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NATIONAL UNIVERSITY OF IRELAND GALWAY

DOCTORAL THESIS

**Coevolutionary Spatial Game Theory:
The Impact of Abstention and Dynamic Networks on the
Evolution of Cooperation**

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for the degree of Doctor of Philosophy

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Abstract

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*Coevolutionary Spatial Game Theory: The Impact of
Abstention and Dynamic Networks on the Evolution of
Cooperation*

Since the dawn of evolutionary game theory, the standard models in evolutionary dynamics have been developed under the assumption of a well-mixed population where agents (individuals) can interact with all the other agents in the population (i.e., unstructured population). However, in real-world scenarios, populations are usually not well-mixed nor unstructured. In this manner, concepts of graph theory to consider structured populations in a network form have been extensively applied to evolutionary game theory, giving birth to the so-called spatial evolutionary game theory domain.

Although there has been a lot of research undertaken into spatial evolutionary game theory over the past number of years, the majority of this research involves the use of static networks in compulsory games such as the prisoner's dilemma game. Recent studies have considered coevolutionary spatial models adopting dynamic networks where both the game strategies and the network itself are subject to evolution, which, in fact, constitute a natural upgrade in the field for being more accurate in describing many social scenarios. For instance, in scenarios involving real biological networks and

social networks, the number of individuals, their connections and the environment are often dynamic. Those studies have been successful in exploring the effects of coevolution in a compulsory game, where coevolution has been recognized as a key mechanism to support cooperative behaviour. However, there has been little success in accounting for the impact of abstention (voluntary/optional participation) in such coevolutionary models.

Thus, within the bounds of multi-agent systems and network science, this thesis aimed to expand the knowledge of evolutionary game theory by bridging the gap between abstention and dynamic networks in social dilemmas. Besides investigating the impact of dynamic networks on the evolution of cooperation on both compulsory and voluntary games, this work also provided a novel perspective for understanding the foundations of cyclic dominance behaviour in the context of the voluntary games.

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Declaration of Authorship

I, Marcos Heli CARDINOT Meira Junior, declare that this thesis is entirely composed of my own work, except where explicitly stated otherwise, and that this work has not been submitted for any other degree or professional qualification.

Marcos Cardinot

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Chapter 1

Introduction

Advances in computer science and information technology have been continuously redefining the way we live, interact and work. With the revolution of the internet and the increasing availability of high computational power in affordable personal computers, studies on artificial intelligence and its applications have been gaining increasing attention. Within the area of artificial intelligence, evolutionary computation arises as a fundamental technique to solve practical problems by formulating and applying algorithms which mimic either human cognitive abilities or the behaviour of living adaptive systems in the execution of a given set of tasks [73, 91].

In this context, advances in artificial intelligence can directly impact several branches of science, including research in the domain of decision sciences. That research, in turn, is not only interested in finding an optimal solution for a particular problem but also in understanding the driving forces behind decision-making. In fact, social decision-making has been extensively explored since the 1950s with the rise of game theory, which is a mathematical theory for decision-making in situations of conflict where the decision of a rational agent (e.g., a person, company or entity) affects the performance/behaviour of the collective [107, 108]. Despite being primarily

developed in the context of economics to model economic competition between entities [176], game theory has been applied across a wide range of disciplines concerning behavioural and social sciences, such as biology, psychology, anthropology, sociology and political science [141, 132, 43].

While game theory has proven to be successful in modelling a number of scenarios involving competition such as auctions and bargains, its applications are limited by the assumption that individuals are rational and selfish actors who are always interested in improving their own performances. However, it is well known that scenarios involving human decision-making usually include the emergence of altruistic behaviour [127]. Moreover, some of the most important analytical tools within game theory, such as the Nash equilibrium, fail in providing significant insights into real-world scenarios, mainly because it does not consider that individuals might not have complete knowledge of the game state. Interestingly, altruism and cooperation are also one of the main puzzles in Darwin's natural selection [40]. After all, why would an individual reduce its own performance to improve the performance of someone else? Considering a population of interacting and reproducing individuals, how can Darwin's natural selection lead to cooperation?

In order to answer those questions and to understand the so-called evolution of cooperation, the concepts of game theory and evolutionary theory have been combined, giving rise to evolutionary game theory, which is another field of research that aims to explain the emergence of altruism and cooperative behaviour among rational individuals in complex environments [72, 142].

This chapter introduces the problem statement (Section 1.1) and gives an overview of the research questions which will be addressed by the work described in this thesis (Section 1.2). The hypotheses (Section 1.3), the main

contributions (Section 1.5) and the publications (Section 1.6) of the work are then outlined. Finally, Section 1.7 gives an overview of the topics discussed in each subsequent chapter.

1.1 Problem statement

Within the areas of artificial life and agent based simulation, evolutionary games such as the classical prisoner's dilemma, and its extensions in the iterated form, have garnered much attention and have provided many useful insights with respect to adaptive behaviours. The prisoner's dilemma game has attained this attention due to its succinct representation of the conflict between individually rational choices and choices that are for the better good. In summary, this game is played by pairs of agents, who simultaneously decide to either cooperate or defect, receiving a payoff associated with their pairwise interaction as follows: R for mutual cooperation, P for mutual defection, S for cooperating with a defector and T for successfully defecting against a cooperator. The dilemma holds when $T > R > P > S$ [132].

However, in many social scenarios that we may wish to model, agents are often afforded a third option — that of abstaining from the interaction. Incorporating this concept of abstention extends the prisoner's dilemma to a three-strategy game such as the optional prisoner's dilemma game (also known as the prisoner's dilemma with voluntary participation), where agents can not only cooperate or defect but can also choose to abstain from a game interaction, receiving the so-called loner's payoff (L). The dilemma is maintained when $T > R > L > P > S$ [147, 13]. There have been a number of recent studies exploring this type of game, which have found that abstention can lead to entirely different outcomes and eventually help cooperators

to avoid exploitation from defectors [81, 102, 137, 177, 111, 130, 18, 55, 38, 99, 79, 34, 188, 152, 67].

In addition to analysing the evolution of different strategies and different outcomes, previous work has also explored the effect of imposing spatial constraints on agent interactions. Traditionally, studies assume no such constraints and agents are free to interact with all other agents in well-mixed populations [9]. However, many models consider restricting interactions to neighbourhoods of agents on some pre-defined topology, i.e., the so-called network reciprocity mechanism, which has gained increasing attention for its role in supporting cooperative behaviour. These more expressive models include lattices, scale-free graphs, small-world graphs, cycle graphs, multi-layer networks and others [127, 125, 155, 94]. In this spatial mechanism, each agent is represented as a node in the network (graph) and is constrained to interact only with its neighbours, which are linked by edges in the network. Studies related to the network reciprocity mechanism constitute the field of spatial evolutionary game theory [116].

Although there has been a lot of research undertaken into spatial evolutionary game theory over the past number of years, the majority of this research involves the use of static networks in compulsory games such as the prisoner's dilemma game. Recent studies have considered coevolutionary spatial models adopting dynamic networks where both the game strategies and the network itself are subject to evolution, which, in fact, constitute a natural upgrade in the field for being more accurate in describing many social scenarios [93, 74, 20, 196, 164, 204, 49, 203]. For instance, in scenarios involving real biological networks and social networks, the number of individuals, their connections and the environment are often dynamic. Those studies have been successful in exploring the effects of coevolution in a compulsory game, where coevolution has been recognized as a key mechanism to

support cooperative behaviour [124]. However, there has been little success in accounting for the impact of abstention in such coevolutionary models. Thus, this thesis aims to expand the knowledge of evolutionary game theory by investigating the impact of dynamic networks on the evolution of cooperation on both compulsory and optional games.

Furthermore, it is known that in many situations involving voluntary participation, such as in human interactions, the use of abstention as a pure strategy may not be ideal to capture the social dilemma. In reality, depending on the context and the type of social relationships we are modelling, abstention can also mean laziness, shyness or lack of proactivity, and all those emotions, feelings or characteristics may exist within a certain range. For instance, in the context of a poll of a number of individuals, there might be some who vote and others who do not. In the latter case, considering all the non-voters as abstainers might be too simplistic. In reality, there might be some who abstain because they do not have a view at all and those who occasionally abstain from convenience, lack of interest or because of some external event. In this way, the present thesis also aims to extend the concept of abstaining to scenarios where abstention is seen as a probabilistic attribute instead of a pure strategy.

1.2 Open research questions

Despite the efforts to understand the conditions necessary for robust cooperation to emerge in coevolutionary spatial models, as well as the efforts to investigate the role of abstention in social dilemmas, the impact of the combination of those two trends in evolutionary game theory has not been well explored. Thus, in order to fill this gap in the literature, the following research questions will be investigated in this thesis.

Q1: What is the impact of abstention on the emergence of cooperation in coevolutionary spatial games?

This central research questions is investigated in Chapter 3. Considering the introduction of the coevolutionary optional prisoner's dilemma game (COPD), the coevolutionary prisoner's dilemma (CPD) and the traditional version of those games; the sub-questions include: What is the difference between the performance of the COPD compared to the CPD and the traditional version of those games? What is the impact of the link update rules and its properties on the emergence of cooperation? How many abstainers would be necessary to guarantee robust emergence of cooperation?

Q2: Can the phenomenon of cyclic competition be observed in a coevolutionary spatial game with voluntary participation?

This central research question is explored in Chapter 4. Sub-questions include: If cyclic competition cannot be observed, then why does it happen in the traditional case? Otherwise, what are the necessary conditions to break the observed scenarios of cyclic dominance?

Q3: What is the role of heterogeneity in coevolutionary spatial games?

Explored in Chapter 5, this central research question involves the following sub-question: Considering a weighted network, how do the link weights evolve over time?

Q4: Can abstention be modeled as a probability instead of a pure game strategy?

This central research question is explored in Chapter 6. Sub-questions include: What is the impact of a probabilistic abstention on the emergence of cooperation? What is the difference between this game and the traditional PD and OPD games?

Q5: Does mobility support the evolution of cooperation in the voluntary prisoner's dilemma game?

Investigated in Chapter 7, this central research question involves the following sub-questions: What is the role of dilution in a population of agents playing the VPD game? Can the cyclic dominance behaviour be observed in a scenario where agents are allowed to move?

1.3 Hypotheses

Considering the aforementioned central open research questions, this thesis aims to investigate the following hypotheses:

- *Hypothesis 1:* The emergence of cooperation is favoured in the presence of abstainers.
- *Hypothesis 2:* Coevolution of game strategies and network play a key role in the sustenance of biodiversity (coexistence) because it allows agents to also adapt the environment against exploitation from predatory strategies.
- *Hypothesis 3:* Coevolutionary spatial models increase the heterogeneity of states, which in turn induces the promotion of cooperation.

- *Hypothesis 4:* Probabilistic abstention enhances the mechanisms of network reciprocity by allowing cooperators to reduce the risk of being exposed to defectors.
- *Hypothesis 5:* Cooperation and cyclic dominance in the voluntary prisoner's dilemma game are biased by the use of the Fermi-Dirac distribution function

1.4 Background literature

This section includes a comprehensive review of the literature relating to game theory, evolutionary game theory, spatial game theory, and Monte Carlo simulations. Finally, the concept of abstention in game theory is discussed.

1.4.1 Game theory

Imagine that you and a colleague have been arrested under the suspicion of committing a crime. The prosecutor is sure that you both committed the crime, but due to the lack of evidence, she can only prove the case if she convinces you to betray your colleague. Thus, considering that you and your colleague are in separate rooms, she decides to make a deal with you both as follows:

- If you testify against your colleague and your colleague remains in silence, then you will go free and your colleague will get three years in prison.
- If you both testify against each other, then each will get two years in prison.
- If you both remain in silence, then each will get one year in prison.

Note that the only way for you to go free is betraying your colleague. However, if your colleague also decides to testify against you, then you both end up in the worst scenario. At the same time, keeping silent is advantageous only if you believe that your colleague will also keep silent. However, if you believe that your colleague will not testify against you, then it would be better for you to betray and go free. What would you do in this case?

This paradox was first proposed by Melvin Dresher and Merrill Flood in 1950, being later formalized as the *prisoner's dilemma* by Albert Tucker in the early 1950s [172, 54]. The mathematical solution for the paradox is also well-known, i.e., considering that you and your partner are in separated rooms and cannot negotiate a collaboration agreement, and considering that you have no reason to be nice to each other, the decision of betraying is always the best choice because it ensures that you will always be in the best scenario (you go free) or at least in a position that is not worse than your opponent (you both get the same time in prison).

As discussed by Dimand & Dimand [42], the so-called theory of strategic games has been studied since the 1710s when James Waldegrave proposed the earliest *min-max* solution to a game [54]. The *min-max* theorem, in turn, was only proved in 1928 by John von Neumann, which states that every finite, zero-sum and two-player game has an optimal mixed strategy [109, 45]. Another key contribution to the theorem was given by John Nash who extended von Neumann's work and proved that in a general sum game with two or more players, there has to be at least one strategy in which none of the players can benefit from changing their strategies, i.e., the so-called Nash equilibrium [107, 108]. The *min-max* theorem is recognized as the fundamental theorem of game theory, which has been established as a field of research only after John von Neumann's contributions to defining the principles of rational decision making [176], as well as for the works of John Nash in proposing that at least one Nash equilibrium will exist in any finite game [107]. Game theory, in turn, can be formally defined as the mathematical theory to model conflicting situations where the payoff of each player depends on the decisions made by the opponents.

The prisoner's dilemma and 2×2 games

The prisoner's dilemma game is the most well-known game in game theory and is still the most-often studied dilemma in the field for its simplicity and ability to capture the complexity of establishing cooperation among self-interested individuals. This game can be generalized as follows: two players have to simultaneously decide to either cooperate or defect; there are four payoffs corresponding to the pairwise interaction between the two players, i.e., reward for mutual cooperation (R), punishment for mutual defection (P), sucker's payoff (S) and temptation to defect (T).

		Prisoner 2	
		C	D
Prisoner 1	C	R,R	S,T
	D	T,S	P,P

TABLE 1.1: The prisoner's dilemma game payoff matrix. Each player can either cooperate (C) or defect (D). Decisions are made simultaneously, where R is the reward for mutual cooperation, S is the sucker's payoff, T is the temptation to defect and P is the punishment for mutual defection. The dilemma holds when $T > R > P > S$.

As shown in Table 1.1 for the generalized payoff matrix of the prisoner's dilemma game, the dilemma is maintained when $T > R > P > S$ [132]. However, since Nowak & May's seminal work [116], studies have also been adopting a less-generalized parametrization where $R = 1$, $P = S = 0$ and $2 > T > 1$, which is usually referred as the weak version of the prisoner's dilemma. In this weak version, P can be equal to S without destroying the nature of the dilemma, i.e., the dilemma is also maintained when $T > R > P \geq S$.

Despite being the standard game in the field, the prisoner's dilemma game is only one example in the class of the so-called 2×2 games, i.e., the two-player games which can be defined with a 2×2 payoff matrix. Other well-known games include the snow-drift dilemma [144] and the hawk-dove

game [141], which, in fact, differs from the prisoner's dilemma only in the payoff constraints, i.e., $T > R > S > P$ [66]. Those games have been also investigated in the iterated form, i.e., where the game is played repeated times with the same opponent [8].

1.4.2 Evolutionary game theory

Analogous to the discussion in Section 1.4.1 for game theory, in order to introduce evolutionary game theory, let's start with a simple example. Inspired by the example provided by Easley & Kleinberg for beetles in their book [48], here we consider a scenario where a colony of ants compete for food. In this colony, there are two types of ants, L and S, which fundamentally differ by their body size. The competition always occurs in a pairwise interaction and the possible scenarios are as follows: the type L is larger and more aggressive; hence it always has an advantage when competing with the type S. When two ants of the same species compete, both get the same amount of food. However, given the aggressivity of the type L ants, they spend too much energy in the competition and end up carrying less food than the case of a competition between two ants of type S. This scenario can be described as a game in game theory, where the payoff matrix is defined in Table 1.2.

		Ant 2	
		Small	Large
Ant 1	Small	4,4	1,7
	Large	7,1	2,2

TABLE 1.2: The payoff matrix of a competing game between ants with different body sizes.

Despite resembling the prisoner's dilemma game for having only one Nash equilibrium, and the stag-hunt game for the social coordination analogy [156], this game differs because the ants cannot choose which body size (strategy) to be in each game interaction. Thus, traditional game theory and

Nash equilibrium fail in providing insights to those scenarios because they consider that the agent has a choice. However, in many real-world scenarios, the agent's fitness is hard-wired to their genes [48]. Thus, started by John Maynard Smith and G. R. Price [141, 140], *evolutionary game theory* aims to solve this problem by applying and combining elementary ideas from evolutionary biology with game theory. In evolutionary game theory, the changing strategies of individuals occur due to an evolutionary process over long time scales. The analogous notion of Nash Equilibrium [107, 108] for evolutionary game theory is called *evolutionary stable strategy* (ESS), which refers to a strategy that cannot be replaced by a competing strategy through natural selection [142]. Following the work of Smith & Price, another remarkable contribution to the field was achieved by Peter Taylor & Leo Jonker [167] for introducing *replicator dynamics*, which provides a simple mathematical model to describe the rate at which strategies evolve over time. Evolutionary dynamics in evolutionary game theory have been systematically explored over the past decades by several researchers [65, 186, 72, 71, 113].

In the context of biological interactions, evolutionary game theory has also been widely explored as a framework to investigate the evolution of cooperation, which remains as one of the main puzzles in natural selection. Namely, in evolutionary terms, altruism occurs when an individual reduces its own performance to improve the performance of another individual. Highly influential studies in this domain include the work of Robert Axelrod to explore the mechanism of direct reciprocity in the iterated (or repeated) prisoner's dilemma game [8, 9], as well as the work from Martin Nowak who argues that the evolution of cooperation is subject to five distinct types of mechanisms, i.e., kin selection, indirect reciprocity, direct reciprocity, group selection and network reciprocity [114].

Spatial evolutionary game theory

Since the dawn of evolutionary game theory, the standard models in evolutionary dynamics have been developed under the assumption of a well-mixed population where agents (individuals) can interact with all the other agents in the population (i.e., unstructured population). However, in real-world scenarios, populations are usually not well-mixed nor unstructured. In this manner, started by Nowak & May in 1992 [116], concepts of graph theory to consider structured populations in a network form have been extensively applied to evolutionary game theory, giving birth to the so-called *spatial evolutionary game theory*. In those models, each agent is represented as a node in the network and is constrained to interact only with its immediate neighbours, which are linked by edges in the network [94].

Research in spatial evolutionary game theory (SEGT) aims to investigate the effects of spatial structures on the evolution of cooperation [120] by considering that, in structured populations, cooperators can help or protect each other from exploitation by forming cooperative network clusters. This, in turn, constitutes the network reciprocity mechanism, which is one of the five mechanisms for the evolution of cooperation proposed by Nowak in 2006 [114].

Over the last decades, SEGt has been applied to a number of different topologies including scale-free graphs [160, 190, 136], small-world graphs [36, 53, 1], cycle graphs [4], multilayer networks [15, 180, 57] and bipartite graphs [129, 63] which have a considerable impact on the evolution of cooperation, and also favour the formation of different patterns and phenomena [125, 155]. However, toroidal lattice grids with *Moore* or *von Neumann* neighbourhoods are still the most often studied topology in the domain. In general, toroidal lattices are used to ensure that all agents have

the same number of neighbours. In the von Neumann neighbourhood, each agent interacts with the four immediate neighbours (i.e., north, east, south and west), while in the Moore neighbourhood, agents can also interact with the other four intermediate directions (i.e., northeast, southeast, southwest and northwest). The difference is that the von Neumann neighbourhood takes twice as long to cover the same distance diagonally, which in turn affects the speed in which a strategy can spread in the lattice [66].

Monte Carlo simulation

Studies in graph theory have been providing mathematical background for network analysis of different topologies [3, 16, 11]. However, despite those advances in the field, when considering graphs in the context of evolutionary game theory (Subsection 1.4.2), there is still a lack of generalized mathematical theories which can solve and predict all the outcomes that could arise from those models. Therefore, computer simulation is still the leading and most reliable method to analyse and explore the emergence of cooperation in structured populations. In particular, Monte Carlo methods are the most important class of computational algorithms used to solve problems in spatial evolutionary game theory [14]. Other research efforts in the domain include the use of mean-field pair approximation [149] and statistical physics techniques to give insights regarding the evolution of cooperation in spatially organized populations [155, 127].

The standard Monte Carlo (MC) approach in SEGT can be summarized as follows. At each Monte Carlo (MC) time step, each agent (x) is selected once on average to update its strategy. Thus, in one MC step, N agents are randomly chosen to perform the subsequent procedures: the agent x accumulates the utility U_x by playing the evolutionary game with all its nearest active (non-empty) neighbours (Ω_x), selects one of them at random (i.e., the

agent y , which also acquires its utility U_y), and considers copying its strategy with a probability given by the Fermi-Dirac distribution function:

$$W = \frac{1}{1 + e^{(U_x - U_y)/K}} \quad (1.1)$$

where K characterizes the amplitude noise to allow irrational decisions. In the $K \rightarrow 0$ limit, the agent x copies the strategy of y if $U_y > U_x$. On the other hand, in the $K \rightarrow \infty$ limit, the utilities do not affect the agent's decision who will swap their strategies as per the flip of a coin [155, 151].

1.4.3 Abstention in game theory

Abstention, or voluntary/optional participation, has been studied in the context of the prisoner's dilemma since Batali and Kitcher, in their seminal work [13], first introduced the optional variant of the game. They proposed the opt-out or "loner's" strategy, in which agents could choose to abstain from playing the game, as a third option, in order to avoid cooperating with known defectors. Using a combination of mathematical analysis and simulations, they found that populations who played the optional games could find routes from states of low cooperation to high states of cooperation. Subsequently, as this extension has grown in popularity and renown, optional participation has been successfully incorporated into models alongside other cooperation enhancing mechanisms such as punishment [68] and reputation [121, 56], and has been applied to probabilistic models [190].

The study of optional participation can be broadly separated into two approaches: one that directly incorporates abstention into the traditional prisoner's dilemma game (the loner's strategy), and another known as conditional cooperation. Models that incorporate the loner's strategy treat the

option to abstain as an alternative strategy for agents to employ [13, 79], separate to the option to cooperate or defect. These models tend to be more grounded in mathematical models with less of an emphasis on experimental simulations, which often-times have been shown to produce unexpected results [67]. On the other hand, conditional cooperation models [7, 85, 77], also known as conditional disassociation, incorporate abstention into cooperation strategies. These models lend themselves more easily to Axelrod-style tournaments [9]. They tend to focus on exit options or partner-leaving mechanisms, and often lack a spatial aspect, which has since been shown to increase the number of abstainer strategies thus increasing the chances of cooperation evolving [79].

Optional participation has been also studied in the context of the public goods games. First introduced by Hauert & Szabó [67], they considered a spatially extended public goods game (PGG), where a population of N agents are arranged and interact on a variety of different geometries, including a regular lattice. Three pure strategies (cooperate, defect and abstain) are investigated using an evolutionary approach. Results showed that the spatial organisation of strategies affected the evolution of cooperation, and in addition, they found that the existence of abstainers was advantageous to cooperators, because they were protected against exploitation.

Voluntary prisoner's dilemma game

The voluntary prisoner's dilemma game, also known as the optional prisoner's dilemma [13] and prisoner's dilemma game with voluntary participation [147], is an extension of the traditional prisoner's dilemma game to include the concept of abstention/optional/voluntary participation. In this game, players can not only cooperate or defect but also abstain from a game interaction, obtaining the so-called loner's payoff (L). The loner's payoff is

awarded to both participants if one or both abstain from the interaction (Table 1.3).

		Prisoner 2		
		C	D	A
Prisoner 1	C	R,R	S,T	L
	D	T,S	P,P	L
	A	L	L	L

TABLE 1.3: The voluntary prisoner's dilemma game payoff matrix. Each player can either cooperate (C), defect (D) or abstain (A). Decisions are made simultaneously, where R is the reward for mutual cooperation, S is the sucker's payoff, T is the temptation to defect and P is the punishment for mutual defection, and L is the loner's payoff. The dilemma holds when

$$T > R > L > P > S.$$

The value of L is set such that: (1) it is not greater than R , otherwise the advantage of not playing will be sufficiently large to ensure that players will always abstain and (2) it is greater than P , otherwise there are no benefits to abstaining. In this way, abstention is defined as a strategy which performs better than groups of defectors but worse than groups of mutually cooperating strategies [67]. The dilemma holds when $T > R > L > P > S$, and similarly to the traditional prisoner's dilemma game, there is also a variant of this game in the weak form where $T > R > L > P \geq S$.

1.5 Research contributions

The most significant contributions of the present research are outlined below.

These are discussed in further detail in Chapter 8.

- Development of *Evoplex*, a free and open-source agent-based modelling platform for spatial evolutionary game theory distributed under a permissive license (Chapter 2).
- Designing, development and extensive analysis of a novel coevolutionary model to account for the concept of abstention on dynamic networks, i.e., a model in which not only the game strategies (i.e., cooperate, defect and abstain) but also the network evolves over time (Chapter 3).
- Extensive analysis on the effects of the concept of abstention on the emergence of cyclic dominance behaviour (Chapter 4).
- Advances on the understanding of the role of heterogeneity on the emergence of cooperation in coevolutionary spatial games (Chapter 5).
- Designing, development and analysis of a novel model to extend the concept of abstention to a probabilistic feature, i.e, abstention seen as a probability instead of a pure strategy (Chapter 6).
- Paradigm shift on the effects of abstention on evolutionary games by revisiting the standard methodology commonly employed in the context of the prisoner's dilemma game with voluntary participation (Chapter 7).
- Designing, development and analysis of a novel model to account for agent mobility and the concept of abstention in a coevolutionary fashion where agents can not only update their strategies but also their positions over time (Chapter 7).

1.6 Publications

A number of peer-reviewed papers were published and presented in prestigious conferences and journals in the domain.

- **Cardinot, M.**, Gibbons, M., O’Riordan, C., and Griffith, J. (2016). “Simulation of an optional strategy in the prisoner’s dilemma in spatial and non-spatial environments”. In *International Conference on Simulation of Adaptive Behavior*, pp. 145-156, vol. 9825, Springer. (Parts of this publication appear in Chapter 1).
- **Cardinot, M.**, O’Riordan, C., Griffith, J., and Perc, M. (2019). “Evoplex: A platform for agent-based modeling on networks”. *SoftwareX*, pp. 199-204, vol. 9, Springer (Chapter 2).
- **Cardinot, M.**, O’Riordan, C., and Griffith, J. (2019). “The impact of coevolution and abstention on the emergence of cooperation”. In *Computational Intelligence. Studies in Computer Science*, pp. 105-122, vol. 792. Springer (Chapter 3).
- **Cardinot, M.**, Griffith, J., and O’Riordan, C. (2016). “Cyclic dominance in the spatial coevolutionary optional prisoner’s dilemma game”. In *24th Irish Conference on Artificial Intelligence and Cognitive Science*, pp. 33-44, Dublin, Ireland (Chapter 4).
- **Cardinot, M.**, Griffith, J., and O’Riordan, C. (2018). “A further analysis of the role of heterogeneity in coevolutionary spatial games”. *Physica A: Statistical Mechanics and its Applications*, pp. 116-124, vol. 493, Elsevier (Chapter 5).
- **Cardinot, M.**, Griffith, J., O’Riordan, C., and Perc, M. (2018). “Cooperation in the spatial prisoner’s dilemma game with probabilistic abstention”. *Scientific reports*, n. 14531, vol. 8, Nature (Chapter 6).

- **Cardinot, M., O’Riordan, C., Griffith, J., and Szolnoki, A. (2019).** “Mobility restores the mechanism which supports cooperation in the voluntary prisoner’s dilemma game”. *New Journal Physics*, n. 073038, vol. 21, IOP (Chapter 7).
- **Cardinot, M., O’Riordan, C., and Griffith, J. (2016).** “The optional prisoner’s dilemma in a spatial environment: coevolving game strategy and link weights”. In *8th International Joint Conference on Computational Intelligence*, pp. 83-93, vol. 1 ECTA, INSTICC, ScitePress.

1.7 Thesis structure

This section provides an overview of each chapter in this thesis. The present chapter includes the justification for this research, the objectives, the hypothesis to be evaluated in this research, and a comprehensive background literature review on game theory and evolutionary game theory. The rest of the thesis is organized as follows:

- Chapter 2 includes a description of the software developed to perform the experiments of this research.
- Chapter 3 presents the experiments considering the coevolutionary optional games, where either the agents' strategies (optional game) and the network (dynamic network) evolves over time.
- Chapter 4 discusses the phenomena of cyclic dominance in the prisoner's dilemma game with voluntary participation.
- Chapter 5 discusses the role of heterogeneity in coevolutionary spatial games.
- Chapter 6 discusses the effects of the concept of optionality when the agents are placed on a diluted network.
- Chapter 7 explores the concept of abstention itself, extending it as an extra attribute of each agent, and not as a pure strategy, i.e., presents the prisoner's dilemma game with probabilistic abstention.
- Chapter 8 provides a summary of the completed work, revisits the hypotheses and outlines a number of possible directions for future work within the domain.

Chapter 2

Evoplex: A platform for agent-based modelling on networks

The work outlined in this chapter was published in:

Cardinot, M., O’Riordan, C., Griffith, J., and Perc, M. (2019). “Evoplex: A platform for agent-based modeling on networks”. *SoftwareX*, pp. 199-204, vol. 9, Springer.

Abstract

Agent-based modelling and network science have been used extensively to advance our understanding of emergent collective behavior in systems that are composed of a large number of simple interacting individuals or agents. With the increasing availability of high computational power in affordable personal computers, dedicated efforts to develop multi-threaded, scalable and easy-to-use software for agent-based simulations are needed more than ever. Evoplex meets this need by providing a fast, robust and extensible platform for developing agent-based models and multi-agent systems on networks. Each agent is represented as a node and interacts with its neighbors, as defined by the network structure. Evoplex is ideal for modelling complex systems, for example in evolutionary game theory and computational social science. In Evoplex, the models are not coupled to the execution parameters or the visualization tools, and there is a user-friendly graphical interface which makes it easy for all users, ranging from newcomers to experienced, to create, analyze, replicate and reproduce the experiments.

2.1 Motivation and significance

Agent-based modelling (ABM) has been used as a framework to simulate complex adaptive systems (CAS) in a wide range of domains such as life sciences, ecology and social sciences [26, 51, 61, 17, 123, 88, 33]. Those systems are composed of a number of interacting agents each of whom have a defined set of attributes and can exhibit specific behaviors based on their interactions with the environment and the other agents [105]. Research in this field usually aims to explore how small changes in individual behavior can both affect and promote collective behavior throughout the system [145, 100, 58].

Given the flexibility of the ABM approach and the increasing computing power of cheap personal computers, efforts to develop reusable, flexible, multi-threaded, scalable and user-friendly software are more than ever required by the scientific community. However, despite the high number of existing ABM toolkits and platforms available [110], due to the heterogeneity and diversity of the areas of research and application domains, most researchers still prefer to implement individual and domain-specific, specialised software from scratch, which is usually not publicly released. Many researchers write MATLAB or *Mathematica* based scripts which, although being complete and well-known scientific platforms, are neither free nor open-source, which therefore reduces the transparency and re-usability of the developed models [128].

In fact, implementing a highly specialized solution from scratch is time-consuming, complex and error-prone. Many projects try to overcome this by implementing a toolkit or platform for a general purpose problem domain. For instance, some projects such as NetLogo [169] and GAMA [60] succeed in providing generic and reusable software; however, they require the user to learn their specific programming language. A wide range of

the ABM solutions including MASON [98] and Repast [112] are written in Java [170], which make them cross-platform and usually faster than some Python or JavaScript alternatives like Mesa [103] and AgentBase [187]. However, they usually require modellers to be highly proficient in the language or they have critical scalability issues. Overall, the main issues with some existing software include the use of old/deprecated technologies, not following state of the art in software engineering, developing single-threaded applications and not being community-friendly.

Furthermore, despite being a common strategy in the field, many ABM projects start with the promising and challenging intention of developing powerful software to meet any requirement in the field, from simple cellular automaton models to complex and realistic geographical information science (GIS) models. Unfortunately, this promising approach usually results in making the code base very complex and hard to both optimise and maintain. In reality, given the small size of the development teams, there is no best strategy for all scenarios, and the user choice is usually guided by their familiarity with the languages or technologies used in the software. In this way, defining a clear and focused scope can help solve those issues.

Thus, in this chapter we present Evoplex, a cross-platform, free and open-source software which aims to mitigate the issues outlined by providing a fast and fully modular ABM platform for implementing and analysing models which impose an explicit graph-theoretical approach, i.e., the agents are represented as nodes (or vertices) and edges (or links) represent connections between agents.

2.2 Software description

Evoplex is a fast, robust and extensible platform for developing agent-based models and multi-agent systems on networks. Here, each agent is represented as a node in the network and is constrained to interact only with its neighbors, which are linked by edges in the network. Although not limited to a particular domain, Evoplex is ideal for tackling problems associated with evolutionary computation, complex adaptive systems, evolutionary game theory and cellular automata.

As shown in Figure 2.1, the Evoplex workflow is very straightforward and intuitive. The engine processes *projects* as inputs. A **project** is a plain table (csv file) where the experiments are listed along the rows, and the inputs to each experiment are placed along the columns. An **experiment** is defined by a set of parameter settings (inputs) necessary to perform one **trial** (simulation) and (optionally) the required data outputs, which can be the result of some statistical function and/or the state of the set of nodes/edges for each time step. Each experiment can run for one or more trials, i.e., repeat the same experiment using different pseudo-random generator seeds. The strategy of having the projects defined in plain text files aims to make it easier for users to replicate and reproduce their results. Furthermore, it allows newcomers to interact with the models without requiring any programming skills.

