The multi-spectral signal properties of multiple reference optical coherence tomography

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The multi-spectral signal properties of multiple reference optical coherence tomography

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A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy,

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Abstract

Efforts to reduce the size and costs of optical coherence tomography systems (OCT) for consumer applications, in general, focus on Fourier-domain OCT, due to its potential to be integrated on an optical chip, implementing the interferometer and the spectrometer together as a compact system. An alternative and near-term solution is multiple reference OCT. This technique utilizes a partial mirror in the reference arm to enhance the axial imaging depth of traditional time-domain OCT.

The motivation of this thesis is to investigate the performance and sensitivity characteristics of multiple reference OCT in comparison to time-domain OCT. Due to the partial mirror, the light is recirculated multiple times and generates additional reference wavefronts reflected on the reference mirror with increasing path delays. A fascinating consequence of this is the frequency-dependence of each reflection, which draws parallels with Fourier-domain OCT. Hence, the spectral properties of the interference signals are studied in more detail.

This thesis guides the reader towards the subject of OCT covering the fundamentals of OCT and the theoretical basis of multiple reference OCT. The spectral properties of the interference signals of MR-OCT are further explained, and it is shown that the increasing path delay in the reference arm of the Michelson interferometer causes an increase of the frequency similar to the increasing frequency vs. depth in the sample for Fourier-domain OCT. The spacing between the partial mirror and the scanning mirror, in conjunction with the scanning range and velocity, play a unique role in controlling a variety of parameters, which are not available in any
other OCT system. Signal simulations are provided for some theoretical aspects of interest and compared with signals measured on a sample mirror.

For each of the multiple interference signals, the sensitivity was measured, and the results are compared to conventional time-domain OCT. The higher orders of reflections show some non-linear characteristics that may reduce dynamic range and sensitivity.

Zemax was used to simulate the beam propagation in the optical system. The impact of the reference mirror scanning parameters on the overall system characteristics was examined. Further, a novel en-face scanning modality is described that can increase the data acquisition speed due to the reduction of the width of the scanned depth layers and demonstrates the flexibility of the scanning protocol of MR-OCT that would otherwise require extensive efforts with other OCT systems.

Some more specific aspects of the signal and image processing are discussed that have relevance for the image reconstruction of the multiple signal segments originating from the recirculated light due to the partial mirror in the reference arm.

So far the results in this thesis have shown that the multiple interference frequencies originate from an actual change of the source spectrum due to the Doppler effect caused by the scanning mirror. Under certain conditions, it is possible that multiple reference waves create phantom signals. So far the phantom signals are difficult to observe and may not be of significant concern for conventional imaging applications with MR-OCT. The increasing axial scanning range causes depth regions to be scanned multiple times, and averaging can improve somewhat the SNR. Whereas, the time delay between the multiple scanned regions has a delay of a few hundred femtoseconds which may open new application areas that require high-speed scanning.
The work in this thesis is based on the research carried out at the Tissue Optics and Microcirculation laboratory (TOMI), School of Physics, National University of Ireland Galway. I, Kai Neuhaus, hereby certify that this thesis has been written by me, that it is the record of work carried out by me and that it has not been submitted in any previous application for a degree or qualification.
Acknowledgments

Foremost of all I like to thank Prof. Martin Leahy for his trust and provision of resources to enable my work as a Ph.D. student in his group. Martin’s fantastic ability to approach any problem with an open and constructive method helped me always to find solutions to questions that appeared otherwise unsurmountable and embrace the challenges of novel ideas.

A significant appreciation must be directed to Dr Hrelesh Subash who guided me during the first two years of my work and was the most resourceful person I have ever met about optical low-coherence tomography leaving no question unanswered. Furthermore, I wish to thank Dr Sergey Alexandrov for his active and critical discussions about most of the details in my work and theoretical aspects of optical systems.

I would especially, like to acknowledge the support of Dr Paul McNamara and Dr Josh Hogan from Compact Imaging Inc. without whom this thesis would never have been brought to completion. I also wish to thank Anand Arangath for reading over the chapters and all the other people of the TOMI team that I had worked with over the years including Dr Roshan Dsouza, Dr Felicity McGrath, James McGrath, Seán O’Gorman, Gillian Lynch, and Cerine Lal who helped me by sharing their time, ideas and resources when I required them.

My work would not have been possible without the support of numerous people working at the NUIG, friends, and others I forgot to mention by name, in which case I hope they forgive me if I address my gratitude to them in an anonymous manner for their great support and their valuable time.

And last but not least I have to thank my sister for her valuable support and my parents for their patience to endure my absence for such prolonged periods of time.
Contributions and collaborative work

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The principle of multiple reference optical coherence tomography is based on patents [1], [2] and based on a study demonstrating the feasibility of the method [3], [4].

Compact Imaging, Inc. provided some optical components that were used in this work to set up different system configurations for sensitivity characterization.

The Centre for Advanced Photonics & Process Analysis at Cork Institute of Technology provided access to its research laboratories for the characterization of the relative intensity noise in an interferometric system.

All work in this document was performed by myself, and all results in this document originate from this work.
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<th>Description</th>
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<tr>
<td>BS</td>
<td>Beam splitter</td>
</tr>
<tr>
<td>PBS</td>
<td>BS polarized</td>
</tr>
<tr>
<td>CNR</td>
<td>Contrast-to-Noise ratio</td>
</tr>
<tr>
<td>CP</td>
<td>Compensation plate</td>
</tr>
<tr>
<td>DC</td>
<td>Direct current</td>
</tr>
<tr>
<td>DR</td>
<td>Dynamic range</td>
</tr>
<tr>
<td>ESA</td>
<td>Electrical spectrum analyzer</td>
</tr>
<tr>
<td>FD</td>
<td>Focal distance of the lens combination L2 and L3</td>
</tr>
<tr>
<td>FD/OCT</td>
<td>Fourier-domain OCT</td>
</tr>
<tr>
<td>FD/SD-OCT</td>
<td>Fourier- and spectral-domain OCT</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier transform or discrete Fourier transform</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full width half maximum</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical user interface</td>
</tr>
<tr>
<td>HWP, QWP</td>
<td>Half and quarter wave plate</td>
</tr>
<tr>
<td>LCI</td>
<td>low-coherence interferometry</td>
</tr>
<tr>
<td>lsMR-OCT</td>
<td>layered scanning MR-OCT</td>
</tr>
<tr>
<td>MI</td>
<td>Michelson interferometer</td>
</tr>
<tr>
<td>MR-OCT</td>
<td>multiple-reference optical coherence tomography</td>
</tr>
<tr>
<td>MS</td>
<td>Mega samples</td>
</tr>
<tr>
<td>MS/s</td>
<td>Mega samples per second</td>
</tr>
<tr>
<td>NEC</td>
<td>Noise equivalent current</td>
</tr>
<tr>
<td>NEP</td>
<td>Noise equivalent power</td>
</tr>
<tr>
<td>NSC</td>
<td>Zemax non-sequential mode</td>
</tr>
<tr>
<td>OCT</td>
<td>optical low-coherence tomography</td>
</tr>
<tr>
<td>ol</td>
<td>Overlap range of sample layers axially in depth z</td>
</tr>
<tr>
<td>PM</td>
<td>Partial mirror</td>
</tr>
<tr>
<td>PIC</td>
<td>Photonic integrated circuit</td>
</tr>
<tr>
<td>PM</td>
<td>Partial mirror</td>
</tr>
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</table>
POC  Point-of-care
POL  Polarizer
PSF  Point spread function
PUH  CD/DVD-ROM pickup head
QSNR Quantization signal-to-noise ratio
RAA Reference arm attenuation
RBW Resolution bandwidth
RIN Relative intensity noise
SAA Sample arm attenuation
SM  Sample mirror
SC  Zemax sequential mode
SD  Single detector
SRM Scanning reference mirror
SLED Superluminescent light emitting diode
SS-OCT swept-source OCT
SN  number of samples
SNR Signal-to-Noise ratio
TD-OCT time-domain OCT
VC Voice coil

Symbols

\( A \)  Amplitude
\( \vec{a} \)  Acceleration vector quantity
\( \alpha \)  Tukey window characteristic: \( \alpha = 0.0 \) rectangular window, \( \alpha = 1.0 \) Hann window
\( \alpha, \beta, \nu \)  Used as triplet for compensation factors
\( B \)  Detector bandwidth
\( bit \)  Gain of bit width due to oversampling
\( c \)  Speed of light, Damping coefficient
\( D \)  Spacing between the partial mirror and the scanning reference mirror
\( \delta(\ldots) \)  Kronecker delta
\( E \)  Electrical field, Energy
\( E_D \)  Detected field e.g. on a detector plane
\( E_s \)  Source field
\( E_R \)  Reference field
\( E_S \)  Sample field
\( \eta \)  Quantum efficiency
\( F_0, F(t) \)  Initial force, Force time dependent as scalar value
\( f, f_0, f_D \)  Frequency [1/s], center frequency, Doppler frequency
\( \vec{F} \)  Force vector quantity
\( G(z), \gamma(z) \)  Gaussian envelope, coherence gate, coherence function vs. distance
\( \gamma(\lambda) \)  Gaussian envelope vs. wavelength
\( \gamma_r, \gamma_s \)  Splitting factors for a beam splitter; reference, sample
\( H(\omega) \)  Sample response function depending on a frequency range
\( h \)  Planck constant
\( I_D \)  Measured detector current
\( I_c \)  Fluence rate
\( I_L \)  Incident fluence
\( i \)  \( \sqrt{-1} \)
\( i^2 \)  Noise current squared
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$j$</td>
<td>index variable, e.g., for sample layers</td>
</tr>
<tr>
<td>KE</td>
<td>Kinetic energy</td>
</tr>
<tr>
<td>$k$</td>
<td>Wave number $k = 2\pi/\lambda$, spring constant</td>
</tr>
<tr>
<td>$l_c$</td>
<td>coherence length</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Wavelength, Center wavelength</td>
</tr>
<tr>
<td>$M_o$</td>
<td>The maximum order to be processed</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$m$</td>
<td>Index variable</td>
</tr>
<tr>
<td>$m_o$</td>
<td>Order of reflection</td>
</tr>
<tr>
<td>$m'_o$</td>
<td>Another order of reflection but different to $m_o$</td>
</tr>
<tr>
<td>$\mu, \mu_a, \mu_s$</td>
<td>Attenuation -, absorption -, scattering coefficient</td>
</tr>
<tr>
<td>$N$</td>
<td>A general number of an upper limit or boundary</td>
</tr>
<tr>
<td>$N_{px}$</td>
<td>Number of pixel of a line camera</td>
</tr>
<tr>
<td>$n$</td>
<td>Refractive index or an index variable</td>
</tr>
<tr>
<td>$n_j$</td>
<td>Refractive index for a sample layer $j$</td>
</tr>
<tr>
<td>$\nu_0$</td>
<td>Center frequency of light source</td>
</tr>
<tr>
<td>$O$</td>
<td>Oversampling factor</td>
</tr>
<tr>
<td>$\omega, \omega_0$</td>
<td>Circular frequency $2\pi f$, circular center frequency</td>
</tr>
<tr>
<td>PE</td>
<td>Potential energy</td>
</tr>
<tr>
<td>$P_t, R_s$</td>
<td>Power reference and sample arm</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Power of light source</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Phase shift</td>
</tr>
<tr>
<td>$\phi'_d$</td>
<td>Phase difference between driving signal and VC motion</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Degree of polarization</td>
</tr>
<tr>
<td>$q_e$</td>
<td>Electron charge</td>
</tr>
<tr>
<td>$R_{PM}$</td>
<td>Reflected power due partial reflection of the partial mirror</td>
</tr>
<tr>
<td>$R_{sp}$</td>
<td>Specular reflected power</td>
</tr>
<tr>
<td>$R_{t}, R_s$</td>
<td>Reflectivity reference and sample arm</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Sample layer reflectivity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Detector responsivity</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>Sensitivity</td>
</tr>
<tr>
<td>$T$</td>
<td>Boundary of a time range</td>
</tr>
<tr>
<td>$t, \Delta t$</td>
<td>Time range or time, time difference</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>coherence time</td>
</tr>
<tr>
<td>$v_M, v_{PM}$</td>
<td>SRM linear scanning velocity</td>
</tr>
<tr>
<td>$v_g, v_p$</td>
<td>Group velocity, phase velocity of waves</td>
</tr>
<tr>
<td>$z, \Delta z$</td>
<td>Axial depth position, axial scan range or path-length mismatch</td>
</tr>
<tr>
<td>$z_s$</td>
<td>Sample layer depth position</td>
</tr>
<tr>
<td>$Z_j$</td>
<td>The sum of sample layers with $j$ layers ($Z_j = \sum_{m=1}^{j} n_m z_m$)</td>
</tr>
<tr>
<td>$\otimes$</td>
<td>Convolution operator</td>
</tr>
<tr>
<td>$(\ldots)^*$</td>
<td>Complex conjugate of the value in brackets</td>
</tr>
</tbody>
</table>
CHAPTER 1

Introduction

Optical coherence tomography (OCT) has risen to become an accepted imaging method in the medical community over the last 25 years [5], filling the gap between imaging methods such as confocal microscopy and ultrasound imaging. The imaging depth of OCT, at about 3 mm in scattering tissue, is similar to that of confocal microscopy while the scan speed is closer to that of ultrasound imaging. The resolution down to 1 µm and high scan speed opens a crucial functional window to monitor biomedical and biological processes in real-time [6], [7] and allows high-resolution images of the retina of living mammals and humans to be obtained. OCT enters biomedical applications as well as areas of non-destructive testing [8]–[11], dentistry [12], [13], biometrics [14], [15], agriculture [16] and possibly others, providing proof of the versatility of the technology. Although OCT systems are still expensive, the affordability is an essential factor in the acceptance among medical practitioners and ophthalmologists enabling screening of the eye for a wider audience. Therefore it is no wonder that the interest to lower the costs of OCT systems is increasing continuously. The demand for low-cost OCT systems is further driven by the abundance of smartphone technology that can provide some part of the image processing, and viewing capabilities and the question of further miniaturization arose in parallel to make OCT systems affordable for end consumers. Not only is end-consumer technology driving interest in OCT, also the need to make health care accessible to an ever-growing population. Governments increasingly realize the need to provide health care access to less privileged regions,
the countryside, or other remote areas. Purpose built low-cost OCT screening devices can, provide the ideal platform to fill the requirements [17]–[19].

