be piecewise deterministic allows it to adapt and learn from its environment. Over-modeling the data can be misleading and dangerous, as it may ignore phase transitions, intermittencies, transients and dynamical pauses. Quantitative risk estimation

techniques need to be accurate, and when they are not accurate, they should not be used. As a general principal, 'known' unknowns should be distinguished from 'unknown' unknowns, that is we should highlight when we are confident in the application of our models and when we are not. This chapter suggests that at times of high uncertainty and variance, the predictability of the markets collapses. Quantitative risk modeling should not be used at these times. To generalize these results, further work needs to be done with other time series, other embedding values and other frequencies. Also the reliability of the RMI should be back-tested by matching failures of stochastic quantitative risk estimation techniques with different values for the RMI.

Chapter 5

Conclusions

5.1 Summary of context

Chapter 1 contextualises the aims of the thesis. In light of the recent financial crisis, there has been widespread recognition of the inefficacy of existing risk estimation techniques (Danielsson 2008). The majority of these techniques are framed in a linear stochastic (usually Gaussian) model. The attractions of these models are that they are relatively easy to derive and to understand. They allow the application of well-known statistical techniques such as correlation and regression analysis. But the models have a number of drawbacks. They assume: that the parameter values are fixed, that the probability distributions are stationary, and thus that all irregularities in the data must be due to the random element in the model (Kantz and Schreiber 2003).

Within the stochastic framework, the Gaussian distribution is the most widely used due to the ease of its analytical tractability; for example, the parameters of a multivariate Gaussian distribution can be estimated from a

linear combination of its components; that is of the univariate Gaussian distributions (including the correlation values). Despite the algebraic ease of the Gaussian distribution, it has been clear for over 40 years that financial data exhibits non-Gaussian characteristics (Mandelbrot 1963, Fama 1965). Financial data has excess kurtosis and is skewed. The introduction of non-Gaussian stochastic distributions can allow these characteristics to be measured as part of the model estimation techniques. But changing from a Gaussian to a non-Gaussian distribution has drawbacks. Most financial models depend on the Gaussian assumption, for example the Markowitz mean-variance model and the Black-Scholes-Merton option pricing model. Other methodologies become prohibitively complex in a non-Gaussian framework, for example the multi-asset copula function, (Jondeau et al. 2007). Much of modern day financial theory will have to be rewritten if the Gaussian framework is no longer imposed.

Despite the inherent complexity of turning to non-Gaussian modeling, the initial stages of this thesis involve applying Student-t and generalized extreme value (GEV) distributions to an ARGARCH value at risk model. The asset chosen here is the credit default swap due to its recognized systemic importance (European Union 2009). This analysis is illustrated in chapter 1. We conclude that Student-t and in particular GEV distributions are improvements on the Gaussian distribution, yet both distributions still under-predict the distribution of returns for the CDS data. We also estimate the tail parameter of CDS data using parametric and non-parametric techniques. We find that in the majority of cases the tail parameter is not a good fit to the data, and that in fact there is evidence of a non-finite variance. A non-finite

variance can indicate a nonstationary distribution i.e. a heterogeneous variance (Patterson 2000). In chapter 2, we show nonlinear dependencies in the residuals of the ARGARCH model for CDS data. These results suggest that stochastic modeling may not be appropriate.

For this reason, chapter 3 and chapter 4 do not focus on Gaussian or non-Gaussian stochastic modeling. Instead, the focus is on the framework of nonlinear time series analysis. This methodological framework allows the dynamic relationship (either univariate or bivariate) to be nonlinear. The methodology is inductive, focusing on interpreting the empirical observations, rather than imposing characteristics on the data. Nonlinear time series analysis and the development of chaos theory have introduced a new theoretical framework to examine financial time series. Methods such as the BDS test and recurrence quantification analysis, present results which could not be seen in the linear framework (Marwan et al. 2009). The drawback of nonlinear time series methodologies is that they often introduce more questions than answers (Sprott 2004). Modeling nonlinear stochastic data is notoriously difficult (Kantz and Schreiber 2003). We do not attempt to model the data in this thesis instead we use a nonlinear framework to describe the characteristics of the financial markets before, during, and after heightened market volatility.