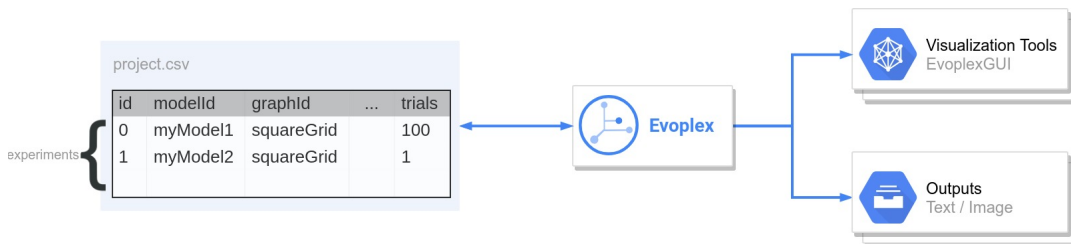


FIGURE 2.1: Simplified overview of the user workflow.

We provide a user-friendly and interactive graphical user interface

(GUI) to allow for creating, opening, running and saving projects. Also, the GUI provides many useful tools including interactive graph and grid views. Moreover, Evoplex allows several experiments to run at the same time. These are automatically distributed in parallel across multiple threads.

2.2.1 Software architecture

Evoplex is simple, user-friendly and was built with performance in mind from the start. It is cross-platform and runs on every major platform, i.e., Linux, Microsoft Windows, and MacOS. Evoplex is developed in modern C++ 14, based on Qt, which is one of the most popular and successful C++ frameworks available to date. Moreover, Evoplex includes CMake scripts to ease the compilation and setup from its source code.

The Evoplex application bundles three main open-source components: the *EvoplexCore* library, the *EvoplexGUI* library and a collection of plugins (example models and graph generators). The *EvoplexCore* library is available under the Apache 2.0 License, which is permissive, free and commercially friendly. The *EvoplexGUI* library is available under the GNU GPLv3 license, which is also free but is conditioned on making the source code of licensed works and modifications available.

Following a common practice in software engineering, the Evoplex architecture is guided by a fully modular approach. The core component, *EvoplexCore*, splits its implementation into both private and public Application Programming Interfaces (APIs). The private API is intended for internal use only and is where the simulations will actually occur; it is responsible for managing the I/O operation, parsing inputs, handling the CPU threads and memory, loading and creating instances of plugins and others. The public

API exposes all the tools and services needed to develop a plugin, which can be either a model or a graph generator.

Figure 2.2 shows a simplified overview of the overall software architecture, which is composed of four major layers: the kernel (i.e., *EvoplexCore* library), the plugins, the data and the applications layers. The current version of the Evoplex application layer includes *EvoplexGUI*, which implements a graphical user interface on top of *EvoplexCore* (kernel) to provide a number of interactive and user-friendly tools. Note that as the kernel is completely independent of the applications layer, Evoplex can be distributed with different user-interfaces but share the same engine (kernel). For instance, one may want to implement an *EvoplexCLI* application to perform simulations via command-line, or an *EvoplexWeb* application to provide visualization tools on a web browser.

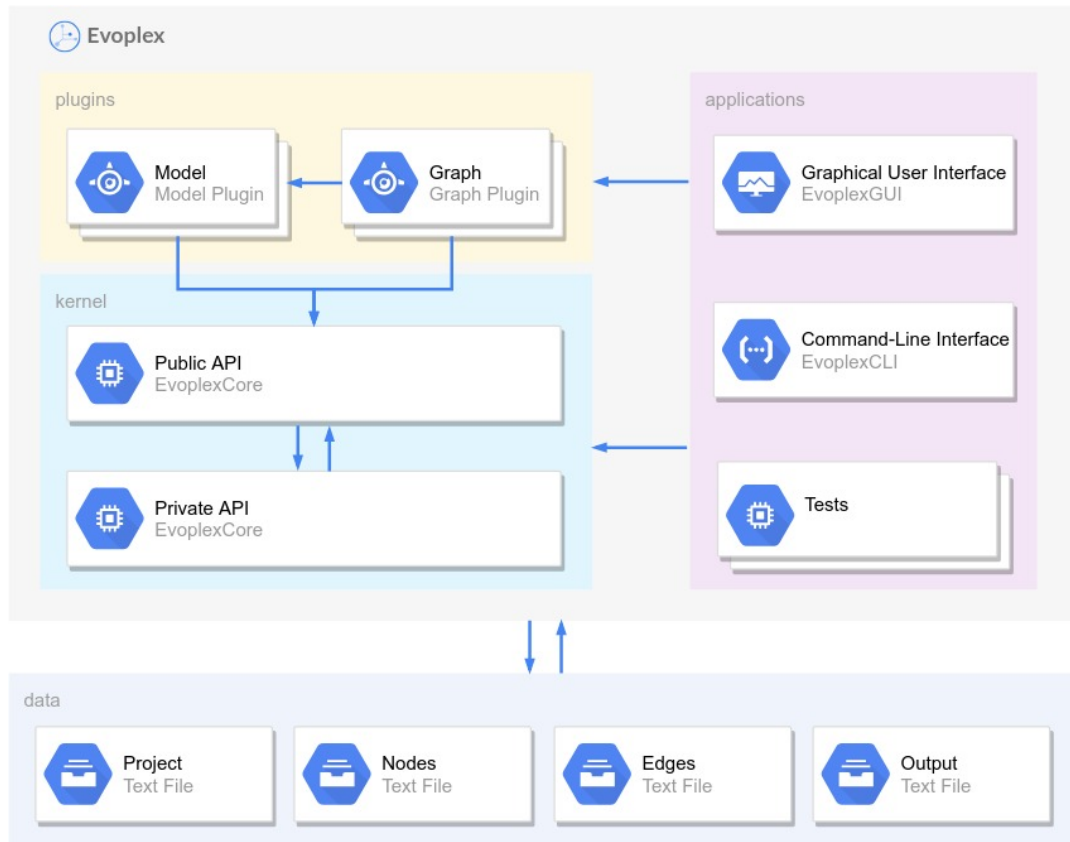


FIGURE 2.2: Simplified illustration of the software architecture.

In Evoplex, every model or graph is a plugin and is compiled independently of the main application. The creation of plugins is very straightforward and requires a very basic knowledge of C++. Given the Evoplex approach of not coupling the visualization tools nor inputs/outputs to the model, the models' code is usually very simple and short. We provide a few examples of plugins of easy reuse and customization¹. In summary, a plugin comprises four files: *CMakeLists.txt* which does not need to be changed by the modeller and is just a CMake script to ease the compilation process and make it portable across different compilers and IDEs, the *plugin.cpp* (source) and *plugin.h* (header) files where the modeller implements the model's algorithm, and *metadata.json* which holds the definition of all the attributes of the model.

Moreover, Evoplex uses automated Continuous Integration (CI) to make sure that the code base works as expected and to allow early detection of problems. After every new commit, the CI system automatically builds Evoplex from the source code, executes regression tests, unit-tests and static code analysis across a range of different versions/distributions of the supported platforms, i.e., Linux, Microsoft Windows, and MacOS.

2.2.2 Software functionalities

The Evoplex application comes with a user-friendly and intuitive GUI that allows loading and unloading of plugins at runtime and provides a bunch of widgets and tools to allow for the creation and running of experiments and for analysing (or visualizing) their outputs. The main tools and widgets are described below:

¹<https://evoplex.org/docs/example-plugins>

- *Project View*: As shown in Figure 2.3, when opening a project, all experiments are listed in a table which is dynamic and customizable and allows running, pausing and queuing multiple experiments at the same time. When running the experiments, Evoplex automatically manages the available resources to run them as fast as possible (in parallel) and use as little memory as possible.
- *Experiment Designer*: This widget is displayed beside the *Project View* in Figure 2.3 and allows creating, removing and editing of experiments.
- *Nodes Generator*: This tool can be accessed in the *Experiment Designer* and provides handy functions to ease the generation of the initial set of nodes.
- *Experiment View*: This widget is opened when the user double-clicks on an experiment in the *Project View*. It allows for the opening of multiple visualization tools at the same time, which can be set to show different trials of the same experiment. For instance, given an experiment with a stochastic model which runs for 10 trials; the user may want to visualize the behavior of the trials side by side to investigate the effects of randomness over time.
- *Graph/Grid View*: Evoplex provides both graph (nodes and edges — Figure 2.4) and grid (cells) views. Those views allow zooming in and out, exporting nodes/edges as a text file, taking screenshots, selecting a node to inspect and change the state of its attributes and others. Also, it allows changing the nodes/edges size and choosing which attribute and colormap will be represented in the nodes/edges.

Differing from most of the other ABM solutions (e.g., NetLogo [169], MASON [98] and GAMA [60]), in Evoplex, the widgets are not statically coupled to the model plugin. That is, the model plugin only defines the entities'

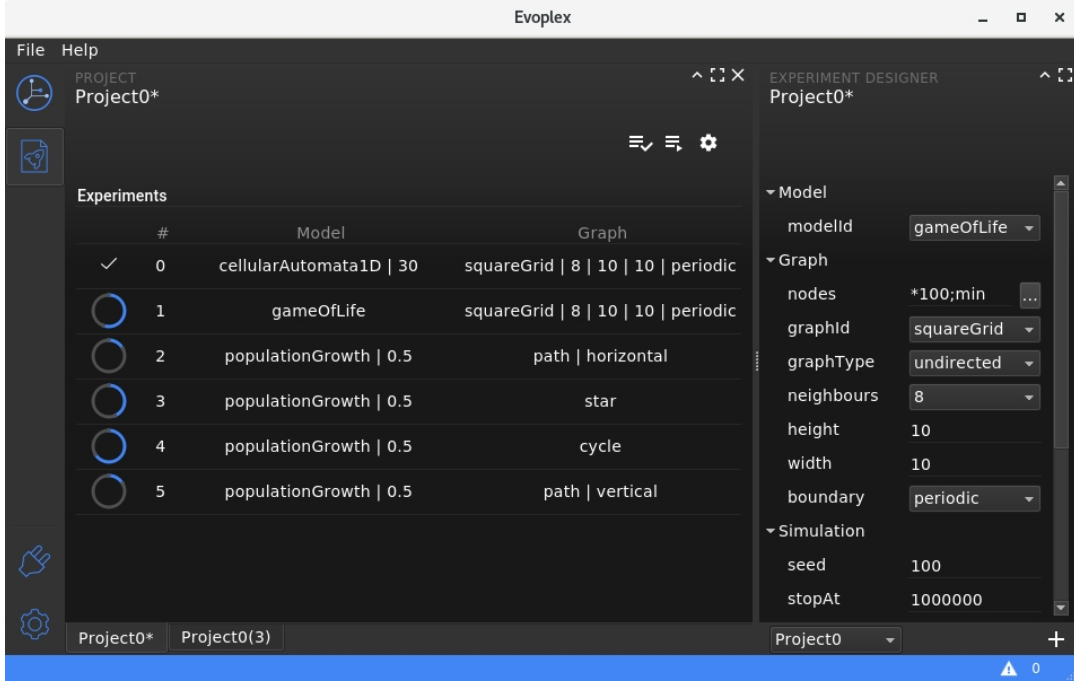


FIGURE 2.3: Screenshot of Evoplex 0.2.1 showing the *Project View* and the *Experiment Designer* tools.

attributes and implements the algorithm to describe the nodes' (agents) behavior for each time step. Then, at runtime and not requiring any programming skill, the users have the freedom to decide which widgets they want to open and where they want to place them. Also, all widgets can be detached from the main window, enabling users to open different views in multiple monitors or attach them at different positions and sizes in the screen.

2.3 Illustrative examples

In order to illustrate the use of Evoplex, we consider an implementation of the widely known model of a spatial prisoner's dilemma (PD) game proposed by Nowak & May in 1992 [116]. In the PD game, agents can be either cooperators or defectors, and receive a fixed payoff based on a pairwise interaction. That is, given two agents, if both are cooperators, both get a reward

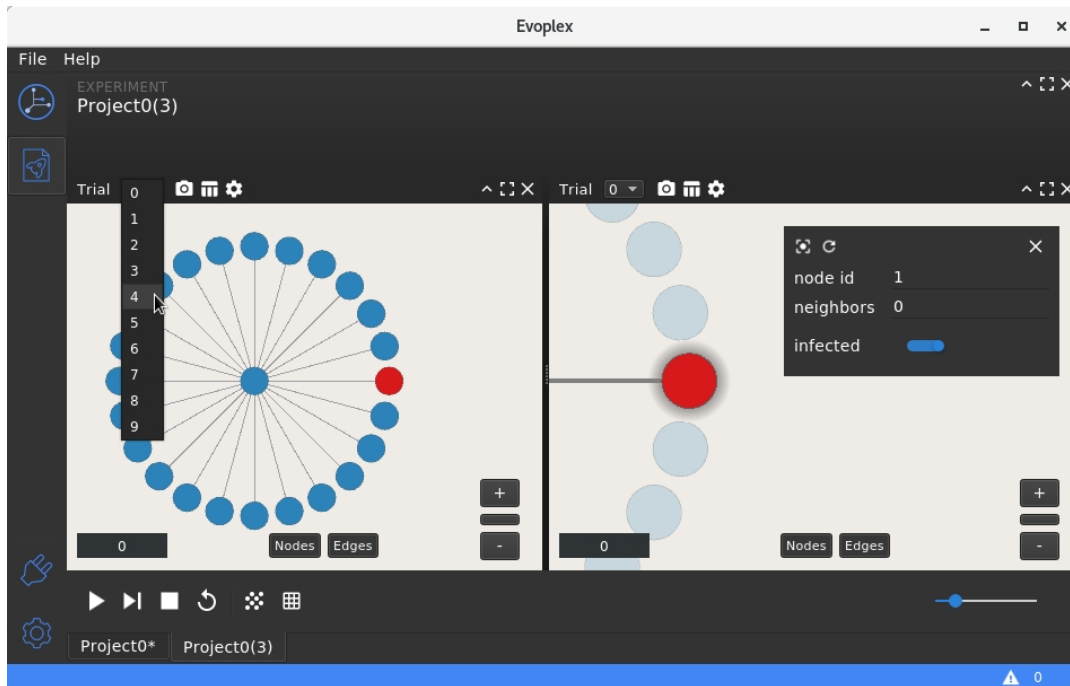


FIGURE 2.4: Screenshot of Evoplex 0.2.1 showing the *Experiment View* docking two instances of the *Graph View* at different positions.

$R = 1$; if both are defectors, both get a punishment $P = 0$; if a cooperator plays with a defector, the cooperator receives $S = 0$, and the defector receives T (temptation to defect) [117].

In this model, agents (nodes) are placed in a square grid, where, in each round: every node accumulates the payoff obtained by playing the PD game with all its immediate neighbors and itself; then, each agent copies the strategy of the best performing agent in its neighborhood, including itself. Note that the model's source code is also freely available online² under the MIT License terms.

Figure 2.5 shows a screenshot of an experiment created with the *Experiment Designer* tool, using an implementation of the PD model in Evoplex. To reproduce this output, run the experiment for one step, open the *Grid View* and place a single defector ($strategy = 1$) in the middle of the grid. Then, when running the experiment for more steps, it is possible to observe the

²<https://github.com/evoplex/model-prisonersDilemma>

emergence of chaotically changing spatial patterns as reported by Nowak & May [116].

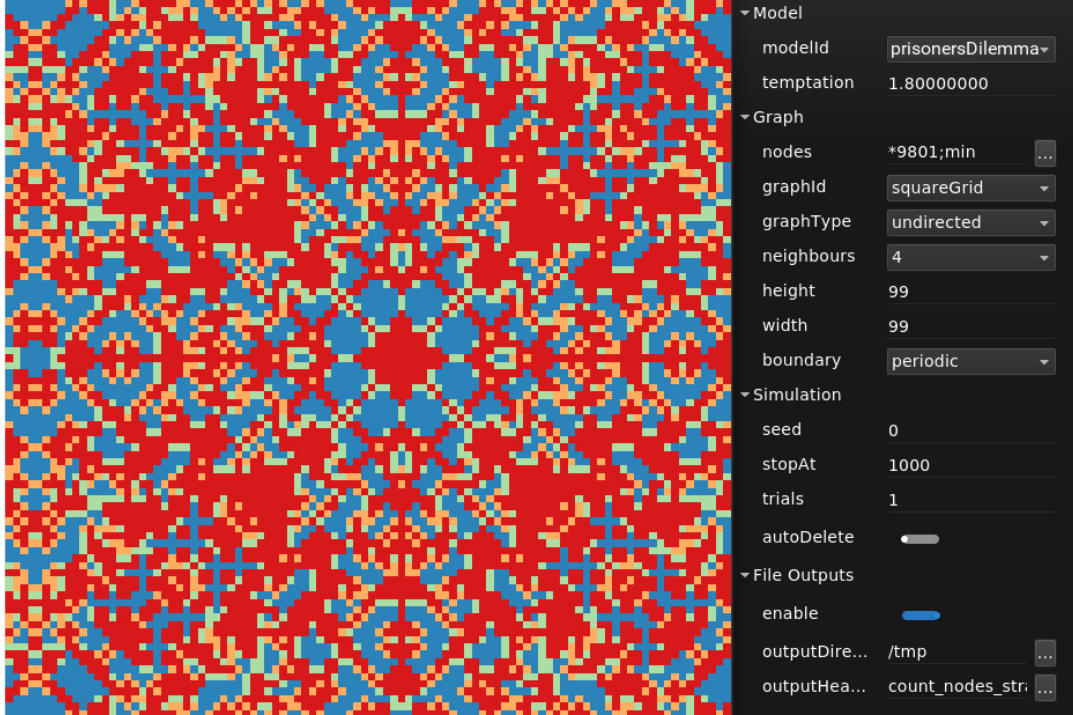


FIGURE 2.5: In this experiment, the model (*prisonersDilemma*) is set with a *temptation to defect* equal to 1.8; the graph is initialised with a single defector (*strategy* = 1) at the centre of a 99×99 *squareGrid* with periodic boundary conditions, fully populated with cooperators (*strategy* = 0), undirected edges and von Neumann neighborhood; the simulation is fed with a pseudo-random generator seed equal to 0 (which does not make any difference in this fully deterministic model), and is set to run for a maximum of 1000 time steps for only one trial; finally, it also stores the frequency of each type of strategy over time. In the *Grid View*, the colors blue, red, green and yellow corresponds to cooperators, defectors, new cooperators and new defectors respectively.

2.4 Impact

Evoplex is intended to address research whose methodology comprises a simulation-based approach to evolve outcomes of populations of autonomous and interacting agents. It has been used to support research in a number of areas, including spatial game theory and evolutionary game

theory [30, 26, 28]. In those scenarios, agents are described in terms of graph theory, i.e., a graph (network) consisting of a set of nodes (agents) and edges (agents' connections).

Despite having a few options of agent-based modelling (ABM) software available, none of them are really suitable for this area of research. Beyond the issues mentioned in Section 2.1, most of the existing simulators have very limited performance and are unable to handle the complexity of the models which are investigated at present, e.g., coevolutionary models with a large number of agents. Thus, one of the main impacts and contributions of Evoplex to this field of research is to provide an easy-to-use and high-performance platform for simulating large-scale experiments.

Moreover, another recurring issue with existing ABM software is that they are designed to run and analyze one experiment at a time. However, research in the field usually needs to explore the outcomes of large populations for a wide range of parameter settings, which in many cases require many Monte Carlo steps to converge. In this case, the user needs to modify the model's source code or write a script on top of it to automate the execution of the experiments, which will usually run in a single thread, one at a time. Some projects like FLAME [89] and OpenMOLE [134] succeeded in allowing efficient parallel processing, but their use and configuration are not straightforward. Thus, in those cases, we observed that for any small interaction with the model, the user ends up having to change the code/script back and forth very often, which is both error prone and difficult for non-experienced users.

Evoplex changes the paradigm of ABM for graphs by allowing nodes and edges to be independent entities. Thus, given a set of nodes (agents), the

user can easily investigate how changes in the topology may affect the population's behavior (and vice versa) without touching the source code or changing the model. Also, the robust and multi-threaded engine combined with the user-friendly GUI makes it easier for users to share, reproduce, replicate and analyze the experiments. Evoplex is free, non-profit and is fully open-source with a permissive license, allowing for both commercial and academic use.

2.5 Conclusions

We have presented Evoplex, a flexible, fast and multi-threaded platform for agent-based modelling imposing an explicit graph-theoretical approach. We discussed that, different to other software, in Evoplex, the models are not coupled to the execution parameters nor the visualization tools. Also, it provides a user-friendly GUI which makes it easy for all users, ranging from newcomers to experienced, to create, analyze, replicate and reproduce experiments. As an open-source project, we encourage users to provide feedback, share models and contribute to improving the software. Evoplex is an ever-evolving project, and future work will involve adding support for multilayer networks, as well as implementing more plugins, and developing more visualization widgets for the GUI.

Chapter 3

The Impact of Coevolution and Abstention on the Emergence of Cooperation

The work outlined in this chapter was published in:

Cardinot, M., O’Riordan, C., and Griffith, J. (2018). “The impact of coevolution and abstention on the emergence of cooperation”. In *Computational Intelligence. Studies in Computer Science*, pp. 105-122, vol. 792. Springer.

Abstract

This chapter explores the coevolutionary optional prisoner's dilemma (COPD) game, which is a simple model to coevolve game strategy and link weights of agents playing the optional prisoner's dilemma game, which is also known as the prisoner's dilemma with voluntary participation. A number of Monte Carlo simulations are performed to investigate the impacts of the COPD game on the emergence of cooperation. Results show that the coevolutionary rules enable cooperators to survive and even dominate, with the presence of abstainers in the population playing a key role in the protection of cooperators against exploitation from defectors. We observe that in adverse conditions such as when the initial population of abstainers is too scarce/abundant, or when the temptation to defect is very high, cooperation has no chance of emerging. However, when the simple coevolutionary rules are applied, cooperators flourish.

3.1 Introduction

Evolutionary game theory in spatial environments has attracted much interest from researchers who seek to understand cooperative behaviour among rational individuals in complex environments. Many models have considered the scenarios where participant's interactions are constrained by particular graph topologies, such as lattices [148, 116], small-world graphs [36, 53], scale-free graphs [160, 190] and, bipartite graphs [63]. It has been shown that the spatial organisation of strategies on these topologies affects the evolution of cooperation [32].

The prisoner's dilemma (PD) game remains one of the most studied games in evolutionary game theory as it provides a simple and powerful framework to illustrate the conflicts inherent in the formation of cooperation. In addition, some extensions of the PD game, such as the optional prisoner's dilemma (OPD) game, have been studied in an effort to investigate how levels of cooperation can be increased. In the OPD game, participants are afforded a third option — that of abstaining and not playing and thus obtaining the loner's payoff (L). Incorporating this concept of abstention leads to a three-strategy game where participants can choose to cooperate, defect or abstain from a game interaction.

The vast majority of the spatial models in previous work have used static and unweighted networks. However, in many social scenarios that we wish to model, such as social networks and real biological networks, the number of individuals, their connections and environment are often dynamic. Thus, recent studies have also investigated the effects of evolutionary games played on dynamically weighted networks [74, 183, 20, 156, 204, 203] where it has been shown that the coevolution of both networks and game strategies can play a key role in resolving social dilemmas in a more realistic

scenario.

In this chapter we explore the coevolutionary optional prisoner's dilemma (COPD) game, which is a simple coevolutionary spatial model where both the game strategies and the link weights between agents evolve over time. In this model, the interaction between agents is described by an OPD game. Previous research on spatial games has shown that when the temptation to defect is high, defection is the dominant strategy in most cases. However, it is been discussed that the combination of both optional games and coevolutionary rules can help in the emergence of cooperation in a wider range of scenarios [29, 27].

Thus, given the coevolutionary optional prisoner's dilemma game (i.e., an OPD game in a spatial environment, where links between agents can be evolved), the aims of the work are to understand the effect of varying the parameters T (temptation to defect), L (loner's payoff), Δ and δ for both unbiased and biased environments.

By investigating the effect of these parameters, we aim to:

- Compare the outcomes of the COPD game with other games.
- Explore the impact of the link update rules and its properties.
- Investigate the evolution of cooperation when abstainers are present in the population.
- Investigate how many abstainers would be necessary to guarantee robust cooperation.

Results show that cooperation emerges even in extremely adverse scenarios where the temptation to defect is almost at its maximum. It can be observed that the presence of the abstainers are fundamental in protecting

cooperators from invasion. In general, it is shown that, when the coevolutionary rules are used, cooperators do much better, being also able to dominate the whole population in many cases. Moreover, for some settings, we also observe interesting phenomena of cyclic competition between the three strategies, in which abstainers invade defectors, defectors invade cooperators and cooperators invade abstainers.

The chapter outline is as follows: Section 3.2 presents a brief overview of the previous work in both spatial evolutionary game theory with dynamic networks and in the optional prisoner's dilemma game. Section 3.3 gives an overview of the methodology employed, outlining the optional prisoner's dilemma payoff matrix, the coevolutionary model used (Monte Carlo simulation), the strategy and link weight update rules, and the parameter values that are varied in order to explore the effect of coevolving both strategies and link weights. Section 3.4 discusses the benefits of combining the concept of abstention and coevolution. Section 3.5 further explore the effect of using the COPD game in an unbiased environment. Section 3.6 investigates the robustness of cooperative behaviour in a biased environment. Finally, Section 3.7 summarizes the main conclusions and outlines future work.

3.2 Related work

The use of coevolutionary rules constitute a new trend in evolutionary game theory. These rules were first introduced by Zimmermann et al. [203], who proposed a model in which agents can adapt their neighbourhood during a dynamical evolution of game strategy and graph topology. Their model uses computer simulations to implement two rules: firstly, agents playing the prisoner's dilemma game update their strategy (cooperate or defect) by imitating the strategy of an agent in their neighbourhood with a higher payoff;

and secondly, the network is updated by allowing defectors to break their connection with other defectors and replace the connection with a connection to a new neighbour selected randomly from the whole network. Results show that such an adaptation of the network is responsible for an increase in cooperation.

In fact, as stated by Perc and Szolnoki [124], the spatial coevolutionary game is a natural upgrade of the traditional spatial evolutionary game initially proposed by Nowak and May [116], who considered static and unweighted networks in which each individual can interact only with its immediate neighbours. In general, it has been shown that coevolving the spatial structure can promote the emergence of cooperation in many scenarios [183, 20], but the understanding of cooperative behaviour is still one of the central issues in evolutionary game theory.

Szolnoki and Perc [156] proposed a study of the impact of coevolutionary rules on the spatial version of three different games, i.e., the prisoner's dilemma, the snow drift and the stag-hunt game. They introduce the concept of a teaching activity, which quantifies the ability of each agent to enforce its strategy on the opponent. It means that agents with higher teaching activity are more likely to reproduce than those with a low teaching activity. Differing from previous research [204, 203], they also consider coevolution affecting either only the defectors or only the cooperators. They discuss that, in both cases and irrespective of the applied game, their coevolutionary model is much more beneficial to the cooperators than that of the traditional model.

Huang et al. [74] present a new model for the coevolution of game strategy and link weight. They consider a population of 100×100 agents arranged on a regular lattice network which is evolved through a Monte Carlo simulation. An agent's interaction is described by the classical prisoner's dilemma with a normalized payoff matrix. A new parameter, Δ/δ , is defined

as the link weight amplitude and is calculated as the ratio of Δ/δ . They found that some values of Δ/δ can provide the best environment for the evolution of cooperation. They also found that their coevolutionary model can promote cooperation efficiently even when the temptation of defection is high.

In addition to investigations of the classical prisoner's dilemma on spatial environments, some extensions of this game have also been explored as a means to favour the emergence of cooperative behaviour. For instance, the optional prisoner's dilemma game, which introduces the concept of abstention, has been studied since Batali and Kitcher [13]. In their work, they proposed the opt-out or "loner's" strategy in which agents could choose to abstain from playing the game, as a third option, in order to avoid cooperating with known defectors. There have been a number of recent studies exploring this type of game [190, 56, 121, 79, 68]. Cardinot et al. [32] discuss that, with the introduction of abstainers, it is possible to observe new phenomena and, in a larger range of scenarios, cooperators can be robust to invasion by defectors and can dominate.

Although recent work has discussed the inclusion of optional games with coevolutionary rules [29, 27], this still needs to be investigated in a wider range of scenarios. Therefore, the present work aims to combine both of these trends in evolutionary game theory in order to identify favourable configurations for the emergence of cooperation in adverse scenarios, where, for example, the temptation to defect is very high or when the initial population of abstainers is either very scarce or very abundant.

3.3 Methodology

This section includes a complete description of the optional prisoner's dilemma game, the spatial environment and the coevolutionary rules for

both the strategy and link weights. Finally, we also outline the experimental set-up.

In the classical version of the prisoner's dilemma (PD), two agents can choose either cooperation or defection. Hence, there are four payoffs associated with each pairwise interaction between the two agents. In consonance with common practice [29, 74, 116], payoffs are characterized by the reward for mutual cooperation ($R = 1$), punishment for mutual defection ($P = 0$), sucker's payoff ($S = 0$) and temptation to defect ($T = b$, where $1 < b < 2$). Note that this parametrization refers to the weak version of the prisoner's dilemma game, where P can be equal to S without destroying the nature of the dilemma. In this way, the constraints $T > R > P \geq S$ maintain the dilemma.

The optional prisoner's dilemma (OPD) game is an extended version of the PD game in which agents can not only cooperate (C) or defect (D) but can also choose to abstain (A) from a game interaction, obtaining the loner's payoff ($L = l$) which is awarded to both players if one or both abstain. As defined in other studies [32, 148], abstainers receive a payoff greater than P and less than R (i.e., $P < L < R$). Thus, considering the normalized payoff matrix adopted, $0 < l < 1$. The payoff matrix and the associated values are illustrated in Table 3.1.

In this work, the following parameters are used: an $N = 102 \times 102$ regular lattice grid with periodic boundary conditions is created and fully populated with agents, which can play with their eight immediate neighbours (Moore neighbourhood). We investigate both unbiased (i.e., initially each agent is designated as C , D or A with equal probability) and biased (i.e., varying the initial percentage of abstainers) environments. Also, each edge linking agents has the same initial weight $w = 1$, which will adaptively change in accordance with the interaction.

TABLE 3.1: The optional prisoner's dilemma game matrix [29].

	C	D	A
C	R	S	L
D	T	P	L
A	L	L	L

(A) Extended game matrix.

Payoff	Value
Temptation to defect (T)	$]1, 2[$
Reward for mutual cooperation (R)	1
Punishment for mutual defection (P)	0
Sucker's payoff (S)	0
Loner's payoff (L)	$]0, 1[$

(B) Payoff values.

Monte Carlo methods are used to perform the coevolutionary optional prisoner's dilemma game. In one Monte Carlo (MC) step, each player is selected once on average. This means that one MC step comprises N inner steps where the following calculations and updates occur:

- Select an agent (x) at random from the population.
- Calculate the utility u_{xy} of each interaction of x with its eight neighbours (each neighbour represented as agent y) as follows:

$$u_{xy} = w_{xy}P_{xy}, \quad (3.1)$$

where w_{xy} is the edge weight between agents x and y , and P_{xy} corresponds to the payoff obtained by agent x on playing the game with agent y .

- Calculate U_x the accumulated utility of x , that is:

$$U_x = \sum_{y \in \Omega_x} u_{xy}, \quad (3.2)$$

where Ω_x denotes the set of neighbours of the agent x .

- In order to update the link weights, w_{xy} , between agents, compare the values of u_{xy} and the average accumulated utility (i.e., $\bar{U}_x = U_x/8$) as follows:

$$w_{xy} = \begin{cases} w_{xy} + \Delta & \text{if } u_{xy} > \bar{U}_x \\ w_{xy} - \Delta & \text{if } u_{xy} < \bar{U}_x \\ w_{xy} & \text{otherwise} \end{cases}, \quad (3.3)$$

where Δ is a constant such that $0 \leq \Delta \leq \delta$.

- In line with previous research [29, 74, 183], w_{xy} is adjusted to be within the range

$$1 - \delta \leq w_{xy} \leq 1 + \delta, \quad (3.4)$$

where δ ($0 \leq \delta < 1$) defines the weight heterogeneity. Note that when Δ or δ are equal to 0, the link weight keeps constant ($w = 1$), which results in the traditional scenario where only the strategies evolve.

- In order to update the strategy of x , the accumulated utility U_x is recalculated (based on the new link weights) and compared with the accumulated utility of one randomly selected neighbour (U_y). If $U_y > U_x$, agent x will copy the strategy of agent y with a probability proportional to the utility difference (Equation 3.5), otherwise, agent x will keep its strategy for the next step.

$$p(s_x = s_y) = \frac{U_y - U_x}{8(T - P)}, \quad (3.5)$$

where T is the temptation to defect and P is the punishment for mutual defection.

Simulations are run for 10^5 MC steps and the fraction of cooperation is determined by calculating the average of the final 10^3 MC steps. To alleviate the effect of randomness in the approach, the final results are obtained by averaging 10 independent runs. The following scenarios are investigated:

- The benefits of coevolution and abstention.
- Presence of abstainers in the coevolutionary model.
- Inspecting the coevolutionary environment.
- Investigating the properties of the parameters Δ and δ .
- Varying the number of states.
- Investigating the relationship between Δ/δ , b and l .
- Investigating the robustness of cooperation in a biased environment.

3.4 The benefits of coevolution and abstention

This section presents some of the main differences between the outcomes obtained by the proposed coevolutionary optional prisoner's dilemma (COPD) game and other models which do not adopt the concept of coevolution and/or abstention. In the COPD game, we also investigate how a population in an unbiased environment evolves over time.

3.4.1 Presence of abstainers in the coevolutionary model

In order to provide a means to effectively explore the impact of our coevolutionary model, i.e., the coevolutionary prisoner's dilemma (COPD) game, in

the emergence of cooperation, we start by investigating the performance of some of the existing models. Namely, the coevolutionary prisoner's dilemma (CPD) game (i.e., same coevolutionary model as the COPD but without the concept of abstention), the traditional prisoner's dilemma (PD) game, and the optional prisoner's dilemma game.

As shown in Figure 3.1, it can be observed that for both PD and CPD games, when the defector's payoff is very high (i.e., $b > 1.7$) defectors spread quickly and dominate the environment. On the other hand, when abstainers are present in a static and unweighted network, i.e., playing the OPD game, we end up with abstainers dominating the environment. Undoubtedly, in many scenarios, having a population of abstainers is better than a population of defectors. However, it provides clear evidence that all these three models fail to sustain cooperation. In fact, results show that in this type of adverse environment (i.e., with a high temptation to defect), cooperation has no chance of emerging.