Combining the properties of low-cost and a small form factor appears to be sufficiently significant to study the integration of OCT systems similarly to available photonic integrated circuitry (PIC) systems as they are used in telecommunication applications [20]. Currently, the OCT community is favoring akinetic detection methods that do not require any mechanically moving parts and have exceptional performance parameters such as speed and sensitivity [21]–[23]. Swept-source OCT (SS-OCT) is using light sources based on newly developed lasers that can change the frequency of the emitted light rapidly over a wide range and is manufactured on wafer level optical systems and are inherently suitable for PIC integration [24]–[26]. On the other hand, the currently most widely available spectral-domain OCT (SD-OCT) systems can use the implementation of spectrometers in integrated photonic circuits promising the integration of a complete OCT on a wafer level optical system [27], [28].

However, no fully integrated miniature OCT systems are commercially available yet, and it is not easy to predict the actual production costs of such systems. Potential challenges for commercially and fully integrated OCT systems could be the thermal dependency of the geometries of the waveguides and also the change of the refractive index of the materials that can cause either change of the spectrum of the light transmitted or path length differences including increased dispersion effects. For the highest integration density, it is undoubtedly desirable to move light sources and high-speed processing electronics closely together with the optically integrated system. It is easy to see that the heat produced by the light sources and electronics must then be managed such that the effect on the integrated optical system is kept to a minimum [29]. Heat management and methods to compensate for changes due to temperature effects will need to be considered concerning the costs of production of photonic integrated OCT systems. Current strategies to integrate an SD-OCT are based on interfacing the line camera with an array waveguide photonic integrated circuit. The connection and alignment between the line camera and the array waveguide can be achieved by different methods such as via microlenses or fiber optics. However, this still requires to align 1024 channels with sufficient accuracy considering a conventional line camera to achieve minimal spectral distortions. For better spectral resolution line cameras with 2048 pixels would require the alignment of the same number of channels [30]. It was not possible to find sufficient literature yet that explains the efficiency of such alignment procedures in real-world production environments and therefore it is difficult to give an accurate forecast about when and at what price OCT on a chip is commercially available.
Despite the highly favored akinetic light sources providing superior detection properties for swept-source OCT, the first OCT systems were based on a kinetic scanning method based on an axially oscillating mirror, scanning the reference arm of the Michelson interferometer (MI) [31], [32]. The interference, in this case, depends on the time the scanning mirror moved along a certain axial distance generating a time-dependent interference signal representing the axial reflection profile of a semi-transparent sample in the sample arm (A-line) and hence the name Time-domain OCT (TD-OCT). Those TD-OCT systems were the first commercially available systems that allowed to scan 100 A-lines per second and achieved about 1.5 mm imaging depth [33]. Of course, those systems would be unable to challenge the performance of nowadays available SD and SS-OCT systems yet they were able to perform scanning of the living eye on human subjects and inform clinical decisions. TD-OCT devices provided cross-sectional images of the retina, optic nerve, nerve fiber layer, and measurements of central corneal thickness and anterior chamber angle (400-Hz, A-scan rate, Stratus OCT; Carl Zeiss Meditec) [34].

The mechanical scanning principle of TD-OCT suffers a reduction of the depth scan range if higher scan speeds are required. At scan frequencies of around 1 kHz, the axial scanning range is limited to a few hundred micrometers effectively limiting the imaging depth to the same range. The scan range limitation can be somewhat alleviated with more elaborate scan systems which however are more complex and challenging if not impossible to reasonably reduce in size and cost.

However, one method to overcome the scanning range limitation is to use a partial mirror in the reference arm of the Michelson interferometer using the effect of the increasing distance light has to travel with each reflection (Figure 1.1).

If a TD-OCT is modified with an additional partial mirror in the sample arm in front of the scanning reference mirror, it is possible to increase the imaging depth up to 1 to 1.5 mm depending on the sample properties, whereas the axial scan distance of the scanning mirror was kept at about 50µm. Because the scanning range of the reference mirror can still be varied depending on the mechanical setup, and the complexity of the system was increased only by the additional partial mirror, it was of interest to prove the feasibility of such a system [3]. The method was named multiple reference OCT (MR-OCT) based on the multiple reference reflections that can create distinct interference signals for regularly spaced depth layers in air or a scattering sample. Further studies showed that sensitivities from 80 dB to 60 dB at imaging depths of about 800µm to 1000µm in human tissue can be achieved [4]. For MR-OCT, the imaging depth is not anymore depending alone on the scattering properties and must include
the position and the splitting ratio of the partial mirror. Depending on the setup the imaging depths can, therefore, slightly vary depending on the sample and the configuration of MR-OCT.

The system was further demonstrated to operate with a repurposed voice coil extracted from a CD/DVD-ROM pickup system. Although, only the voice coil was used as an electro-mechanical driver for the scanning mirror underlining the low-cost potential of the MR-OCT scanning system, the striking similarity between the required optical arrangement and the pickup system was sufficient to ask the question if it is possible to achieve a similar sized OCT system for the same price.

Considering that CD/DVD-ROM pickup systems [35] have a purchasing price of around 10 USD and later iteration cycles of those components incorporated a miniature optical bench including a beam splitter, mirrors, and light sources [36] such a technology would open a secondary path to building low-cost and miniature OCT systems with off-the-shelf components and well-known production methods.

Therefore, a growing interest in the performance characteristics of MR-OCT directed the attention of scientific research towards the origin and propagation of the reference wavefronts and the generation of the multiple interference frequency bands.

Understandably, image quality and sensitivity are dependent on multiple other factors that are either simpler for TD-OCT or not required. Due to the more challenging optical noise properties, it is clear that MR-OCT does not try to challenge existing medical grade FD-OCT systems. It is undoubtedly not possible to surpass high-end clinical systems in ophthalmology
in imaging quality but the resolution and sensitivity are well within range to perform single point measurements of layers in the eye’s retina [37]–[39]. For monitoring selected eye related properties in ophthalmology, single shot measurements may be sufficient, and adequate resolution and sensitivity are readily achievable with MR-OCT.

The MR-OCT technology can provide valuable support for remote healthcare and can be used in conjunction with abundantly available smartphones devices. Other applications include spoof and liveness detection for biometric systems and the improvement of existing biometric databases for fingerprint recognition by adding subsurface information [40]–[42]. In the area of non-destructive testing, MR-OCT has potential to serve as a surface monitoring system that can include spectroscopic data for each depth point for semi-transparent and scattering materials for a chosen wavelength of light. This has been demonstrated previously using time-domain [43] and spectral domain interferometry [44]. Those applications would benefit from the larger detection bandwidth of the photo-diode compared to charge-coupled detectors in other OCT technologies [45]–[48].

Another unique feature of MR-OCT is the increasing axial imaging range per order which causes specific depth regions to be scanned multiple times, since successive orders will eventually (with increasing order) overlap (Figure 3.1). Due to the very small time delay determined by the path delay in the interferometer arms and the speed of light, there is potential (in theory at least) to measure high-speed motion or vibrations of optically reflecting interfaces or particles within the overlapping regions.

The research effort in this manuscript is therefore directed towards those additional properties introduced by the partial mirror (PM) in the reference arm that have not been studied yet before and are not found in other OCT systems.

More in-depth knowledge of the theory of MR-OCT can help to increase the prospect of a purpose built and low-cost miniature OCT system that makes use of available manufacturing concepts and to construct sensing units for specific purposes that can complement other high-end OCT systems. Therefore, the added knowledge in this work contributes to the enhancement of MR-OCT systems and the potential to make OCT systems accessible for people in need.
1.1 Objectives

The overall objectives are to examine the properties of the multiple interference signals and their interaction in the optical system, during detection and processing. This study aims to provide answers to the following key questions:

**Research Question 1** Can the theoretical description of MR-OCT show that the frequency of the multiple interference signals is equivalent to the frequency change vs. depth in SD-OCT?

**Research Question 2** How does the position of the scanning mirror vs. the partial mirror affect the wavefront on the detector plane using Zemax simulation?

**Research Question 3** Can sacrificing of scan range and promoting higher A-line scanning speeds still be used to obtain images of sufficient quality using a low-cost voice-coil resonant scanning system at 1 kHz?

**Research Question 4** Do the signal quality and filter characteristics improve by using a Gaussian window in the frequency domain vs. a Chebyshev or elliptic filter?

**Research Question 5** Do the higher orders of reflections follow a linear reduction matching the signal strength characteristics for the first order?

**Theory of MR-OCT and numerical computation**

The first description of the properties of the interference signals was partially based on empirical observations [4]. The Doppler effect is the obvious candidate to describe the increase of frequency of the higher order interference signals. However, the enlarged scanning range in conjunction with the Doppler effect is explained in Chapter 2 describing the theory based on known concepts of TD-OCT. Consequently, it was concluded that the path delay in air due to the multiple reflections is based on the same principle as that for multiple regular highly reflecting layers in the Fourier domain. Because, TD-OCT can not detect the frequency spectrum the change of frequency is directly measured in the time-domain revealing the equivalence of the principles between TD and FD/SD-OCT, which differ merely in the detection scheme.

The numerical computation provides for an in-depth discussion of artifacts that need to be considered for the transition from continuous valued problems to discrete valued problems. The
simulation includes a detailed analysis between the computation of the sample response function to obtain the ideal reflection from a sample vs. the entire generation of interference including artifacts for SD-OCT and MR-OCT.

**Layered scanning MR-OCT (lsMR-OCT) and performance**

The distance between the partial mirror and the scanning reference mirror can be arbitrarily chosen. Usually, the spacing is chosen in such a way that already after the second or third order of reflection the scan ranges overlap to produce a continuous B-frame. Chapter 3 section 3.4 describes a novel configuration that allows increasing the scanning frequency by reducing the scanning range, scanning only thin layers of interest. For this particular configuration, a crude bar-spring mirror mount for the voice coil was constructed, increasing the resonance frequency by selecting a bar geometry and material with a larger spring constant. Such a configuration appears to be counter-intuitive considering the usual goal to obtain a continuous B-frame image. However, for simple screening scans of the eye’s retina, the vitreous body contains only limited information and the layer-scanning method allows one to use the most sensitive interference to obtain most of the information from the retinal layers.

Furthermore, a ray-tracing simulation with Zemax shows that using lens diameters as small as 5 mm diameter produce a negligible distortion of the wavefront simulated for an equivalent of four orders of reference reflections with a spacing between the partial mirror and the reference mirror of 100 µm. The simulation was performed by sequentially simulating each order of reflection separately and creates the basis to use a newer version of Zemax allowing one to simulate the reversal of rays that can lead to a full ray-tracing between the partial mirror and the scanning mirror.

Another major question was how the sensitivity characteristics of the higher orders of reflections are changing vs. the first order reflection. It was hypothesized that the reduction in reference arm power due to the partial mirror splitting ratio should shift the maximum of the sensitivity for each order. However, no significant shift was observed. The first order reflection relates to an ordinary TD-OCT mode and the performance was compared to the measured performance of published results. The measurements were performed for a single and balanced detection with a special beam separation configuration using polarized optical components. For the measurements, an optical setup was prepared to enable nearly identical conditions for single detection and ordinary TD mode, MR mode, and balanced detection for both modes.
Signal and Image processing aspects

An acquisition and processing framework was implemented in Python to access low-cost hardware drivers and open up the opportunity to transfer code fragments to other languages or hardware implementations.

Particularly a low-cost digitizer from Picoscope was used over USB 3.0 to demonstrate the feasibility to acquire B-frames at scan speeds of 300 A-lines per second.

Due to the availability of the ctypes module from Python it was possible to access the underlying C-API to obtain full access to all configuration functions of the Picoscope digitizer.

The logging module from Python allowed the convenient recording of arbitrarily defined configuration settings and would assure repeatability of measurements if desired.

1.2 Synopsis

Chapter 2 reviews the theory of TD-OCT and FD/SD-OCT and provides a detailed analysis of the theory of MR-OCT and how it is related to TD-OCT. Numerical computation of different variations of MR-OCT signals demonstrate the validity and tries to reproduce artifacts as observed in calibration measurements.

Chapter 3 analyzes in detail the noise and sensitivity characteristics MR-OCT compared to TD-OCT and balanced detection scheme. A Zemax simulation is used to measure the wavefront distortion on the detector plane depending on the position of the scanning mirror to reproduce the impact of the change of the spot size. The lsMR-OCT scanning configuration is demonstrated and compared to other solutions. The scanning configuration is unique insofar that the scanning layers are available instantaneously using a single photodetector.

Chapter 4 discusses different aspects of the signal and image processing. Notably, it was shown that the use of a Gaussian window in the frequency domain could have a better filter performance with reduced artifacts compared to an elliptic or Chebyshev type filter. Furthermore, the efficiency of other programming frameworks was investigated in the relation of the non-optimized processing speed vs. availability of processing libraries. So far the results show that Python can converge different hardware libraries, or code from different languages such as Java, Fortran, and C that helps to maintain legacy scientific
libraries. All of the computation and the acquisition framework were programmed in Python with one exception of legacy code in LabView.

Chapter 5 describes materials and hardware components and the characterization of signal artifacts. It is shown that particular artifacts do not originate from signal distortions and are part of the optically detected signal. Furthermore, the roll-off control for MR-OCT is described that can be used as a particular alignment method to improve visibility for higher orders of interference signals and some image results demonstrating the capabilities are also shown.