The central research question of the thesis is: Can we improve our understanding of risk and volatility in the financial markets, particularly during times of heightened turbulence, by using applied nonlinear time series techniques? In short the answer to this question is yes. During times of heightened risk and volatility, nonlinear tools indicate nonlinear dependencies and

structural instabilities in univariate data; as well as nonlinear convergence and synchronization in bivariate data sets. Analysis of these characteristics will improve our understanding of how the market behaves during periods of heightened turbulence and this will improve the accuracy of risk estimation techniques.

5.2 Summary of chapters

This thesis includes three papers on various topics in nonlinear dynamics, they are presented as chapters 2, 3, and 4. They are summarized individually in the following section.

Chapter 2 - Nonlinear dependence

Contribution

This chapter's first contribution is to apply the BDS test to the GARCH residuals of CDS data, and to discover the nonlinearities. Neither the linear nor the nonlinear GARCH models applied remove the dependencies in the data. The chapter also contributes to the literature by applying the nonlinear tools of delay plots and the correlation dimension test to CDS data. The delay plots indicate that the CDS data lies on the x and y axes. Delay plots are used to model nonlinear data and suggest certain modeling techniques (see future research below). The results of the correlation dimension test indicate that the CDS data is not low-dimensional deterministic, but could be either high-dimensional deterministic or stochastic. Finally, the chapter shows that the equity and bond residuals are independent, thus a linear

stochastic GARCH model can be applied to these asset classes. The chapter concludes that CDS data must be remodeled to ensure that these nonlinear dependencies are removed before a stochastic distribution can be applied.

Limitations of the research

A limitation of the BDS test is that a rejection of the null hypothesis does not distinguish between a nonlinear deterministic and a nonlinear stochastic process. The application of the delay plots and the correlation dimension test indicates that the data is either high dimensional deterministic or stochastic. These tests cannot estimate the size of a high dimensional process. In general, this is notoriously difficult to estimate (Kantz and Schreiber 2003).

Future research

Analysis of the delay plots indicates that the data settles on the x and y axes. This occurs as the CDS data fluctuates from very low returns (or no change in the spread value) to large jumps. Although not discussed in the thesis, we have considered two models to reflect this characteristic. Firstly, a nonlinear threshold autoregressive model would allow for the dynamic jumps. Some initial work has been done with the model, but further examination (including goodness of fit and other residual tests) need to be applied. Alternatively a linear stochastic model including two stochastic probability distributions may fit the CDS data well.

The application of recurrence quantification analysis to univariate CDS data would allow the examination of the trajectories of the data in more detail and should elucidate the data's characteristics. For example, this would

distinguish between a stochastic and a deterministic process. Applying recurrence quantification analysis to the CDS residuals of the GARCH process would allow the examination of the remaining nonlinearities.

Chapter 3 - The arbitrage-free parity theory

Contribution

This chapter has four original contributions to the analysis of equivalence between CDS and bond spreads. Firstly, the chapter supports chapter 2 and provides further evidence that linear Gaussian or stochastic models may not be reliable for CDS data. A statistical analysis of the descriptive statistics of the CDS and bond data indicates that the variances are statistically different and that neither asset's distribution is Gaussian. The BDS test is applied to the ARGARCH residuals of CDS (and bond) data. The sovereign CDS data exhibits nonlinear dependencies whereas the sovereign bond data does not. Linear Gaussian models are commonly used when pricing and estimating risk for CDS or bond data. Also bonds and CDS spreads are often assumed to have the same distribution in bivariate models. As well as supporting the conclusions of chapter 2, this chapter questions the application of Gaussian modeling for CDS or bond data analysis and also questions the use of equivalent bivariate stochastic modeling for CDS and bond data.

Secondly, the application of the nonlinear nonstationary framework, the cross recurrence plot (CRP) is used to examine the arbitrage-free parity theory for sovereign CDS and bond spreads. This chapter is the first to apply CRPs to CDS data. The plots and the CRP measures illustrate dynamical

convergence and synchronization between the two assets. Analysis of the CRP plots indicates that all the markets swing from periods of equivalence to periods of nonequivalence. The Greek market is highlighted as having particularly extreme variability. This variability indicates nonstationarity in the data particularly around the time of the Greek bailout i.e. May 2010. We suggest that this is evidence that a stationary model – be it linear or nonlinear, may not provide accurate estimates during a crisis.