Figure 3.2 shows a typical phase diagram for both CPD and COPD games for a fixed value of $\delta = 0.8$ and $l = 0.6$ (on the COPD game). It can be observed that if a given environmental setting (i.e., b , Δ and δ) produces a stable population of cooperators in the CPD game, then the presence of abstainers will not change it. In other words, the COPD game does not affect the outcome of scenarios in which cooperation is stable in the absence of abstainers. Thus, the main changes occur in scenarios in which defection dominates or survives ($b > 1.5$).

Surprisingly, as shown in Figure 3.2, when considering the coevolutionary optional prisoner's dilemma (COPD) game for the same environmental settings of Figure 3.1 (i.e., $l = 0.6$, $\Delta = 0.72$ and $\Delta = 0.72$), with the temptation of defection almost at its peak (i.e., $b = 1.9$), it is possible to reach high levels of cooperation.

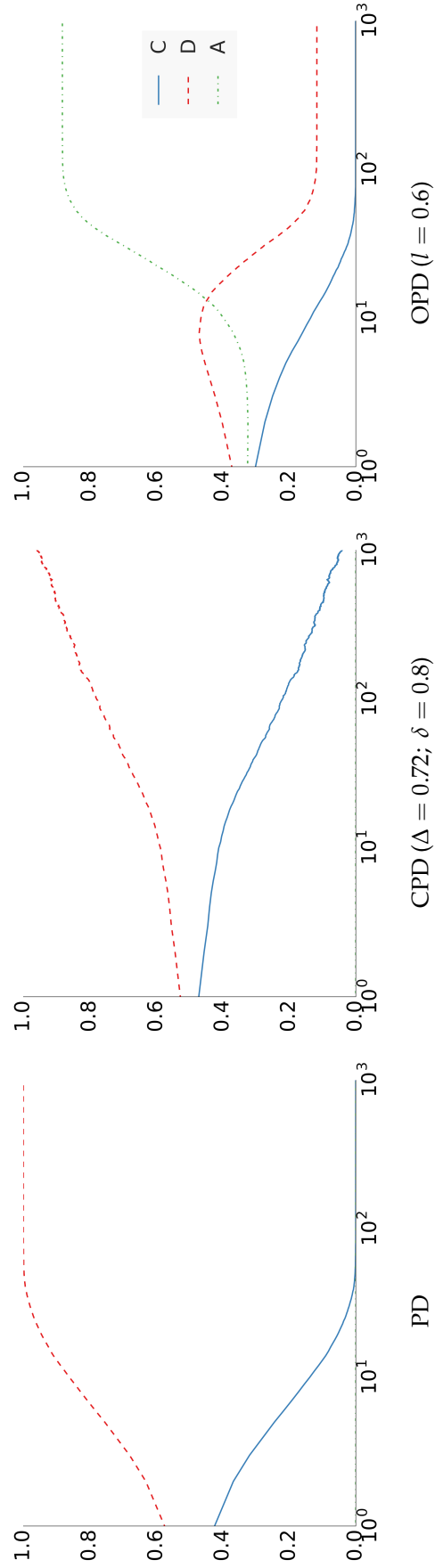


FIGURE 3.1: Comparison of the prisoner's dilemma (PD), the coevolutionary prisoner's dilemma (CPD) and the optional prisoner's dilemma (OPD) games. All with the same temptation to defect, $b = 1.9$. It can be observed that defectors spread quickly and dominate the environment for both PD and CPD games. However, when agents play the OPD game, abstention becomes the dominating strategy.

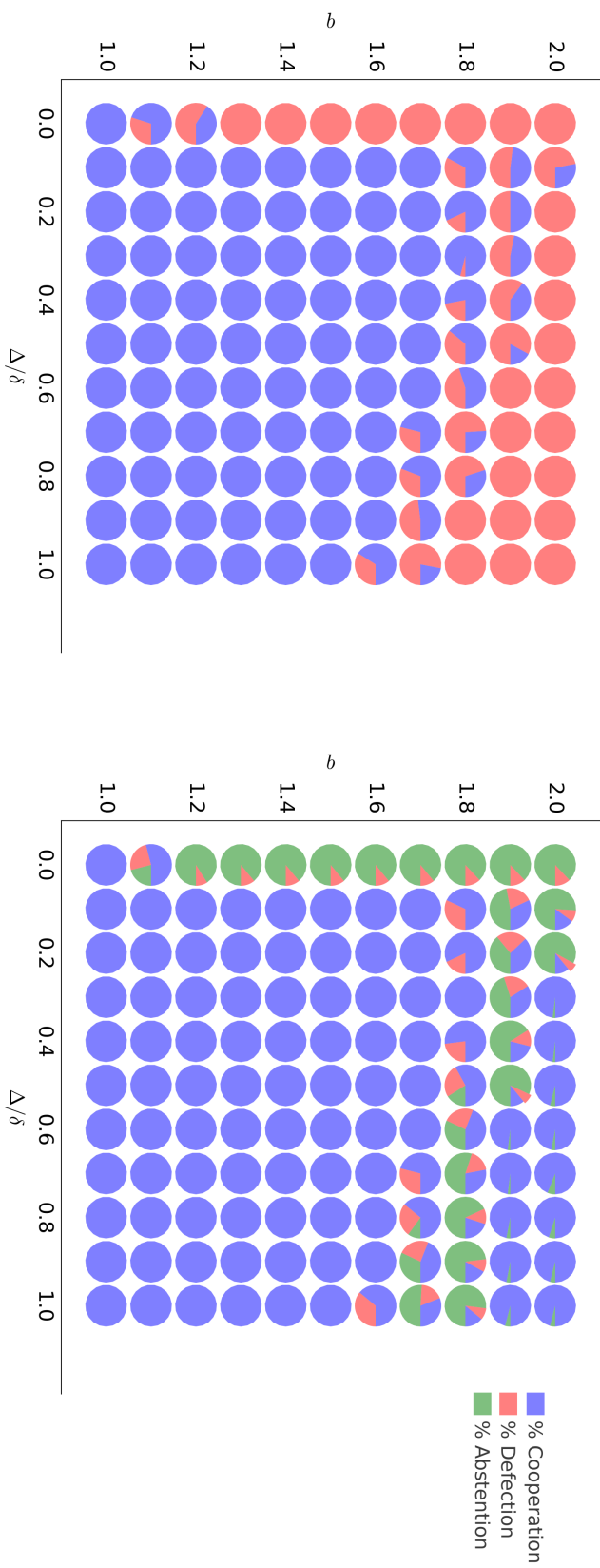


FIGURE 3.2: Typical phase diagram for an initial balanced population playing the coevolutionary prisoner's dilemma game (left) and the coevolutionary optional prisoner's dilemma game with loner's payoff $l = 0.6$ (right), both with $\delta = 0.8$.

To summarize, despite the fact that the coevolutionary prisoner's dilemma (CPD) game succeeds in the promotion of cooperation in a wide range of scenarios, it is still not able to avoid the invasion by defectors in cases where $b > 1.5$, which does not happen when the abstainers are present (i.e., COPD game).

3.4.2 Inspecting the coevolutionary environment

In order to further explain the results witnessed in the previous experiments, we investigate how the population evolves over time for the coevolutionary optional prisoner's dilemma game. Figure 3.3 features the time course of cooperation for three different values of $\Delta/\delta = \{0.0, 0.2, 1.0\}$, which are some of the critical points when $b = 1.9$, $l = 0.6$ and $\delta = 0.8$. Based on these results, in Figure 3.4 we show snapshots for the Monte Carlo steps 0, 45, 1113 and 10^5 for the three scenarios shown in Figure 3.3.

We see from Figure 3.4 that for the traditional case (i.e., $\Delta/\delta = 0.0$), abstainers spread quickly and reach a stable state in which single defectors are completely isolated by abstainers. In this way, as the payoffs obtained by a defector and an abstainer are the same, neither will ever change their strategy. In fact, even if a single cooperator survives up to this stage, for the same aforementioned reason, its strategy will not change either. In fact, the same behaviour is noticed for any value of $b > 1.2$ and $\Delta/\delta = 0$ (COPD in Figure 3.2).

When $\Delta/\delta = 0.2$, it is possible to observe some sort of equilibrium between the three strategies. They reach a state of cyclic competition in which abstainers invade defectors, defectors invade cooperators and cooperators invade abstainers.

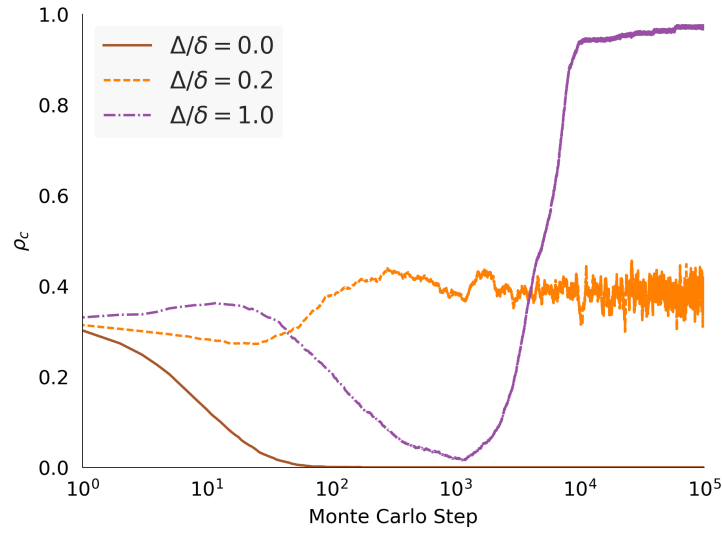


FIGURE 3.3: Progress of the fraction of cooperation ρ_c during a Monte Carlo simulation for $b = 1.9$ (temptation to defect), $l = 0.6$ (loner's payoff) and $\delta = 0.8$ [29].

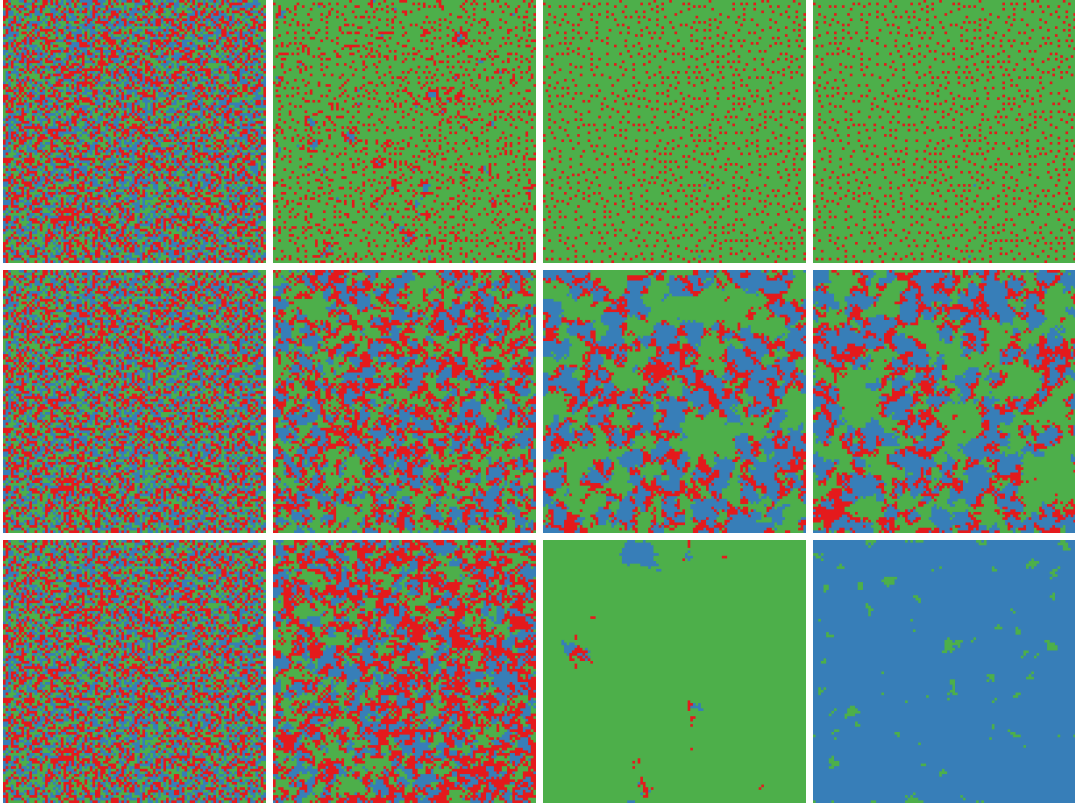


FIGURE 3.4: Snapshots of the distribution of the strategy in the Monte Carlo steps 0, 45, 1113 and 10^5 (from left to right) for Δ/δ equal to 0.0, 0.2 and 1.0 (from top to bottom). In this Figure, cooperators, defectors and abstainers are represented by the colours blue, red and green respectively. All results are obtained for $b = 1.9$ (temptation to defect), $l = 0.6$ (loner's payoff) and $\delta = 0.8$ [29].

This behaviour, of balancing the three possible outcomes, is very common in nature where species with different reproductive strategies remain in equilibrium in the environment. For instance, the same scenario was observed as being responsible for preserving biodiversity in the neighbourhoods of the *Escherichia coli*, which is a bacteria commonly found in the lower intestine of warm-blooded organisms. According to Fisher [52], studies were performed with three natural populations of this bacteria: (i) produces a natural antibiotic but is immune to its effects, (ii) is sensitive to the antibiotic but can grow faster than the third population, which (iii) is resistant to the antibiotic. They observed that when these populations are mixed together, each of them ends up establishing its own territory in the environment. It happens because the first population kill off any other bacteria sensitive to the antibiotic, the second population uses their faster growth rate to displace the bacteria which are resistant to the antibiotic, and the third population could use their immunity to displace the first population.

Another interesting behaviour is noticed for $\Delta/\delta = 1.0$. In this scenario, defectors are dominated by abstainers, allowing a few clusters of cooperators to survive. As a result of the absence of defectors, cooperators invade abstainers and dominate the environment.

3.5 Exploring the coevolutionary optional prisoner's dilemma game

In this section, we present some of the relevant experimental results of the Monte Carlo simulations of the coevolutionary optional prisoner's dilemma game in an unbiased environment. That is, a well-mixed initial population with a balanced amount of cooperators, defectors and abstainers.

3.5.1 Investigating the properties of Δ and δ

This section aims to investigate the properties of the presented model (Sect. 3.3) in regard to the parameters Δ and δ . These parameters play a key role in the evolutionary dynamics of this model because they define the number of possible link weights that an agent is allowed to have (i.e., they define the number of states).

Despite the fact that the number of states is discrete, the act of counting them is not straightforward. For instance, when counting the number of states between $1 - \delta$ and $1 + \delta$ for $\Delta = 0.2$ and $\delta = 0.3$, we could incorrectly state that there are four possible states for this scenario, i.e., $\{0.7, 0.9, 1.1, 1.3\}$. However, considering that the link weights of all edges are initially set to $w = 1$, and due to the other constraints (Equations 3.3 and 3.4), the number of states is actually seven, i.e., $\{0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3\}$.

In order to better understand the relationship between Δ and δ , we plot Δ , δ and Δ/δ as a function of the number of states (numerically counted) for a number of different values of both parameters (Figure 3.5). It was observed that given the pairs (Δ_1, δ_1) and (Δ_2, δ_2) , if Δ_1/δ_1 is equal to Δ_2/δ_2 , then the number of states of both settings is the same.

Figure 3.5 shows the ratio Δ/δ as a function of the number of states. As we can see, although the function is non-linear and non-monotonic, in general, higher values of Δ/δ have less states.

3.5.2 Varying the number of states

Figure 3.6 shows the impact of the coevolutionary model on the emergence of cooperation when the ratio Δ/δ varies for a range of fixed values of the

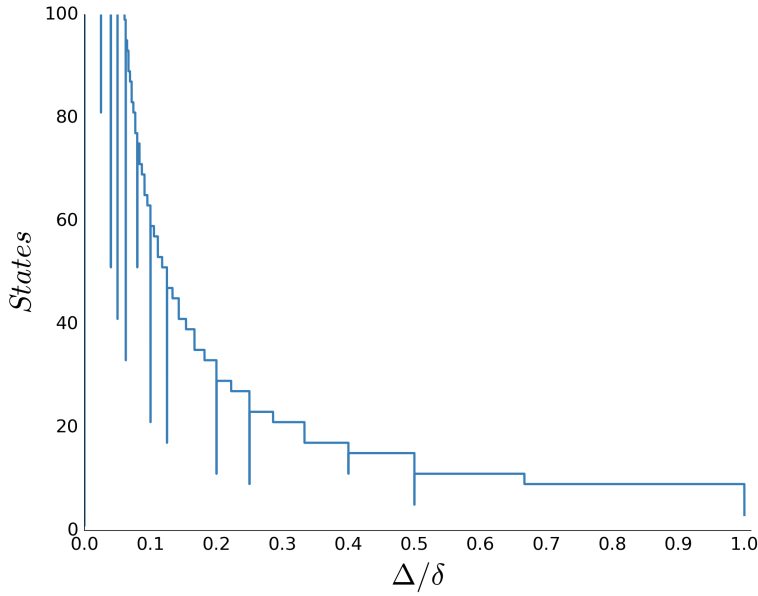


FIGURE 3.5: The ratio Δ/δ as a function of the number of states. For any combination of Δ and δ , the ration Δ/δ will always have the same number of states.

loner's payoff (l), temptation to defect (b) and δ . In this experiment, we observe that when $l = 0.0$, the outcomes of the coevolutionary optional prisoner's dilemma (COPD) game are very similar to those observed by Huang et al. [74] for the coevolutionary prisoner's dilemma (CPD) game. This result can be explained by the normalized payoff matrix adopted in this work (Table 3.1). Clearly, when $l = 0.0$, there is no advantage in abstaining from playing the game, thus agents choose the option to cooperate or defect [29].

Results indicate that, in cases where the temptation to defect is very low (e.g, $b \leq 1.34$), the level of cooperation does not seem to be affected by the increment of the loner's payoff, except when the advantage of abstaining is very high (e.g, $l > 0.8$). However, these results highlight that the presence of the abstainers may protect cooperators from invasion. Moreover, the difference between the traditional optional prisoner's dilemma (i.e., $\Delta/\delta = 0.0$) for $l = \{0.0, 0.6\}$ and all other values of Δ/δ is strong evidence that our coevolutionary model is very advantageous to the promotion of cooperative

behaviour.

Namely, when $l = 0.6$, in the traditional case with a static and unweighted network ($\Delta/\delta = 0.0$), the cooperators have no chance of surviving; except, of course, when b is very close to the reward for mutual cooperation R , where it is possible to observe scenarios of quasi-stable states of the three strategies or between cooperators and defectors. In fact, in the traditional OPD ($\Delta/\delta = 0.0$), when $l > 0.0$ and $b > 1.2$, abstainers are always the dominant strategy. However, as discussed in previous work [29], when the coevolutionary rules are used, cooperators do much better, being also able to dominate the whole population in many cases.

It is noteworthy that the curves in Figure 3.6 are usually non-linear and/or non-monotonic because of the properties of the ratio Δ/δ in regard to the number of states of each combination of Δ and δ (Sect. 3.5.1).

3.5.3 Investigating the relationship between Δ/δ , b and l

To investigate the outcomes in other scenarios, we explore a wider range of settings by varying the values of the temptation to defect (b), the loner's payoff (l) and the ratio Δ/δ for a fixed value of $\delta = 0.8$.

As shown in Figure 3.7, cooperation is the dominant strategy in the majority of cases. Note that in the traditional case, with an unweighted and static network, i.e., $\Delta/\delta = 0.0$, abstainers dominate in all scenarios illustrated in this ternary diagram. In addition, it is also possible to observe that certain combinations of l , b and Δ/δ guarantee higher levels of cooperation. In these scenarios, cooperators are protected by abstainers against exploitation from defectors.

Complementing previous findings [29], another observation is that defectors are attacked more efficiently by abstainers as we increase the loner's

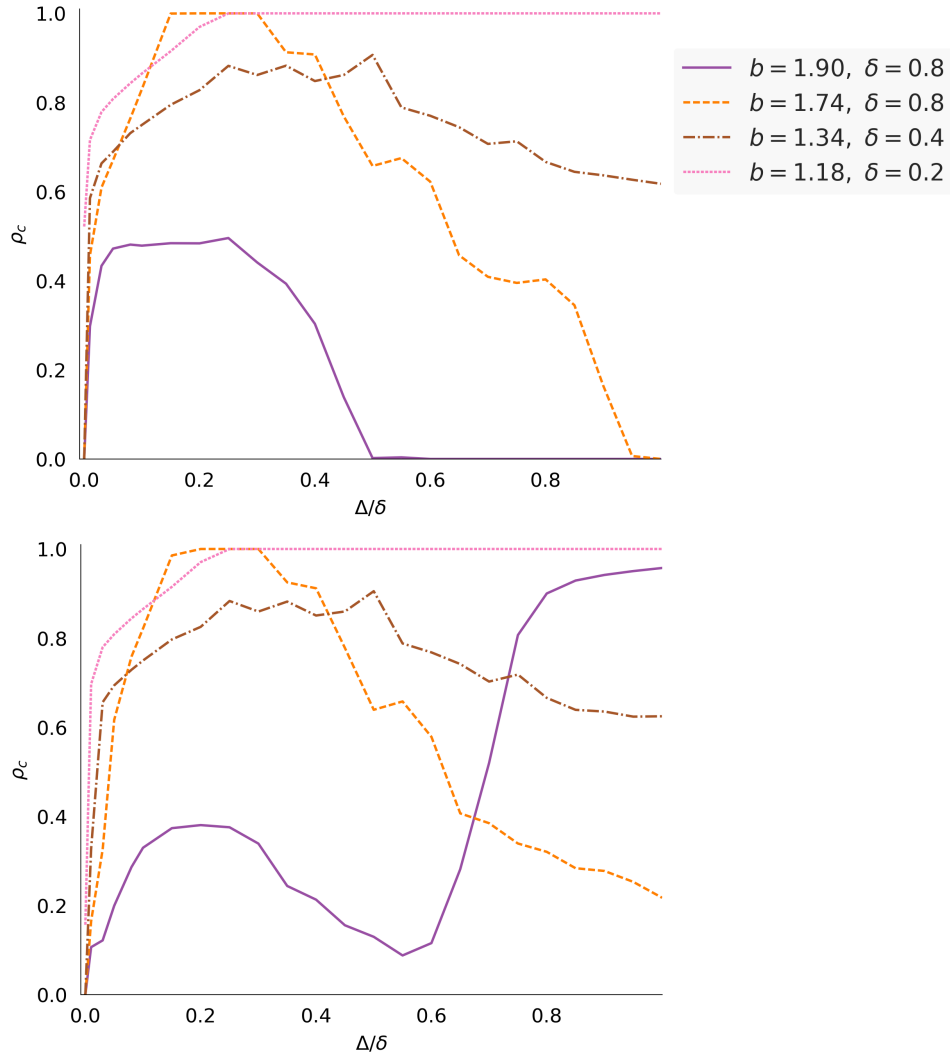


FIGURE 3.6: Relationship between cooperation and the ratio Δ/δ when the loner's payoff (l) is equal to 0.0 (top) and 0.6 (bottom) [29].

payoff (l). Simulations reveal that, for any scenario, if the loner's payoff is greater than 0.7 ($l > 0.7$), defectors have no chance of surviving. However, the drawback of increasing the value of l is that it makes it difficult for cooperators to dominate abstainers, which might produce a quasi-stable population of cooperators and abstainers. It is noteworthy that it is a counter-intuitive result from the COPD game, since the loner's payoff is always less than the reward for mutual cooperation (i.e., $L < R$), even for extremely high values of L . This scenario (population of cooperators and abstainers) should always lead cooperators to quickly dominate the environment.

In fact, it is still expected that, in the COPD game, cooperators dominate abstainers, but depending on the value of the loner's payoff, or the amount of abstainers in the population at this stage, it might take several Monte Carlo steps to reach a stable state, which is usually a state of cooperation fully dominating the population.

An interesting behaviour is noticed when $l = [0.45, 0.55]$ and $b > 1.8$. In this scenario, abstainers quickly dominate the population, making a clear division between two states: before this range (defectors hardly die off) and after this range (defectors hardly survive). In this way, a loner's payoff value greater than 0.55 ($l > 0.55$) is usually the best choice to promote cooperation. This result is probably related to the difference between the possible utilities for each type of interaction, which still needs further investigation in future.

Although the combinations shown in Figure 3.7 for higher values of b ($b > 1.8$) are just a small subset of an infinite number of possible values, it is clearly shown that a reasonable fraction of cooperators can survive even in an extremely adverse situation where the advantage of defecting is very high. Indeed, our results show that some combinations of high values of l and δ , such as for $\delta = 0.8$ and $l = 0.7$, can further improve the levels of cooperation, allowing for the full dominance of cooperation.

3.6 Investigating the robustness of cooperation in a biased environment

The previous experiments revealed that the presence of abstainers together with simple coevolutionary rules (i.e., the COPD game) act as a powerful mechanism to avoid the spread of defectors, which also allows the dominance of cooperation in a wide range of scenarios. However, the distribution

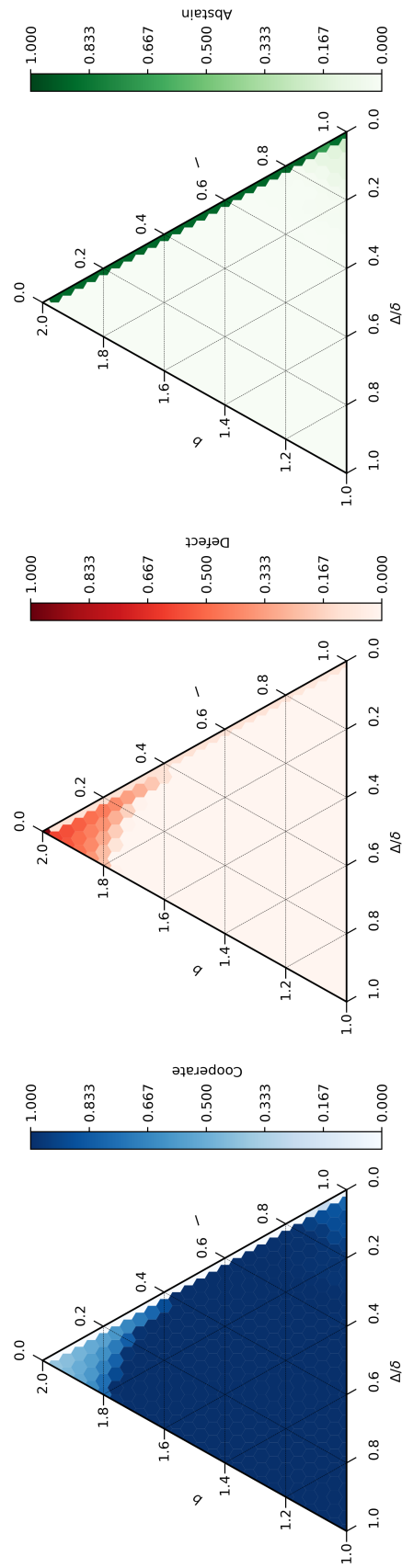


FIGURE 3.7: Ternary diagrams of different values of b (temptation to defect), l (loner's payoff) and Δ/δ for $\delta = 0.8$ [29].

of the strategies in the initial population used in all of the previous experiments was uniform. That is, we have explored cases in which the initial population contained a balanced amount of cooperators, defectors and abstainers. Thus, in order to explore the robustness of these outcomes in regard to the initial amount of abstainers in the population, we now aim to investigate how many abstainers would be necessary to guarantee robust cooperation.

Figure 3.8 features the fraction of each strategy in the population (i.e., cooperators, defectors and abstainers) over time for fixed values of $b = 1.9$, $\Delta = 0.72$ and $\delta = 0.8$. In this experiment, several independent simulations were performed, in which the loner's payoff (l) and the number of abstainers in the initial population were varied from 0.0 to 1.0 and from 0.1% to 99.9%, respectively. Other special cases were also analyzed, such as placing only one abstainer into a balanced population of cooperators and defectors, and placing only one defector and one cooperator in a population of abstainers. For the sake of simplicity, we report only the values of $l = \{0.2, 0.6, 0.8\}$ for an initial population with one, 5%, 33% and 90% abstainer(s), which are representative of the outcomes at other values also. Note that, for all these simulations, the initial population of cooperators and defectors remained in balance. For instance, an initial population with 50% of abstainers, will consequently have 25% of cooperators and 25% of defectors.

Experiments reveal that the COPD game is actually extremely robust to radical changes in the initial population of abstainers. It has been shown that if the loner's payoff is greater than 0.55 ($l > 0.55$), then one abstainer might alone be enough to protect cooperators from the invasion of defectors (see Figures 3.8a, 3.8b and 3.8c). However, this outcome is only possible if the single abstainer is in the middle of a big cluster of defectors.

This outcome can happen because the payoff obtained by the abstainers is always greater than the one obtained by pairs of defectors (i.e., $L < P$).

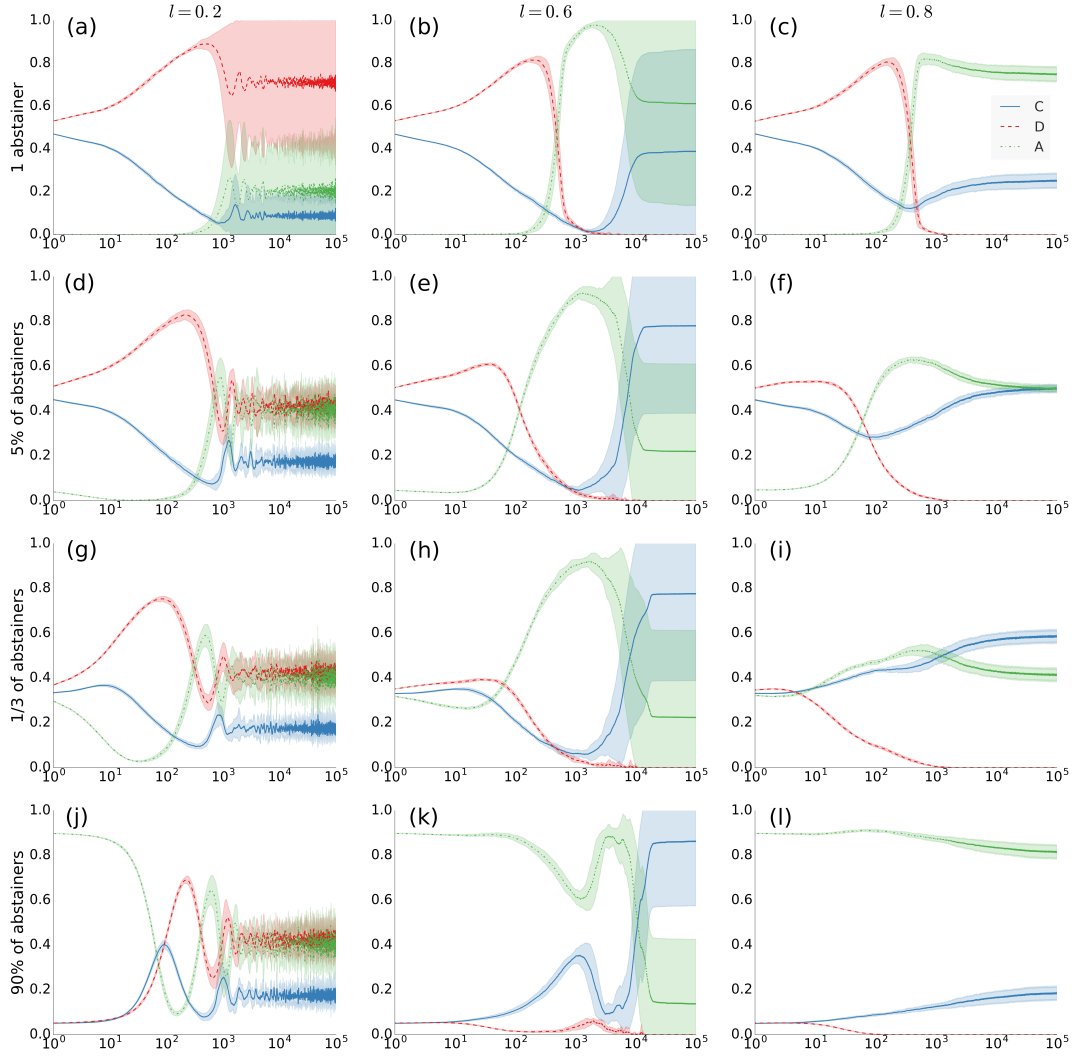


FIGURE 3.8: Time course of each strategy for $b = 1.9$, $\Delta = 0.72$, $\delta = 0.8$ and different values of l (from left to right, $l = \{0.2, 0.6, 0.8\}$). The same settings are also tested on populations seeded with different amount of abstainers (i.e, from top to bottom: 1 abstainer, 5% of the population, 1/3 of the population, 90% of the population).

Thus, in a cluster of defectors, abstention is always the best choice. However, as this single abstainer reduces the population of defectors, which consequently increases the population of abstainers and cooperators in the population, defection may start to be a good option again due to the increase of cooperators. Therefore, the exploitation of defectors by abstainers must be as fast as possible, otherwise, they might not be able to effectively attack the population of defectors. In this scenario, the loner's payoff is the key parameter to control the speed in which abstainers invade defectors. This explains why a single abstainer is usually not enough to avoid the dominance of defectors when $l < 0.55$.

In this way, as the loner's payoff is the only parameter that directly affects the evolutionary dynamics of the abstainers, intuition might lead one to expect to see a clear and perhaps linear relationship between the loner's payoff and the initial number of abstainers in the population. That is, given the same set of parameters, increasing the initial population of abstainers or the loner's payoff would probably make it easier for abstainers to increase or even dominate the population. Despite the fact that it might be true for high values of the loner's payoff (i.e., $l \geq 0.8$, as observed in Figure 3.8), it is not applicable to other scenarios. Actually, as it is also shown in Figure 3.8, if the loner's payoff is less than 0.55, changing the initial population of abstainers does not change the outcome at all. When $0.55 \leq l < 0.8$, a huge initial population of abstainers can actually promote cooperation best.