Chapter 6 summarizes the key findings of this thesis and provides suggestions of aspects that should be further investigated in future work.
CHAPTER 2

Time-domain, Fourier-domain, and multiple-reference optical coherence tomography

2.1 Introduction to multiple reference optical coherence tomography

The idea of using multiple reflections to enhance an optical pathway is not particularly new and can be found in different reports published. Perhaps the most well-known so called folded optical delay line is the Herriott cell [49], [50] which is using two spherical or aspherical mirrors. A beam of light is injected into the mirror system at an angle or through a pin-hole in one of the mirrors, and depending on the angle multiple reflections occur until the back reflection exits the mirror system. Another method was reported by Sharon, Friedman, and Abdulhalim [51] using a stack of semi transparent mirrors in the reference arm as a slow scanning focal compensation for the full field TD-OCT system. Although, the mirror stack approach was not intended for high scanning rates it can be assumed that the larger mass of such a mirror stack would not only limit the scan rate but also reduce the pointing stability at higher rates. Besides the problems with the pointing stability, manufacturing accurate partial reflectivities and avoiding artifacts due to the reflections between the mirror interfaces are aspects that would need to be considered for the implementation of such a system.

Similar to the Herriott cell, multiple reflections can be achieved with two plane mirrors where one mirror is semi transparent, allowing beam injection and extraction at normal incidence
Figure 2.1: MR-OCT optical system and imaging ranges (ellipses) for seven orders of reflections. Superluminescent diode (SLD), photodetector (PD), beam splitter (BS), partial mirror (PM), scanning reference mirror (SRM), piezo or voice coil (PZT/VC), lenses (L), turning mirror (TM).

to the mirror surfaces. Using a partial mirror in the reference arm of an MI was proposed by Hogan and Wilson [1], [2] comprising an OCT system called thereafter multiple reference OCT (MR-OCT).

More importantly, it was shown by Dsouza and others [3], [4], [52] that for MR-OCT the scanning frequency with a conventional mirror was sufficiently fast to be able to perform in-vivo imaging of human skin. The multiple imaging ranges or layers have sufficient sensitivity to deliver a signal up to a depth of about 1.5 mm.

Figure 2.1 shows a stylistic representation of the imaging ranges for MR-OCT as blue ellipses. And indeed the imaging ranges overlap partially depending on the axial position of the partial mirror and the scanning range of the scanning mirror.

It should be noted that the indicated imaging ranges do not represent any photon path within the sample. The more accurate situation is explained in the subsequent chapters of this thesis and should resemble more like a single beam through the sample with a multitude of ballistic photons reflected from different sample structures, whereas on the detector multiple beams from the multiple reflections are superimposed and interfere with all photons from a particular depth.

2.2 Introduction to Michelson interferometer and MR-OCT

MR-OCT is based on an optical delay in the reference arm using a mechanical axially scanning reference mirror (SRM) which is the fundamental principle of a time-domain OCT (TD-OCT) system (Figure 2.2a). Related to the principle of OCT a low coherence light-source with a
Gaussian spectrum is used that allows to detect reflecting interfaces based on narrow Gaussian peaks (Figure 2.2c) with an FWHM = \(2 \ln 2 / \pi \cdot \lambda_0^2 / \Delta \lambda\) (see section 2.6) as opposed to a coherent laser light source (Figure 2.2b).

The distinctive feature of MR-OCT is the partial mirror in the reference arm that generates multiple further delays which have different distinct frequencies and axial lengths (Figure 2.7). The frequency components occur on the detector as a summation of all reflections from the numerous delays and are separated after digitizing by digital filtering. At closer inspection, it can be found that the deterministic frequency components of MR-OCT increase in frequency with increasing axial depth. While FD-OCT is understood to provide a continuous range of frequencies that increase with the depth of the reflecting sample layer, for MR-OCT the increased frequencies originate from the reference arm for each deeper scan range.

Therefore it was of interest to investigate the precise properties of the signal generation of MR-OCT which are analyzed in this chapter. This chapter reviews the theoretical aspects of TD-OCT and FD-OCT and is discussing overlapping properties related to MR-OCT, and enhances the current theory of MR-OCT.

Follow-on sections discuss in detail the numerical simulation of OCT signals and mainly adding numerical results for MR-OCT calibration lines and the numerical computation of the MR-OCT signal in the digitizer buffer.
2.3 The Michelson interferometer

Figure 2.2: A conventional Michelson interferometer (a) with a moving reference scanning mirror moved over a distance $\Delta z$. The recorded intensity on the detector using a laser diode as light source (b) with $\Delta z = 6 \mu m$ and $\lambda_D = 665 \text{nm}$ which is half of the source wavelength due to the double pass of the reference beam. A recorded intensity using an SLED (c) with a Gaussian spectrum ($\Delta z = 50 \mu m$, FWHM $\approx 13 \mu m$).

Fundamentally MR-OCT is operating in a time-domain mode based on the Michelson interferometer (see Figure 2.3).

The field originating from a monochromatic light source (e.g. laser) is split at the BS and each reflected part from the reference scanning mirror (SRM) and the sample mirror are recombined after passing again through the BS and superimposed on the detector

$$E = E_S + E_R.$$ (2.1)

The reference and sample fields can be described as

$$E_R = E_0 R e^{-i2k_R l_R} \text{ and } E_S = E_0 S e^{-i2k_S z_S}.$$ (2.2)
where $E_{0R}$ and $E_{0S}$ are the field strengths reflected from the reference and sample respectively. The propagation constant or wave number in vacuum $k = \frac{2\pi}{\lambda} = \omega c$ describes the spatial propagation of the wave with wavelength $\lambda$. The propagation constant can also be equated to the wave vector as it can be used to describe the spatial wavelength within a sample for different refractive index values [53]. Consequently, as extensively described by literature [54]–[58] the summing of the fields on the detector generating photo detection current $i_D$ can be expressed as

$$i_D \propto E_{R}^2 + E_{S}^2 + 2\text{Re}\{E_{R}E_{S}^*\}$$

where $E_{R}^*$ is the complex conjugate wave from the reference arm. The Eq. 2.3 can then be separated into a DC-part and the interference part. The DC-part is usually rejected on the detection system by a high-pass filter and only the interference part is passed through for digitizing and further processing.

Furthermore the equations Eq. 2.1 can be substituted [54], [59], [60] into interference part of Eq. 2.3 such that

$$E_D = \text{Re}\{E_{S}E_{R}^*\} = E_{S}E_{R}\cos(2k_{R}z_{R} - 2k_{S}z_{S}).$$

Assuming wave propagation in air
\[ k_R = k_S = \frac{2\pi}{\lambda} \]  

(2.5)

where \( \lambda \) is the wavelength of the used light source. Using Eq. 2.5 and substituting into Eq. 2.4 the relation to the path-length difference \( \Delta z = |z_S - z_R| \) is expressed with

\[ E_D = E_{0R}E_{0S}\cos\left(\frac{2\pi}{\lambda} \cdot 2\Delta z\right). \]  

(2.6)

It should be noted here that the meaning of interference is related to the sinusoidal oscillation on the photodetector \( D \). To observe some sinusoidal signal (Doppler frequency) on the detector the parameter \( \Delta z \) must be changing over time and has to be in some time interval correlating to the sampling time of an oscilloscope or digitizer. Typically this means that the SRM needs to be moving with a particular velocity \( v_M \) then the Doppler frequency of the sinusoidal interference signal can be calculated as

\[ f_D = \frac{2v_M}{\lambda} \]  

(2.7)

with \( \lambda \) the used wavelength of the light source [61]. It appears intuitively that the doubling of the interference frequency can be expected but a more rigorous treatment can be deduced using Huygens wavelets [62] and the relative speed of mirror vs. the speed of light [63]. In so far it may be noted that in case of the incident angle (\( \Theta_0 \)) larger than zero will cause that the angle of reflection \( \Theta \) is larger than the incident angle \( \Theta > \Theta_0 \), which is of some relevance for MR-OCT using multiple reflections.

The simulated interference signal can be calculate and plotted with a suitable computational method (Figure 2.4). The example plot shows the mapping of space and time. The length of the time axis depends on the digitizer buffer length and sample rate and the wavenumbers vs. time depend on the mirror velocity. The length of the space axis depends on the moved distance of the scanning mirror \( \Delta z \) and the wavenumbers vs. space depend on the wavelength of the light source. So it is easy to confirm that at a mirror velocity \( v_M = 0.003 \text{ m s}^{-1} \) and acquisition time \( \Delta t_{\text{scan}} = 1 \text{ ms} \) the moved distance is 3 \( \mu \text{m} \). The plot in Figure 2.4 shows that the time base of the digitizer should be chosen in such a way that at least one cycle of the scanning mirror frequency can be accommodated. In practice, the digitizer will be synchronized with the scanning mirror to digitize one or multiple oscillation cycles depending on the digital buffer.
Figure 2.4: Interference signal properties of monochromatic coherent light source using a
digitizing sample rate of 20 MHz, $\lambda = 1330$ nm, and $v_M = 0.003 \text{ m s}^{-1}$ (see A.3-2).

Each oscillation cycle of the scanning mirror represents a forward and reverse motion
which each generates an interference signal or one A-line. Any increase in sample rate can be
used to increase the time resolution of the interference signal that could be useful to detect
higher frequency artifacts. Some more notes are provided in the appendix (A.3-2) in relation of
different ways to compute the mapping of the x-axis which can be either by calculating with the
appropriate x-values or by just using the precalculated x-values and assign those to the x-axis
of the plot as shown here between time and space.

In Figure 2.4 the digitized wavelength $\lambda_{dig}$ is based on the Doppler effect can be directly
calculated as half of the wavelength of the light source with $\lambda/2 = 665.5$ nm.

2.4 Low coherence interferometry

Optical interferometry is related to the superposition of two light waves. Assuming a Michelson
interferometer using a laser light source with high coherence the absolute position of a sample
mirror could not be determined based on the interference signal alone. If using a low coherence
light source, such as a super-luminescent light emitting diode (SLED), then the maximum
amplitude of the interference can only be observed if the position of the reference mirror vs. the
sample mirror position has an equal distance relative to the beam splitter. If one moves the
reference mirror over a known distance relative to the thickness of a semi transparent specimen
with some partially reflecting structures that was placed in the sample arm, then the reflection
profile along depth can be measured. This method is called low-coherence interferometry (LCI),
and optical coherence tomography (OCT) is fundamentally using the same principle while
adding the capability to scan laterally to obtain a tomographic-like cross-sectional intensity image of the specimen's structure in depth.

The low-coherence of the light source can be visualized by the summation of multiple frequencies of the emitted spectrum [64]. Only if all frequencies are in coherence a maximum intensity of the interference occurs, which is principally true for OCT for all scattering depth layers that are located at the coherence gate for a particular frequency. Preferably the power distribution of the spectrum of the light source is chosen to be Gaussian since the autocorrelation function of a Gaussian is Gaussian, or in other words the application of the Fourier-transform on a Gaussian is again a Gaussian function [58].

Consequently, the introduced plane waves (section 2.3) need to consider now not only one frequency of one wavelength but a frequency range $\omega$

$$E_R(\omega) = E_{0_R}(\omega)e^{-j2k_R(\omega)z_R} \quad \text{and} \quad E_S(\omega) = E_{0_S}(\omega)e^{-j2k_S(\omega)z_S} \quad (2.8)$$

and if interference occurs the intensity $I$ (neglecting constants for simplicity) is the sum of all frequencies in coherence

$$I \propto \text{RE} \left\{ \int_{-\infty}^{\infty} E_R(\omega)E^*_S(\omega)\,d\omega \right\} . \quad (2.9)$$

If the spectrum $S(\omega)$ has a Gaussian distribution, then the FFT on this spectrum will again result in a signal with a Gaussian envelope [58], [65, p. 2.2.3]. Because the autocorrelation function is equal to the inverse Fourier-transform and we can assign the spectrum $S(\omega)$ and the phase mismatch $\Delta\phi$ such that

$$S(\omega) = E_{0_R}E^*_{0_S} \quad (2.10)$$
$$\Delta\phi(\omega) = 2k_S(\omega)z_S - 2k_R(\omega)z_R \quad (2.11)$$

and write intensity as

$$I \propto \text{RE} \left\{ \int_{-\infty}^{\infty} S(\omega)e^{-j\Delta\phi(\omega)}\frac{d\omega}{2\pi} \right\} . \quad (2.12)$$
2.5 Non-dispersive medium

The dependency of the phase difference $\Delta \phi$ in terms of a propagation constant $k$ and for a center frequency $\omega_0$ vs. a frequency range $\omega$ must be approximated using a Taylor expansion

$$ks(\omega) = kR(\omega) = k(\omega) + k'(\omega_0)(\omega - \omega_0)$$

assuming both the sample and reference arm are situated in the same linear, uniform and nondispersive medium. The phase velocity in a medium is defined as $v_p = \omega_0/k(\omega_0)$ meaning that $k$ can change for different frequencies. The group velocity is then the inverse change of $k$ with respect to the frequency change and defined as $v_g = 1/(\delta k/\delta \omega(\omega_0))$. Due to the definition of the phase difference $\Delta \phi = k2\Delta z$ it can then be substituted with

$$\Delta \phi(\omega) = k(\omega)2\Delta z + k'(\omega)(\omega - \omega_0)2\Delta z$$

and relating phase delay $\Delta \tau_p$, and phase group delay $\Delta \tau_g$ using the group velocity $v_g$ and the phase velocity $v_p$ then

$$\Delta \tau_p = \frac{k(\omega_0)}{\omega_0}2\Delta z = \frac{2\Delta z}{v_p}$$

and

$$\Delta \tau_g = \frac{k'(\omega_0)}{v_g}2\Delta z$$

Eq. 2.12 can be rewritten as

$$I \propto \text{RE} \left\{ \exp[-j\omega_0 \Delta \tau_p] \int_{-\infty}^{\infty} S(\omega - \omega_0) \exp[-j(\omega - \omega_0)\Delta \tau_g] \frac{d(\omega - \omega_0)}{2\pi} \right\}. \quad (2.17)$$

This shows that the interference signal consists of a carrier oscillating with increasing path delay $2\Delta z$ and spatial frequency $k(\omega_0)$, and an envelope which is essentially the inverse Fourier transform of the power spectrum $S(\omega - \omega_0)$ of the source.