The third contribution is the estimation and statistical analysis of the dynamical probability of convergence, which varies significantly through time and across countries. There is evidence to show non-mean reversion in the Irish, the Spanish, and the Greek markets. This result contradicts the arbitrage-free parity theory, which assumes mean reversion. The Greek market shows strong evidence of a trend in the probability to converge, suggesting non-mean reversion and a trend in the arbitrage-trading in this market. During 2010, it was suggested that the Greek CDS market was being manipulated by hedge-fund traders (Oakley 2010) although no evidence of this was found by the EU Commission investigation (Tait 2010). This chapter provides regulators with an alternative framework for the analysis of arbitrage-trading across markets.

The fourth contribution is the estimation of the nonlinear synchronization between the CDS and bond data in each country. The results in the chapter indicate that (in general) the level of synchronization is equivalent across countries though it changes significantly through time. Of particular note is the sharp rise in synchronization in the Greek market in March 2010 (prior to the bailout), which suggests an increase in arbitrage trading at this time.

Time varying arbitrage trading suggests that a time-varying common influence (possibly macroeconomic or trading related) is affecting both of these markets. In conclusion, the chapter notes significant problems with linear stochastic modeling for CDS and bond data and introduces a new approach, the CRP. This framework allows a model-free examination of the variability in the equivalence between two markets and can contribute significantly to the supervision and regulation of financial markets.

Limitations of the research

In this chapter the Granger causality model is applied to replicate the results of existing literature. This is only one of a number of linear tests which could have been applied. Considering that the result of the linear test is similar to that in existing literature, it was felt unnecessary to apply further linear tests. We suggest that this limitation is not significant.

The main limitation of the chapter is the dependency of the results on the parameters chosen. Although the embedding parameters (m, τ, ϵ) were chosen with the up-most care and by following existing guidelines, the sensitivity of results to parameter values is noted. We note that Aparicio et al. (2008) suggest a solution to the problem of embedding, but this solution is only relevant if RQA measures are used to test for general dependence in a time series. As that is not the case here, the solution is not applied.

The CRP allows us to analyze data in a model-free framework. This framework in itself cannot forecast future values for markets although it can be used to recognize changing dynamics. Comparison of CRP measures to known systems (e.g., the Lorenz system of equations) can lead to recognition

of similar characteristics and facilitate modeling of the data. The possibility of modeling the data in this way was not explored in this chapter.

Future research

Future research could extend the methodology to include a wider data base. For example, other countries' pairs of corporate CDS and bond, or pairs of CDS and equity data could be included. We note evidence of horizontal and vertical line structures in the CRPs in this chapter, which could be evidence of lagged equivalence. This result could be examined further through the examination of other CRP measures - laminarity, trapping time and maximum vertical line length (Marwan and Kurths 2002).

The chapter suggests that there is time varying synchronicity between the two assets, which suggests that the financial markets are a complex system. This assertion could be further examined through the estimation of dynamic invariants, such as K2 entropy (Marwan et al. 2007). This chapter focuses on a bivariate analysis of CDS and bond data. The framework can also be applied to CDS data as a univariate framework, through the application of recurrence plots (RP) and recurrence quantification analysis (RQA) measures (see chapter 4). It would be particularly interesting to analyze the changing dynamics in a number of CDS data sets (corporate as well as sovereign) during a financial crisis. This would be a similar approach to that taken in chapter 4 for equity indices.

Chapter 4 - Dynamical transitions

Contribution

This chapter has five original contributions to the analysis of financial market dynamics during a transition from a bull to a bear market. Firstly the chapter applies the concept of a time-based ‘control’ sample. The idea of comparing financial data pre and post crisis is common in the literature (Fontana and Scheicher 2010). The contribution here is to use a control sample to analyze the recurrence quantification analysis measures for equity data prior to a financial collapse. The method allows the statistical comparison of the recurrence quantification analysis measures for both samples.

The second contribution is the statistical analysis of L_{max} and V_{max} across time for the same sample of financial data. This method allows for the examination of the time series for phase transitions, bifurcations points and intermittencies. Marwan et al. (2002) use this methodology to indicate the onset of a life threatening heart attack. We replicate this to indicate the onset of a market collapse, specifically focusing on statistical tests of L_{max} and V_{max} . The test results indicate evidence of chaotic to chaotic transition and evidence of periodic to a chaotic transition in the 1929 and 2007 samples. The chapter provides evidence of similarities in these two economically significant market collapses. Basto and Caiado (2011) use a similar statistical test to indicate differences in the recurrence quantification analysis (RQA) measures across countries during one time period. The contribution of this chapter is to apply the statistical test to the same data series during different time periods in search of recognizable nonlinear transitions.