As discussed in Section 3.5.3, populations of cooperators and abstainers tend to converge to cooperation. In this way, the scenario showed in Figure 3.8 for $l = 0.8$ will probably end up with cooperators dominating the population, but as the loner's payoff is close to the reward for mutual cooperation, the case in Figure 3.8i will converge faster than the one shown in Figure 3.8l.

Another very counter-intuitive behaviour occurs in the range $l = [0.45, 0.55]$ (this range may shift a little bit depending on the value of b), where the outcome is usually of abstainers quickly dominating the population (Sect.3.5.3). In this scenario, we would expect that changes in the initial population of abstainers would at least change the speed in which the abstainers fixate in the population. That is, a huge initial population of abstainers would probably converge quickly. However, it was observed that the convergence speed is almost the same regardless of the size of the initial population of abstainers.

In summary, results show that an initial population with 5% of abstainers is usually enough to make reasonable changes in the outcome, increasing the chances of cooperators surviving or dominating the population.

3.7 Conclusions and future work

This chapter studies the impact of a simple coevolutionary model in which not only the agents' strategies but also the network evolves over time. The model consists of placing agents playing the optional Prisoner's dilemma game in a dynamic spatial environment, which in turn, defines the coevolutionary optional Prisoner's dilemma (COPD) game [29].

In summary, based on the results of several Monte Carlo simulations, it was shown that the COPD game allows for the emergence of cooperation in a wider range of scenarios than the coevolutionary Prisoner's dilemma (CPD) game [74], i.e., the same coevolutionary model in populations which do not have the option to abstain from playing the game. Results also showed that COPD performs much better than the traditional version of these games, i.e., the prisoner's dilemma (PD) and the optional prisoner's dilemma (OPD) games, where only the strategies evolve over time in a static and unweighted

network. Moreover, we observed that the COPD game is actually able to reproduce outcomes similar to other games by setting the parameters as follows:

- CPD: $l = 0$.
- OPD: $\Delta = 0$ (or $\delta = 0$).
- PD: $l = 0$ and $\Delta = 0$ (or $\delta = 0$).

Also, it was possible to observe that abstention acts as an important mechanism to avoid the dominance of defectors. For instance, in adverse scenarios such as when the defector's payoff is very high ($b > 1.7$), for both PD and CPD games, defectors spread quickly and dominated the environment. On the other hand, when abstainers were present (COPD game), cooperation was able to survive and even dominate.

Furthermore, simulations showed that defectors die off when the loner's payoff is greater than 0.7 ($l > 0.7$). However, it was observed that increasing the loner's payoff makes it difficult for cooperators to dominate abstainers, which is a counter-intuitive result, since the loner's payoff is always less than the reward for mutual cooperation (i.e., $L < R$), this scenario should always lead cooperators to dominance very quickly. In this scenario, cooperation is still the dominant strategy in most cases, but it might require several Monte Carlo steps to reach a stable state.

Results revealed that the COPD game also allows scenarios of cyclic dominance between the three strategies (i.e., cooperation, defection and abstention), indicating that, for some parameter settings, the COPD game is intransitive. That is, the population remains balanced in such a way that cooperators invade abstainers, abstainers invade defectors and defectors invade cooperators, closing a cycle.

We also explored the robustness of these outcomes in regard to the initial amount of abstainers in the population (biased population). In summary, it was shown that, in some of the scenarios, even one abstainer might alone be enough to protect cooperators from the invasion of defectors, which in turn increases the chances of cooperators surviving or dominating the population. We conclude that the combination of both of these trends in evolutionary game theory may shed additional light on gaining an in-depth understanding of the emergence of cooperative behaviour in real-world scenarios.

Future work will consider the exploration of different topologies and the influence of a wider range of scenarios, where, for example, agents could rewire their links, which, in turn, adds another level of complexity to the model. Future work will also involve applying our studies and results to realistic scenarios, such as social networks and real biological networks.

Chapter 4

Cyclic Dominance in the Spatial Coevolutionary Optional Prisoner's Dilemma Game

The work outlined in this chapter was published in:

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Abstract

This chapter studies scenarios of cyclic dominance in a coevolutionary spatial model in which game strategies and links between agents adaptively evolve over time. The optional prisoner's dilemma (OPD) game is employed. The OPD is an extended version of the traditional prisoner's dilemma where players have a third option to abstain from playing the game. We adopt an agent-based simulation approach and use Monte Carlo methods to perform the OPD with coevolutionary rules. The necessary conditions to break the scenarios of cyclic dominance are also investigated. This work highlights that cyclic dominance is essential in the sustenance of biodiversity. Moreover, we also discuss the importance of a spatial coevolutionary model in maintaining cyclic dominance in adverse conditions.

4.1 Introduction

Competition is one of the most fundamental concepts in the study of the interaction between individuals in an ecosystem. Competition occurs when there is a contest for resources, such as food, mates or territories. Competition also favours a selection process, which reaches its peak with the dominance of better-adapted individuals and the extinction of less-adapted individuals. In many cases, rather than one individual dominating, the system can reach an equilibrium where individuals can coexist [84].

Scenarios of coexistence may occur when two or more individuals, or species, form a cycle of dominance. For instance, considering a population of three species: X , Y and Z ; cyclic dominance occurs when X dominates Y , Y dominates Z , and Z dominates X , forming a closed loop, which is also known as an intransitivity [165].

In nature, cyclic dominance plays a key role in the sustenance of biodiversity. For example, the male side-blotched lizard shows an intransitive behaviour when guarding their mates. This kind of lizard can be divided into three categories based on their throat colours:

- Blue-throated males guard small territories with a single female. They are efficient in defending their mate from yellow-throated lizards.
- Yellow-throated males do not guard territories at all, but they move around in search of mates.
- Orange-throated males guard larger territories, keeping harems of females. Consequently, as they have to split their efforts defending several territories, they are less efficient in defending them from yellow-throated lizards. However, they are more aggressive and can steal mates from blue-throated lizards.

Therefore, a cyclic competition exists because the blue-throated males beat the yellow-throated males, the yellow-throated males beat the orange-throated males, and the orange-throated males beat the blue-throated males [139].

It is noteworthy that as the number of species, or the population size, increases, the collective behaviours of a system subject to these sort of scenarios may become much harder to analyse and predict. In this way, frameworks like evolutionary game theory, which employs game theory to evolve populations of rational agents, have been widely applied by researchers as it provides many useful insights to explain such a complex scenario [142].

Particularly, games such as the Rock-Paper-Scissors game [194, 106, 37, 133, 87] and the prisoner's dilemma game [193, 79, 166, 147] have been studied in the context of cyclic competition. In these games, a participant's interactions are generally constrained by particular graph topologies [116], where it has been shown that the spatial organisation of strategies may also affect the outcomes [32]. Recent studies have also explored dynamically weighted networks, where it has been shown that the coevolution of both the game strategies and the spatial environment can further help in understanding real-world systems [29, 74, 183, 41, 203].

In this chapter, we employ extensive agent-based Monte Carlo simulations to perform the coevolutionary optional prisoner's dilemma game in a population of agents placed on a lattice grid where, game strategies, and the edges linking agents, adaptively evolve over time. We aim to investigate:

- scenarios of cyclic dominance in the optional prisoner's dilemma, and
- the necessary conditions needed to break the scenarios of cyclic dominance.

Specifically, the experiments performed are:

- Finding scenarios of cyclic dominance.
- Exploring the sustenance of coexistence after the extinction of one strategy.
- Investigating the robustness of the coexistence of three strategies when some portion of the strategies mutate into another strategy.
- Investigating the impact of the coevolutionary rules in the sustenance of coexistence.

The chapter outline is as follows: Section 4.2 presents a brief overview of the previous work in cyclic dominance in the context of evolutionary and coevolutionary game theory. Section 4.3 introduces the optional prisoner's dilemma game, describes the coevolutionary model adopted, and outlines the experimental set-up. In Section 4.4, some scenarios of cyclic competition between three strategies are investigated. Section 4.5 explores the necessary conditions to break the cyclic competition. Lastly, Section 4.6 summarizes the main results and outlines future work.

4.2 Related work

Started by John Maynard Smith, evolutionary game theory has been studied since the 1980s where ideas from evolutionary theory have been applied to game theory. Game theory models situations of conflict between rational agents, i.e., individual players make decisions, in which the outcome will depend on the other players' decisions [142].

Evolutionary game theory has been used as an important framework to explore and study the phenomena of cyclic competition, or intransitivity, which can be found in many real-world systems in different domains such as biology [133, 87] and physics [90, 37]. Moreover, it has been widely studied

at a higher level of abstraction, providing insights into the understanding of oscillatory and stochastic systems [194, 106].

Despite the fact that cyclic competition has been observed in two-strategy games [166], it is more likely to happen in games involving three or more strategies [165]. The rock-paper-scissors (RPS) game remains one of the most oft-studied games in scenarios of cyclic dominance due to its intransitive nature, in which the loop of preference between pairs of strategies is very straightforward — paper covers rock, rock crushes scissors and scissors cuts paper [201, 86]. It is noteworthy that such an intransitive behaviour has also been noticed in other evolutionary games such as the optional prisoner’s dilemma game [79, 147] and the voluntary public goods game [68]. However, they have been much less explored in this specific context of cyclic dominance.

For instance, Yu et al. [193] proposed a study of the influence of the population size and the level of individual rationality on the evolutionary dynamics of the voluntary prisoner’s dilemma (VPD) game, which is very similar to the optional prisoner’s dilemma (OPD) game, in which a third type of strategy is considered. In their chapter, scenarios of cyclic dominance in the VPD game are discussed. It was shown that these scenarios prevent the full dominance of a specific strategy in the population.

Recent studies have also explored the use of coevolutionary rules in game theory. First introduced by Zimmermann et al. [203], those rules propose a new model in which agents can adapt their neighbourhood during a dynamical evolution of graph topology and game strategy. As discussed by Perc and Szolnoki [124], the coevolutionary games constitute a natural upgrade of the well-know spatial evolutionary games [116], where dynamic spatial environments are taken into consideration. In fact, the coevolution of

strategies and spatial environment has given ground to a new trend in evolutionary game theory due to its wider applicability in the understanding of more realistic scenarios [74, 183].

The inclusion of coevolutionary rules in the optional prisoner's dilemma game has been recently proposed by Cardinot et al. [29], who identified that coevolutionary rules may favour the emergence of cyclic competition. However, many questions remain unanswered, such as its robustness against frozen states, i.e., when strategies become extinct because of some disturbance in the system.

4.3 Methodology

In this section, we will describe the optional prisoner's dilemma game, the Monte Carlo methods and the coevolutionary rules adopted. Finally, the spatial environment and the experimental set-up are outlined.

4.3.1 The optional prisoner's dilemma

The optional prisoner's dilemma (OPD) game is an extension of the classical version of the prisoner's dilemma (PD) game. This extension incorporates the concept of abstinence, where agents can abstain from playing the game. It leads to a three-strategy game in which agents can not only defect or cooperate, as in the classical PD, but can also choose to abstain from a game interaction. Consequently, there are nine payoffs associated with each pairwise interaction between strategies. However, as defined in other studies [32, 68], in this work we also assume that if one or both players abstain, both will obtain the same payoff, which is called the loner's payoff (L). Hence, as illustrated in Table 4.1, the OPD game is actually characterised by five payoffs,

where the other four payoffs are known as the reward for mutual cooperation (R), punishment for mutual defection (P), sucker's payoff (S) and temptation to defect (T).

TABLE 4.1: The optional prisoner's dilemma game matrix.

	C	D	A
C	R,R	S,T	L
D	T,S	P,P	L
A	L	L	L

In order to establish the dilemma of the OPD, it is important to consider that the loner's payoff (L) obtained by abstainers is greater than P and less than R , and that the traditional constraints of the prisoner's dilemma still hold, i.e., $T > R > P > S$. Thus, in this extension the dilemma arises when the payoff values are ordered such that $T > R > L > P > S$. In consonance with common practice [193, 74, 116], as the evolutionary rule depends on the payoff differences between agents, the payoff values can be rescaled to $R = 1$, $P = 0$, $S = 0$, $T = b$ and $L = l$, where $1 < b < 2$ and $0 < l < 1$, which, in turn, maintain the dilemma.

4.3.2 Monte Carlo simulation

This work considers a population of N agents placed on a square lattice with periodic boundary conditions, i.e., a torus topology (upper-bottom and left-right borders must match each other exactly). In this lattice, each agent interacts only with its eight immediate neighbours (Moore neighbourhood) by playing the optional prisoner's dilemma game with coevolutionary rules. In our experiments, initially, each agent is designated as an abstainer (A), cooperator (C) or defector (D) with equal probability. Each edge linking agents has the same weight $w = 1$, which will adaptively change in accordance with the agents' interactions.

Monte Carlo simulations are performed to investigate the dynamics of the coevolution of both game strategy and link weights. In one Monte Carlo (MC) step, each player is selected once on average, that is, one MC step comprises N inner steps where the following calculations and updates occur: an agent (x) is randomly selected from the population; its utility $u_{xy} = w_{xy}P_{xy}$ is calculated for each of its eight neighbours (represented as y), where w_{xy} is the link weight between agents x and y , and P_{xy} corresponds to the payoff obtained by agent x on playing the game with agent y ; the average accumulated utility, i.e. $\overline{U}_x = \sum u_{xy}/8$, is calculated and used to update the link weights (Eq. 4.1); as the link weights have been updated, the utilities must be recalculated; finally, strategies are updated based on the comparison of the accumulated utilities U_x and U_y (obtained from a randomly selected neighbour) (Eq. 4.2).

As shown in Equation 4.1, the link weight (w_{xy}) between agents is updated by comparing the utility (u_{xy}) and the average accumulated utility (\overline{U}_x),

$$w_{xy} = \begin{cases} w_{xy} + \Delta & \text{if } u_{xy} > \overline{U}_x \\ w_{xy} - \Delta & \text{if } u_{xy} < \overline{U}_x \\ w_{xy} & \text{otherwise} \end{cases} , \quad (4.1)$$

where Δ is a constant such that $0 \leq \Delta \leq \delta$, where δ ($0 < \delta \leq 1$) defines the weight heterogeneity. Moreover, as done in previous research [29, 74, 183], the link weight w_{xy} is also adjusted to be within the range of $1 - \delta$ to $1 + \delta$. In this way, when $\Delta = 0$ or $\delta = 0$, the link weight keeps constant ($w = 1$), which results in the traditional scenario where only the strategies evolve.

In order to update the strategy of the agent x , the accumulated utilities U_x and U_y are compared such that, if $U_y > U_x$, agent x will copy the strategy of agent y with a probability proportional to the utility difference (Eq. 4.2),

otherwise, agent x will keep its strategy for the next step.

$$p(s_x = s_y) = \frac{U_y - U_x}{8(T - P)}, \quad (4.2)$$

where T is the temptation to defect and P is the punishment for mutual defection [29, 74].

4.3.3 Experimental set-up

In this work, the population size is constant, $N = 102 \times 102$, in all simulations, which are run for 10^6 Monte Carlo steps. In order to alleviate the effect of randomness in the approach, each specific experimental set-up is run 10 times.

Initially, we identify scenarios of intransitivity in the optional prisoner's dilemma game, i.e., the values of b , l and Δ which promote the coexistence of the three strategies (Sec. 4.4). After that, the stable population is tested to find the necessary conditions to break the equilibrium. To do this, the following experiments are performed: investigating the outcomes of populations with only two types of strategies (Sec. 4.5.1); exploring the effects of different mutation rates (Sec. 4.5.2); and investigating the importance of the coevolutionary model in the sustenance of cyclic competition (Sec. 4.5.3).

4.4 Cyclic competition with three strategies

Given an initial population with the same number of abstainers, cooperators and defectors uniformly distributed, we start by investigating some parameter settings in which a state of cyclic competition can be observed. Specifically, we look for combinations of the loner's payoff l , the temptation to

defect b , and the link amplitude Δ/δ , which promote an equilibrium between the three strategies. This experiment is based on the work described by Huang et al. [74], where it is shown that some parameters settings may promote cyclic dominance. However, it is noteworthy that this phenomenon is not discussed in their paper and their methods consider only the classical prisoner's dilemma.

In the traditional case of the optional prisoner's dilemma game, i.e., for a static and unweighted network ($\Delta = 0.0$), results show that abstainers or defectors dominate in most scenarios and the dominance is closely related to the payoff values.

In comparison, the population rarely reaches a state of cyclic competition between the three strategies. In fact, it was only noticed when the temptation to defect is in the range $b = [1.1, 1.2]$. Certainly, it is more likely to happen in this scenario because there is no big advantage in choosing a specific strategy and consequently the strategies tend to remain in equilibrium. However, this behaviour is not very stable.

When it comes to the cases of $\Delta > 0.0$, results show that a wide number of different parameter settings can spontaneously promote the intransitive behaviour, in which cooperators, defectors and abstainers remain in equilibrium. For instance, Figure 4.1 shows the progress, over the Monte Carlo time steps, of each strategy for $\Delta/\delta = 0.3$, $\delta = 0.8$, $l = 0.5$ and $b = 1.9$. We can observe that agents quickly organise in the population in a way that the fraction of each strategy remains about 33% ($\pm 7\%$).

In order to investigate what spatial patterns emerge in this scenario, we also took some snapshots of the population at different Monte Carlo steps. Particularly, the MC steps 0 and 10^6 are illustrated in Figure 4.2.

An interesting phenomenon in this simulation is that abstainers tend

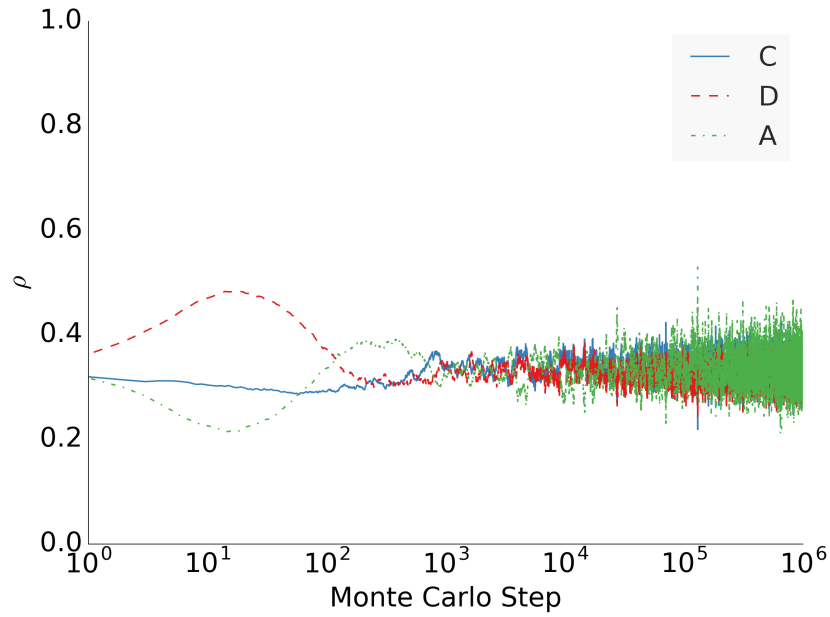


FIGURE 4.1: Progress of the fraction of cooperators, defectors and abstainers during a Monte Carlo simulation for $\Delta/\delta = 0.3$, $\delta = 0.8$, $l = 0.5$ (loner's payoff) and $b = 1.9$ (temptation to defect). It is observed that agents quickly reach a state where the fraction of each strategy remains in balance.

to form bigger clusters by dominating defectors. However, abstainers ensure that a small fraction of defectors remain in its surrounding area as a mechanism to protect them against an invasion of cooperators. Defectors, in turn, attempt to encircle cooperators, disconnecting them from abstainers, as a way to isolate and easily dominate them. These dynamics explain the reason why the population is never fully dominated by any one strategy.

4.5 Breaking the cyclic competition: extinction of species

In real-world systems, species may become extinct due to a variety of causes such as climate change, habitat degradation, diseases, genetic factors, etc. This sort of scenario is also present in many other domains, for example, in business with the extinction of companies caused by a pricing war.

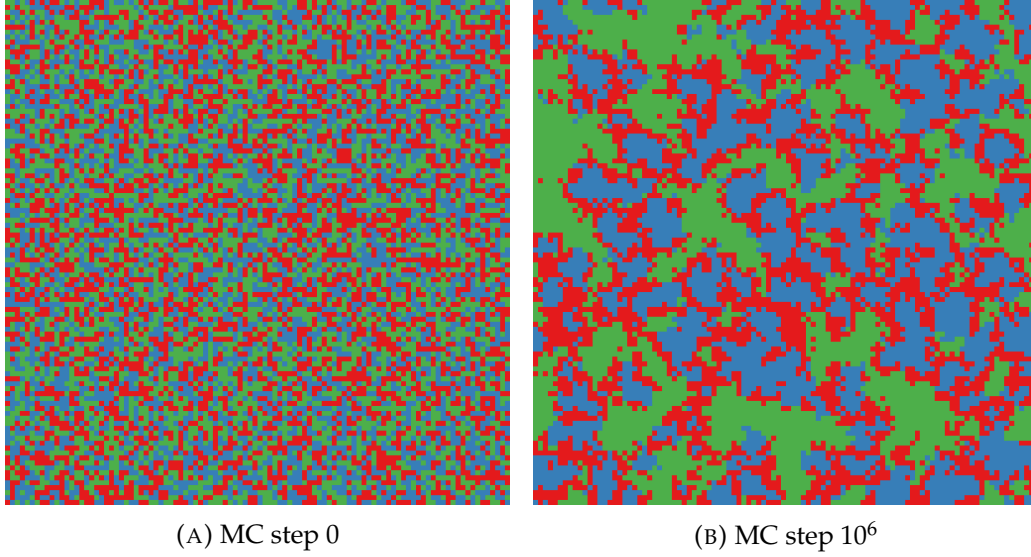


FIGURE 4.2: Snapshots of the distribution of the strategy in the Monte Carlo simulation for $\Delta/\delta = 0.3$, $\delta = 0.8$, $l = 0.5$ (loner's payoff) and $b = 1.9$ (temptation to defect). In this Figure, cooperators, defectors and abstainers are represented by the colours blue, red and green respectively.

In other cases, species may mutate in order to avoid extinction. For example, in the case of the common side-blotched lizards, it is known that yellow-throated males can, in specific instances, mutate into blue-throated males and once they transform, it cannot be reverted. Moreover, the yellow-throated male is the only species able to undergo mutation [139].

Inspired by the behaviour of some species in nature such as the side-blotched lizards and the *Escherichia Coli* bacteria, we investigate scenarios in which a species can mutate and we investigate the effect that this phenomenon can have on the population and its evolutionary mechanisms. These set of experiments aim to explore what are the necessary conditions to maintain the cyclic competition between the three strategies, even in adverse scenarios in which the fraction of agents of a given strategy is reduced in the population, up to its complete extinction.

4.5.1 Two species

For the experiments involving only two strategies, or species, we use the evolved population (strategies and spatial structure) obtained in our first experiment (Sec. 4.4), i.e., the outcome of 10^6 Monte Carlo steps. All other parameter settings are kept the same ($\Delta/\delta = 0.3$, $\delta = 0.8$, $l = 0.5$ and $b = 1.9$).

Before the simulation, we replace all individuals of a strategy by another, obtaining a population of two strategies. It was found that a state of cyclic competition between only two strategies cannot be reached and that the outcomes are always a full dominance of a specific strategy. The results can be summarised as follows:

- With an initial population of C and A , C will dominate.
- With an initial population of D and A , A will dominate.
- With an initial population of C and D , D will dominate.

As well as mimicking outcomes that we observe in nature, these results highlight the importance of cyclic competition in the sustenance of biodiversity. For instance, in our abstract model, the state of complete dominance of a strategy does not necessarily incur advantages. In other words, some species may prefer to live in smaller numbers in the environment in order to give more opportunity for prey to develop. Thus, although defectors prefer to stay away from abstainers, a population fully occupied by defectors lacks resources ($P = 0$). Thus, defectors need abstainers to keep the cooperators alive, which in turn will enable defectors to increase their payoffs.

4.5.2 Three species

Following the same procedures performed for the pairwise simulation (Sec. 4.5.1), and for the same parameter settings, we now explore the behaviour of a population with three strategies, or species, after an adverse scenario occurs in which most of the agents of a particular strategy undergo mutation.

This analysis is important in order to understand the necessary conditions for a population to maintain the stable coexistence of strategies in cyclic competition. Thus, considering that the initial population has $1/3$ of each strategy, we run several Monte Carlo simulations varying the mutation rate from 1 to 99 percent. Surprisingly, results show that the state of cyclic competition is very robust and for all simulations the population quickly returns to the equilibrium of about $1/3$ of each strategy with the same spatial pattern as shown in Figure 4.2.

In order to further explain the results witnessed in these experiments, in which the vast majority of our simulations with extremely high mutation rates, i.e. 99.9%, converged to the equilibrium; we decided to analyse cases where only one agent of a specific strategy does not mutate in the population (i.e., maximum mutation rate). This scenario, in turn, is illustrated in Figure 4.3, which shows the snapshots of the mutated population at the initial Monte Carlo step.

Results show that only one individual of a species needs to be kept in the population in order for the cyclic competition to remain. All scenarios in Figure 4.3 quickly reverted back to the spatial pattern illustrated in Figure 4.2. However, it is only possible if the single agent is linked with a sufficient number of subordinate agents. Namely, a cooperator must have abstainers in its neighbourhood; similarly abstainers must have defectors,

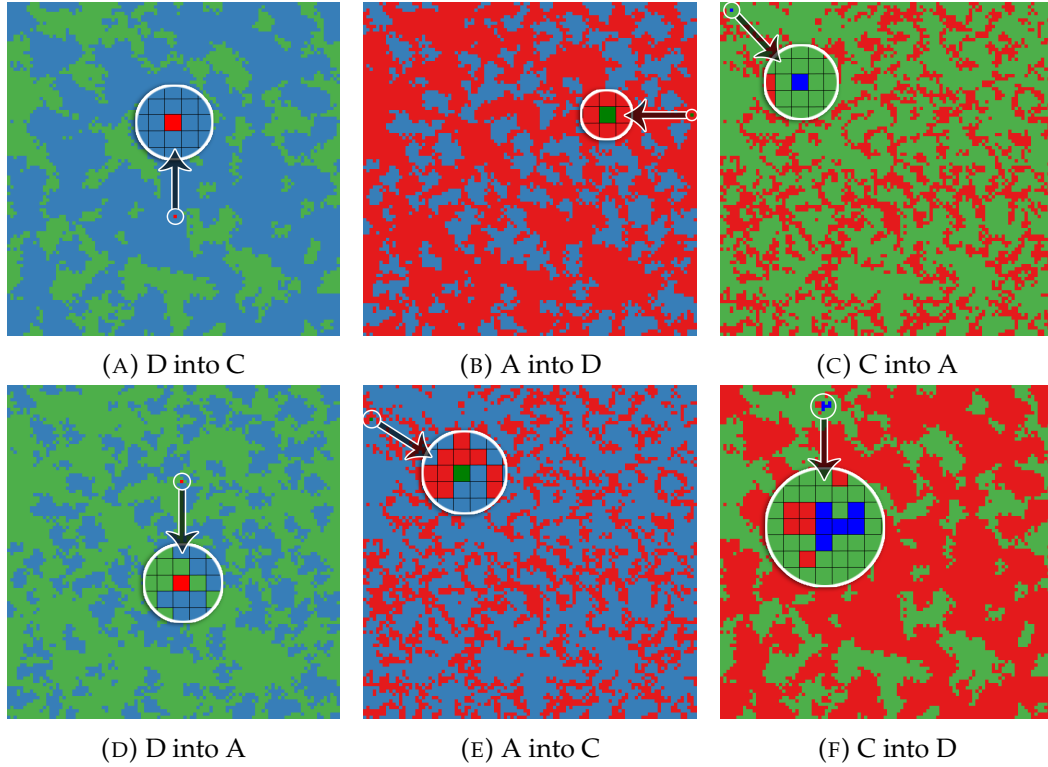


FIGURE 4.3: Initial population after mutation of strategies. All scenarios recover the balance between the three strategies, showing a spatial pattern similar to the one illustrated in Figure 4.2. In this Figure, cooperators, defectors and abstainers are represented by the colours blue, red and green respectively.

and defectors must have cooperators. This condition explains the robustness of our previous experiment, in which the balance was recovered even when 99% of the agents of a strategy were mutated. In this situation, as we have a population of 10404 agents, the chances of at least one of 104 agents being connected to a satisfactory number of subordinated strategies is very high.

It can be seen that when mutating D s into C s, we unbalance the final population to about $2/3$ C s and $1/3$ A s. We observe that this difference is not too relevant for the final outcome and that the key role in sustaining the cyclic competition is in the hands of the single agent (D in this example). Results show that balance between the three strategies is always recovered when the single agent is completely surrounded by its subordinate strategy.

For instance, a single defector surrounded by cooperators will never be replaced by a cooperator because its utility will be greater than the utility of any neighbour.

When the mutation occurs from a dominant to a subordinate strategy such as, from defector to cooperator, abstainer to defector or cooperator to abstainer, the chances of the single agent being kept in the middle of a suitable cluster (subordinate strategy) are very high. This scenario can be observed in Figures 4.3a, 4.3b and 4.3c. However, in the opposite case it is not generally possible because, during the evolutionary dynamics, a subordinate strategy will almost never survive alone in the middle of a cluster of a dominant strategy.

Therefore, the special aforementioned cases are not as stable as the case in which the single agent is surrounded by subordinated neighbours. Its stability will depend on the number of links with subordinated strategies and the respective values of the link weights. In our experiments, it was observed that when the single agent is a defector, the cyclic competition is recovered more often when the number of cooperators in the neighbourhood is greater than or equal to four (Fig. 4.3d). When the single agent is an abstainer, at least five of the eight neighbours need to be defectors (Fig. 4.3e).

Finally, as shown in Figure 4.3f, it is usually impossible to sustain cyclic competition when the single agent kept after mutation is a cooperator. It happens because cooperators are very sensitive to the presence of defectors and, in this sort of scenario, they need at least one partner to be able to be rewarded for mutual cooperation (R). Thus, better results are obtained as the number of cooperators kept together increases.

4.5.3 Destroying the environment

Previous experiments discussed in Sections 4.5.1 and 4.5.2 have considered the mutation of species (strategies) applied to an already coevolved population (species and spatial environment). Namely, strategies are transformed, but the spatial environment is still the same.

In this section we are interested in exploring the impacts of changing the spatial environment at the mutation step such that all link weights between agents are reset back to $w = 1$ as existed in the initial settings of the first experiment (Sec. 4.4).

Results show that the robustness witnessed in previous simulations, in which only one species was needed to recover the balance between the three species, is actually only possible because the spatial environment is kept unchanged. The coevolutionary rules adopted enables agents to constantly strengthen beneficial connections and weaken harmful ones, adapting the environment to fit individual needs. Hence, in the previous experiments, as the non-mutated agent is probably the most adapted in the neighbourhood, its strategy spreads quickly, allowing the population to recover the balance.

This finding highlights the importance of the coevolutionary model in allowing agents to adapt the environment to sustain the diversity of strategies.

4.6 Conclusions and future work

In this chapter, we have investigated the phenomenon of cyclic dominance in a coevolutionary optional prisoner's dilemma, in which both game strategies and edges linking agents adaptively evolve over time. An agent-based Monte Carlo simulation approach was adopted to perform the evolutionary

game in a population of agents placed on a lattice grid with a Moore neighbourhood.

Despite the fact that the rock-paper-scissors game remains one of the most oft-studied games in scenarios of cyclic dominance, in this chapter, we showed that the same behaviour can also be noticed in the optional prisoner's dilemma game, which allows the investigation of more complex scenarios that may lead to a variety of outcomes.

Results show that cyclic dominance between the three strategies can emerge spontaneously in a wide range of parameter settings, i.e., l , b , Δ and δ , including the traditional case ($\Delta = 0.0$) for an unweighted and static network. However, it was observed that populations of only two strategies can quickly lead to dominance of one strategy, which may lead to a much lower performance.

Experiments revealed that the equilibrium between the three strategies is maintained even in adverse scenarios, in which the mutation rate is extremely high. It was observed that having only one agent of a strategy is often enough to enable the population to revert back to a balanced state. However, this single agent must be surrounded by subordinated agents, i.e., cooperators surrounded by abstainers, abstainers surrounded by defectors and defectors surrounded by cooperators.

Moreover, it was shown that the coevolutionary spatial method adopted plays a key role in the sustenance of coexistence because it allows agents to also adapt the environment, which is reasonable in more realistic scenarios. For instance, in real life, a population is often changing the environment over time in order to improve their performance and welfare. Thus, in adverse scenarios, it is much easier for an individual to overcome and survive in such an evolved environment.

Future work will involve the mathematical analysis of the necessary conditions to sustain the coexistence of three competing strategies of the optional prisoner's dilemma game, allowing us to further explain the results obtained by Monte Carlo simulations.

Chapter 5

A Further Analysis of The Role of Heterogeneity in Coevolutionary Spatial Games

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Abstract

Heterogeneity has been studied as one of the most common explanations of the puzzle of cooperation in social dilemmas. A large number of papers have been published discussing the effects of increasing heterogeneity in structured populations of agents, where it has been established that heterogeneity may favour cooperative behaviour if it supports agents to locally coordinate their strategies. In this chapter, assuming an existing model of a heterogeneous weighted network, we aim to further this analysis by exploring the relationship (if any) between heterogeneity and cooperation. We adopt a weighted network which is fully populated by agents playing both the prisoner's dilemma or the optional prisoner's dilemma games with coevolutionary rules, i.e., not only the strategies but also the link weights evolve over time. Surprisingly, results show that the heterogeneity of link weights (states) on their own does not always promote cooperation; rather cooperation is actually favoured by the increase in the number of overlapping states and not by the heterogeneity itself. We believe that these results can guide further research towards a more accurate analysis of the role of heterogeneity in social dilemmas.