With TD-OCT and MR-OCT we can use Eq. 2.16 to describe the time-dependent formation of the high-frequency carrier and the Gaussian envelope, considering that the scanning mirror introduces a time-dependent interference signal which is reproduced in the digitizer buffer.
Digitizing could be visualized as a scanning process of the sample wave with the reference wave from the scanning mirror. Although the frequency of the light waves is much too high for the used detector, the time-dependent change of the superposed signal of sample and reference waves depends on the scan velocity of the scanning reference mirror. The resulting interference frequencies will be in the kilohertz range and the digitizer takes one snapshot for each of the scanning cycles of the reference mirror. Depending on the sample rate of the digitizer, the interference can be resolved multiple times better than the Nyquist limit would require (oversampling). The interference pattern with a Gaussian envelope will then occur if, for example, a sample mirror and reference mirror have the same distance to the beam splitter.

### 2.6 Gaussian power spectrum

Signal processing for OCT usually requires in most of the cases some digital processing steps that include the computation of the FFT or the Hilbert transform. The FFT applied on a non-Gaussian signal would cause the generation of higher order signal components that distort the PSF and reduces the resolution. Consequently, for OCT it is preferable to use a light source with a spectral distribution close to a Gaussian. The Gaussian shape is then simply described by

\[
S(\omega - \omega_0) = \sqrt{\frac{2}{\pi \sigma_\omega}} e^{-\frac{(\omega - \omega_0)^2}{2\sigma_\omega^2}} \quad (2.18)
\]

with \(2\sigma_\omega\) as the standard deviation.

The Eq. 2.18 can be substituted into Eq. 2.17 [58] and the integral can be solved resulting in

\[
I \propto \text{RE} \left\{ \exp[-j\omega_0 \Delta \tau_p] \exp \left[ -\frac{\Delta \tau_g^2}{2\sigma_\omega^2} \right] \right\}. \tag{2.19}
\]

Eq. 2.19 shows two terms that describe the high frequency carrier depending on the phase delay \(\Delta \tau_p\) and the Gaussian envelope depending on the group phase delay \(\Delta \tau_g\) [65, p45].

Due to the inverting characteristics of the Fourier-transform the standard deviation of the Gaussian envelope \(2\sigma_\tau\) is inversely proportional to the power spectral bandwidth \(\sigma_\omega\).
The group delay $\Delta \tau_g$ in conjunction with $\sigma_\tau$ and $\sigma_\omega$ consequently describes the full-width at half maximum (FWHM) of the Gaussian envelope. The inverse relationship of the standard deviation of a Gaussian function after an FFT operation implies that a larger spectral bandwidth of the light source reduces the FWHM of the detected interference envelope, effectively improving the axial resolution [66]. Best axial resolutions of 1 µm are reported for OCT systems [67].

Using Eq. 2.16 and Eq. 2.20 the spatial width expressed as the standard deviation $\Delta z_{SD}$ can be expressed as

$$\Delta z_{SD} = \frac{1}{k'(\omega_0)\sigma_\omega} = \frac{v_g}{\sigma_\omega}$$

(2.21)

and assuming the light propagating in air / vacuum the phase velocity and group velocity equal the speed of light $c$

$$\Delta z_{SD} = \frac{c}{\sigma_\omega}$$

(2.22)

With FWHM = $2\sigma\sqrt{2\ln 2}$ this can then be expressed as

$$\Delta z_{FWHM} = \frac{2 \ln 2 \lambda_0^2}{\pi \Delta \lambda}$$

(2.23)

The Eq. 2.19 and Eq. 2.17 can be implemented in a computer script language and allow the calculation of the high-frequency carrier and the Gaussian envelope. A plot for a computed interference signal with Gaussian envelope is shown in Figure 2.2c.

### 2.7 Signal simulation

#### 2.7.1 Introduction

The simulation of the OCT signal can be performed with varying degree of detail depending on the interest of physical effects that are anticipated to be investigated. Although most of
the theoretical understanding of the OCT signal formation is well described in the literature [57], [58], [60], [64], [68] numerical computation can provide additional information on signal characteristics. Although numerical modeling will always be an approximation of nature, a model can be designed for particular interests. The current literature suggests different methods of simulation for OCT signals such as the use of mathematical models [69]–[72], Huygens-Fresnel principle [73]–[77], scattering properties in samples [73], [78]–[80], and speckle pattern analysis [81] among others [82]–[86].

In this manuscript only scattering of a plane wave in one dimension on a smooth interface is considered.

An example of simulating MR-OCT with the mathematical model is shown in section 4.4 Signal simulation and signal processing for MR-OCT and the results allow the visualization of the signal characteristics to some degree.

While the mathematical treatment of physical phenomena can provide exact solutions for problems, it may not always be possible to find such a solution. In some cases, it is impossible to find an exact solution, and the application of numerical methods can provide approximations for possibly all problems only limited by the computational power and resources. However, the numerical computation may require additional attention to aspects of the limited accuracy of numbers that can be represented in a computing machine. That means for numerical models some estimation of the expected errors or the growth of errors should be included to obtain repeatable results [87]. Modern computer languages of which some prominent ones are Mathematica, R, or Matlab replicate mathematical rules avoiding computational errors as much as possible. In other cases to simplify the manipulation of matrices, the handling of numbers is supported by algorithms managing memory and storage of values as well as representation of results [88]–[90]. However, in some cases, the use of hardware resources need to be managed more directly which can be challenging if the software tools limit access too much by abstraction and are not providing sufficient functionality [69]–[71]. Often scientists like to resort to languages such as Fortran, C, or Java to obtain more direct access to the machine’s hardware resources [91]–[93]. It is certainly beyond the scope of this manuscript to give a complete answer to the question of why scientists cannot agree on a unique computing languages. However, for the purpose in this manuscript the choice of language was Python in conjunction with the SciPy and Matplotlib packages which provided the most efficient way to combine hardware accessibility with mathematical formalism [94]–[96] (see section A.1).

The following sections will discuss the known mathematical concepts of the Michelson interfer-
ometer, low-coherence interferometry (LCI), and the OCT principle [56]–[58], [64], [72] followed by replicating some of the concepts using numerical modeling.

The mathematical model for MR-OCT is discussed further including the numerical construction of the MR-OCT signals. The spectral properties of MR-OCT are compared to that of FD-OCT and it is shown that some overlap exists, that at least in the numerical domain is related to spectral signal properties.

Finally, the simulation of the MR-OCT signal within the digitizing buffer is described and the effect of the refractive index of the PM on the phase delay is discussed.

2.7.2 Conceptual analysis of the origin of interference

This section starts with a quick overview of the most important points of mathematical concepts of electromagnetic fields and origin of interference in an interferometer.

Figure 2.5 and Figure 2.6 are faithful replications from Izatt, Choma, and Dhalla which appear to be best to visualize the different mathematical components to model low-coherence interferometry and OCT [64]. The complex field originating from the light source is described as

\[ E_i = s(k, \omega)e^{(kz - \omega t)} \]

assuming that \( k \) and \( \omega \) include multiple frequency components of a broad-band light-source. The use of \( k \) and \( \omega \) depends on how the waveform is acquired or if the light-source itself is moving. In relation to OCT the light-source will be static but the reference mirror may be scanning (TD-OCT) or static and the spatial frequency is detected with a spectrometer (FD-OCT). A mixed use of \( k \) and \( \omega \) occurs if the path is further delayed due to the light travelling through a medium with a refractive index other than air. For TD-OCT a simple spatial delay occurs and for FD-OCT a temporal delay occurs that causes a change of the spectrum.

The spatial frequency and temporal frequency are connected over the wave number \( k \)

\[ k\lambda = 2\pi \quad (2.24) \]

whereas \( \lambda \) is related to frequency \( f \) by the speed of light \( c \)

\[ c = \lambda f. \quad (2.25) \]
The refractive index $n$ relates temporal and spatial frequency to the speed of light in a material other than vacuum (air)

$$n = c/v$$  \hspace{1cm} (2.26)

$$k\lambda = n(\lambda)2\pi.$$  \hspace{1cm} (2.27)

The refractive index may also have dependency on the wavelength ($n(\lambda)$) which is important to consider dispersion effects.

Considering $N$ discrete reflecting sample layers and a reference mirror the fields are accordingly

$$E_R = 2^{-1/2}E_i r_R \exp[i2kz_R]$$  \hspace{1cm} (2.28)

$$E_S = 2^{-1/2}E_i \sum_{n=1}^{N} r_{S_n} \exp[i2kz_{S_n}].$$  \hspace{1cm} (2.29)

The summation of the fields on the detector are then described by
Figure 2.6: A simple multi-layer structure with semi-transparent smooth interfaces and the equations for the layer reflectivity \( r_S \) at the depth \( z_S \) as well as the resulting sample field \( E_S(z_S) \) from each depth. Three reflecting rays are shown at an angle to distinguish them. However, the model considers only the incident and reflected rays perpendicular to the layer boundaries, meaning ballistic scattering of photons.

\[
E_S = \frac{E_s}{\sqrt{2}} r_s(z_s) \otimes e^{i2kz_s} \\
r_s(z_s) = \sum_{n=1}^{N} r_{s_n} \delta(z_s - z_{s_n})
\]

\[
I_D(k,\omega) = \frac{\rho}{2} \langle |E_R + E_S|^2 \rangle = \langle (E_R + E_S)(E_R + E_S)^* \rangle \\
= \frac{\rho}{2} \langle E_S E_S^* \rangle + \langle E_R E_R^* \rangle + 2\Re \{ \langle E_S E_R^* \rangle \} 
\]

(2.30)

where \( \rho \) is the detector’s responsivity and \( I_D(k,\omega) \) is the detector photo current. To obtain the detector current vs. the wavenumber \( I(k) \) the FFT is applied which corresponds to integrating with respect to \( \omega \) and substitute \( R = |r|^2 \) and \( S(k) = \langle |s(k,\omega)|^2 \rangle \)

\[
I_D(k) = \frac{\rho}{4} \left[ S(k)(R_R + \sum_{n=1}^{N} R_{S_n}) \right] \\
+ \frac{\rho}{2} \left[ S(k) \sum_{n=1}^{N} \sqrt{R_R R_S} (\exp[i2k(z_R - z_{S_n})] + \exp[-i2k(z_R - z_{S_n})]) \right] \\
+ \frac{\rho}{4} \left[ S(k) \sum_{n \neq m=1}^{N,M} \sqrt{R_{S_n} R_{S_m}} (\exp[i2k(z_{S_n} - z_{S_m})] + \exp[-i2k(z_{S_n} - z_{S_m})]) \right]
\]

DC

CC

AC

(2.31)

Equation 2.31 is a mathematical model predicting three different signal components on the detector. The three different components are the DC, the cross-correlation (CC) and the autocorrelation relation (AC), where the cross-correlation term (CC) is the interference between sample and reference.

The light source is usually chosen to have a Gaussian spectrum as its Fourier transform is
Gaussian again and the transform develops minimal or no side lobes. Assuming here $k$ for the high frequency spatial content of the light source then the power spectral density is $S(k) = |s(\omega, k)|^2$ and the mathematical model of the Gaussian characteristic is

$$S(k) = \frac{1}{\Delta k \sqrt{\pi}} e^{-\left[\frac{(k - k_0)}{\Delta k}\right]^2}. \quad (2.32)$$

Assuming further the validity of the Wiener-Khinchin theorem (see appendix B.1) the FFT of the power spectrum results then in

$$\text{FFT}(S(k)) = \gamma(z) = e^{-z^2 \Delta k^2}, \quad (2.33)$$

and using Eq. 2.33 the coherence length can be obtained with

$$l_c = \frac{2\sqrt{\ln 2}}{\Delta k} = \frac{2 \ln 2}{\pi} \frac{\lambda_0^2}{\Delta \lambda}, \quad (2.34)$$

which corresponds to FWHM of the Gaussian envelope from Eq. 2.23. Although Eq. 2.34 is a well known and important parameter describing the axial resolution of OCT which has inverse relationship to the spectral bandwidth $\Delta k$ or $\Delta \lambda$ it may be not immediately obvious that for numerical computation the number of samples have also an inverse relation, which is discussed in some more detail in the next sections.

Eq. 2.31 is simplified using the normalized Euler’s formula

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2} \quad (2.35)$$

the cosine function can be directly substituted

$$I_D(k) = \frac{\rho}{4} \left[ S(k)(R_R + \sum_{n=1}^{N} R_{S_n}) \right]$$

$$+ \frac{\rho}{2} \left[ S(k) \sum_{n=1}^{N} \sqrt{R_R R_S} \cos(2k(z_R - z_{S_n})) \right]$$

$$+ \frac{\rho}{4} \left[ S(k) \sum_{n \neq m=1}^{N,M,S} \sqrt{R_{S_n} R_{S_m}} \cos(2k(z_{S_n} - z_{S_m})) \right]. \quad (2.36)$$
The detection current $I_D(k)$ is the frequency domain representation of an sample reflector at a depth of $z_S$ and describes the concept of FD-OCT. In practice the frequency domain signal is captured by using a spectrometer to detect $I_D(k)$ which requires one further conversion using FFT to convert the signal into the spatial domain.