Instability in L_{\max} for financial data has been noted previously (Zbilut 2004). This chapter's third contribution is that of linking the collapse in L_{\max} to the market transition. The collapsing L_{\max} indicates a rising Lyapunov exponent, which is further evidence of a chaotic to chaotic transition. The second and third contributions indicate that the financial market data is structurally unstable around the time of the market peak.

Collapsing RQA measures around the time of a market crisis has been noted in previous literature (see chapter 1.3). We show this result again and point out that this refutes the LPPL model of Johansen and Sornette (2000).

The fourth contribution of the chapter is to link the collapsing RQA with rising volatility clustering. By simulating the GARCH process we show that as variance rises the RQA collapse. This shows a link between variance and the RQA measures. We show that DET, L_{\max} and V_{\max} all collapse as the scaled variance rises. We suggest this link can be explained economically by the noisy trader theory. As information becomes more uncertain, rational traders chase noise. This causes the market's deterministic characteristics to be lost and the market is characterized by random variability in returns.

The final contribution of the chapter is to apply principal component analysis to the RQA measure DET and the scaled variance (VAR) to develop a principal component series, the random market indicator (RMI). The chapter proposes that this series can be used as an indicator of oncoming market collapse as the market becomes nonstationary and random. During these times stationary quantitative risk estimation techniques should not be used. The chapter suggests that this framework and the RMI indicator would facilitate monitoring of the financial markets by regulators.

Limitations of the research

As with chapter 3, the main limitation of the chapter comes from the dependency of the results on the parameters chosen. Although the embedding parameters (m, τ, ϵ) were chosen by following existing guidelines, the sensitivity of results to parameter values is noted. Webber & Zbiliut (2005) show that noise inflates m , as noise is shown to vary in terms of influence and size in this chapter, its influence on the results may be of particular importance. Some works transform the data for example by removing outliers (Basto and Caiado 2011) but as the data is being analyzed for changes during extreme events, this is not seen as an appropriate transformation.

The use of time based control samples has some success although it is noted that the 1971 sample is reduced in size by 90 observations to remove the 20% collapse in the index during this time. This implies that for the 1970's samples, the Mann-Whitney U test is comparing two data sets with varying number of observations that is 1971 has 321 observations and 1973 has 421 observations. This does not appear to have a significant effect on the overall results and contributions of the chapter.

It is also noted that the 1998 sample may not be an appropriate 'control' as during this time the market was turbulent due to the Russian currency crisis and the LTCM collapse. The objective of using control samples is to test the indicators of an oncoming collapse that is a rising L_{max} and V_{max} followed by collapsing RQA measures concurrent with a rising VAR. The difficulty in choosing control samples is finding samples which are less turbulent than the peak sample and are also close in time to the peak sample. This limitation does not prevent the discovery of the patterns which occur

during the peak samples, particularly during the economically important 1929 and 2007 samples.

Future research

The main focus of future research would be to further validate the indicators of an oncoming nonstationary random regime that is, peaking L_{\max} and V_{\max} followed by the RMI series rising above 3. This could be done by back-testing the RMI against failures in quantitative risk estimation techniques, such as value at risk. As was shown in chapter 1, value at risk models generally underestimates the risk of holding an asset during times of heightened turbulence. Matching failures in value at risk with values of the RMI index would test the efficacy of the indicator. The methodologies outlined in this chapter could also be used for other financial data such as individual equities, other indices, bonds, CDS, derivatives etc. Also higher and lower frequencies could be analyzed. We note in Strozzi et al. (2007) high frequency data i.e. 30 minutes, are used, whereas in Holyst et al. (2001) weekly and monthly frequencies are analyzed. The reliability of the results could be further verified by altering the embedding parameters. The results of transformed (e.g., with outliers removed) and untransformed data could be compared. This would indicate the influence of extreme events on the results. Finally the residuals of known series, such as the ARGARCH model could be analyzed for dynamical deterministic characteristics.

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