5.1 Introduction

Issues regarding the emergence of cooperation and altruism in structured populations have puzzled scientists in a large range of domains. In this context, methods of statistical physics combined with concepts of both graph theory and evolutionary game theory [113, 142] have been used as simple and powerful tools to describe and analyse the conflict of interest between individuals and groups [127]. In those models, agents are arranged on graphs in such a way that their interactions are restricted to their immediate neighbours [118, 94]. Over the last two decades, it has been shown that different topologies such as lattices [116], scale-free graphs [160, 190, 136], small-world graphs [36, 53], cycle graphs [4], star-like graphs [168] and bipartite graphs [129, 63] have a considerable impact on the evolution of cooperation, which also favours the formation of different patterns and phenomena [155, 125]. However, the vast majority of these studies adopt static networks, which are not suitable for modelling scenarios in which both the game strategies and the network itself are subject to evolution [195, 197, 198, 156, 204, 203]. Thus, the use of dynamic networks represents a natural upgrade of the traditional spatial games [124].

The prisoner's dilemma (PD) is still the most often used game in this field. In this game, an agent can either cooperate (C) or defect (D), obtaining a payoff that depends on the other's agent choice [132]. However, in many scenarios, agents have the freedom to decide whether to participate in the game. Games such as the optional prisoner's dilemma (OPD) [147, 13] and the voluntary public goods game [97, 68] incorporate this concept of voluntary participation by adding a third strategy to the game, allowing agents to not only cooperate or defect but also to abstain (A) from a game interaction. Research has shown that the presence of abstainers in the population can actually protect cooperators against exploitation [32, 67].

Studies on weighted networks have attracted much attention as such networks enable the representation of the strength of each connection, which is essential information in a wide range of real-world scenarios including biological networks and social media. Recently, both the prisoner's dilemma and the optional prisoner's dilemma games have been explored in the context of dynamic weighted networks, which lead to a coevolutionary scenario where not only the game strategies, but also the link weights, evolve over time [28, 27, 74, 20, 93]. Moreover, it has been shown that the use of dynamic weighted networks can increase heterogeneity of states (i.e., the number of possible utilities in the network), which in turn induces the promotion of cooperation. In fact, previous work has also discussed the effects of heterogeneity on the evolution of cooperation [50, 76, 5, 126, 185, 157, 163, 158, 136], however, the specific conditions that increase the diversity of link weights in the dynamic weighted networks remain unclear. Also, a number of questions regarding the evolutionary dynamics of the network itself remain to be answered, such as:

- How the link weights between agents evolve over time?
- How two parameters of the model (Δ and δ) affect the link weight variance?
- Is there an optimum value of the two parameters Δ and δ ?
- Why higher values of δ promote cooperation best?
- Why the coevolutionary optional prisoner's dilemma game performs better than the coevolutionary prisoner's dilemma game in adverse scenarios?
- Does the value of Δ affect the convergence speed in scenarios of full dominance of cooperation?

Thus, this work aims to answer these questions by analysing the micro-macro behaviour of a population of agents playing both the coevolutionary prisoner's dilemma (CPD) and the coevolutionary optional prisoner's dilemma (COPD) game, i.e., the classical PD and OPD games in a dynamic weighted network. The remainder of this chapter is organized as follows. Section 5.2 describes the Monte Carlo simulation and the coevolutionary games adopted. Section 5.3 features the results. Finally, Section 5.4 summarizes our findings and outlines future work.

5.2 Methodology

This work adopts a weighted lattice grid with periodic conditions (i.e., a toroid) fully populated with $N = 102 \times 102$ agents playing a coevolutionary game. Each agent on site x interacts only with its eight immediate neighbours (i.e., $k = 8$, Moore neighbourhood). Both the coevolutionary prisoner's dilemma (CPD) game [74] and the coevolutionary optional prisoner's dilemma (COPD) game [28] are considered.

Initially, each edge linking agents has the same weight $w = 1$, which will adaptively change according to their interaction. Also, each agent (x) is initially assigned to a strategy with equal probability. For the CPD game, each agent can be designated either as a cooperator ($s_x = C$) or defector ($s_x = D$), while in the COPD game, agents can also be designated as abstainer ($s_x = A$). Thus, strategies ($s_x = C, D, A$) can be denoted by a unit vector respectively as follows:

$$C = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, D = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, A = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \quad (5.1)$$

The games are characterized by the payoff obtained according to the pairwise interaction of agent x and its neighbour y . Accordingly, the agent x may receive a reward $\pi_{xy}(C, C) = R$ for mutual cooperation; a punishment $\pi_{xy}(D, D) = P$ for mutual defection; $\pi_{xy}(D, C) = T$ for successful defection (i.e., there is a temptation to defect); $\pi_{xy}(C, D) = S$ for unsuccessful cooperation (well-known as the sucker's payoff); or the loner's payoff (L), which is obtained when one or both agents abstain (i.e., $\pi_{xy}(A, C) = \pi_{xy}(A, D) = \pi_{xy}(C, A) = \pi_{xy}(D, A) = \pi_{xy}(A, A) = L$). We adopt a weak version of both games, where the payoff $R = 1$, $T = b$ ($1 < b < 2$), $L = l$ ($0 < l < 1$) and $S = P = 0$ without destroying the nature of the dilemma [116]. Thus, the payoff matrix $\boldsymbol{\pi}$ is given by:

$$\boldsymbol{\pi} = \begin{pmatrix} 1 & 0 & l \\ b & 0 & l \\ l & l & l \end{pmatrix}, \quad (5.2)$$

where:

$$\pi_{xy}(s_x, s_y) = s_x^T \boldsymbol{\pi} s_y. \quad (5.3)$$

The utility u_{xy} of agent x with its neighbour y is calculated as follows:

$$u_{xy} = w_{xy} \pi_{xy}, \quad (5.4)$$

where w_{xy} represents the symmetric link weight of their interaction, i.e., $w_{xy} = w_{yx}$.

A number of Monte Carlo (MC) simulations are carried out to explore the micro-macro behaviour of both the strategies and the weighted network itself. Each MC simulation comprises the following elementary steps. First an agent x is randomly selected to play the coevolutionary game with its

$k = 8$ neighbours, obtaining an accumulated utility expressed as:

$$U_x = \sum_{y \in \Omega_x} u_{xy}, \quad (5.5)$$

where Ω_x denotes the set of neighbours of the agent x . Second, the agent x updates all the link weights in Ω_x by comparing each utility u_{xy} with the average accumulated utility (i.e., $\overline{U}_x = U_x/k$) as follows:

$$w_{xy} = \begin{cases} w_{xy} + \Delta & \text{if } u_{xy} > \overline{U}_x, \\ w_{xy} - \Delta & \text{if } u_{xy} < \overline{U}_x, \\ w_{xy} & \text{otherwise,} \end{cases} \quad (5.6)$$

where Δ is a constant such that $0 \leq \Delta \leq \delta$. In line with previous work [28, 74, 183], the link weight is corrected to satisfy $1 - \delta \leq w_{xy} \leq 1 + \delta$, where δ ($0 \leq \delta < 1$) defines the weight heterogeneity. Note that when $\Delta = 0$ or $\delta = 0$, the link weight remains constant ($w = 1$), which decays in the classical scenario for static networks, i.e., only the strategies evolve. Finally, the agent x updates its strategy by comparing its current accumulated utility U_x (i.e., considering the updated weights) with the accumulated utility of one randomly selected neighbour (U_y) such that, if $U_y > U_x$ agent x copies s_y with a probability proportional to the utility difference as follows:

$$p(s_x \leftarrow s_y) = \frac{U_y - U_x}{k(T - P)}, \quad (5.7)$$

otherwise, agent x keeps its strategy for the next step.

In one Monte Carlo step (MCS), each agent is selected once on average, which means that the number of inner steps in each MCS is equal to the population size. Simulations are run for a sufficiently long thermalization time (10^6 MCS). Furthermore, to alleviate the effect of randomness and

to ensure proper accuracy in the approach, the final results are obtained by averaging 10 independent runs. It is noteworthy that due to the introduction of the weight factor (w) and the quenched heterogeneities via δ , the model is prone to evolve into frozen patterns which represent quenched spatial randomness where a Griffiths phase [59] can emerge. As has been discussed in previous studies [126, 44], the evolutionary dynamics in these scenarios tend to be very slow, which introduces some technical difficulties in classifying the final stationary state. This is because the transition of the clusters of the subordinate strategy into the dominant strategy requires that a large number of the subordinate agents swap their strategies in a short period of time, which is an occurrence that is very difficult.

5.3 Results

In this section, we present some of the relevant experimental results obtained when simulating a population of agents playing both the prisoner's dilemma and the optional prisoner's dilemma game on weighted networks.

5.3.1 Exploring the coevolutionary rules

As discussed in previous research [28], one interesting property of the ratio Δ/δ is that for any combination of both parameters Δ and δ , if their ratio is the same, then the number of states is also the same. For instance, the pairs $(\Delta = 0.02, \delta = 0.2)$ and $(\Delta = 0.08, \delta = 0.8)$ have both 21 possible link weights (states).

Despite the fact that the interval between the maximum and minimum link weights increases as we increase δ , intuition may lead us to believe that given two scenarios with the same payoff matrix (i.e., Eq. 5.2 for the same

temptation to defect and loner's payoff) and the same number of states (i.e., the same ratio Δ/δ), the outcome would be the same. Surprisingly, previous research has shown that it is not true [28, 74]. Actually, it has been discussed that higher values of δ promote cooperation best, even if the number of states remains the same. Figure 5.1 illustrates this scenario, where the average fraction of cooperation of the last 10^3 steps for a wider range of settings (i.e., the ratio Δ/δ , the loner's payoff l and the temptation to defect b) are considered for different values of δ (i.e., $\delta = \{0.2, 0.4, 0.8\}$). In fact, the outcomes of the different scenarios are very different to each other and we can observe that as δ increases, the number of cases in which cooperation is the dominant strategy also increases. This phenomenon is still unexplained and exploring the properties that cause this discrepancy may lead to a complete understanding of the presented coevolutionary model. In any case, it is noteworthy that even for small values of δ , the coevolutionary optional prisoner's dilemma (COPD) game is still much more beneficial for the emergence of cooperation than the traditional OPD [32] or the coevolutionary prisoner's dilemma (CPD) [74].

As is also shown in Figure 5.1, we investigate the influence of the ratio Δ/δ in the link weight variance (i.e., link weight heterogeneity) at the last Monte Carlo steps. Note that as the link weight (w) is always within the range $[1 - \delta, 1 + \delta]$, the maximum link weight variance is defined by δ^2 . These experiments reveal that the link weight variance is not uniform for all environmental settings. Also, although higher values of δ promote higher heterogeneity, we can see that the correlation between cooperation and heterogeneity is not necessarily true for all values of δ . For instance, the link weight variance for $\delta = 0.8$ is usually maximum when cooperation is the dominant strategy. However, this does not hold for both $\delta = 0.4$ and $\delta = 0.2$.

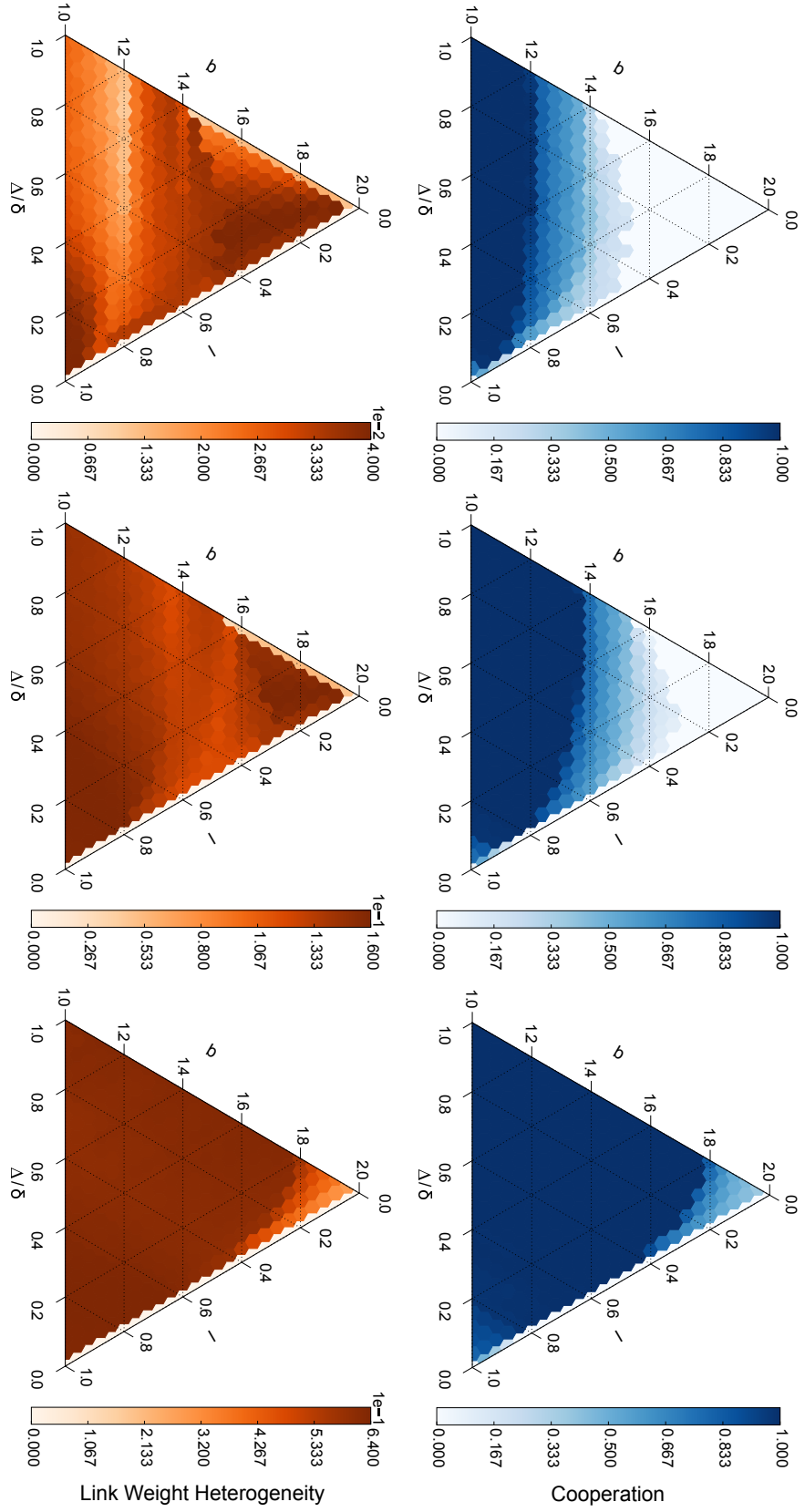


FIGURE 5.1: Ternary diagrams showing the average fraction of cooperation (top) and the link weight heterogeneity (bottom) for different values of b (i.e., temptation to defect), l (i.e., the loner's payoff) and Δ/δ for $\delta = \{0.2, 0.4, 0.8\}$ (from left to right). The diagrams allow us to investigate the effects of different parameter settings for fixed values of δ . The correlation between cooperation and heterogeneity does not seem to hold for all values of δ .

Figure 5.2 shows the typical distributions of the pairs of strategies (i.e., the edges) in the stationary or quasi-stationary states. In these snapshots we use colors to differentiate the types of edges and opacity to differentiate the link weights, where, for each scenario, a transparent edge means that the weight is at minimum (i.e., $w = 1 - \delta$) and a bright edge means that the weight is at maximum (i.e., $w = 1 + \delta$). As expected, considering that $l > 0$, the pattern *all D* (Fig. 5.2a) is only possible in the CPD game. Moreover, for the CPD game, it is also possible to observe the patterns *all C* (Fig. 5.2b) and *C+D* (Fig. 5.2d). For the COPD game, all other patterns are also possible, i.e., *all A* (Fig. 5.2c), *C+A* (Fig. 5.2e) and *C+D+A* (Fig. 5.2f) phases can also be observed. Of course, the size of the clusters and the average link weight at the stationary state will depend on the parameter settings. However, for any scenario, it was observed that the population always evolves to one of these patterns. Further analysis on the effects of varying the parameter settings have been shown in previous studies [28, 27, 74].

Another interesting result is that, although previous research has claimed that “intermediate link weight amplitude can provide best environment for the evolution of cooperation” [74], our experiments reveal that there is no global optimal value of Δ/δ (defined by Huang et al. [74] as the link weight amplitude) nor δ for all environmental settings. Moreover, despite the fact that high δ usually leads to more cooperation, it does not mean that high δ is always the best option. Fig. 5.3, for example, illustrates a scenario in which high δ is actually a bad choice. In fact, as already expected (Section 5.2), in many cases it is possible to observe that the population evokes the existence of Griffiths-like phases, which makes it very difficult for the system to converge to a stationary state. For instance, the population evolves into a frozen pattern in the scenarios shown in Fig. 5.2d and Fig. 5.2e; moreover, the curves for $\{\Delta = 0.45, \delta = 0.5\}$, $\{\Delta = 0.63, \delta = 0.7\}$

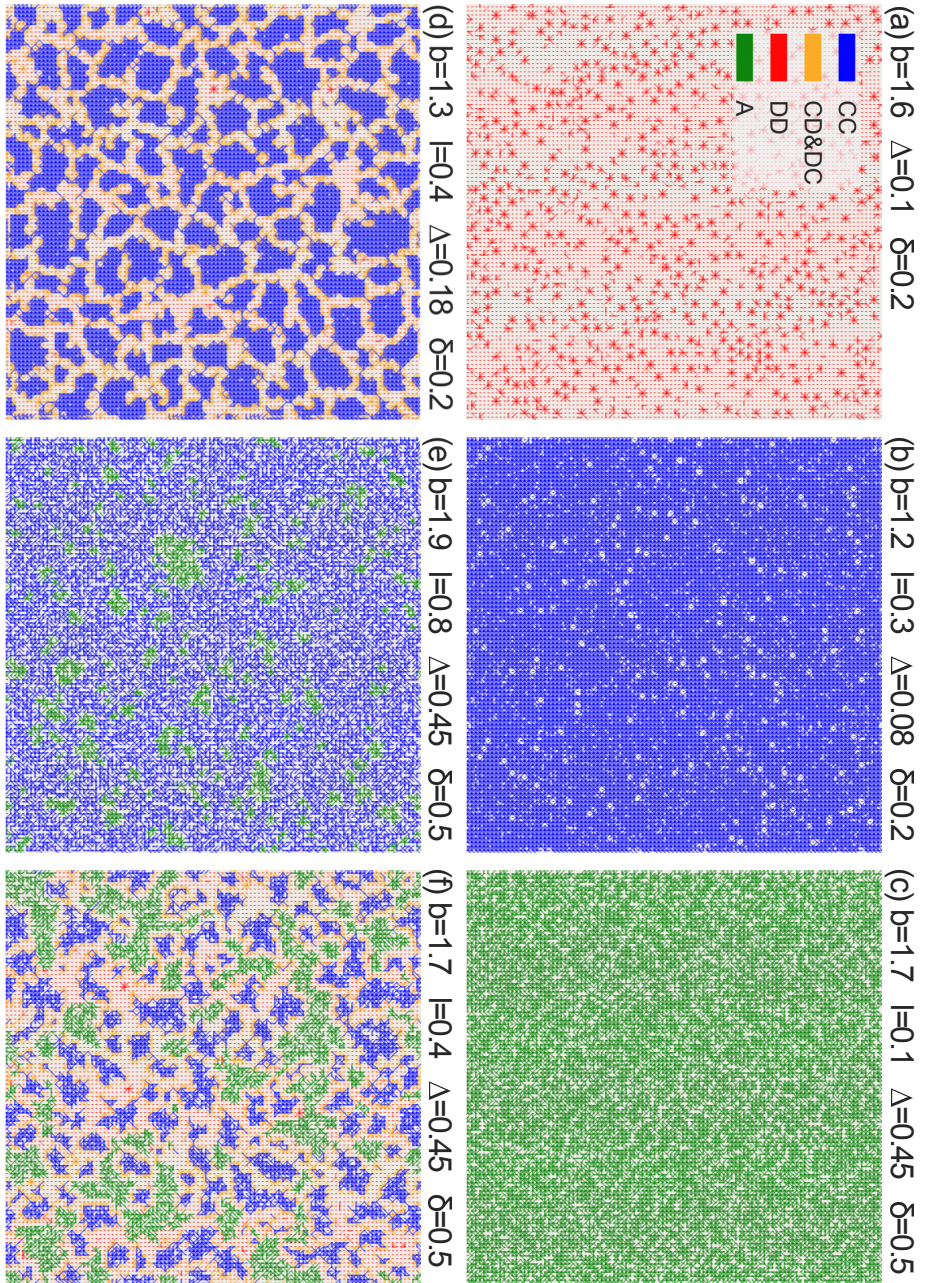


FIGURE 5.2: Typical distribution of edges after a sufficiently long thermalization time (10^6 MCS). The opacity of each edge represents its current weight (w) in the network and is adjusted to be transparent at minimum (i.e., $w = 1 - \delta$) and bright at maximum (i.e., $w = 1 + \delta$). The pattern **a** is only possible for the CPD game, which can also exhibit the patterns **b** and **d**. The patterns **b-f** can be observed in the COPD game.

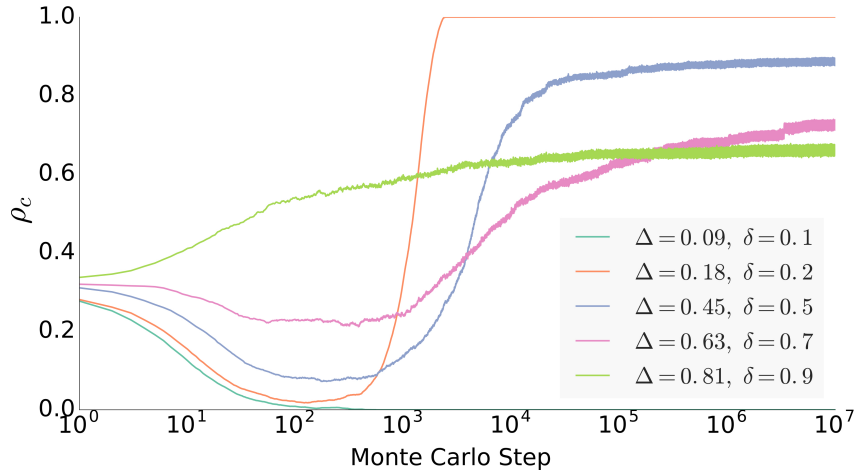


FIGURE 5.3: Time course of the fraction of cooperation for different values of Δ and δ when $b = 1.9$ (temptation to defect), $l = 0.8$ (loner's payoff) and $\Delta/\delta = 0.9$. Contrary to what Fig. 5.1 may suggest, here we see that high δ is not always the best option to promote cooperation.

and $\{\Delta = 0.81, \delta = 0.9\}$ in Fig. 5.3 are also evidence of the same technical difficulties in classifying the stationary state. Note that, in some scenarios, the presence of the cyclic dominance for the COPD (i.e., coexistence of the three strategies as observed in Fig. 5.2f) may eliminate the emergence of the frozen patterns when the population has only two strategies, i.e., C+D or C+A. This phenomenon has been discussed in the literature of evolutionary games [127, 126, 44]. Furthermore, the dynamical behaviour observed in Fig. 5.3 illustrates the nature of enhanced network reciprocity [114] promoted when $\delta > 0.1$. In these scenarios, we can see that defectors are dominated by abstainers, allowing a few clusters of cooperators to survive; as a result of the absence of defectors, cooperators invade most (or all) of the abstainers in the population, which explains the initial drop, and the subsequent recovery, of the fraction of cooperators in Fig. 5.3. Similar behaviour has also been observed in previous work [157].

These results motivate the search for a better understanding of the evolutionary dynamics of the link weights. In the following sections, we will discuss how the link weights evolve over time.

5.3.2 Understanding how the link weights evolve

In order to better understand how the link weights between agents evolve over time, we investigate the distribution of link weights for different values of b (temptation to defect), Δ and δ , for both the coevolutionary prisoner's dilemma (CPD) [74] and the coevolutionary optional prisoner's dilemma (COPD) [28] games, where the latter also involves the variation of the loner's payoff (l).

Figure 5.4 shows the distribution of link weights for each type of agent interaction when $b = 1.6$, $l = 0.2$, $\Delta = 0.2$ and $\delta = 0.8$ which is representative of the outcomes of other values as well. As discussed previously, we know that the ratio Δ/δ can be used to determine the number of link weights that an agent is allowed to have, which is actually evidenced when the link weight distribution is plotted over time. Despite the fact that the percentage of each type of link varies according to factors such as the total number of states and the value of b and l (for the COPD game), which will consequently affect the final outcome, it was observed (as shown in Figure 5.4) that for any ratio Δ/δ the initial dynamics of all types of links is exactly the same for both games, that is:

- **Observation 1** Defector-Defector (DD) tends to move to states of lowest link weight.
- **Observation 2** Cooperator-Defector (CD) and Defector-Cooperator (DC) move to the extremes, keeping a small amount of intermediate states.
- **Observation 3** Cooperator-Cooperator (CC) tends to move to states of highest link weight, but will also occupy the state of lowest link weight as the DCs and ACs will eventually become CCs.

- **Observation 4** Abstainers (AC, AD, CA, DA or AA) move to the extremes.

Considering that utility is obtained by the product of link weight and payoff (Eq. 5.4), and that the payoff of DD and CD is equal to zero (Eq. 5.2), the utility (u_{xy}) associated with these link types will always be equal to zero, which is always the worst case as $u_{xy} \geq 0.0$. In this way, these agents will always be punished by Δ (Eq. 5.6) and consequently, occupy states of lowest link weight (Observation 1).

Also, note that CD and DC are unstable configurations as the first will always get $u_{xy} = 0.0$ and the second is prone to get higher utilities as the temptation to defect (b) is always the highest payoff (Eq. 5.2). Thus, these agents are constantly receiving $\pm\Delta$, which explains the phenomenon of having a small number of them along the intermediate states (Observation 2).

Although the payoff obtained by mutual cooperators (CC) is smaller than the one obtained by a defector-cooperator interaction (DC), i.e., $T > R$, the mutual cooperators are much more stable than DCs as both agents always get the same payoff (i.e., R). That is exactly the reason why these agents tend to a maximum link weight (Observation 3). Also, note that as the link weight is updated based on the comparison of the local utility of each connection with the average utility of the eight neighbours, when a cluster of nine cooperators is formed (i.e., one cooperator surrounded by eight cooperators), their links will remain in equilibrium, where the average link weight will tend to the value of R . In this way, for most scenarios of full dominance of cooperation, approximately half of the links will have a minimum weight and the other half will have a maximum weight.

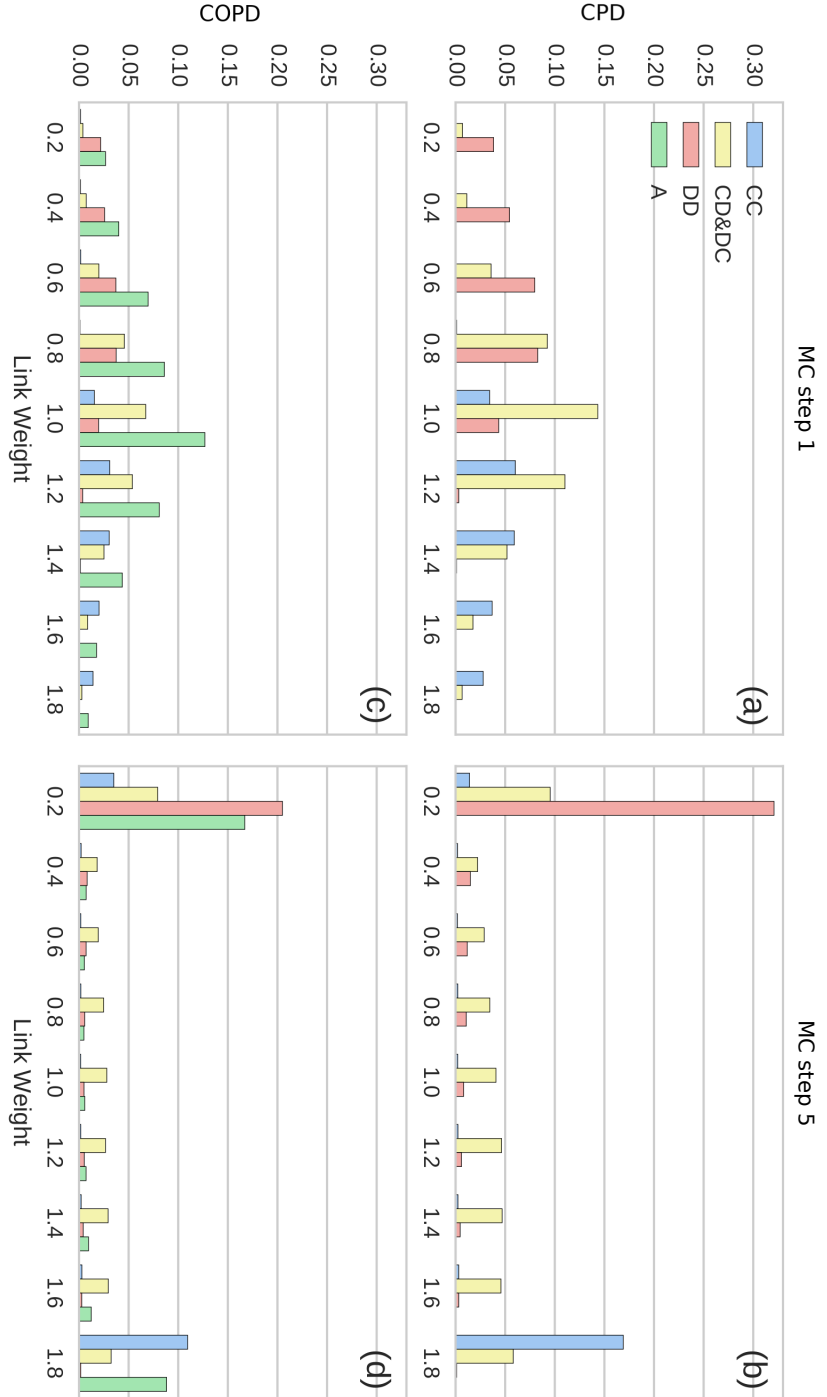


FIGURE 5.4: Distribution of link weight at the Monte Carlo steps 1 (left) and 5 (right) for a population of 102×102 agents playing the coevolutionary prisoner's dilemma game (top) and the coevolutionary optional prisoner's dilemma game (bottom) for $l = 0.2$ (loner's payoff), $b = 1.6$ (temptation to defect), $\Delta = 0.2$ and $\delta = 0.8$. It was observed that the initial behaviour of all types of links is the same for both games regardless of the ratio Δ/δ , i.e., the initial link weight distribution looks the same for all scenarios, where the only difference is in the proportion of each type, which depends on the values of all parameter settings.

It is noteworthy that we count all types of Abstainer's connections (AC, CA, AD, DA and AA) together because in the optional prisoner's dilemma game, when a agent abstains, both agents receive the same payoff (l). In this way, the main reason why abstainers move to the extremes in Figure 5.4 is that DD and CD agents ($u_{xy} = 0.0$) tend to abstain to increase their local utility ($u_{xy} > 0.0$) becoming ADs, which consequently are allocated in states of lowest link weight. For the same reason, abstention might be the best option in mixed clusters of C's and D's, where the chances of getting $u_{xy} = 0.0$ increases, then the agents may tend to abstain, eventually going to states of highest link weight (Observation 4).

Moreover, we point out that, as we force all link weights to be within the range $1 - \delta$ to $1 + \delta$, the phenomenon of having more agents occupying the maximum and minimum states is clearly expected. However, the observation of the initial dynamics of both games being the same for any combination of the parameters (i.e., b, l, Δ and δ) is a counter-intuitive result, which in turn shows that the observations discussed above are valid for both models.

5.3.3 Investigating the role of heterogeneity

The reason why higher values of δ promote cooperation best remains one of the central open questions in this model. Based on the results discussed in previous sections, we know that the link weights usually evolve heterogeneously, which makes the effective payoff matrix unpredictable, adding a new layer of complexity to the model. For instance, in the traditional prisoner's dilemma game, any defector who plays with a cooperator will always get the value of the constant b ; however in the coevolutionary model, this is unpredictable and heterogeneous as each defector-cooperator interaction might be in a different state.

Considering that the boundary states and the set of possible link weights is determined by the parameters Δ and δ , we can calculate all the possible utilities for each type of edge (i.e., CD, DC, DD, CC and A), which may allow us to better understand how the parameter settings affect the interplay between the evolution of strategies and their possible utilities. In this way, Figure 5.5 shows the shape of all possible utilities for four different scenarios, all for the same temptation to defect ($b = 1.6$) and the same number of states ($\Delta/\delta = 0.2$, i.e., 11 states). Monte Carlo simulations revealed that cooperation is the dominant strategy in the scenarios of Fig. 5.5b-d; and that abstention dominates in Fig. 5.5a.

In fact, when we plot the possible utilities side by side (Fig. 5.5) we can see that the outcomes obtained through Monte Carlo simulations were actually expected. For instance, in Fig. 5.5a, the DC connections will always be the most profitable option in the initial steps, which in turn make the population of cooperators die off. After that, with the lack of cooperators in the population, DC is not possible anymore and abstention starts to be the best option as its payoff is always greater than the punishment for mutual defection (i.e., $lw_a > 0$).

However, notice that when the value of δ is increased (i.e., Fig. 5.5b), the overlap between the possible utilities for each type of connection also increases. In this case, we see that DC is the best option only in 5/11 of the cases, which enable cooperators to survive and as DC tends to minimum, CC will tend to maximum and abstention is sometimes better than DC (Section 5.3.2). Thus, the dominance of cooperators is also expected. We can also observe that due to the huge overlap of DCs and CCs, even if the loner's payoff is very low, i.e., Fig. 5.5d, or if abstention does not exist (i.e., CPD game), cooperation would still be expected.