Using the Fourier transform pairs

$$\frac{1}{2} [\delta(z + z_0) + \delta(z - z_0)] = \text{FFT}(\cos(kz_0)) \quad (2.37)$$

and

$$x(z) \otimes y(z) = \text{FFT}(X(k)Y(k)) \quad (2.38)$$

the FFT($I_D(k)$) from Eq. 2.36 can be written as to describe the signal after the grating on the line camera

$$I_D(z) = \frac{\rho}{8} \left[ \gamma(z)(R_R + \sum_{n=1}^{N} R_{S_n}) \right]$$

$$+ \frac{\rho}{4} \left[ \gamma(z) \otimes \sum_{n=1}^{N} \sqrt{R_R R_S} \delta(z \pm 2(z_R - z_{S_n})) \right]$$

$$+ \frac{\rho}{8} \left[ \gamma(z) \otimes \sum_{n \neq m=1}^{N,M} \sqrt{R_{S_n} R_{S_m}} \delta(z \pm 2(z_{S_n} - z_{S_m})) \right]. \quad (2.39)$$

Using the sifting properties of the delta function removes the requirements to calculate the Kronecker delta for all $z$ and instead obtain values for only the sample reflectors $z_{S_n}$

$$I_D(z) = \frac{\rho}{8} \left[ \gamma(z)(R_R + \sum_{n=1}^{N} R_{S_n}) \right]$$

$$+ \frac{\rho}{4} \left[ \sum_{n=1}^{N} \sqrt{R_R R_S} \gamma(2(z_R - z_{S_n})) + \gamma(-2(z_R - z_{S_n})) \right]$$

$$+ \frac{\rho}{8} \left[ \sum_{n \neq m=1}^{N,M} \sqrt{R_{S_n} R_{S_m}} \gamma(2(z_{S_n} - z_{S_m})) + \gamma(-2(z_{S_n} - z_{S_m})) \right], \quad (2.40)$$

which means basically just that the Kronecker delta convoluted with a Gaussian becomes a Gaussian with the center position that was the Kronecker delta. So each Gaussian can now be directly calculated based on all sample layer positions $z_{S_n}$. For numerical computation those simplifications are often not required but are useful for the validation process.
To summarize the equations for FD-OCT, it is explained that the line camera of the spectrometer used for SD-OCT virtually scans the frequency domain along $k$ and at a sample rate of a line camera is in the kilohertz range it virtually captures a full spectrum with all detector pixels at once. The captured signal is therefore detected by the number of detector pixels in which case a better discrimination between noise and signal can be achieved effectively increasing sensitivity.

The TD-OCT, on the other hand, is scanning the sample field with the reference field due to a scanning reference mirror, and based on Eq. 2.31 the spatial distribution of the sample field is recovered sequentially in time. In other words TD-OCT is directly recovering the source field $E_i = s(\omega,k)e^{i(kz-\omega t)}$ which is encoded in the sample field $E_S$ by scanning $z_R$ of the reference field $E_R$. That means TD-OCT directly recovers the coherence function without the need to apply the FFT as required in FD-OCT in relation to Eq. 2.33 and the integration of the source spectrum ($S_0 = \int_0^{\infty} S(k)dk$) provides the amplitude of the time-domain signal ($S_0\gamma(z) = S_0\exp[-(\Delta z \Delta k)^2]$).

For TD-OCT the relative position of the reference mirror to the reflecting layer in the sample arm measures the path-delay. Due to the scanning motion of the reference mirror at each particular position in time only one sample layer matches the path-delay and is generating interference. Although the light from all sample layers is present on the detector as a DC component there is no particular interaction as for SD/FD-OCT that could interfere. The mathematical model for TD-OCT has therefore no autocorrelation terms (Eq. 2.31) and the detector signal depending on the scanning reference mirror position $z_R$ is expressed as

\[
I_D(z_R) = \frac{\rho}{4} \left[ S_0(R_R) + \sum_{n=1}^{N} R_{S_n} \right] + \frac{\rho}{2} \left[ S_0 \sum_{n=1}^{N} \sqrt{R_R R_{S_n}} \gamma(z) \cos(2k_0(z_R - z_{S_n})) \right].
\]

Although, in real scattering samples some internal reflections can occur that can cause multiple path-delays (not shown in Eq. 2.42) however those reflections are usually small and often negligible. Furthermore, the random nature of the internal reflections makes it difficult to describe these without sufficient knowledge about the internal structure of the sample.

The equations described in this section from Izatt, Choma, and Dhalla [64] refer briefly to the Kronecker delta function in conjunction with the Fourier transform pair Eq. 2.37. From the view of numerical computation the Kronecker delta function can be seen as a sequence of values.
of unity each at a depth position relating to an element of the sequence or a numerical array. The array of unity values or reflecting layers in this case reside directly in the spatial domain.

On the other hand literature often describes the reflecting structure using the sample response function which appears to be more intuitive related to FD-OCT and SD-OCT describing the integration over a reflection profile in depth using a continuous function for reflectivity and refractive index (Eq. 2.43).

Therefore, it appears to be important to include the approach from Tomlins and Wang [72] that mathematically describe the same model for FD-OCT and TD-OCT but use a different mathematical approach which is used here to evaluate the validity of the numerical model.

So far the equations for the reference and the sample field (Eq. 2.28, Eq. 2.29) are introduced in the same manner using field amplitudes based on the intensity transmittance of the beam splitter \((T_S + T_R = 1)\) on the peak amplitudes. This is related to the reflected power \(R_r = T_S T_R R = 1/4R\) with \(R = |r|^2\) and the factor \(1/\sqrt{2}\) is implicitly used.

Of more relevance is the introduction of the sample response function \(H(\omega)\) and the discrete sample response function \(H[\omega]\) that allows to describe the Kronecker delta function in space [97]

\[
H(\omega) = \int_{-\infty}^{\infty} r(\omega, z) e^{i2n(\omega,z)\omega z/c} dz
\]

\[
H[\omega] = \sum_{j=1}^{n_0} r_j \exp \left\{ i2\omega/c \sum_{m=1}^{j} n_m z_m \right\}.
\]  

(2.43)

(2.44)

The connection is found by investigating equation Eq. 2.29 and Eq. 2.43 and acknowledging that the integral can be approximated by a sum

\[
E_s = \frac{1}{\sqrt{2}} E_i H[\omega]
\]

\[
= \frac{1}{\sqrt{2}} E_i \sum_{n=1}^{N} r_{S_n} \exp[i2kz_{S_n}].
\]  

(2.45)

(2.46)

However, it should be noted that Eq. 2.43 applied for discrete values for \(r\) and \(z\) is changing the operation from convolution to multiplication which results in Eq. 2.47 being equal to Eq. 2.46 if Eq. 2.47 is using a single reflecting layer and Eq. 2.46 is expressed for \(N = 1\) omitting the sum

\[
E_S = \frac{1}{\sqrt{2}} E_i r_S(z) \otimes \exp[i2kz].
\]  

(2.47)
That means the equation Eq. 2.43 multiplied by \( \frac{1}{\sqrt{2}} E \) is equal to Eq. 2.47.

The difference between the continuous and discrete equation is of relevance to construct a valid numerical model which is possibly not immediately visible from the mere mathematical model.

### 2.7.3 Conceptual analysis of the origin of interference for multiple reference optical coherence tomography

MR-OCT was described by Dsouza [4] and most of the theory was based on a modified TD-OCT which incorporates a partial mirror (PM) in the reference arm (Figure 2.7). The multiple reflections from the PM generate multiple reference mirror interfaces increasing the scan depth and the scan range (Figure 2.8).

![Figure 2.7: Schematic of the modified Michelson interferometer with a partial mirror in the reference arm. The wave \( E_i \) from the light source is split on the beam splitter (BS) into equal parts. The sample beam and the reference beam travel respectively a distance \( z_s \) and \( z_r \) relative to the zero position (\( z = 0 \)) and the reflection of the sample and the scanning reference mirror (SRM). An additional partial mirror (PM) generates additional path delays that cause the reference beam to travel increased distances \( z_r(m_o) \) depending on the order of reflection \( m_o \). The corresponding sample wave \( E_S \) and reference wave \( E_R \) are combined on the detector and recorded as a detector current \( I_D \).](image)

For MR-OCT the Eq. 2.28 for the reference arm now contains as in the equation of the sample arm multiple layers of reflections

\[
E_R = 2^{-1/2} E_i \sum_{m_o=1}^{M_o} r_{Rm_o} \exp[i2kz_{Rm_o}],
\]

(2.48)
Figure 2.8: Schematic representation of rays creating multiple reflections. The angle $\alpha$ is shown exaggerated to allow visibility of the ray tracing and is close to zero for practical systems. The shaded areas indicate the scan range for each higher order of reflection.

The transmission for each reflection $m_o$ between the PM and the SRM are then described based on the transmission or reflection ratio of the PM

$$t(m_o) = t^2 (1 - t)^{(m_o - 1)}$$  \hspace{1cm} (2.49)

$$r(m_o) = (1 - r)^2 r^{(m_o - 1)}.$$  \hspace{1cm} (2.50)

A splitting ratio of 80/20 was empirically found to be suitable for human tissue imaging, and it was still possible to be manufactured [4], [98]. The boundary conditions of possible splitting ratios are determined based on the desired number of reference reflections. Each higher order reflection provides a dedicated reference power determining the achievable sensitivity. A smaller ratio does decrease the power of higher orders of reflections quickly, such that their sensitivity is not suitable for imaging anymore. Although, a larger splitting ratio increases the power of higher orders somewhat the increase of the power above 80/20 becomes negligible, and manufacturing partial mirrors with such splitting ratios becomes more difficult without achieving any further improvement.

The multiple reference reflections from the reference arm are distinct in air, meaning that they can be treated right away as discrete for numerical modeling. However, as each order $m_o$ of reflection can create a superimposed signal with any other order $m'_o$ of reflection the description of the Kronecker delta function becomes

$$r_R(m_o, m'_o) = \sum_{m_o \neq m'_o = 1}^{M_o} r_R(m_o, m'_o) \delta(z_R - Dm_o - D(m'_o - 1)).$$  \hspace{1cm} (2.51)

Due to the geometrical increase of the round trip distance the scan range increases by $m_o$ and
$m'_o$ for each reference mirror interface (Figure 2.7). That means that MR-OCT is a TD-OCT with multiple scanning mirrors at increasing distance $D$.

The power reflected by each virtual mirror is described with Eq. 2.50 whereas the superposition of two reference reflections is the sum of each reference and sample. In practice, the superposition of two reference reflections itself does not generate any signal for MR-OCT as for TD-OCT. However, in the presence of a sample reflector and sufficient reference arm power, additional signal terms and artifacts are observable. The artifacts are additional phantom signals of the sample and are described in more detail in section 5.5. For scattering samples, the reference arm power must be reduced for optimal sensitivity, and the phantom signals are below the noise threshold and not visible.

Due to the increased round trip distance between PM and SRM the scan range of each virtual reference mirror increase by a factor $m_o$. If the distance $D$ is chosen small enough the increase of the scan ranges will cause overlaps, which means that effectively some layers in depth are scanned multiple times (Figure 3.1). Each virtual mirror interface must traverse the increased scan range in the same time as the SRM and consequently the detected interference frequency during digitizing is also increased by a factor $m_o$, which originates from the Doppler effect and the constancy of the speed of light.

Consequently, the interference term for the sample interference (S-I) for MR-OCT contains now multiple scanning layers originating from the SRM

\[
I_D(z_{R_{mo}}) = \frac{\rho}{4} \left[ S_0 \left( \sum_{m_o=1}^{M_o} R_{R_{mo}} \right) + \sum_{n=1}^{N} R_{S_n} \right] \quad \text{DC}
\]

\[
+ \frac{\rho}{2} \left[ S_0 \sum_{m_o \neq m'_o}^{M_o} \sum_{n=1}^{N} \sqrt{R_{R_{mo}} R_{S_n}} \gamma(z_{R(m_o, m'_o)}, \Delta k) \cos(\phi(m_o, m'_o)) \right] \quad \text{Fringes.}
\]

(2.52)

The coherence function $\gamma(z_{R_{mo}}, \Delta k(m_o))$ obtained from the Kronecker delta function (Eq. 2.51) consequently multiple reflectors that occur at $z$ positions that are multiples of the PM-SRM distance $D$ while the frequency increases depending on which one is interfering with the sample layer $z_{S_n}$

\[
\gamma(z_R(m_o, m'_o), \Delta k) = \exp(- (z - D(m_o) - D(m'_o - 1))^2 \Delta k^2)
\]

(2.53)

\[
\cos(\phi(m_o, m'_o)) = \cos(2k_0(D(m_o) - D(m'_o - 1) - z_{S_n})).
\]

(2.54)
The interference terms for a sample mirror are not expected to overlap due to their PSFs occurring delayed in time. Nevertheless, as it was mentioned, some artifacts can be observed with a sample mirror and a sufficiently large reference arm power (see section 5.5). The observation of artifact signals would support the theory that the superposition of multiple reference signals can generate phantom signals that occur at spatial positions that are displaced by a distance corresponding to the separation between the PM and SRM. It was not possible to observe the artifacts consistently, and it is assumed that they are only visible for a narrow range of the reference arm power and an optimally aligned sample mirror. More investigation of the artifacts is required in which case the subsequently described simulation framework may help.

If $z_S$ is large depending on $D$ and $\Delta z_R$ then multiple reference reflections can create interference with the same sample layer at $z_S$, meaning the sample layer is scanned multiple times instantaneously only delayed by the optical path difference.

Another property of the MR-OCT signal is the frequency dependency ($k_0(m_o)$) that increases with each higher order of reflection $m_o$. The origin of the path delay for SD-OCT is due to the refractive index of the sample. However, for MR-OCT the increasing frequencies originate from a repeated Doppler effect due to multiple reflections. As shown in Figure 2.8 due to the partial mirror multiple virtual scanning mirrors are generated, whereas each subsequent mirror interface has increased path delay and moves a larger axial distance within the same time. The increasing path delay effectively allows scanning the sample at increasing depths, and the increasing scan velocity is responsible for the higher frequencies of each order of reflection.

That leads to an interesting thought experiment to ask what happens if the spacing between the PM and the SRM is gradually reducing to zero ($\lim_{D \to 0} \Delta z_R = 0$)? The scan range will gradually reduce to zero as well, which means that the scanned layers reduce in size, but due to the reduction of $D$, the number of layers increases to infinity. That leads to an interesting thought experiment to ask what happens if the spacing between the PM and the SRM is gradually reducing to zero ($\lim_{D \to 0} \Delta z_R = 0$)? The scan range will gradually reduce to zero as well, which means that the scanned layers reduce in size, but due to the reduction of $D$, the number of layers increases to infinity. That would mean that MR-OCT would be able to scan a continuous depth range made up of an infinite number of scanning layers with infinitesimal small layer thickness. Because the interference frequency increases correspondingly with each deeper scanning layer, corresponding to a higher order of reflection, it can be concluded that MR-OCT encodes the depth as frequency into the interference signal. The ability to encode a depth by frequency is similar to a spectral domain OCT (SD-OCT) where each sample reflection is encoded with an increasing frequency component vs. increasing depth position. The only difference would be
that the detected spectrum is continuous for SD-OCT while for MR-OCT discrete frequency interference signals are generated.

This thought experiment is not intended to consider that it would be ever possible to reduce the spacing between the PM and the SRM to achieve a continuous detection of frequency components like for SD-OCT and should instead propose the question about the connection of the theoretical aspects between MR-OCT and SD-OCT.

It further may help as well to offer a second explanation of the frequency increase of higher order interference signals and raise the question if there is any potential to use for MR-OCT in conjunction with a spectral detection method as used for SD-OCT. For instance, if the discrete interference frequency components can be separated optically and detected by separate detectors, it would be possible at least partially to gain access to the noise reducing advantages of SD-OCT.

2.7.4 Numerical simulation of the source

The light source is described by its spectral content which can be expressed either as power vs. wavelength or power vs. frequency $\omega = 2\pi c/\lambda$ and was introduced with Eq. 2.18

$$S(\omega - \omega_0) = \sqrt{2\pi} e^{-\frac{(\omega - \omega_0)^2}{2\sigma_0^2}}$$

(2.55)

where $\omega$ is the according frequency range and $\omega_0$ the center frequency or center wavelength.

For numerical computation therefore, the parameter $\omega$ becomes a numerical array with a finite number of frequency components and from frequency zero to a maximum frequency with $N$ frequency components which can be generated with Python using $w = \text{linspace}(0, w_{\text{max}}, N)$.

Then the numerical source spectrum $S$ with a center frequency $w_0$ and the standard deviation $s$ can be computed with the command:

$$S = \text{sqrt}(2*\text{pi}/s_w^2) * \exp(-\frac{(w-w_0)^2}{2*s_w^2})$$

Consequently, to avoid digital artifacts, the digital spectrum $S$ needs to contain a sufficient number of sample points. Each sample point corresponds then to the amplitude of the power vs. a frequency bin (Figure 2.14).

It is essentially possible to model the light source in the time-domain to obtain the field values, which is numerically easily achieved by applying the Fourier transform ($\text{fft}(S)$). Although, in
practice most or all detectors won’t be able to detect the high frequency content it allows to investigate the contribution to the final signal.

As shown above, the mathematical model calculates the time-average of the field sum before rearranging into the three terms of DC, autocorrelation, and cross-correlation. This avoids carrying around the high frequency term to show only the relevant interference signal resulting from the cross-correlation term.

The low coherence spectrum of some SLED can be calculated by summing an infinity of frequency components with normal distribution \[99\] which was introduced in Eq. 2.17 integrating over all spectral components \(\omega\) with intensity \(I\) resulting in a time domain signal \(I(t)\)

\[
I(t) = \int_{-\infty}^{\infty} I(t, \omega) d\omega. \tag{2.56}
\]

This can be numerically approximated (listing 2.1) by summing multiple frequency components with a frequency probability distribution

\[
P(\lambda[n]) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(\lambda_0 - \Delta \lambda)^2}{2\sigma^2} \right) \tag{2.57}
\]

which is the spectral power density of the light source for a discrete wavelength \(\lambda[n]\) and summing those wavelengths with

\[
y[n] = \sum_{n=0}^{N-1} \exp[-i2\pi k[n]z] \tag{2.58}
\]

with \(k[n] = 2\pi/P(\lambda[n])\), \(N\) of the number of wavelength components, \(z\) the simulated spatial range.

Using Eq. 2.58 the results can be visualized in Figure 2.10 that shows an emerging Gaussian shaped signal from the sum of only a few wavelength components.

For numerical simulation targeting particular center wavelength of 1330 nm and bandwidths of 60 nm after summing about 5000 frequency components create a signal that is close to the theoretical expected signal (Figure 2.10).

In relation to Figure 2.10 the Gaussian shapes were compared to the mathematically computed shape for different numbers of frequency components. Figure 2.11 shows multiple Gaussian
Figure 2.9: A few monochromatic waves (colored) with different frequencies are summed (black). With increasing number of frequencies the amplitude of the envelope of the sum converges to a Gaussian shape.

Figure 2.10: The match confirms the expected characteristics that is theoretically predicted using 5000 frequency components.
Listing 2.1: Constructing a light source response using normal distributed frequency components.

```python
space = linspace(-100e-6, 100e-6, N).astype(complex)  # spatial range in [m]
tau_p = linspace(0, 200e-6 / v_M, N).astype(complex)  # scan time for 200 um based on mirror speed
wavelength = 1330e-9  # meter
wavelengthBW = 60e-9
FWHM = 2 * log(2) / pi * wavelength ** 2 / wavelengthBW  # [m]
wavelength = 1330e-9

# use normal distribution for random frequencies
wl_N = normal(loc=wavelength, scale=wavelengthBW/sqrt(2*log(2)), size=number_wl_components)

for wl, i in zip(wl_N, range(len(wl_N))):
    k_n = 2*pi/wl
    t_0 = 100e-6/self.v_M
    spectral_wave = (exp(-1j*k_n*space))
    spectral_wave_sum += spectral_wave
```

envelopes using different numbers of frequency components $N = 500, 1000, 5000, 10000, \text{and} 15000$. The deviation from the ideal Gaussian shape is shown in the error plots below in Figure 2.11.

One interesting aspect in Figure 2.12 is that the digital noise (Figure 2.12(a)) does not reduce linearly. Also the difference between of the FWHM shows a slight increase for larger number of used frequency components which is most likely caused by numerical errors becoming dominant. The SNR of the generated Gaussian reaches about 42 dB after 15000 frequency components which is for a numerical simulation a rather large error and appears not to increase linearly with increasing numbers of frequency components.

Some fundamental consideration related to the sample points for numerical computation is related to the Nyquist limit and it is sometimes easy to overlook that the representation of dense numbers of sample points are in most plotting frameworks automatically connected. This often creates the impression that even with a limited number of sample points a smooth signal shape is achieved. As expected if the frequency is shifted below the Nyquist limit more sample points are available representing the expected field oscillation more closely (Figure 2.13a, 2.13b). Although, the number of pixels is limiting the maximum range in depth [72, eq.21] it is possible to detect frequencies due to aliasing that represent reflective interfaces beyond the maximum range. Also, if multiple internal reflections increase the pathlength in the sample they may occur at deeper depths with frequencies close or larger than the Nyquist limit. The considerations
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Figure 2.11: Comparison ideal Gaussian spectrum ($\lambda=1330$ nm, $\Delta\lambda=60$ nm, FWHM=13 nm) to the spectrum generated by the sum of different numbers of frequency components used ($N$). The error plot shows the difference between the ideal Gaussian and the generated Gaussian shapes.

Figure 2.12: Related to the error plot in Figure 2.11 the deviation from an ideal Gaussian shape was evaluated with three error metrics vs. different numbers of frequency components used ($N$). The noise power (a) is the median of the error fluctuations, the FWHM error (b) is the difference from the ideal FWHM in fractions of sample points, and the SNR (c) is the maximum Gaussian amplitude divided by the noise power in dB values.
Figure 2.13: All units are given in sample points to demonstrate the relation of the number of samples and center frequency. The frequency is varied by 8, 4, 2, and 1.125 times of the number of samples. If the center frequency needs to be kept constant then the number of samples can be increased instead of moving the center frequency.

The sample range in a numerical array containing the spectral values of the frequency components of frequencies beyond the Nyquist limit are of interest to study effects of parasitic frequency components originating from layer widths beyond the resolution limit of OCT systems.

Literature often appears to use a cropped frequency or wavelength range of the source spectrum with components of power above some threshold to reduce computational complexity with reduced number of data points. This introduces digital noise and should be avoided if simulation accuracy is of interest.

For instance, the Hilbert transform already produces spurious results if one selects just about the Nyquist condition Figure 2.14c. The error of the reconstructed PSF was calculated vs. the according oversampling factor and plotted in Figure 2.15 and it can be shown that at least 2.2 times the Nyquist condition must be selected.
Figure 2.14: The power spectral density of the light source for 1024 sample points, frequency and wavelength range of 451 THz and 2660 nm respectively. The application of the Hilbert transform is exemplary of errors of intermediate computational operations for a wrong selection of sample range and frequency/wavelength range.

Figure 2.15: Deviation of the FWHM relative to an ideal Gaussian envelope due to the computation using the Hilbert transform for different number of sample points (SN) and selected range of frequencies or wavelengths normalized to the center values of the frequency or wavelength. The inset shows the point close to the Nyquist limit relating to two times the center frequency/wavelength. The error reduces quickly and reaches a minimum already at $2.2 \times \omega_0, \lambda_0$. Increasing the number of samples reduces the minimal error further to about 0.003% for SN=65536.
is limited by the computer’s memory size. Again if in relation to the Nyquist theorem the frequency range is chosen too small vs. the number of sample points then large deviations of the generated Gaussian distribution are to be expected. In Figure 2.15 the deviation of the FWHM from an ideal Gaussian is plotted vs. the frequency/wavelength range and for different number of sample points $SN$. The range of frequencies and wavelengths are directly related by the relation $c = \lambda f$ and the range is normalized with the center frequency or center wavelength on the x axis of Figure 2.15. The immediate reduction of the error of the FWHM relative to an ideal Gaussian shape with increasing sample points is to be expect as indicated with different marker labels for $SN$ values of 1024, 2048, 4096, 16384, 65536. However, it is perhaps not obvious why the error should change if the frequency range is increased. Although, the change is small Figure 2.15 shows a further reduction of the error if the frequency or wavelength range is increased. But also the error reduction becomes somewhat stronger with an larger number of sample points $SN$. The residual error for frequency and wavelength ranges above ten times the center value is important to acknowledge as it can build up to large errors if a simulation performs a large number of numerical operations.

Conveniently, the simulation allows direct access to the high frequency time-domain signal of the light source which is difficult or impossible to measure in practice (Figure 2.14) due to limited detector response times.

It mathematical models for TD-OCT the high frequency term is usually not carried around and converted directly to the low Doppler frequency.

Furthermore, it is interesting to see that the direct reconstruction of the source spectrum into the time-domain has an arbitrary centered time-location which depends only on the speed of light, although the mere mathematical effect is the result of the double sided centering of the FFT shifting the field oscillation to the middle of the data range in space or time.

However, it should be remarked again that strictly speaking the numerical model can limit the number of samples to the Nyquist limit, for visual representation it would be required nevertheless to increase the sample rate to reconstruct the sinusoidal shapes and more importantly the image characterizing PSF.

### 2.7.5 Numerical model of the Kronecker delta function

For the mathematical model of the OCT interference the sample response function is the most fundamental description of a continuous intensity profile of the sample along the depth axis. The Kronecker delta function occurs during the description to convert the interference equations
from the frequency domain to the spatial domain and although not essential provides a more intuitive description of a reflection profile constructed by a sequence of infinitesimal small slices. On the other hand, the Kronecker delta function can describe the intensity profile in the spatial domain directly which may be helpful for numerical analysis to investigate the impact of the FFT operation and if direct numerical computation is required.

A discrete number $N$ of reflectivities $r_{s_n}$ can be described by

$$
E_S = \frac{1}{\sqrt{2}} E_i \sum_{n=1}^{N} r_{s_n} \delta(z_s - z_{s_n}) \exp[i2kz_{s_n}],
$$

(2.59)

Fundamentally the Kronecker delta can be expressed with the sample response function (SRF) and vice versa based on the Fourier transform pair

$$
\frac{1}{2} [\delta(z + z_0) + \delta(z - z_0)] \leftrightarrow \exp[2i\omega z_0/c].
$$

The values computed using the Kronecker delta function can be stored with machine accuracy in a sparse array alongside with arrays for reflectivity $r_s$ and depth positions $z$. However, for further computation and final superposition, the interaction with the field from the light source requires the distribution of the reflectivities in an array that is equal to the number of sample points of the source field.

For example, for $j$ number of elements or layers, an array of depth positions $z_{s_j}$ is prepared in conjunction with an array of reflectivities $r_{s_j}$

$$
r_s(z_s) = \sum_{j=1}^{N} r_{s_j} \delta(n_{s_j}z_s - n_{s_j}z_{s_j}).
$$

(2.60)

Including the refractive index $n_{s_j}$ of the layer $s_j$ allows also to correct for the depth position due to the optical path delay. The values for $r_{s_j}$ are computed using the Fresnel equation [54] assuming that each layer is perpendicular to the beam from the light source

$$
r_{s_j} = \frac{(n_{j-1} - n_j)}{(n_{j-1} + n_j)}.
$$

(2.61)

The mathematical equations can be rewritten using the SciPy package from Python with the values listed in Table 2.1.

To account for the refractive index of each layer an additional value for air was added. The actual position of each layer boundary is stored in $z_{locs}$. The total width is calculated based on twice
Table 2.1: Parameters to compute a three layer sample. For each layer index $j$ a refractive index $n$ and a thickness $z$ is defined.