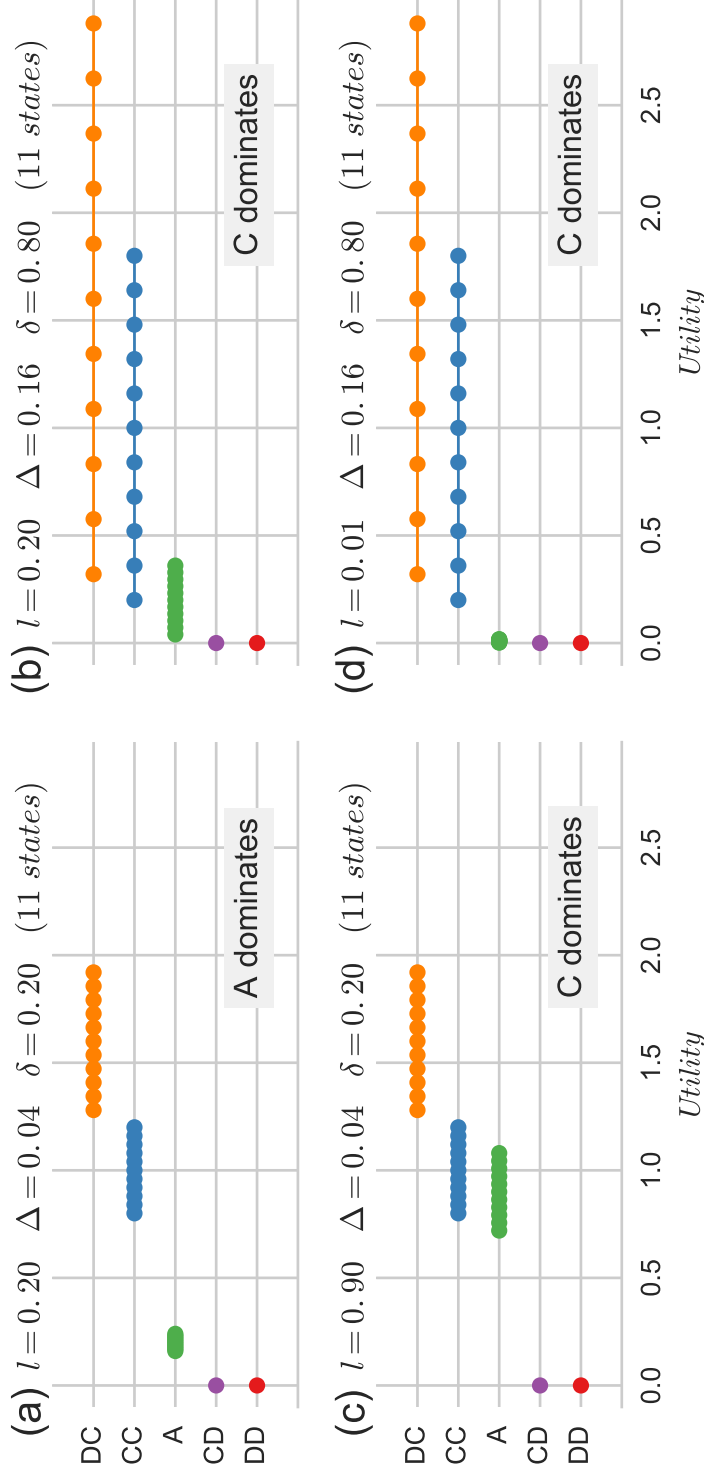


FIGURE 5.5: Possible utilities (i.e., the product of payoff and link weight) for each type of edge, i.e., Defector-Cooperator (DC), Cooperator-Cooperator (CC), Cooperator-Defector (CD), Defector-Defector (DD) and the ones with one or two Abstainers (A). We observe that there is a connection between the overlap of possible utilities and the final outcome.

Moreover, Fig. 5.5c illustrates interesting evidence of how abstention can support cooperation. The only difference between this scenario and Fig. 5.5a is the value of l . At a first glance, intuition may lead us to believe that if Fig. 5.5a with $l = 0.2$ resulted in full dominance of abstainers, increasing the value of l would just make the option to abstain more profitable, which consequently would not change the outcome. Surprisingly, this does not occur. Actually, abstainers only dominate the whole population when the population of cooperators is decimated. In this way, despite the fact that DC is still the best option and that the population of cooperators tend to decrease, they will not die off. Then, when the population of defectors become too high, they will prefer to turn into abstainers and with the increase of abstention, mutual cooperation will now be the best option, which allows abstainers to fully dominate the environment.

Thus, results show that if any of the possible DC, CC and A utilities do not overlap, then abstention will be the dominant strategy for the COPD game and defection will be the dominant strategy for the CPD game, except of course, when b is too low (i.e. $b < 1.1$), which usually promotes the coexistence of the available strategies. In general, we can observe that the greater the overlap between DC and CC utilities or/and CC and A utilities, then the more chances cooperators have to survive and dominate.

Notice that both the loner's payoff (l) and the link weights (which are controlled by the parameters Δ and δ) are actually mechanisms to weaken the benefits of defecting (i.e., effective utility of DC). As the parameter δ will act in the expansion of the utility boundaries, the greater the value of δ , the greater are the number of cases in which CC overlaps DC, which in turn promote cooperation best. The same scenario occurs when CC and A overlap, which will work as an extra mechanism to strengthen cooperators. That is, when the overlap of DC and CC is scarce or absent, overlapping CC and A

can help cooperators to survive. This also explains why COPD is better than CPD in adverse scenarios [28].

The drawback of a large overlap of utilities is that the population may evolve into a frozen pattern in which a Griffiths-like phase can occur. In these scenarios, it might be very difficult to reach the full dominance of cooperative behaviour (Fig. 5.3). Thus, higher values of δ may promote cooperation best in a wider range of scenarios, but it might evoke the presence of frozen patterns of C+A. It is noteworthy that as the utility overlaps are also dependent of the values of b and l , all parameter settings may, in fact, influence the emergence of these frozen patterns.

5.4 Conclusions

This work investigates the role of heterogeneity in a population of agents playing the prisoner's dilemma (PD) game and the optional prisoner's dilemma (OPD) game on a weighted square network with boundary conditions. coevolutionary rules are adopted, enabling both the game strategies and the network to evolve over time, leading to the so-called coevolutionary prisoner's dilemma (CPD) and the coevolutionary optional prisoner's dilemma (COPD) games respectively. A number of Monte Carlo simulations are performed in which each agent is initially assigned to a strategy with equal probability (i.e., random initial distribution of strategies). Echoing the findings of previous research [28], we show that independently of the link weight heterogeneity, the COPD game is still much more beneficial for the emergence of cooperation than the traditional OPD or the CPD games. Moreover, although previous research has claimed the opposite [74], we show that there is no global optimal value of the parameters Δ and δ for all environmental settings.

Experiments revealed that the correlation between the emergence of cooperation and heterogeneity does not hold for all scenarios, indicating that heterogeneity itself does not favour cooperation. Actually, it was observed that the higher the heterogeneity of states, the greater the chance of overlapping states, which is the actual mechanism for promoting cooperation. Namely, when considering the COPD game, if any of the possible *Defector-Cooperator* (DC), *Cooperator-Cooperator* (CC) and *Abstention* (A) utilities do not overlap, then abstainers dominate the environment; while for the CPD game, defection will be the dominant strategy. In general, we observed that the greater the overlap between DC and CC utilities or/and CC and A utilities, the more chances cooperators have to survive and dominate.

Finally, we highlight that both the loner's payoff and the link weights are actually mechanisms that weaken the benefits of defecting. In addition, abstention also works as an extra mechanism to strengthen cooperators, which explains why COPD is better than CPD in adverse scenarios. We believe that it might be possible to analytically define, through the analysis of utility overlap, which is the best value of δ for a given payoff matrix. Also, considering this model for regular graphs, it might be interesting to consider pair approximation techniques to describe the evolutionary dynamics of weighted networks [10, 119]. To conclude, this chapter provides a novel perspective for understanding cooperative behaviour in a dynamic network, which resembles a wide range of real-world scenarios. We hope this chapter can serve as a basis for further research on the role of utility overlap to advance the understanding of the evolution of cooperation in coevolutionary spatial games.

Chapter 6

Cooperation in the spatial prisoner's dilemma game with probabilistic abstention

The work outlined in this chapter was published in:

Cardinot, M., Griffith, J., O'Riordan, C., and Perc, M. (2018). "Cooperation in the spatial prisoner's dilemma game with probabilistic abstention". Scientific reports, n. 14531, vol. 8, Nature.

Abstract

Research has shown that the addition of abstention as an option transforms social dilemmas to rock-paper-scissor type games, where defectors dominate cooperators, cooperators dominate abstainers (loners), and abstainers (loners), in turn, dominate defectors. In this way, abstention can sustain cooperation even under adverse conditions, although defection also persists due to cyclic dominance. However, to abstain or to act as a loner has, to date, always been considered as an independent, third strategy to complement traditional cooperation and defection. Here we consider probabilistic abstention, where each player is assigned a probability to abstain in a particular instance of the game. In the two limiting cases, the studied game reverts to the prisoner's dilemma game without loners or to the optional prisoner's dilemma game. For intermediate probabilities, we have a new hybrid game, which turns out to be most favorable for the successful evolution of cooperation. We hope this novel hybrid game provides a more realistic view of the dilemma of optional/voluntary participation.

6.1 Introduction

Over the last decades, the prisoner's dilemma game has been adopted in a variety of studies which seek to explore and resolve the dilemma of cooperation [127, 142, 8]. These studies include the use of the network reciprocity mechanism [114], which has gained increasing attention for its support of cooperative behaviour. In this mechanism, each agent is represented as a node in the network (graph) and is constrained to interact only with its neighbours, which are linked by edges in the network [118, 94]. Research concerning network reciprocity includes the use of different topologies such as lattices [116], scale-free graphs [160, 190, 136], small-world graphs [36, 53, 1], cycle graphs [4], multilayer networks [15, 180, 57] and bipartite graphs [129, 63] which have a considerable impact on the evolution of cooperation, and also favour the formation of different patterns and phenomena [125, 155]. Moreover, approaches adopting coevolutionary networks, where both game strategies and the network itself are subject to evolution have also been investigated [26, 93, 74, 20, 196, 124, 164, 204, 49, 203].

In essence, evolutionary game theory and its most-often used game, the prisoner's dilemma (PD) game, provides a simple and powerful framework to study the conflict between choices that are beneficial to an individual and those that are good for the whole community. The game is played by pairs of agents, who simultaneously decide to either cooperate (C) or defect (D), receiving a payoff associated with their pairwise interaction as follows: R for mutual cooperation, P for mutual defection, S for cooperating with a defector and T for successfully defecting a cooperator. The dilemma holds when $T > R > P > S$ [132]. In addition to theoretical research, there is also a lot of work using experimental games. The experimental prisoner's dilemma has been used by several researchers to find mechanisms to promote cooperative behaviour, including the benefit-to-cost ratio of cooperation [23], group

size [12, 22], dynamic spatial structure [131, 146], just to name a few examples.

Despite the overwhelming amount of scenarios that can be described as a PD game, it has been discussed that in many scenarios agents' interactions are not compulsory, and in those cases, the PD game would not be suitable. Thus, extensions of this game such as the optional prisoner's dilemma (OPD) game, also known as the prisoner's dilemma game with voluntary participation, have been explored in order to allow agents to abstain from a game interaction, that is, do not play the game and receive the so-called loner's payoff (L), which is the same regardless of the other agent's strategy (i.e., if either one or both agents abstain, both agents will get L). The dilemma is maintained when $T > R > L > P > S$ [147, 13]. Studies reveal that the concept of abstaining can lead to entirely different outcomes and eventually help cooperators to avoid exploitation from defectors [28, 81, 102, 137, 177, 111, 130, 18, 55, 38, 27, 99, 79, 32, 34, 188, 152, 67]. Of relevance for our research is also the literature on games with an exit strategy. For example, research has been done on the dictator game with an exit strategy [92, 21].

However, we believe that in many situations involving voluntary participation, such as in human interactions, the use of abstention as a pure strategy may not be ideal to capture the social dilemma. In reality, depending on the context and the type of social relationships we are modelling, abstention can also mean laziness, shyness or lack of proactivity, and all those emotions, feelings or characteristics may exist within a certain range. Thus, we propose that in a round of interactions, some agents might be interested in interacting with all of its neighbours (i.e., never abstain), while others may be willing to interact with only a few of them and abstain from interacting with others. To give another example, in the context of a poll of a number of individuals,

there might be some who vote and others who do not. In the latter case, considering all the non-voters as abstainers might be too simplistic. In reality, there might be some who abstain because they do not have a view at all and those who occasionally abstain from convenience, lack of interest or because of some external event. In this way, we believe that abstention should be seen and explored as an extra attribute of each agent, and not as a pure strategy.

Given this motivation, in this chapter, we introduce a prisoner's dilemma with probabilistic abstention (PDPA), which is a hybrid of two well-known games in evolutionary game theory: the PD and the OPD game (also known as the PD game with voluntary participation). As occurs in the PD game, in the hybrid game each agent can choose either to cooperate or defect. The only difference is that in the PDPA game, in addition to the game strategy, each agent is defined by a value $\alpha = [0, 1]$ to denote a probability of abstaining from any interaction.

This work aims to investigate the differences between the PDPA game and the classic PD and OPD games. A number of Monte Carlo simulations are performed to investigate the effects of α in the evolution of cooperation. In order to have a more complete analysis of the evolutionary dynamics, both synchronous and asynchronous updating rules [66, 115, 75] are explored.

6.2 Methods

This work considers the prisoner's dilemma game with probabilistic abstention (PDPA), which is an evolutionary theoretical-game with two pure competing strategies: cooperate (C) and defect (D). In this game, each agent is characterized by two different attributes: game strategy s and the probability of abstaining α , which determines how likely it is that an agent will interact in each pairwise play. When an agent abstains from a game interaction, both

agents acquire the same loner's payoff L . In this way, α is a number from zero to one where $\alpha = 0$ denotes an agent who never abstains (always plays the game), and $\alpha = 1$ denotes an agent who always abstains (never plays the game). When both agents play the game, their payoffs follow the same structure of the classic prisoner's dilemma game, i.e., the reward for mutual cooperation $R = 1$, punishment for mutual defection $P = 0$, T for the temptation to defect and the sucker's payoff $S = 0$. To ensure the proper nature of the dilemma, $1 < T < 2$ and $0 < L < 1$ [147].

Without loss of generality, we discretize the values of α to $|\alpha| = 2\kappa$ in equal intervals, where κ is the agent's degree. We adopt a regular square lattice grid with periodic boundary conditions (i.e., a toroid) fully populated with $N = 102 \times 102$ agents playing the PDPA game. Each agent interacts with its four immediate neighbours (von Neumann neighborhood) and is initially assigned a strategy $s = \{C, D\}$ and a probability of abstaining $\alpha = \{0, 0.125, 0.250 \dots 0.750, 0.875, 1.0\}$ with equal probability. The evolution process is performed through a number of Monte Carlo (MC) simulations[31] in both synchronous and asynchronous fashion as follows [66]:

- **Synchronous updating:** at each time step, all agents x in the population play the game once with each of their four neighbours y acquiring the payoff p_{xy} for each interaction. After that, for the current time step, each agent copies the strategy and the value of α of the best performing agent in the neighbourhood. In case of ties, or if x is the best in the neighbourhood, its strategy and α remains the same.
- **Asynchronous updating:** at each time step, each agent is selected once on average to play the game and update its strategy and α immediately. That is, in one time step, N agents are randomly selected to perform the following elementary procedures: the agent x plays the game with all neighbours y , acquiring the payoffs p_{xy} for each play (i.e., obtaining

the utility of $u_x = \sum p_{xy}$; one randomly chosen neighbour of x (y) also acquires its payoffs p_{yz} by playing with all its neighbours z (i.e., obtaining the utility of $u_y = \sum p_{yz}$); finally, if $u_y > u_x$, agent x copies the strategy and the value of α from its neighbour y with a probability:

$$W = \frac{1}{1 + e^{(u_x - u_y)/(\kappa K)}}, \quad (6.1)$$

where $K = 0.1$ denotes the amplitude of noise [155].

In our experiments, all Monte Carlo simulations are run for 10^5 steps, which is a sufficiently long thermalization time to determine the stationary states. Furthermore, to ensure proper accuracy and alleviate the effect of randomness in the approach, the final results are obtained by averaging 100 independent runs.

It is noteworthy that the PDPA game allows us to perform both classic games (PD and OPD). That is, by setting all agents to have $\alpha = 0$, we ensure that they will always play the game, which is essentially the same as considering the classic PD game. Similarly, by setting agents to have $\alpha = \{0, 1\}$ we ensure that some agents will purely abstain, while others will play the game, which is the same as considering the OPD game.

6.3 Results

In order to increase the understanding of the outcomes associated with the hybrid game proposed in this chapter (i.e., the prisoner's dilemma game with probabilistic abstention – PDPA), in the following experiments we adopt $\epsilon = (1 - s)(1 - \alpha)$ to denote the effective cooperation rate of an agent, where $s = \{0, 1\}$ and $\alpha = [0, 1]$ correspond to the agent's strategy and its probability of abstaining from a game interaction respectively. Note

that here $s = 0$ means cooperator, $s = 1$ means defector, $\alpha = 0$ indicates that the agent never abstains and $\alpha = 1$ indicates that the agent always abstains. In this way, we can have two types of agents for each strategy: the pure-cooperators and the pure-defectors (i.e., the agents who always play the game, $\alpha = 0$); and the agents who sporadically play the game (i.e., the sporadic-cooperators and sporadic-defectors, $0 < \alpha < 1$). Thus, the value of ϵ is very important to easily distinguish between a cooperator who always abstains (i.e., $\{s = 0, \alpha = 1\} \implies \epsilon = 0$), from the sporadic-cooperators (i.e., $\{s = 0, \alpha = (0, 1)\} \implies \epsilon > 0$), and the pure-cooperators (i.e., $\{s = 0, \alpha = 0\} \implies \epsilon = 1$).

We start by comparing the outcomes of the PDPA game with those obtained for the classic prisoner's dilemma (PD) and optional prisoner's dilemma (OPD) games for both synchronous and asynchronous updating rules (Figure 6.1). We test a number of randomly initialized populations of agents playing the PDPA game with three different setups:

- $\alpha = 0$ for all agents (equivalent to the PD game);
- α is either 0 or 1 with equal probability (equivalent to the OPD game);
- $\alpha = [0, 1]$ uniformly distributed.

For all setups, we investigate the relationship between the fraction of effective cooperation ϵ and the probability of abstaining α for different values of the temptation to defect T and the loner's payoff L .

As shown in Figure 6.1, for the synchronous rule, it is possible to observe that the PDPA sustains higher levels of cooperation even for large values of the temptation to defect T . The difference between the outcomes of the synchronous and asynchronous versions in the classic games occur as expected: cooperation has more chance of surviving when the updating rules

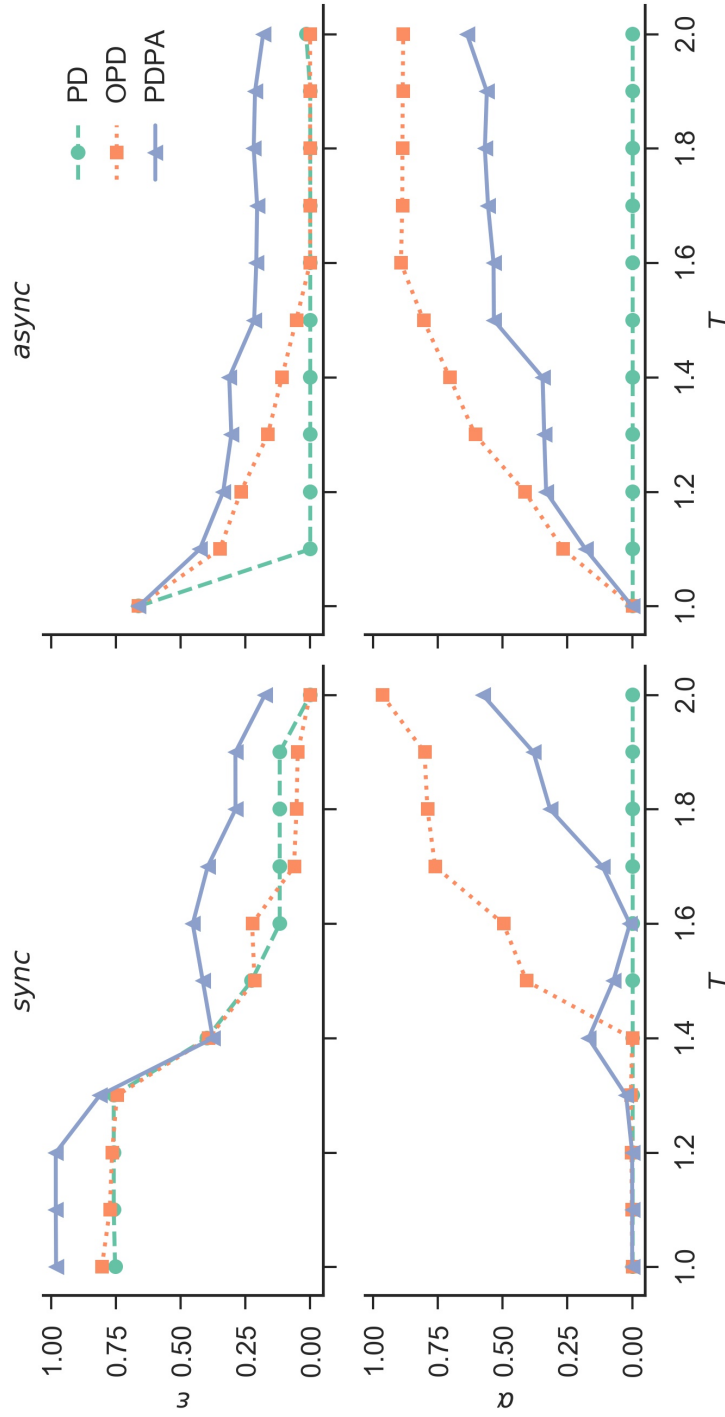


FIGURE 6.1: Comparing the average fractions of α (i.e., the probability of abstaining frequency) and ϵ (i.e., the effective cooperation frequency $-\epsilon = (1-s)(1-\alpha)$), where s denotes the agent's strategy) for different values of the temptation to defect T . The results are obtained by averaging 100 independent runs at the stationary state (after 10^5 Monte Carlo steps) of the classic prisoner's dilemma (PD), the optional prisoner's dilemma (OPD) and the hybrid of them, i.e., prisoner's dilemma with probabilistic abstention (PDPA). All games are tested in both synchronous and asynchronous updating fashions with a regular square lattice grid populated with $N = 102 \times 102$ agents, for a fixed reward for mutual cooperation $R = 1$, punishment for mutual defection $P = 0$, sucker's payoff $S = 0$ and loner's payoff $L = 0.4$.

are synchronous, with less stochasticity and more awareness of the neighbourhood's behaviour, i.e., the agent knows who is the best player in its neighbourhood. Surprisingly, results indicate that when the PDPA is considered, this enhancement also holds for the asynchronous updating model, which is a well-known adverse scenario for both classic games [75]. In general, it is clear that irrespective of the updating rule, the PDPA game is most beneficial for the evolution of cooperation. Moreover, when comparing the OPD with the PDPA game, we see a correlation between their levels of abstention, showing that abstention may act as an important mechanism to maintain cooperation and avoid defector's dominance, which is also supported by Figure 6.2, which features the time course of the fraction of ϵ and α for both updating rules of agents playing the PDPA game for three values of the temptation to defect (i.e., $T = \{1.1, 1.4, 1.9\}$).

Given the nature of the classic PD and OPD games, it is known that in a well-mixed population, defection and abstention are usually the dominant strategies respectively. As discussed in previous work [26, 157], this happens because cooperators need to form clusters to be able to protect themselves against exploitation from defectors, and if we consider a randomly initialized population, it takes a few steps for cooperators to cluster. Meanwhile, the defection rate increases quickly in the initial steps until the agents reach a stage where defectors have more chance of finding another defector than a cooperator. Consequently, defection starts to be a bad strategy and if abstention is an option, the agents prefer to abstain; otherwise defectors will hardly become cooperators as they do not have the incentive to change their strategies. Interestingly, as shown in Figure 6.2, a similar pattern can be observed in the PDPA game, i.e., in the initial steps, the rate of pure-defectors ($\alpha = 0$) increases more quickly, causing the sporadic-cooperators ($\alpha > 0$) to have a better performance. Then, with the increase of the pure-defectors,

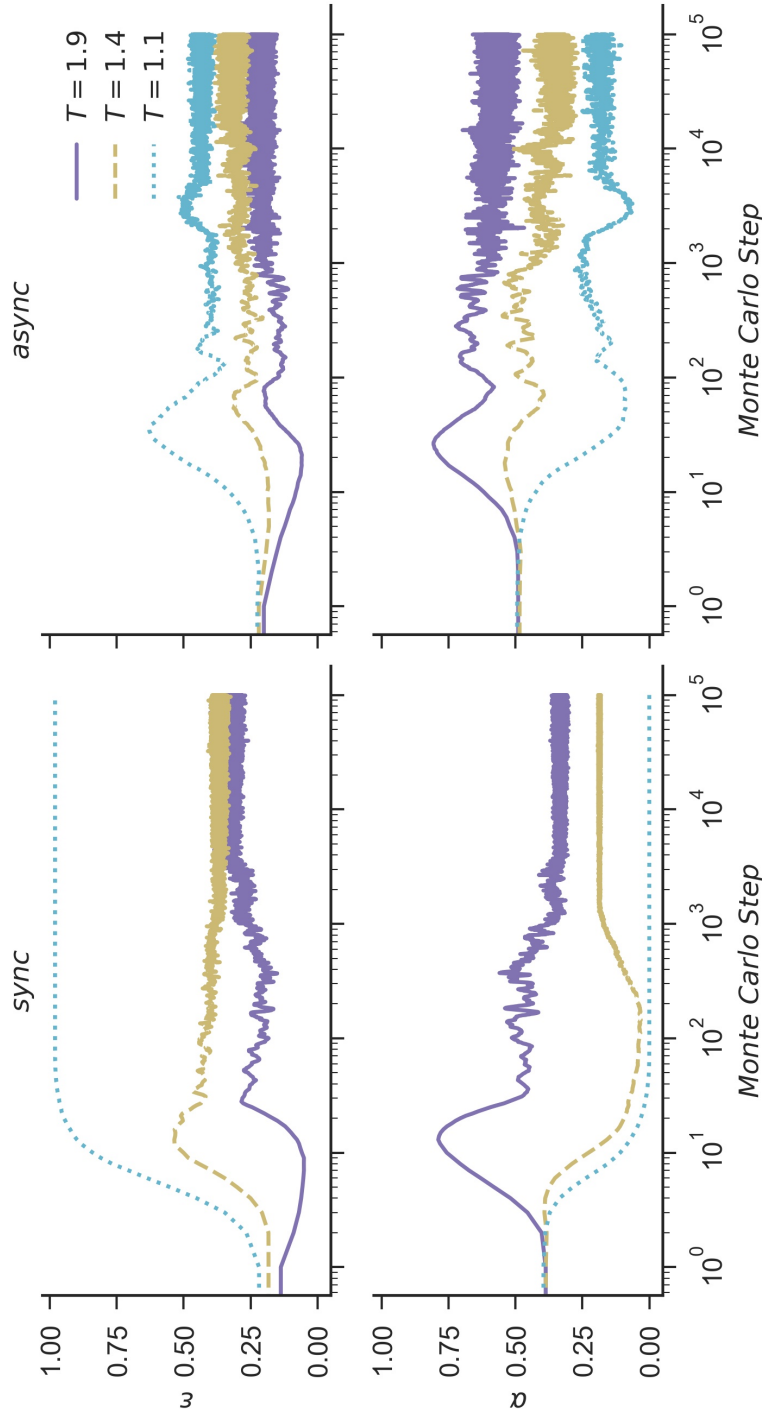


FIGURE 6.2: Time course of the effective cooperation ϵ and the probability of abstaining α for different values of the temptation to defect T . All curves refer to the prisoner's dilemma with probabilistic abstention (PDPA) game for a regular square lattice grid populated with $N = 102 \times 102$ agents, for a fixed reward for mutual cooperation $R = 1$, punishment for mutual defection $P = 0$, sucker's payoff $S = 0$ and loner's payoff $L = 0.4$.

sporadic-defectors ($\alpha > 0$) start to be a better choice. At this point, with less pure-defectors in the population, cooperators with smaller values of α start to perform better, producing a wave towards the decrease of α . This simple mechanism explains the initial bell-shaped curve in the average fraction of α in Figure 6.2.

In order to further investigate the results obtained for the PDPA game, some typical distributions of the strategies, probability of abstaining α , and the effective cooperation rate ϵ are shown in Figure 6.3. In addition to the similarities with the classic games, other interesting phenomena can be observed in the PDPA game, such as robust coexistence of cooperation and defection for different values of T and L . Results show that agents who always refuse to interact ($\alpha = 1$) are wiped out in most scenarios when $T < 1.9$ and $L < 0.8$. That is, agents who interact at least once will usually have a better performance. Moreover, it was observed that irrespective of the high heterogeneity of values of α in the initialization, the population usually converges to two values of α for the synchronous model, and three distinct values of α for the asynchronous model. However, the higher heterogeneity of states in the initial steps plays a key role in increasing the performance of cooperators in the PDPA game. This happens because the intermediate values of α help to reduce the exposure of cooperators to the risk of being exploited by defectors too quickly.

Finally, Figure 6.4 shows the average fraction of ϵ and α on the plane $T - L$ (i.e., temptation to defect vs loner's payoff) for both PDPA and OPD games, with synchronous and asynchronous updating rules. It is possible to observe that the PDPA acts like an enhanced version of the OPD game. In addition, its performance with the asynchronous updating rules is remarkable; we see that when the concept of optionality is given in levels, i.e., the introduction of the probability of abstaining α , the population succeeds in

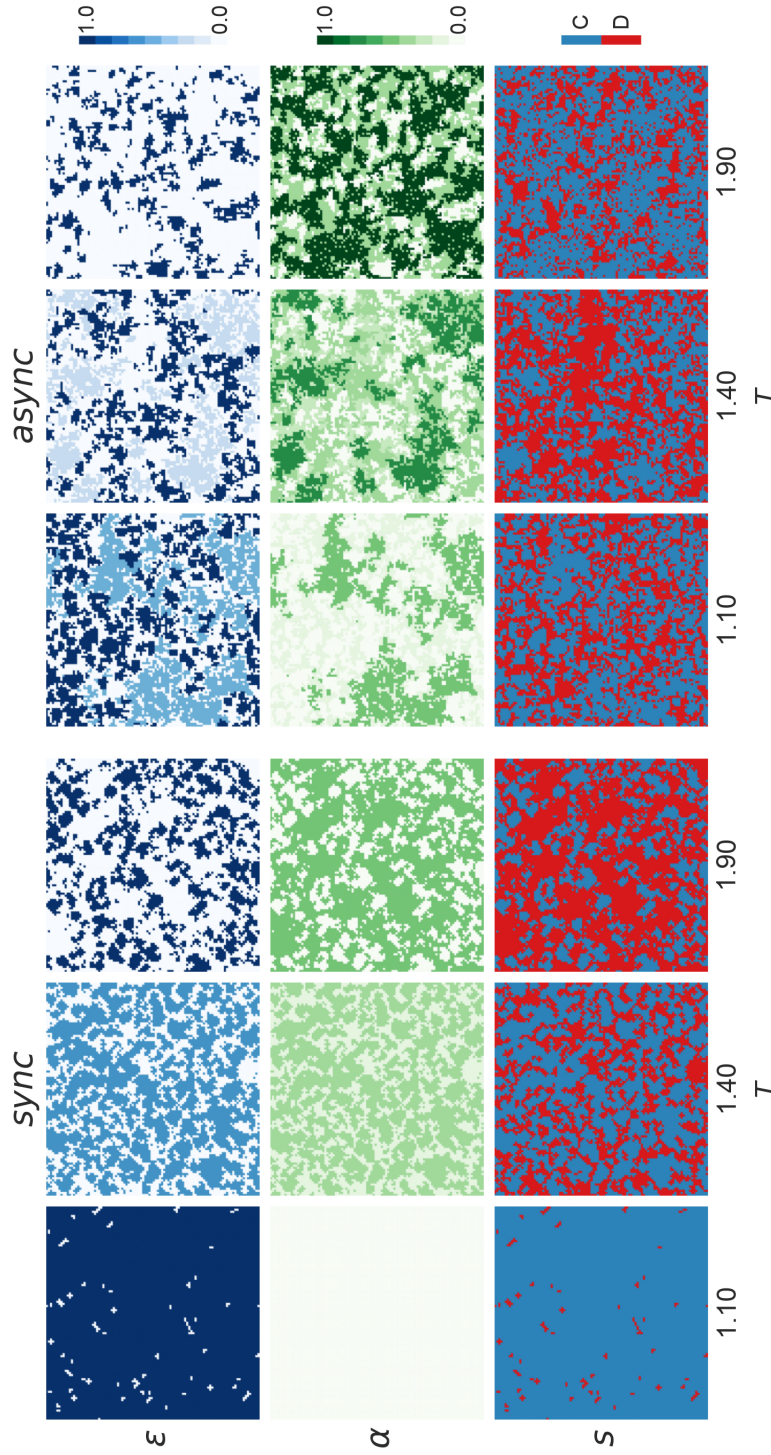


FIGURE 6.3: Typical distributions of the effective cooperation ϵ , probability of abstaining α , and game strategy s for different values of the temptation to defect T . All screenshots refer to the prisoner's dilemma game with probabilistic abstention (PDPA) at the 10^5 Monte Carlo step for a regular square lattice grid populated with $N = 102 \times 102$ agents, with a fixed reward for mutual cooperation $R = 1$, punishment for mutual defection $P = 0$, sucker's payoff $S = 0$ and loner's payoff $L = 0.4$.

controlling the dominance of abstention behaviour, which maintains the diversity of strategies and also helps to promote cooperation.

Furthermore, as discussed previously, despite being more effective in promoting cooperation than the classic games, we observed that cooperation is the dominant strategy only if T is relatively small in a synchronous updating fashion. In summary, for both updating rules, the possibility of not interacting with all neighbours ($\alpha > 0$) helps cooperators to decrease the risk of being exposed to defectors in the initial steps (when most of them could not yet cluster), which consequently allows them to survive even when T is very high. However, this possibility also hampers them from dominating the environment afterwards, which results in the promotion of a robust state of coexistence of both strategies.