<table>
<thead>
<tr>
<th>j</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>1.001</td>
<td>1.002</td>
<td>1.003</td>
</tr>
<tr>
<td>z (µm)</td>
<td>15</td>
<td>30</td>
<td>60</td>
</tr>
</tbody>
</table>

Listing 2.2: Python construction of the Kronecker arrays.

```python
1 air = 1.0
2 ns = array([air, 1.001, 1.002, 1.003])
3 z_widths = array([15, 60, 90])
4 z_widths = z_widths * ns[0:-1]  # correct with ref. index
5 z_locs = z_widths.cumsum()
6 total_width = z_widths.sum()
7 z_rng_max = total_width * 2
8 z_rng = linspace(0,z_rng_max,SN)  # use sample length of E_i
9 getlocidx = interpolate.interp1d([0, z_rng_max], [SN, SN])
10 rs_kd = zeros(SN)  # create empty Kronecker delta array -> all zeros
11 rjs = array([(np−nn)/(np+nn) for np, nn in zip(ns[0:-1], ns[1:]))].squeeze())
12 rs_kd[getlocidx(z_locs).astype(int)] = 1 * rjs  # We indicate the Kron.−
```

the width of the sum of all layers for cosmetic reasons. To allow arbitrary layer positions the values in `z_locs` are linearly interpolated with the function `getlocidx(z_locs)` which was defined by `interpolate.interp1d([0, z_rng_max], [0, SN])` using the function `interp1d` from the SciPy module `interpolate` and parameters of the range of the z and the number of samples `SN`. The computation of the Fresnel reflectivities `rjs` is using NumPy comprehension syntax which is equal to the somewhat more explicit form in listing 2.3 below.

Listing 2.3: Python computation of reflectivities.

```python
1 rjs = []
2 for np, nn in zip(ns[0:-1], ns[1:]):
3 rjs.append((np−nn)/(np+nn))
```

The zip function allows convenient traversal of multiple arrays at once, in this case `np` and `nn`. The values chosen for the refractive index create only small changes, possibly smaller than detectable by real systems. It is however unlikely that any uncertainties due to numerical computation are recognizable considering a default 64 bit floating point operations on a reasonably up-to-date computer or laptop. This a coarse error evaluation and not supposed to be representative, but allows for a quick sanity check of the values.
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Plotting the Kronecker deltas results in the reflectivity for each layer boundary (Figure 2.16) which can be easily evaluated manually from the values in Table 2.1.

![Kronecker delta](image)

Figure 2.16: The computed values of the array of Kronecker delta functions based on the parameters in Table 2.1.

If the Kronecker delta function is convolved directly with the coherence function then we can immediately reconstruct the so called A-line (Figure 2.17). Please note that for the particular plot (Figure 2.17) the coherence function amplitude was normalized to recreate the reflectivities. The coherence function was computed based on the mathematical model from Eq. 2.33 and therefore the mere convolution with the Kronecker delta functions remains a mathematical replication of interference term only. Therefore, this representation is not strictly a numerical model but may otherwise be helpful for testing.

2.7.6 Numerical model of the sample response function

The sample response function was introduced in section section 2.7.2 Eq. 2.43 and the numerical properties are analyzed in some more detail in this section. The values obtained from the sample response function depend on the frequency $\omega$ and therefore it appears obvious that the depth of a reflecting layer is encoded in frequency. Reviewing the numerical sample response function from Eq. 2.44 introducing a parameter $Z_j$ it is easier to see that the integral was replaced with summation operation for $N$ discrete layers with a finite width $Z_j$.
The second sum now assigned to $Z_j$ has therefore no direct relation to the integral of sample response function but assures only that the optical path delay related to the refractive index is incorporated as well. The values of the Fresnel reflectivities $r_j$ for each layer $Z_j$ are calculated as explained in Eq. 2.61.

The equations Eq. 2.62 and Eq. 2.63 can be written as a Python script (listing 2.4).

Listing 2.4: Python computation of the sample response function

```python
1 air = 1.0
2 ns = array([air, 1.001, 1.002, 1.003])
3 z_widths = array([15, 60, 90])
4 z_widths = z_widths * ns[0:-1]  # correct with ref. index
5 Z_j = z_widths.cumsum()
6 rjs = array([(n1-n2)/(n1+n2) for n1,n2 in zip(ns[0:-1],ns[1:])] )
7 Hj = []
8 for r_j, z_j in zip(rjs,Z_j):
9    Hj.append(r_j * exp( 1j * 2 * w_rng / c * z_j))
10 H = sum(Hj, axis=0)
```

If compared to the computation of the Kronecker delta function in listing A.3-7 an equal correction of the optical path delay is found (line 4 of listing 2.4) and $Z_j$ is finally computed on
line 5 using the `cumsum` function representing the summation from Eq. 2.63. Also the Fresnel reflectivities are computed using NumPy list comprehension (line 6) which was explained in listing 2.3. The computation of the sample response function should be easily recognizable in line 9 including the final summing in relation to Eq. 2.62. It should be obvious at this point that $H$ is an array with number of elements equal to the chosen sample rate and represents each depth-layer as a frequency. In Figure 2.18 it can be seen that the interpretation of the frequency representation is rather limited but it shows at least one minor problem of the sharp cut-off at the sample point 0 and $2^{16}$ whose effects are discussed later on. The representation in the

![Figure 2.18: The response of three reflecting interfaces computed with the values in table 2.1 in the frequency domain over the full sample range (top) and partial range (bottom). Each frequency component corresponds to one layer.](image)

time domain by applying the FFT on $H$ is shown in Figure 2.19. It is apparent that using the sample response function to construct the reflectivity values generates a larger error compared to the Kronecker delta function. This is plausible because the number and range of frequencies is limited and the rectangular window causes artifacts after application of the FFT. A window function can be applied to reduce the artifacts and improve the reconstruction of the Fresnel values.

Therefore, the sample field can be constructed based on the FFT\{ $S(\omega)H(\omega)$ \} which is equal to the convolution of the coherence function with the Kronecker deltas, except that the Kronecker deltas were computed from the fast Fourier transform of the sample response function (Figure 2.20). The errors on an A-line computed using the sample response function is already recognizable.

The discontinuous frequency spectrum as mentioned above creates a large amount of high frequencies that can be compensated with some window function. For simplicity a Tukey window ($\alpha = 0.48$) was chosen, where the value for $\alpha$ allows to change the window shape gradually.
Figure 2.19: Converting the sample response function into the time domain using FFT should result in the Kronecker delta functions.

Figure 2.20: The plot shows the result of the sample field $I_S = \text{FFT}\{S(\omega)H(\omega)\}$
from a rectangular ($\alpha = 0.0$) to a Hann window $\alpha = 1.0$. The selection of the most suitable window function is beyond the scope of this thesis and would require an in depth analysis of the mathematical properties related to limited spectral ranges. For most practical applications windows are selected based on their efficiency to suppress side lobes which is not fully applicable in this case. One could assume that the source spectrum provides a natural given window with Gaussian shape, but this would require to scale the source spectrum arbitrarily to the sample length of the sample response function. The application of the Tukey window with $\alpha = 1.0$ after sample response function ($H(\omega)$) is combined with the source field, provides a reasonable approximation within 5 percent of the theoretically calculated values (Table 2.2), although it is possible to improve the accuracy further by optimization of window parameters.

2.7.7 Numerical properties of the sample field $E_S$

The sample field $E_s$ is the convolution of the reflectivities along depth with the source field $E_i$ in the time domain or the product of the sample response function in the spectral domain.

In the case of TD-OCT, the scanning of the reference field is the superposition at each scanning position. This builds up the interference related to the time of the scanning. The sample structure along the depth is then generated by the amplitude of the envelope of the interference vs. time.

Although, we can numerically implement the scanning reference field the numerical computation allows direct access to the sample field and does not need to use the frequency down conversion to make the high frequency components accessible (listing 2.5).

Consequently, to compute the detectable interference signal it is sufficient to combine the Kronecker delta or the sample response function with the source field listing 2.5.

Listing 2.5: Generation of the sample field using the Kronecker delta function (see Figure 2.22a).

```python
1 um = 1e6
2 c = speed_of_light
3 smp = Sample(ns=[1.3,1.5,1.0],z_widths=[5,15,30])
4 src = Source(center_wavelength=800e-9, bandwidth=50e-9)
5 E_t_i, ez_rng = src.get_E_i_td(mode=src.SRF, sigma_x=2, w_x=2)
6 rs_kd, kz_rng, z_rng_max = smp.kronecker_deltas(src,new_z_rng=ez_rng)
7 E_t_s = convolve(E_t_i, rs_kd, mode='same')
```

In listing 2.5 the sample response function $H$ is before the application of the FFT multiplied by a Hann window ($\alpha=1.0$) using the `tukey` function to correct for the parasitic high frequency
Figure 2.21: The Tukey-window with $\alpha = 0.48$ compensates for refractive index $n = \{1.001, 1.002, 1.003\}$ (a,b), $n = \{1.01, 1.02, 1.03\}$ (c,d), and $n = \{1.1, 1.2, 1.3\}$ (e,f).
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Listing 2.6: Generation of the sample field using the sample response function (Figure 2.22b).

```python
1 um = 1e6
2 c = speed_of_light
3 smp = Sample(ns=[1.3, 1.5, 1.0], z_widths=[5, 15, 30])
4 src = Source(center_wavelength=800e-9, bandwidth=50e-9)
5 E_t_i, ez_rng = src.get_E_i_td(mode=src.SRF, sigma_x=1, w_x=1)
6 H, hz_rng = smp.generate_H(src)
7 # Tukey window to reduce error of reflectivity
8 alpha = 0.48  # 1.0 is a Hann window
9 tukwin = tukey(len(H), alpha=alpha, sym=False)
10 H = tukwin * H
11 E_t_s = convolve(E_t_i, abs(fftshift(fft(H))/src.SN, mode='same')
```

<table>
<thead>
<tr>
<th>n</th>
<th>z(μm)</th>
<th>Theoretical</th>
<th>$H(\omega, z)$</th>
<th>$r_S\delta(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.3</td>
<td>5.0</td>
<td>0.1304</td>
<td>0.1368</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>15.0</td>
<td>0.0714</td>
<td>0.0737</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>30.0</td>
<td>0.2</td>
<td>0.2094</td>
</tr>
</tbody>
</table>

Table 2.2: List of theoretical reflectivities vs. computed by the simulation framework. The first value is assumed the outer medium air before the beam enters the first sample layer.

components of a finite frequency spectrum. Especially, without the Hann window the relative reflectivity values generated by the sample response function converted into the time domain would show differences compared to the theoretical expected values.

The reflectivities are based on the list of refractive index from [72] (Table 2.2).

For the purpose of comparison the plotted results are here provided using values for light source parameters of wavelength with $\lambda = 800 \text{ nm}$ and a bandwidth of $\Delta \lambda = 50 \text{ nm}$ which can be directly compared to [72].

The two plots in Figure 2.22 show the same sample structure for layer widths 5, 10, and 30 μm and index of refractions of 1.3, 1.5, and 1.0 respectively. The theoretical calculated FWHM of the PSF would be 5.648 27 μm and the simulated values using the Kronecker delta computation were 5.648 44 μm while for the SRF computation it was 5.648 49 μm. Although the differences in the theoretical values appear to be negligible, they may become increasingly significant with increasing number of samples. For instance, after 1000 summing iterations, the error would have increased up to 4 percent of the theoretical value regarding numerical precision which could lead to distortions in numerically generated signals.

As opposed to the results from Tomlins and Wang [72] the computation used normalized field
Figure 2.22: A graphical validation confirms the computed results in comparison to the data presented by [72] (solid black line) with the computational model (blue shaded) in (c). The generation of the sample signal was performed using the Kronecker delta function (a) and the sample response function (b).

strength and removed the DC in which case it is possible to relate the reflectivity to the reflected power directly.

2.7.8 Numerical properties of the reference field $E_R$

The reference field is only of secondary interest for the numerical computation because it is an intermediate practical step to overcome the problem that detectors are not able to detect the high frequency variations of the electro-magnetic field of the light source. The resulting interference detected is however the sample response by the incident field which is directly accessible numerically. Although it may be possible to simulate also the superposition of the sample field with the scanning reference field, it does not strictly contribute to any additional knowledge about the sample field. Unless, the reference field has numerically added noise components one wants to investigate there is no need to perform this operation just to get access to the sample field. Furthermore, for the numerical modeling of both fields the number of sample points will need to be increased by the ratio between the source and interference frequency. For instance for a light source with a the center wavelength $\lambda_0 = 1330$ nm and assuming an interference frequency of $f_D = 61$ kHz (see section 2.8.6) and the center frequency $f_0$ of the light source the minimal number of samples required for simulation is $f_0/f_D = 4$ Mega samples, and including five times oversampling for reconstruction of the PSF $f_0/f_D = 18$ Mega samples would be required. This increases somewhat the data management if the complexity of structural information increases and multiple reflections are to be included. However, due to the limited gain of additional information of such a model it was not further explored here.
2.7.9 The detection field for TD-OCT

The detection field for TD-OCT can be computed by summing the reference field $E_R$ with the sample field $E_S$ as it has been described by the mathematical model

$$E_D = E_R + E_S. \quad (2.64)$$

However, especially for TD-OCT the summation is only one part of the operation that does not show that the scanning of the interference mirror is crucial to allow the high frequency of the light source to be converted into a much lower frequency that can be detected by presently available detectors. The low frequency is also referred to as the beat frequency originating from the heterodyne mixing of the reference Doppler frequency with the sample frequency. The detector current can then be expressed by

$$I_D = \frac{\rho}{2} \langle E_R E_R^\ast \rangle + \langle E_S E_S^\ast \rangle + 2 \text{Re}\{\langle E_R E_S^\ast \rangle\} \quad (2.65)$$

based on Eq. 2.3 and Eq. 2.31 where the angled brackets indicate the time-average. The scanning mirror motion $z_R(t)$ then is generating a time-dependent scanning field $E_R$ (see Eq. 2.28)

$$E_R = \frac{1}{\sqrt{2}} E_i r_R \exp[i2kz_R(t)]. \quad (2.66)$$

The oscillatory term $\exp[ikz_R(t)]$ is using the wave number $k$ from the light source converted into the time-domain and the traveled distance of the scanning mirror is connected to the scan time $\Delta z_R(t) = \Delta t c/n_{\text{air}}$.