6.4 Discussion

We have studied a novel evolutionary game called the prisoner's dilemma with probabilistic abstention (PDPA), which is essentially the merger of two well-known games: the prisoner's dilemma (PD) game and the optional prisoner's dilemma (OPD) game. A number of Monte Carlo simulations with both synchronous and asynchronous updating rules were carried out, where it was shown that the PDPA game is much more beneficial for promoting cooperation than the classic PD and OPD games.

It was discussed that in most evolutionary scenarios (i.e., $T < 1.9$ and $L < 0.8$), the agents who interact at least once ($\alpha < 1$) usually have a better performance. This indicates that intermediate values of α are a better option for promoting both cooperative behaviour and diversity of strategies (cyclic dominance) in the population. Moreover, results suggest that the higher heterogeneity of states in the initial steps play a key role in slowing down the

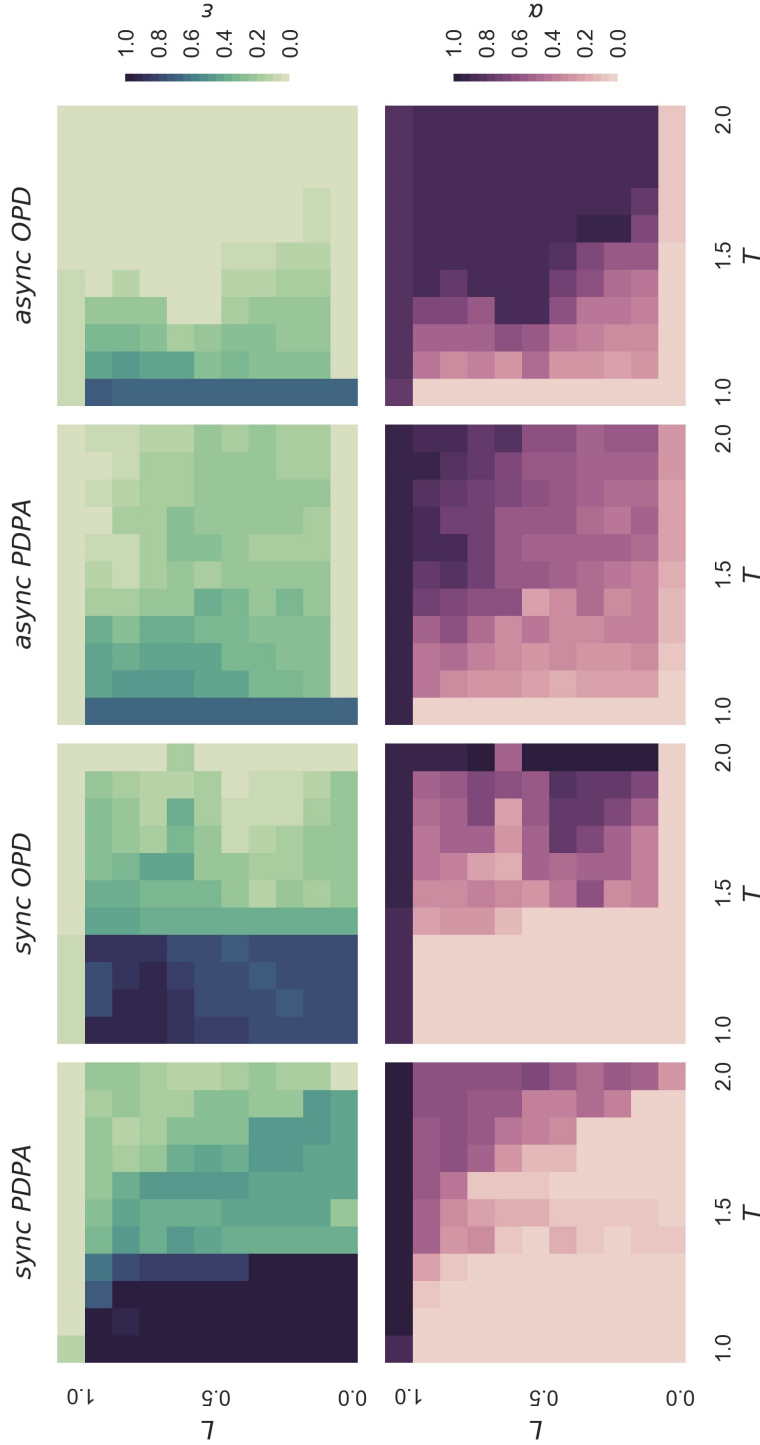


FIGURE 6.4: Heat map of the average effective cooperation ϵ and probability of abstaining α in the $T - L$ plane (i.e., temptation to defect vs loner's payoff) for 100 independent runs at the stationary state (i.e., after 10^5 Monte Carlo steps). Both synchronous and asynchronous fashions of the optional prisoner's dilemma game (OPD) and the prisoner's dilemma game with probabilistic abstention (PDPA) are explored. A regular square lattice grid is adopted, populated with $N = 102 \times 102$ agents, for a fixed reward for mutual cooperation $R = 1$, punishment for mutual defection $P = 0$, sucker's payoff $S = 0$ and loner's payoff $L = 0.4$.

evolution of defection, which increases the chance of the formation of cooperative clusters. It is noteworthy that the precise role of heterogeneity in the PDPA game needs to be further explored. To conclude, it was observed that PDPA is, in fact, an enhanced version of the OPD game, which provides a more realistic representation of the concept of voluntary/optional participation.

Chapter 7

Mobility restores the mechanism which supports cooperation in the optional prisoner's dilemma game

The work outlined in this chapter was published in:

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Note: This chapter adopts the voluntary prisoner's dilemma game terminology, which is synonymous to the optional prisoner's dilemma game.

Abstract

It is generally believed that in a situation where individual and collective interests are in conflict, the availability of optional participation is a key mechanism to maintain cooperation. Surprisingly, this effect is sensitive to the use of microscopic dynamics and can easily be broken when agents make a fully rational decision during their strategy updates. In the framework of the celebrated prisoner's dilemma game, we show that this discrepancy can be fixed automatically if we leave the strict and frequently artifact condition of a fully occupied interaction graph, and allow agents to change not just their strategies but also their positions according to their success. In this way, a diluted graph where agents may move offers a natural and alternative way to handle artifacts arising from the application of specific and sometimes awkward microscopic rules.

7.1 Introduction

Despite extensive research efforts, the evolution of cooperation remains a puzzle in a wide range of domains [127, 142]. In this context, two-strategy games such as the prisoner's dilemma (PD) game have been widely studied for many years from different perspectives with mechanisms [114] such as group selection [125] and network reciprocity [155, 94, 116] investigated. Traditionally, the agents' interactions in those games are compulsory, i.e., the agent has to opt between cooperation or defection, where the dilemma arises because individual selfishness leads to a collective disaster [8, 132]. However, in many real-world scenarios, the agents' participation in the game is voluntary (optional). Thus, in order to account for the concept of voluntary participation (abstention), researchers have been exploring the voluntary prisoner's dilemma (VPD) game, also known as the optional prisoner's dilemma game, which extends the PD to a three-strategy game where agents can also choose to abstain from playing the game [147, 13, 122]. In particular, abstention has attracted attention both for acting as a mechanism to support cooperation and for promoting cyclic behaviour [30, 82, 80, 62, 79, 67]. The cyclic dominance behaviour is often studied within the bounds of the rock-paper-scissors game, which, different to the VPD game, imposes the cyclic dominance in the payoff matrix [165, 27, 162, 153, 150, 47].

In addition to the discussion about the game strategies, studies concerning agent mobility are also of interest because, in many real ecological systems, individuals are usually on the move to improve their performance [133]. In this sense, research has shown that in a spatial environment, mobility and percolation thresholds have a critical impact on the sustenance of biodiversity in nature [165, 192, 191, 35, 179, 182, 96]. Interestingly, despite a large number of papers discussing the effects of mobility in the prisoner's dilemma [174, 173, 138, 202, 6, 171], the rock-paper-scissors [133, 161, 178]

and the optional public goods games [175, 200, 189], the impact of mobility in the context of the VPD game is still almost unknown. Indeed, some effort has also been made to explore contingent movement strategies modelling the so-called “win-stay, lose-move” rule, which, as also argued by Szabó and Fáth [155], might capture the concept of abstention in the sense that agents abstain by moving away from their opponents [2, 64, 104, 24]. Although this is a valid way to account for voluntary participation, we highlight that in many scenarios there must be a cost (payoff) associated with the act of not playing the game, i.e., abstention defined in terms of the set of game strategies rather than the movement strategies. In other words, defining abstention as a strategy rather than a movement ensures that all agents have the right to abstain from a game interaction, independently of having a way to walk away (space permitting) or not.

Despite the very recent introduction of the VPD game in a diluted network with a purely random mobility scenario [19], many questions regarding the impact of mobility, in both the sustenance of biodiversity and the potential for widespread cooperation, remain unanswered. For instance, given the recent advances in the understanding of coevolutionary models [26, 159, 29, 74, 124, 199, 95], what happens to the population when considering agent mobility in a coevolutionary fashion? Thus, without loss of generality, this research introduces the VPD game with a coevolutionary model where not only the agents' strategies but also their movement is subject to the evolutionary process, which provides a more realistic representation of mobility within the domain of voluntary/optional participation.

Furthermore, we investigate the foundations of the emergence of cyclic dominance for the VPD game in both the fully populated (without mobility) and diluted networks. We discuss that the emergence of the cyclic dominance behaviour, which is commonly associated with the VPD game,

is very sensitive to the chosen imitation rule. Results show that when using other imitation rules, the cyclic dominance can be broken easily, but this difference diminishes when we use a more general diluted model where mobility can repair the missing chain that is necessary to support cyclic dominance.

The remainder of the chapter is organised as follows. Section 7.2 describes the model and the experimental settings. Section 7.3 presents the results of the extensive Monte Carlo simulations, which allow us to unveil the reason why mobility and optionality favour cooperation and cyclic dominance. Finally, Section 7.4 outlines the main conclusions.

7.2 Methods

In order to account for the features of the concept of voluntary participation (abstention) and agent mobility, we consider a set of N rational agents playing the voluntary prisoner's dilemma game (also known as the optional prisoner's dilemma game) on a $M \times M$ diluted square lattice network with von Neumann neighbourhood and periodic boundary conditions, i.e., a toroid where sites are either empty or occupied by an agent. In this way, to describe the lattice occupation, we define the lattice's density as $\rho = N/M^2$ ($0 < \rho < 1$), where $\rho = 1$ means that the lattice is fully populated.

In the voluntary prisoner's dilemma (VPD) game, agents can be designated as a cooperator (C), defector (D) or abstainer (A). Considering a pairwise interaction, the payoffs are defined as follows: D gets $P = 0$ for mutual defection, C gets $R = 1$ for mutual cooperation, $T = b$ for defection against a cooperator, and $S = 0$ for cooperation against a defector. Regardless of whether one or two agents abstain, both agents get the loner's payoff

$L = \sigma$, where $R > L > P$. Note that we adopt a weak version of the game, where $T > R > L > P \geq S$ maintains the nature of the dilemma [116, 147].

We consider a randomly initialized population in which $N/3$ of each strategy (C , D and A) is distributed at random in the network. Following the standard procedures of an asynchronous Monte Carlo (MC) simulation in this context [31, 66], at each MC time step, each agent (x) is selected once on average to update its strategy and position immediately. Thus, in one MC step, N agents are randomly chosen to perform the subsequent procedures: if the agent x has no neighbours, it moves to one of the four nearest empty sites (von Neumann neighbourhood) at random; otherwise, the agent x accumulates the utility U_x by playing the VPD game with all its nearest active (non-empty) neighbours (Ω_x), selects one of them at random (i.e., the agent y , which also acquires its utility U_y), and considers copying its strategy with a probability given by the Fermi-Dirac distribution function:

$$W = \frac{1}{1 + e^{(U_x - U_y)/K}} \quad (7.1)$$

where $K = 0.1$ characterizes the amplitude noise to allow irrational decisions [155, 151]. In this research, we also consider the scenario in which agents do not make irrational choices in the strategy updating process (Equation 7.1), i.e., the agent x only considers copying y if $U_y > U_x$.

After the agent x updates its strategy, U_x is recalculated, and x considers moving to a random empty site (if any) in its neighbourhood with probability:

$$W = \frac{1}{1 + e^{(u_x - v_x)/K}} \quad (7.2)$$

where $K = 0.1$, $u_x = U_x/k_x$ is the agent x 's average utility, k_x is the number of active neighbours in x 's neighbourhood, and $v_x = (u_x + \sum_{y \in \Omega_x} u_y)/(k_x + 1)$ is the average utility of x 's neighbourhood including itself. Thus, the agents

that are performing worse (better) than their neighbours have more (less) incentive to move.

Note that to make this research comparable with previous works, we consider the absolute payoff during the strategy imitation process (Equation 7.1). Nevertheless, it is noteworthy that our key results remain unchanged qualitatively if we apply a degree-normalized payoff in this function. However, in the case of mobility, the application of an absolute payoff in Equation 7.2 would cause an artifact effect. More precisely, it would result in the erosion of a cooperative cluster because agents at the periphery, who have fewer neighbours, would always be unsatisfied and move, i.e., the mentioned cluster would shrink gradually.

In order to avoid finite size effects, results are obtained for different network sizes, ranging from $M = 200$ to $M = 1000$. Simulations are run for a sufficiently long relaxation time (10^5 or 10^6 MC steps), where the final level of each strategy is obtained by averaging the last 10^4 MC steps.

7.3 Results

In this section, we present some of the relevant experimental results obtained when simulating a population of agents playing the voluntary prisoner's dilemma (VPD) game on diluted square lattice networks, i.e., a coevolutionary model where not only the agents' strategies but also their positions evolve over time. Firstly, we consider the case in which the population is fully populated, i.e., density $\rho = 1$, and we demonstrate that the emergence of cyclic dominance in the VPD game is sensitive to the chosen dynamical rule because by using other imitation rules the cyclic dominance can be broken easily. Secondly, we investigate the case in which $\rho < 1$ (diluted network), where we show that mobility and dilution can repair the mechanisms

necessary for supporting cyclic dominance. Thirdly, we further investigate the micro-level evolutionary dynamics for a diluted network both with and without mobility.

7.3.1 Fully populated network ($\rho = 1$): fragile cyclic dominance

In order to validate our coevolutionary model and provide grounds to explore the effects of mobility on a diluted square lattice, we start by investigating how the population evolves when there is no space for the agents to move. Figure 7.1 (upper panel) features the time course of the average frequency of each pure strategy, i.e., cooperation, defection and abstention, for a density $\rho = 1$, temptation to defect $b = 1.4$, and the loner's payoff $\sigma = 0.5$. The lower panel of Figure 7.1 shows the typical spatial patterns of the strategies at different Monte Carlo steps. Note that as $\rho = 1$, the model collapses to the traditional and well-known scenario in which only the strategies evolve. As expected, the results are qualitatively the same as those reported in previous studies [147, 19]. In this case the three strategies coexist because of the emergence of cyclic dominance behaviour where defectors beat cooperators, cooperators beat abstainers, and abstainers beat defectors [165, 27, 69].

To gain deeper insights into the mechanisms which underlie the cyclic dominance behaviour in the context of a spatial voluntary prisoner's dilemma game, we perform the same experiments as above but for the case in which an agent (x) only considers copying the opponent's strategy if the opponent (y) is performing better than itself, i.e., applies the Fermi-Dirac distribution function (Equation 7.1) if and only if the utility of y is greater than the utility of x , $U_y > U_x$. Interestingly, Figure 7.2 shows that when this

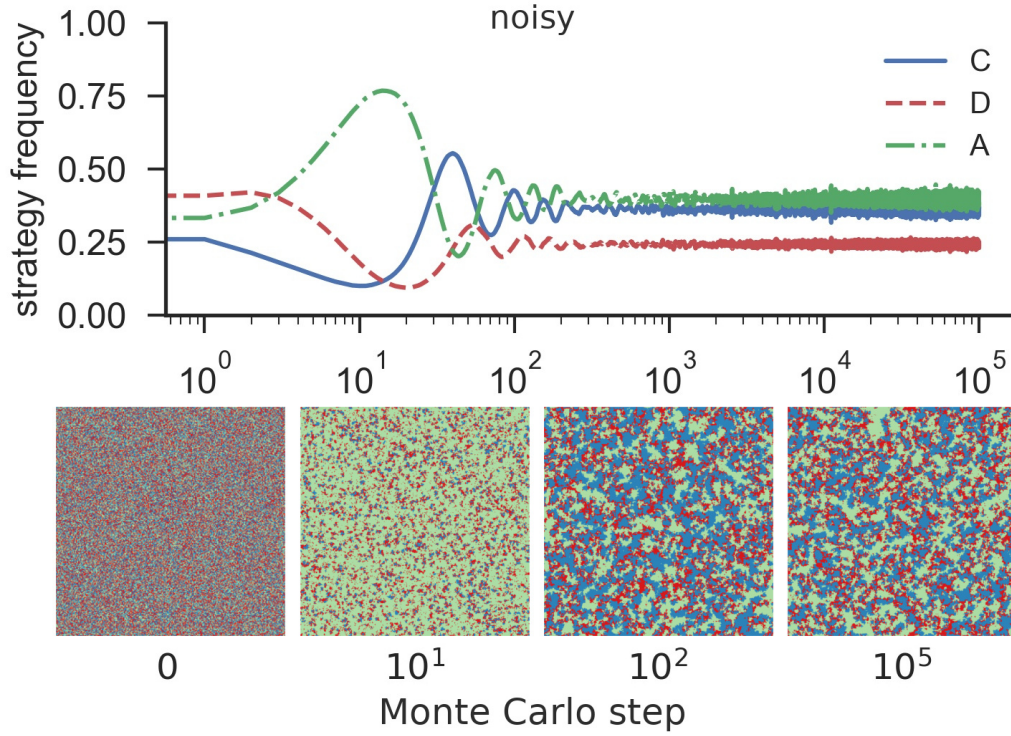


FIGURE 7.1: Time course of the average frequency of cooperation (blue), defection (red) and abstention (green) for a fully populated network (density $\rho = 1$) with $N = 400^2$ agents, temptation to defect $b = 1.4$, and loner's payoff $\sigma = 0.5$ (top panel). Typical evolution of spatial distribution of the strategies (bottom panel). Results are obtained for the case in which agents are allowed to make irrational decisions, i.e., applies the noisy Fermi-Dirac imitation rule (Eq. 7.1) for any value of U_y and U_x .

simple modification in microscopic dynamics is imposed, the cyclic dominance behaviour is broken and the population converges to a frozen state where only defection and abstention are present, but the cooperator strategy becomes extinct. Note that the idea of employing different imitation rules such as Equation 7.1 for both rational and irrational decisions have been systematically investigated in previous studies for two-strategy games [6, 118, 135], and it is well-known that different imitation rules, as well as the adoption of different values of K (amplitude noise) in the Fermi-Dirac rule may affect the outcome [151]. However, there is an unexplored gap in the literature regarding the possible consequences of the adoption of the Fermi-Dirac rule in the context of the VPD game, and our results suggest that the

cyclic behaviour commonly associated with the VPD game may be related to the use of this function, which also supports strategy change when the utility values are equal.

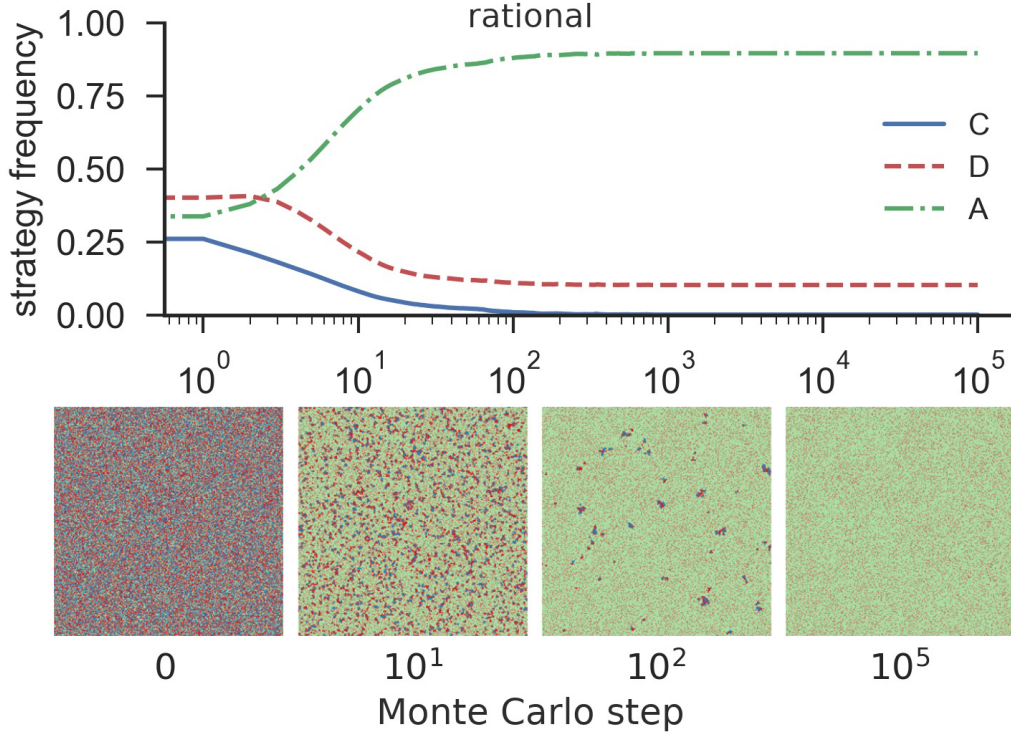


FIGURE 7.2: Time course of the average frequency of cooperation (blue), defection (red) and abstention (green) for a fully populated network (density $\rho = 1$) with $N = 400^2$ agents, temptation to defect $b = 1.4$, and loner's payoff $\sigma = 0.5$ (top panel). Typical evolution of spatial distribution of the strategies (bottom panel). Results are obtained for the case in which agents make rational decisions only, i.e., applies the Fermi-Dirac imitation rule (Eq. 7.1) if and only if $U_y > U_x$. As compared to Fig. 7.1, note that cyclic dominance is quickly broken and cooperators die out soon because of the slight change in the imitation rule.

Figure 7.3 depicts the average frequency of the three strategies (C , D and A) in the full $b - \sigma$ plane when agents are allowed to make irrational (top panels) and rational (bottom panels) decisions. Note that while cyclic dominance is maintained for almost any combinations of b and σ values in the traditional case (top), the same does not occur when the imitation rule is slightly changed (bottom). Thus, contrary to previous observations, our results highlight that the use of noisy imitation, dictated by Equation 7.1, is

an essential condition for promoting cyclic behaviour in the context of the VPD game. The reason for this discrepancy can be summarized as follows:

- Considering a random initial population (see the early MC steps in Figures 7.1 and 7.2), the typical trajectory predicts the advantage of defectors which is then followed by the rise of abstainers on both cases.
- Next, checking (or not) for the $U_y > U_x$ condition can be decisive to allow (or not) the subsequent rise of cooperators, which in turn supports the cyclic dominance phenomenon seen in Figure 7.1.
- At a micro level, if one cooperator/defector (x) is mostly surrounded by abstainers (y), its utility U_x will be mostly equal to U_y . Remember that in the voluntary prisoner's dilemma game, if one or two agents abstain (A), both will get the same loner's payoff σ , i.e., for any pair of strategies CA, AC, DA, AD, AA both agents get an identical σ value.

Thus, if we impose the $U_y > U_x$ condition, as the utilities of x and y are the same, the population is not able to curb the spreading of abstainers, which consequently produces the pattern observed in Figure 7.2, i.e., a few isolated defectors stuck in a sea of abstainers. Otherwise, if Equation 7.1 is applied for any value of $U_x - U_y$, as the number of abstainers increase, W will be approximately equal to 0.5 for most agents, which is one of the main mechanisms to keep the three strategies alive as observed in Figure 7.1.

7.3.2 Diluted network ($\rho < 1$): recovering cyclic dominance and promoting cooperation

As we already argued, a fully occupied interaction graph seems to be a specific rather than a generally valid real life situation, hence this section discusses the coevolutionary cases for a diluted lattice network where not only

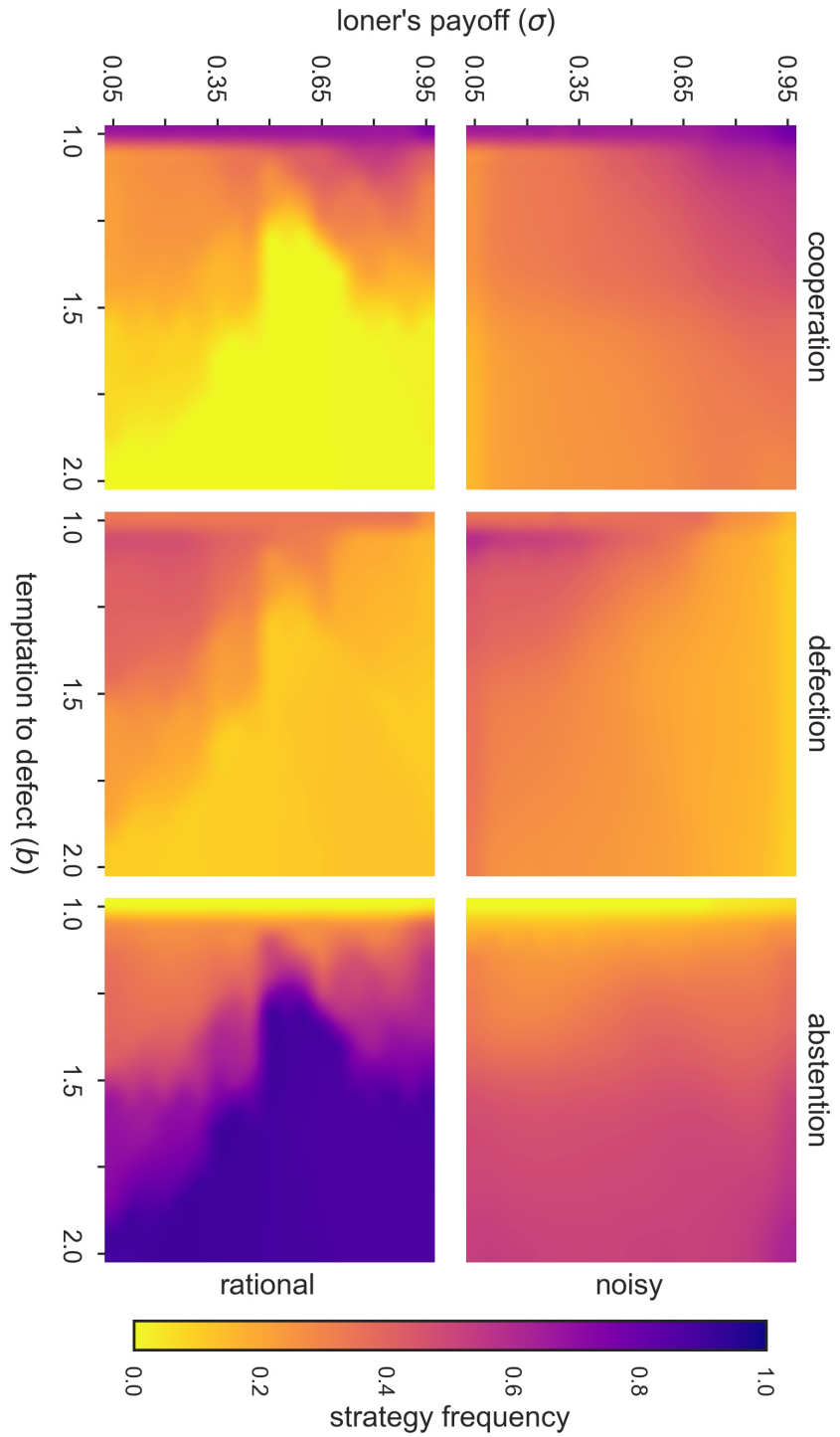


FIGURE 7.3: Heat maps of the average frequency of the strategies in the full $b - \sigma$ plane at the stationary state for a fully populated network (density $\rho = 1$). Results for the noisy imitation rule on the top, and for exclusive rational imitation rule on the bottom row. Note that while cyclic dominance is maintained for almost any combination of b and σ values on the top, this behaviour is easily broken when changing the imitation rule to “only copy if the opponent is performing better” (bottom). In the latter case abstention becomes the dominant strategy for most $b - \sigma$ pairs.

the strategies but also the agents' positions evolve over time.

At a macro-level, we start by analysing the influence of the density ρ on the evolutionary process for the noisy Equation 7.1 (i.e., agents are allowed to make irrational decisions) after a sufficiently long relaxation time. In line with previous research for two-strategy games such as the prisoner's dilemma game [174, 138, 70], experiments with our coevolutionary model reveal that mobility and dilution also play a key role in promoting cooperation in the VPD game. Figure 7.4 shows the average frequency of the three strategies in the full $b - \sigma$ plane for some representative densities. As compared to the traditional case ($\rho = 1.0$ regime i.e., Figure 7.3 top), we observe that the cyclic dominance behaviour still emerges for most $b - \sigma$ settings for $\rho \geq 0.59$. Interestingly, results show that scenarios of full cooperation arise monotonously when $\rho < 0.59$, i.e., the more diluted the network is, the easier it is for cooperators to dominate the population. However, when the density is too low ($\rho < 0.10$) the cooperators become too vulnerable to invasion by abstainers due to the increasing difficulty of forming clusters. Also, experiments show that $0.10 \geq \rho > 0.05$ quickly produces very unstable $C + A$ states which either converge to full C or full A . Notably, this behavior cannot be seen directly from the heat map because the average of full C and full A destinations results in around 0.5 density for both strategies. The latter may also suggest a coexistence of these strategies, but as we stressed, not in the present case because either C or A prevails at these global concentration values. Furthermore, when $\rho \leq 0.05$ cooperators always die out and abstainers dominate in all scenarios.

Note that the percolation threshold (ρ_p) for this square lattice network with von Neumann neighbourhood is approximately equal to 0.59 [101, 143]. Thus, this result is of particular interest because cooperation is favoured when the density is below the percolation threshold, which is known to be

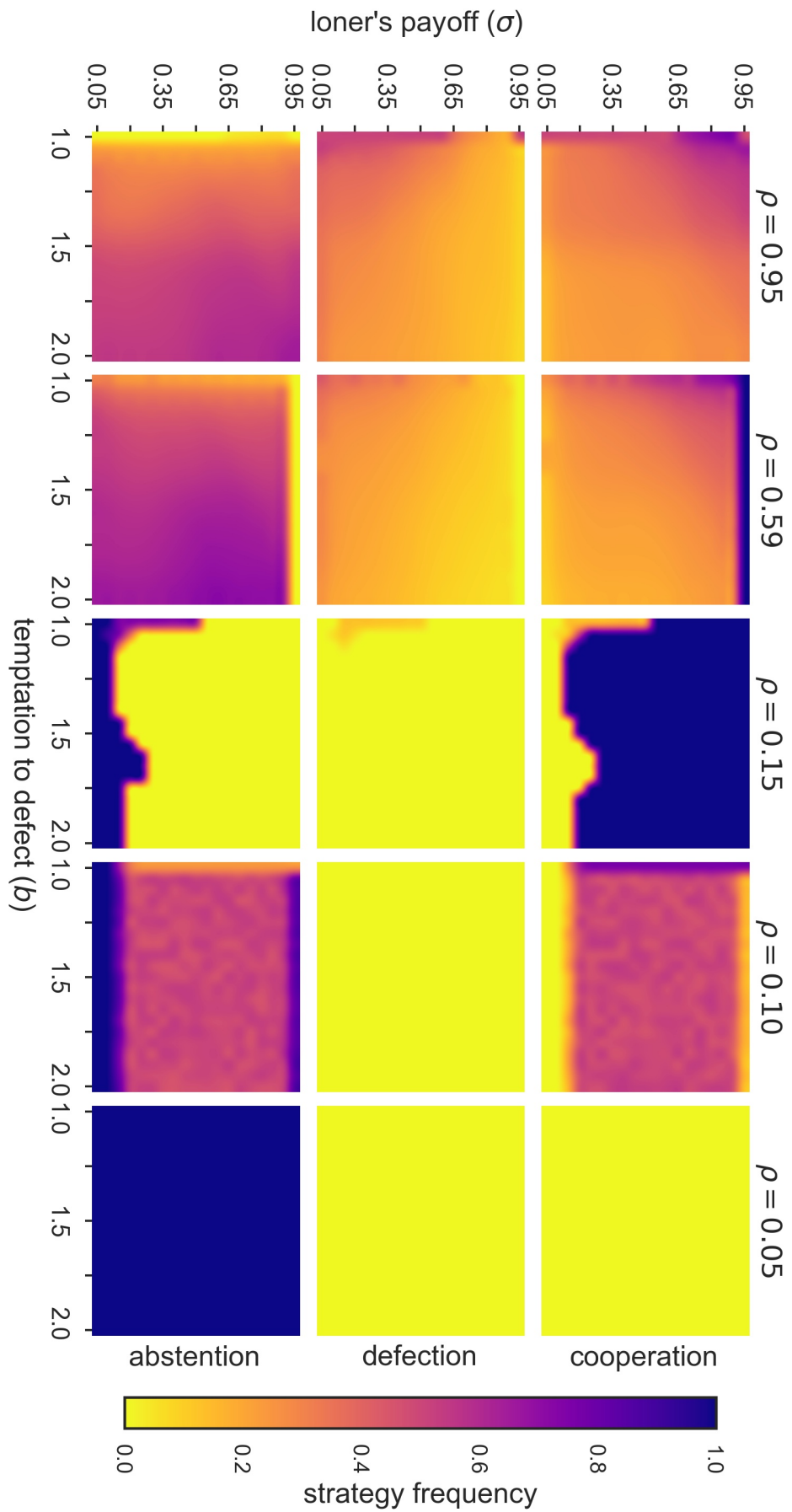


FIGURE 7.4: Heat maps of the average frequency of cooperation (top), defection (middle) and abstention (bottom) in the full $b - \sigma$ plane at the stationary state for a diluted network. All results are obtained for the noisy imitation rule. Note that at $\rho = 0.10$ global concentration, the average frequencies of C and A are approximately 0.5 as a result of a bistable destination of the evolutionary process, where the population either converges to full dominance of cooperators or abstainers.

an adverse situation for maintaining cooperation [191, 179, 182]. Moreover, results in Figure 7.4 also highlight the importance of exploring the outcomes of the VPD game across the whole loner's payoff (σ) spectrum, and not only for a specific $\sigma = 0.3$ value, as was used earlier [147, 19].

Considering the discrepancy observed in Figure 7.3 for $\rho = 1$, we now repeat the same experiments as above but for the case where an agent only applies Equation 7.1 if the opponent is performing better than itself, i.e., the case of a fully rational imitation rule. Surprisingly, Figure 7.5 shows that the previously observed difference for both imitation rules diminishes when we consider a diluted network ($\rho < 1$) with mobile agents. More importantly, results show that when $1 > \rho > \rho_p$ the mechanisms which support cyclic dominance in the traditional case (i.e., for the noisy Equation 7.1 and $\rho = 1$) are recovered for a wide range of $b - \sigma$ scenarios. In fact, results for both imitation rules and $\rho < 1$ are qualitatively the same for most settings. However, as seen in figures 7.4 and 7.5, when the density is below the percolation threshold $\rho < \rho_p$, it is possible to observe a small shift of $\rho \approx 0.05$ in the boundaries of the region in which full C occurs. For instance, results for $\rho = 0.15$ in Figure 7.4 are similar to those when $\rho = 0.10$ in Figure 7.5. Note that the bistable outcomes, where the population either converges to a full C or a full A state, observed for $\rho \approx 0.10$ in the first case happens at $\rho \approx 0.05$ in the later case.

7.3.3 Micro-level analysis of the effects of dilution and mobility

In order to further explore the aforementioned phenomena, we extend our analysis of the evolutionary process to a micro perspective. Figure 7.6 shows the average time course of the three strategies for a fixed temptation to defect

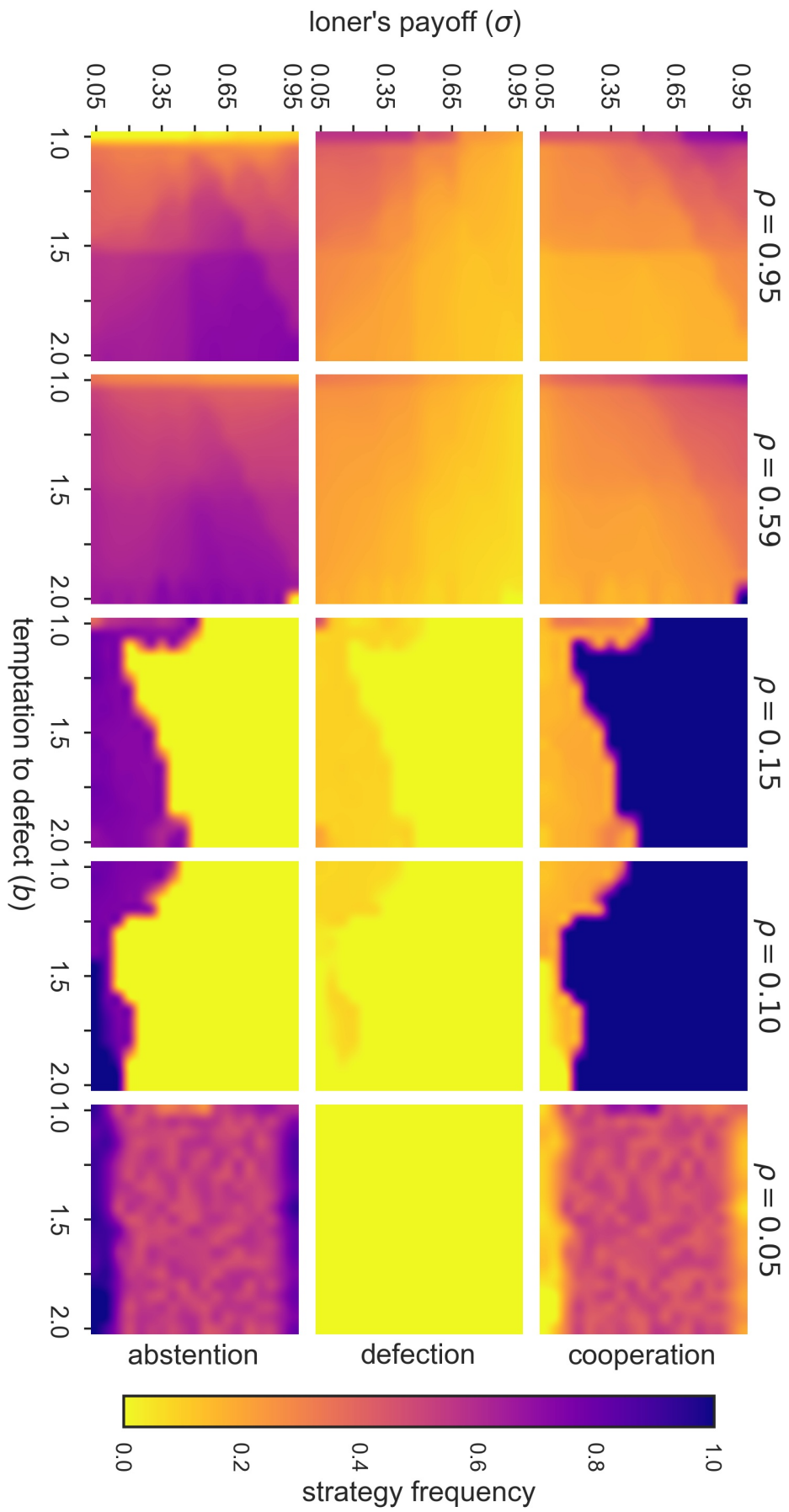


FIGURE 7.5: Heat maps of the average frequency of cooperation (top), defection (middle) and abstention (bottom) in the whole $b - \sigma$ plane at the stationary state for a diluted network. All results are for the fully rational imitation rule.

$b = 1.65$ and loner's payoff $\sigma = 0.55$, which is representative of the outcomes of other parameters as well. For this scenario, when $\rho = 1$, cyclic dominance is maintained for the traditional case with the noisy imitation rule, but it is easily shattered when considering a rational rule. However, the difference diminishes when $\rho < 1$.

Results show that the profile of the curves for the initial 10^2 MC steps are very similar to scenarios which support cyclic dominance, i.e., an initial drop followed by a quick recovery of the frequency of cooperators. This phenomenon has also been observed in previous work for dynamic networks [26, 157], where it was discussed that defectors are quickly dominated by abstainers, allowing a few clusters of cooperators to remain in the population, then with the lack of defectors, those cooperative clusters expand by invading the abstainers. Note that it also explains the reason that higher values of σ are more beneficial in promoting cooperation (as seen in Figures 7.3, 7.4 and 7.5), i.e., abstainers have to be strong enough to protect cooperators against invasion from defectors in the initial steps. Moreover, Figure 7.6 (right) shows a clear correlation between the density ρ and the speed of the initial inflation of abstention.

In order to distinguish between the impact of mobility and dilution on the emergence of cooperative behaviour and cyclic dominance, we have also investigated the case in which the agents are not allowed to move. That is, the same model described in Section 7.2, but without the movement updating process. As shown in Figure 7.7, when $\rho \leq \rho_p$ the frequency in which the agents change their strategies is extremely low, i.e., the population quickly reaches a frozen pattern which is very dependent on the initial configuration. Also, in line with preceding research [179, 182], we observe that when considering the traditional noisy imitation rule (Figure 7.7 top), dilution alone can improve the level of cooperation, where the optimal value of ρ is always

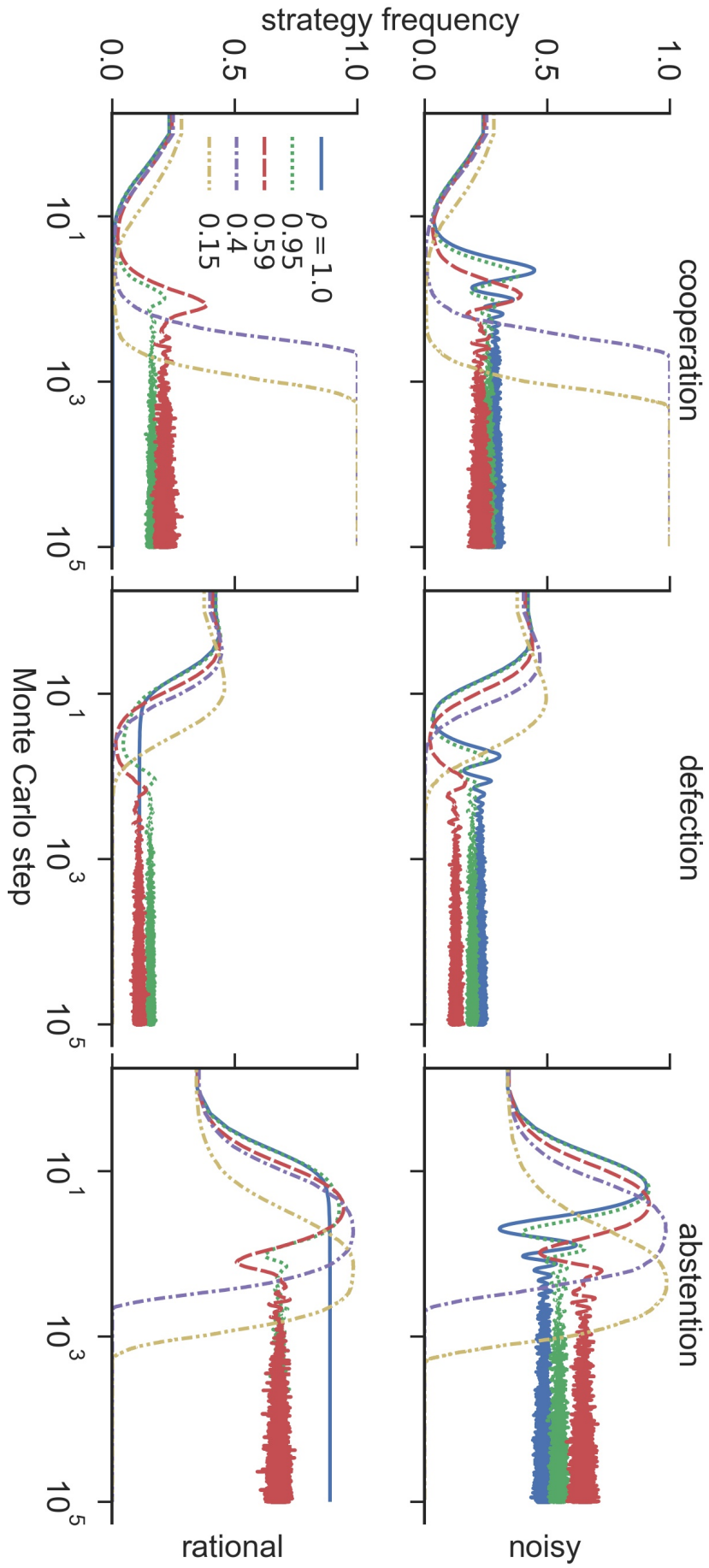


FIGURE 7.6: Time course of the average frequency of cooperation (left), defection (center) and abstention (right) for different densities ρ , temptation to defect $b = 1.65$, and loner's payoff $\sigma = 0.55$. Results for the noisy imitation rule on the top, and the rational imitation rule on the bottom. Note that the difference between both rules diminishes when $\rho < 1$.

above the percolation threshold ($1 > \rho > \rho_p$). In another perspective, the emergence of cyclic dominance behaviour is diminished when the agents do not move (e.g., compare the top panels of the Figures 7.6 and 7.7).

Interestingly, different phenomena occur when we consider the fully rational imitation rule (Figure 7.7 bottom). Note that dilution alone is not able to fix the evolutionary mechanisms which support either the emergence of cyclic dominance and the evolution of cooperation. In other words, results show that mobility plays a key role in diminishing the difference on the outcomes of both imitation rules (as seen in Figure 7.6 for $\rho < 1$). Moreover, it is noteworthy that mobility allows for the full dominance of cooperation for lower values of ρ , as well as the robust emergence of cyclic dominance for a wider range of scenarios.

To advance the understanding of mobility and dilution in the context of the VPD game, we also analyse the spatio-temporal dynamics of the strategies for both the noisy and the rational imitation rules. Figure 7.8 provides an animation for a prepared initial state where the strategies are arranged in stripes. This prepared configuration allows us to separate cooperators from defectors, making it easier to observe the mechanisms which are responsible for breaking the cyclic chain where A beats D , D beats C and C beats A . In summary, results show that the key difference between the dynamical rules is that, when applying the fully rational rule, defectors in the middle of abstainers do not have the incentive to become abstainers. Hence, as discussed in Section 7.3.1, the rational rule produces frozen $D + A$ states (as seen in Figure 7.2) which cannot be observed in the noisy Fermi-Dirac case. As a consequence, the isolated defectors trapped in the sea of abstainers inhibit the formation of larger cooperative clusters, which in turn breaks the cyclic chain. However, when mobility is introduced for $\rho < 1$, the $D + A$ states are

not a stable phase anymore. Here, there is a small stir which causes a random drift of defectors. Consequently, when two defectors meet they become vulnerable against invasion from abstainers. This process would lead to a homogeneous A phase, but the latter is sensitive to the attack of cooperators. In this way, abstainers are now able to support the emergence of cooperation, which in turn restores the mechanism to maintain the coexistence of all competing strategies.

Furthermore, regarding the phenomenon of cyclic dominance observed when $\rho < 1.0$, although using a different scenario and methodology, our results are compatible with previous research concerning mobility in the rock-paper-scissors game, where it is discussed that mobility can jeopardise cyclic dominance [133, 83]. However, in the context of the VPD game, the enhancement of cooperation for $\rho < \rho_p$ is counter-intuitive because it diminishes the cooperators' ability to form larger clusters [19]. Besides, results show that when the agents are allowed to abstain, the population of mobile agents will never converge to full defection. Finally, it is noteworthy that results also echo the findings of previous research concerning the PD and VPD games on weighted networks [26, 29, 74], i.e., a coevolutionary model in which the link weights are also subject to evolution. In parallel, the ability of avoiding interactions either by weakening the link weight or by moving to another position acts as an important mechanism to strengthen cooperators against exploitation.

7.4 Discussion and conclusions

This work investigates the role of mobility and dilution in a population of agents playing the voluntary prisoner's dilemma (VPD) game, also known as the optional prisoner's dilemma game, in a diluted square lattice network.

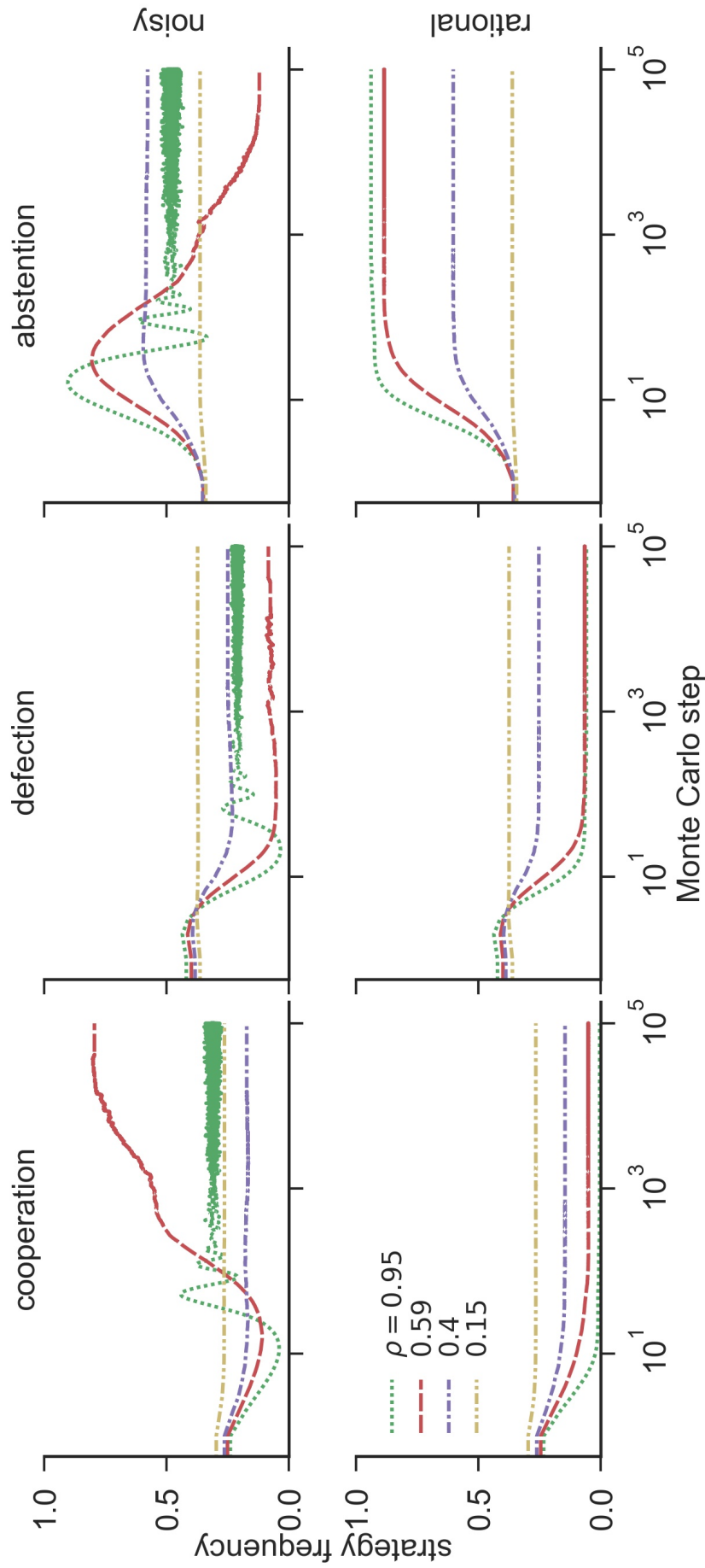


FIGURE 7.7: Time course of the average frequency of cooperation (left), defection (center) and abstention (right) for different densities ρ , temptation to defect $b = 1.65$, and loner's payoff $\sigma = 0.55$. Results obtained for a diluted network *without* mobility. The panels compare the noisy imitation rule (top) and the rational imitation rule (bottom).

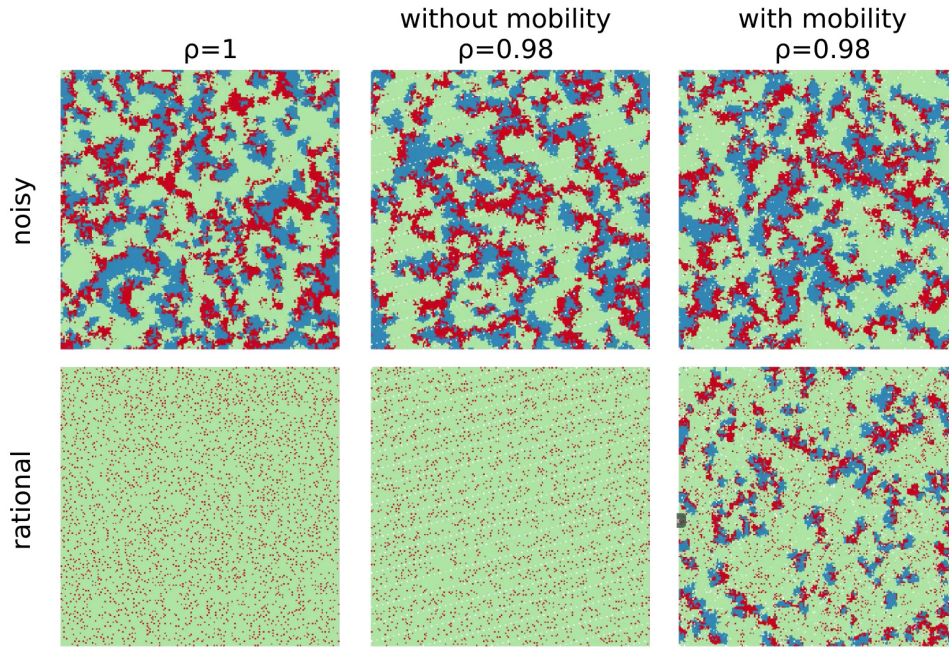


FIGURE 7.8: Snapshots of the spatial evolution of cooperation (blue), defection (red) and abstention (green) in the final stationary state with different scenarios. All simulations run for the same loner's payoff $\sigma = 0.55$, and temptation to defect $b = 1.65$. Results for the noisy imitation rule on the top, and the fully rational imitation rule on the bottom. An animation of the evolution of the strategies is provided [25].

We propose a coevolutionary model where both the agents' strategy and position are subject to evolution. In this model, in addition to the commonly applied imitation rules for the strategies [147], we also adopt a mobility rule in which agents who are performing worse (better) than their neighbours have more (less) chance to move. Thus, without loss of simplicity, this coevolutionary and asynchronous model is more realistic than the previous ones which consider random mobility with synchronous updating rules [19].

Research in this domain has claimed that the addition of abstention in the prisoner's dilemma game leads to a rock-paper-scissors type game, in which cooperation dominates abstention, abstention dominates defection, and defection, in turn, dominates cooperation, which describes the so-called cyclic dominance behaviour [165]. Interestingly, the present study shows

that, in the context of the traditional VPD game for a fully populated network [147], the emergence of cyclic behaviour is biased by the use of the Fermi-Dirac distribution function (sigmoid) in the strategy adoption process. This sigmoid function is often employed to allow for irrational or unjustified decisions where agents occasionally copy the strategy of a worse or an equally performing neighbour [155, 151, 154, 46, 78]. We show that when agents make fully rational decisions such as only copying the strategy of better performing neighbours, the outcome changes drastically, making cyclic behaviour unsustainable in most cases. However, the present study shows that the mechanism that supports cyclic behaviour is fixed when agents are allowed to move due to a diluted interaction space.

In fact, the noisy strategy updating rule has been applied to avoid artifact or frozen outcomes. However, in the present study we show that it is also possible to avoid such frozen states in a more realistic way, where, for instance, agents are allowed to move and change their connections over time. Hence, a deterministic rule can be as efficient as the noisy Fermi-Dirac function if we assume a partly diluted system. Furthermore, by means of robust and systematic Monte Carlo simulations, results show that mobility plays a crucial role in promoting cooperation in the VPD game for a wide range of values of the temptation to defect b , and loner's payoff σ , including for scenarios of high b and density below the percolation threshold $\rho < \rho_p$, which are known to be adverse for maintaining cooperative behaviour [191, 179, 182].

To conclude, this chapter aims to bridge the gap between agent mobility and the concept of voluntary/optional participation in social dilemmas. In addition, it provides a novel perspective for understanding the foundations of cyclic dominance behaviour in the context of the prisoner's dilemma game with voluntary participation (VPD game). We hope this work can serve

as a basis for further research on the role of abstention to advance the understanding of the evolution of cooperation in coevolutionary spatial games.

Chapter 8

Conclusion

As stated in the introduction, this thesis aims to advance the understanding of the role and impact of abstention in a population of interacting rational agents. Hence, within the bounds of evolutionary game theory, multi-agent systems and network science, the voluntary prisoner's dilemma game has been applied as the main framework to model the social dilemma of cooperation and abstention.

This chapter provides a summary of the main findings discussed in the previous chapters of this thesis (Section 8.1). Moreover, it revisits the hypotheses postulated in Chapter 1 (Section 8.2), and outlines some of the main limitations of this research (Section 8.3). Finally, the concluding remarks (Section 8.4) and some directions for future work and are presented (Section 8.5).

8.1 Summary of thesis achievements

As part of the methodology to conduct the experiments proposed in chapters 3–7, Chapter 2 presented Evoplex, which is software produced as part of this thesis. Evoplex is a flexible, fast and multi-threaded platform for agent-based modelling imposing an explicit graph-theoretical approach. Evoplex, differs from other software, in that the model is not coupled to the execution

parameters nor the visualization tools. Also, it provides a user-friendly GUI which makes it easy for all users, ranging from newcomers to experienced, to create, analyze, replicate and reproduce experiments.

In the context of the role and impact of abstention in a population of interacting rational agents, each experimental chapter of this thesis explored a different gap found in the literature. Namely, Chapter 3 investigated the effects of evolutionary games played on a dynamically weighted network, where it was discussed that the coevolution of both game strategies and network can play a key role in maintaining cooperative behaviour.

Chapter 4 further explored the phenomenon of cyclic dominance observed in Chapter 3 and discussed that the equilibrium between the three strategies can be maintained even in adverse scenarios, in which the replacement rate of the individuals is extremely high. Also, the experiments showed that the coexistence (biodiversity) is more robust in this coevolutionary model because the adoption of a dynamic network introduces another evolutionary mechanism to the system, which in turn allows the agents to adapt the environment and protect them against exploitation.

Giving the findings discussed in the chapters 3 and 4, as well as the state-of-the-art of coevolutionary spatial games, it is well known that recent studies in this field has been claiming that increasing heterogeneity favours cooperative behaviour, which in turn is one of most common explanations of the puzzle of cooperation in social dilemmas. In other words, this statement is equivalent to saying that social diversity favours cooperation. However, despite being valid for many scenarios, experiments in Chapter 5 revealed that this claim was too vague to really explain the observed emergence of cooperative behaviour. In this way, Chapter 5 aimed to further investigate the role of heterogeneity in coevolutionary spatial games for both the compulsory and the optional games. This chapter discussed that heterogeneity itself

is not enough to promote cooperation. In fact, it was found that promotion of cooperation is actually induced by the increase in the number of overlapping states.

While investigating the role of the concept of optionality in chapters 3, 4 and 5, it was found that in many situations involving voluntary participation, such as in human interactions, the use of abstention as a pure strategy may not be ideal to capture the social dilemma. In reality, depending on the context and the type of social relationships we are modelling, abstention can also mean laziness, shyness or lack of proactivity, and all those emotions, feelings or characteristics may exist within a certain range. In this way, to capture those scenarios, Chapter 6 introduced a novel evolutionary game called the prisoner's dilemma with probabilistic abstention to further explore the concept of abstention itself, extending it as an extra attribute of each agent (α), and not as a pure strategy. Despite representing a natural upgrade of the concept of abstention, interestingly, results showed that the proposed game is much more beneficial for promoting cooperation than the traditional prisoner's dilemma and voluntary prisoner's dilemma games. In summary, it was discussed that the possibility of not interacting with all neighbours (i.e., $\alpha > 0$) helps cooperators to decrease the risk of being exposed to defectors in the initial steps (when most of them could not yet cluster), which consequently allows them to survive even when temptation to defect is very high.

In addition to the discussion about the concept of optionality and dynamic networks in chapters 3–6, it was discussed that studies concerning agent mobility are also of interest because, in many real ecological systems, individuals are usually on the move to improve their performance. Thus, in order to bridge the gap between agent mobility and the concept of voluntary/optional participation in social dilemmas, Chapter 7 proposed a novel

coevolutionary model where both the agents' strategy and position are subject to evolution. Similarly to the case in which agent's are placed on a fully populated dynamic network (without mobility) discussed in chapters 3– 5, it was found that dilution and mobility in conjunction with abstention can serve as a key mechanism to not only promote cooperative behaviour but also to sustain social diversity in a wide range of scenarios.

Finally, beyond the discussion on dilution and mobility, Chapter 7 also revisits the role of the Fermi-Dirac distribution function in the strategy adoption process, which in turn provides a novel perspective for understanding the foundations of cyclic dominance behaviour in the context of the voluntary prisoner's dilemma game. Surprisingly, results showed that the phenomenon of cyclic dominance observed for this game is sensitive to the use of microscopic dynamics and can easily be broken when agents make a fully rational decision during their strategy updates. In particular, it was shown that this discrepancy can be fixed automatically if we leave the strict and frequently artifact condition of a fully occupied interaction network, and allow agents to change not just their strategies but also their positions according to their success. In this way, a diluted network where agents may move offers a natural and alternative way to handle artifacts arising from the application of specific and sometimes awkward microscopic rules.

8.2 Revisiting the hypotheses

Hypothesis 1: *The emergence of cooperation is favoured in the presence of abstainers*

The impact of abstention is explored in all experimental chapters of this thesis (i.e., chapters 3– 7), where it has been shown that abstention not only plays

a key role in resolving the dilemma of cooperation, but also allows for modelling more accurate representations of social dilemmas, because, in many real world scenarios, agents are usually afforded an option of not participating in an instance of the game.

Hypothesis 2: *Coevolution of game strategies and network play a key role in the sustenance of biodiversity (coexistence) because it allows agents to also adapt the environment against exploitation from predatory strategies*

Results in chapters 3 and 4 showed that abstention itself only supports the emergence of cooperative behaviour for a very limited set of evolutionary scenarios. In practice, when considering the traditional model for a static network, in most cases the system either converges to full dominance by abstainers, or a full dominance by defectors.

In this scenario, it has been shown that the coevolutionary model proposed in Chapter 3 outperforms the traditional models (i.e., the prisoner's dilemma and the voluntary prisoner's dilemma games, where only the strategies evolve over time in a static and unweighted network), allowing cooperation to emerge even in extremely adverse scenarios where the temptation to defect is almost at its maximum.

Hypothesis 3: *Coevolutionary spatial models increase the heterogeneity of states, which in turn induces the promotion of cooperation*

Although recent studies in coevolutionary spatial game theory corroborate this hypothesis, which in other words is equivalent to saying that social diversity favours cooperation, interestingly, this hypothesis has proven to **not** be valid in Chapter 5. This chapter discusses that this claim might be too vague to really explain the observed emergence of cooperative behaviour

on those coevolutionary models. In fact, results showed that cooperation is favoured by the presence of overlapping utilities and not by the heterogeneity itself. Thus, it is more accurate to state that social diversity *may* favour cooperation *when* different types of individuals have the chance of getting the same utility.

Hypothesis 4: *Probabilistic abstention enhances the mechanisms of network reciprocity by allowing cooperators to reduce the risk of being exposed to defectors*

Chapter 6 introduced a prisoner's dilemma with probabilistic abstention (PDPA), which is a hybrid of two well-known games in evolutionary game theory: the prisoner's dilemma (PD) and the voluntary prisoner's dilemma (VPD) games. As occurs in the PD game, in the hybrid game, each agent can choose either to cooperate or defect. The only difference is that in the PDPA game, in addition to the game strategy, each agent is defined by a value $\alpha = [0,1]$ to denote a probability of abstaining from any interaction. Results showed that the possibility of not interacting with all neighbours is favourable for the evolution of cooperation as it helps cooperators to decrease the risk of being exposed to defectors in the initial steps. Metaphorically, it is equivalent to saying that sometimes "sitting on the fence" is a good way for cooperators to protect themselves against exploitation.

Hypothesis 5: *Cooperation and cyclic dominance in the voluntary prisoner's dilemma game are biased by the use of the Fermi-Dirac distribution function*

Research in this domain has claimed that the addition of abstention in the prisoner's dilemma game leads to a rock-paper-scissors type game, in which

cooperation dominates abstention, abstention dominates defection, and defection, in turn, dominates cooperation, which describes the so-called cyclic dominance behaviour. In this context, Chapter 7 discussed that the emergence of cyclic behaviour is biased by the use of the Fermi-Dirac distribution function (sigmoid) in the strategy adoption process. This sigmoid function is often employed to allow for irrational or unjustified decisions where agents occasionally copy the strategy of a worse or an equally performing neighbour.

However, results in Chapter 7 showed that when agents make entirely rational decisions such as only copying the strategy of better performing neighbours, the outcome changes drastically, making cyclic behaviour unsustainable in most cases. Interestingly, it was shown that the mechanism that supports cyclic behaviour is fixed when agents are allowed to move due to a diluted interaction space.

8.3 Limitations

Despite the ongoing advances in the application of evolutionary game theory in the investigation of complex social dilemmas, such as the ones explored in this thesis, it is well known that game theory, evolutionary game theory and agent-based modelling might not be accurate enough to represent all the variants of a real and highly complex adaptive system. However, it is noteworthy that this limitation does not discard evolutionary game theory from being one of the most important, solid and well-explored frameworks to investigate the puzzle of cooperation in social dilemmas.

Another known limitation of this work includes the lack of mathematical analysis on the role of abstention and coevolution on the emergence of cooperation and social diversity. However, considering the several layers of

complexity involved in the models proposed in this thesis, this challenge was (partially) overcome by intensive use of sophisticated computational techniques to simulate populations of interacting agents in a large scale. Moreover, in order to avoid finite-size effects, results were obtained for different network sizes, and simulations were run for a sufficiently long relaxation time (e.g., 10^5 or 10^6 Monte Carlo steps). Also, to ensure proper accuracy and alleviate the effect of randomness in the approach, the final results are obtained by averaging a number of independent runs (e.g., 10^2 to 10^3 depending on the scenario).

8.4 Future work

This thesis discussed the impact of abstention on the evolution of cooperation and its effects on increasing and sustaining social diversity in a population of rational agents. Despite the advances in exploring more complex scenarios, there are a number of potential future avenues of research stemming from this thesis, including the following:

- Exploration of different topologies and the influence of a wider range of scenarios, where, for example, agents could rewire their links, which in turn, adds another level of complexity to the model. Other potential avenues include exploring the impacts of abstention in interdependent and multilayer networks [181, 184], and co-evolutionary models taking into account the information from past interactions [39].
- Applying our studies and results to realistic scenarios, such as social networks and real biological networks.
- Mathematical analysis of the necessary conditions to sustain the coexistence of three competing strategies of the voluntary prisoner's dilemma

game, allowing us to further explain the results obtained by Monte Carlo simulations.

8.5 Concluding remarks

To conclude, the goal of this thesis was to bridge the gap between abstention and dynamic networks in social dilemmas. We believe that the combination of both of these trends in evolutionary game theory may shed additional light on gaining an in-depth understanding of the emergence of cooperative behaviour in real-world scenarios. Besides, this thesis provides a novel perspective for understanding the foundations of cyclic dominance behaviour in the context of the prisoner's dilemma game with voluntary participation (VPD game). We hope this work can serve as a basis for further research on the role of abstention to advance the understanding of the evolution of cooperation in coevolutionary spatial games.

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