The reference term $E_R$ Eq. 2.66 is required in practice to generate beating between $E_R$ and $E_S$ with a lower frequency suitable for the detector. However, this is not required to extract the sample response with the numerical model which is able to access directly the sample field $E_S$ with the frequency of the source.

This discussion also underpins that the detection of the signal sum depends only on the detector response relative to the frequency of light and the response time to build up a signal for that light frequency. That means as long as the digitizer clock rate is larger than the minimal response time of the detector the power of the sum signal is linearly converted to a detector current.
Naturally, the interference frequency will be less than the clock rate to obey the Nyquist limit and therefore any change of interference frequency due to changing scan speed of the scanning mirror should have a linear power to signal relationship on the detector.

The detected field on the detector is the sum of the sample and the reference field. The light that is reflected back from the scattering samples is naturally heavily attenuated, and the total contribution of the sample field is tiny. The interferometric method is shown to be able to detect intensities from the sample that can be one hundred thousand times smaller than the incident light or around 100 dB. This sensitivity is achieved since the light that is reflected back from the reference arm is increasing the average intensity of the sample arm due to the field-sum which allows shifting the level of the sample interference into the most low-noise region of the detector’s detection range. That means the much larger reference arm power is boosting the weak sample signal and providing a significant gain for detecting weak reflecting structures in scattering samples. The sensitivity further depends on the scan speed and the sample rate of the digitizer determining the detection bandwidth. By increasing the number of samples (resolution bandwidth) and averaging multiple scans the noise power could be reduced which corresponds to an reducing the detection bandwidth but increased acquisition time. However, in the case of MR-OCT no averaging is performed due to the slow scan speed.

2.7.10 The detection field for SD-OCT

As in the case of TD-OCT the detection field for SD-OCT is computed by summing the reference field $E_R$ with the sample field $E_S$ in the frequency domain (listing 2.7).

It should be noted that some of the code is hidden in functions and objects. This makes it very easy to generate any number of source objects (Source()), sample objects (Sample()) and access the right parameters to obtain the sample response function (smp.generative_H()) or to generate a source (src.S(...)). The simplifications can be even further refined by hiding the source spectrum in the object Source() or the sample response function in the sample object but it appears here of advantage to show it for clarity in the code.

The difference is that the reference field is not scanning or is not required to scan because it can capture the interference in the frequency domain by using a grating projecting all frequency components onto a line camera.

Also, the summation with the reference field is included to be able to observe the autocorrelation terms. The field names in listing 2.7 should be easily recognizable relating to Figure 2.7.
The use of a grating allows to split the frequency components and the sum with the reference field creates the interferogram $E_{w_d}$ (Figure 2.23a). The grating acts like an optical FFT operation and the summation on the line camera is the summation of the spectrum of the light source and the spectrum from the sample. Consequently, this operation must be reversed by computing the FFT of $E_{w_d}$ to obtain the spatiotemporal representation of the signal or the depth profile of the reflecting interfaces.

The numerical model can also directly access the sample field without summing with the reference field, which is often presented with mathematical models to avoid the autocorrelation terms. However, to investigate the origin of artifacts and to be able to study noise statistics it appears of advantage to compute also the signal including the autocorrelation terms.

### 2.7.11 The detection field for MR-OCT

Autocorrelation cannot occur in TD-OCT because a single scanning mirror can only generate one unique beat-frequency, the interference pattern (Eq. 2.42). That means any summing with the same frequency would merely have effects on the amplitude of the interference signal but can not generate additional frequency terms. Because the group delays within the sample are random they would merely cause extinction at random points in time and depth.

For MR-OCT multiple virtual scanning mirrors are generated due to the multiple reflections on
the PM. Those virtual scanning mirrors generate multiple different beat-frequencies which if in superposition can indeed generate artifacts. So far it does not appear evident from Equation 2.52 that parasitic beat-frequencies could occur and generate artifacts, such as multiple phantom images from a single reflecting interface. Equation 2.52 only states that multiple interference patterns are present as a sum signal on the detector. Also, for a single reflecting interface, the multiple PSFs due to the multiple reference fields are well separated, and superposition appears to be unlikely or negligible. Measurements, however, suggest that some artifacts occur (Figure 2.27b) that may be due to some effects that allow superposition of adjacent PSFs. The patterns of the artifacts suggest that additional frequency terms are generated at exact multiples of the beat frequencies of the orders of reflections as predicted with Eq. 2.54.

The numerical framework described in this work can easily be configured to arbitrarily generate additional reflecting layers giving the axial position, if the signals are computed in the time-domain. But this method can not predict the artifacts alone by summing the reference $E_R$ and the sample field $E_S$.

On the other hand summing or mixing of the fields $E_R$ and $E_S$ computing them in the frequency-domain provides more readily artifacts (Figure 2.26). However, this is not plausible if only optical effects of the field in the time-domain are considered. There is currently no obvious explanation that similar to SD-OCT mixing of frequencies can occur.

Therefore, both numerical methods for time-domain and frequency domain are discussed in more detail compared to the used code segments.
The simulation for MR-OCT is described based on a particular B-frame image called calibration line (see Figure 2.27b).

The calibration line is simply based on a series of A-lines for a sample mirror moved along depth positions with regular spaced intervals. The principle is also explained in section 4.3 and the resulting image shows a diagonal line that allows to calibrate the position of the orders within the B-frame and the evaluation of other processing aspects. Qualitatively it also allows to evaluate the intensity roll-off for an OCT system using the color or gray scale encoded values. Therefore, each calibration line shows the sample mirror depth-position on the x-axis and the reconstructed depth-position on the y-axis.

The first code segment (listing 2.8) demonstrates the initial assumed theory of the signal formation that does not yet consider artifact generation (Figure 2.24) [4].

The second code segment is computing the artifacts directly by placing arbitrary signal at precomputed positions (listing 2.9). As it was already mentioned, this method can merely serve as a artificial reproduction of signals and does not allow predicting artifacts based on the fundamental input fields.

The third code segment on the other hand (listing 2.10) (Figure 2.26) is using spectral mixing which generates the artifacts based on the input field $E_R$ and $E_S$ directly.

For simplicity the simulation assumes that the SRM is moving at constant velocity away from the PM beginning with a spacing $D = 0$ mm, in which case all reference wave-fronts cover the whole scan range.

In practice the spacing must be $D > 0$ mm due to the scanning oscillation of the SRM and signals from separate scan ranges at increasing depths are obtained, which may become visible due to noise. For instance in Figure 2.27a about 17 scan ranges were processed.

In listing 2.8 line 4 a source with 1300 µm center wavelength and 60 µm bandwidth is generated. The spectrum of the light source $S_w$ is generated at line 9. The spectrum is converted into the input field $E_t_i$ and assuming an ideal reference mirror and no Doppler effect then $E_{t\_r} = E_{t\_i}$ which is further normalized on line 15. On line 24 a loop for the step positions of the sample mirror is prepared for a full range of 1500 µm in steps of 10 µm.

Then the position of the sample mirror is calculated using the Kronecker delta and refractive index from air to $n = 1.04$ on line 25 and the convolution with the input field creates the sample field $E_{t\_s1}$ (see section 2.7.3). A further convolution simulates the scanning and the field $E_{t\_d1}$
Listing 2.8: Computation of the MR-OCT calibration B-frame without beat frequency artifacts.

```python
1 um = 1e6
2 nm = 1e9
3 c = speed_of_light
4 src = Source(center_wavelength=1300e-9, bandwidth=60e-9)
5 w_rng = src.w_rng
6 w_0 = src.w_0
7 sigma_w = src.sigma_w
8 S_w = src.S(w_rng, w_0, sigma_w**2)[0]
9 WL_rng = src.WL_rng * nm
10 CWL = src.CWL * nm
11 BW = src.BW * nm
12
13 E_t_i = fftshift(fft(S_w))
14 E_t_r = E_t_i / E_t_i.max()  # ideal scanning mirror
15
16 # initial z range of TD source field
17 f_max = 2*w_rng[-1]/2/pi
18 ez = 4/f_max*src.SN * c  # set scan range
19 ez_rng = linspace(0,ez,src.SN)
20
21 bframe = zeros((1,src.SN))
22
23 for z in arange(0,1500,10):
24     rs_kd1, zr = Sample(ns=[1.04],z_widths=[0.0+z]).kronecker_deltas(src,
25         new_z_rng=ez_rng)[0:2]
26     E_t_s1 = convolve(E_t_i, rs_kd1, mode='same')  # sample field
27     E_t_d1 = convolve(E_t_r, E_t_s1, mode='same')  # scanning
28     E_t_d1 /= E_t_d1.max()
29     a_line = log10(abs((E_t_d1)**2))
30     bframe = concatenate((bframe, [a_line]))
```

which is then stored in an `a_line` and subsequently added to `bframe`. The content of `bframe` is shown in Figure 2.24.
Figure 2.24: The simulation was performed for 150 mirror steps of an arbitrary distance. The intensity roll-off was not incorporated here as it would not contribute to more information related to the artifact evaluation. Some familiar aliasing noise is visible in the log scaled plot (b).

The listing listing 2.9 demonstrates the addition of parasitic SRM reflections that generate image signals displaced by $z_{m_o} - z_{m'_o}$ (see Eq. 2.53). Compared to listing 2.8 the listing 2.9 is configured equally with regard to the light source parameters. Beginning from line 30 (listing 2.9) additional z-values for parasitic SRM motions are calculated. Because the displacement of the scanning wave front can be directly expressed with the displacement of the scanning mirror the Sample object is used to generate two additional image artifacts. Please take note that the values for the refractive index was arbitrarily adjusted by factors $1 \times 10^{-1}$, $1 \times 10^{-11}$, and $1 \times 10^{-15}$ to match the appearance for the first artifact in Figure 2.27a. The convolution with the source field $E_{t_i}$ is performed for all additional artifacts, indicated with suffix _ac, as well as the scanning with the reference field. Finally all fields are summed and added as A-lines with scaling $\log_{10}$. The results of the simulation with explicit artifacts added are shown in Figure 2.25.

The last simulation listing 2.10 use mixing of the fields in the frequency domain using one scan field $S_{w1}H1$ and a second one $S_{w2}H2$ assuming to originate from a higher order reflection. This generates multiple artifact lines (Figure 2.26).

Figure 2.26 shows multiple artifact lines that are not immediately recognizable in real measurements (Figure 2.27b). However, in some calibration images multiple artifacts remain visible which may be related to repeated mixing of a some of the lower order interference signals (see Figure 4.4). Currently, it is assumed that those are indeed artifact lines and die out quickly due to the lack of overlapping to generate beat frequencies with sufficient intensity.
Figure 2.25: The simulation (listing 2.9) for 150 mirror steps of an arbitrary distance. No intensity roll-off was simulated (see Figure 2.24). The second artifact line is usually not visible but was added for evaluation. Other artifacts such as the faint parallel lines are similar to the artifacts in Figure 2.27a.

Figure 2.26: The simulation (listing 2.10) for 150 mirror steps of an arbitrary distance. No intensity roll-off was simulated (see Figure 2.24). The noise floor raises more realistically. Interestingly many more phantom lines are visible due the lack of attenuation.

2.7.12 Conclusion

The numerical simulation based on the mathematical analysis of the MR-OCT signal generation is partially able to recreate artifacts observed in some B-frame images showing the calibration lines that are otherwise difficult or impossible to explain. The simulation up to this extent used two different methods to recreate the interaction of two or more interference signals. The first method used a direct calculation of a phantom calibration line, and the new displacement
was computed based on the theoretically assumed shift into a different frequency band. This method, however, depends on the arbitrary construction of a signal and cannot generate the artifacts based on only using the fundamental waves $E_R$ and $E_S$ as it is for example possible for the computation of the autocorrelation terms for the SD-OCT signals. The second method assumes that some spectral mixing occurs before and during detection. Consequently, similar to the first method, one phantom line was injected based on the mathematically described concepts assuming such a signal must occur due to the beating of two neighboring signal orders. This method generates multiple further phantom lines which are not immediately plausible. However, at closer inspection of further calibration lines, it was possible to find images that indeed showed another phantom line alongside the initially observed single phantom line (see Figure 2.27b).

Filter artifacts are discussed in section 5.5 and section 4.3 that show that the origin of the phantom signals is unlikely to be caused by the filter process. Therefore the proposed theory offer another possible explanation of the origin. Because the phantom signals are difficult to reproduce and usually do not occur in scattering samples, it was not possible to purpose-build a system within the time frame of the project to investigate the artifact generation in more detail. Consequently, the results shown in this section must remain indicative and would need further confirmation before a definite conclusion can be given. Nevertheless, the mathematical aspects
have been provided in this detail the first time for MR-OCT and provide the basis for further work that may reveal new information that can better suppress noise or make use of particular features of the MR-OCT signal. Future enhancement of the numerical model can include the T-Matrix method using spheres or cylindrical dipole shapes to investigate the scattering on structured reflecting layers [100]–[103].
Listing 2.9: Computation of the MR-OCT calibration B-frame including artifacts.

```python
um = 1e6
nm = 1e9
scale = 30  # arbitrary to fill plot
c = speed_of_light
src = Source(center_wavelength=1300e-9, bandwidth=60e-9)  # Simulate for Tomlins data
w_rng = src.w_rng
w_0 = src.w_0
sigma_w = src.sigma_w
S_w = src.S(w_rng, w_0, sigma_w**2 * scale/10)[0]
f_max = 2 * w_rng[-1]/2/pi  # initial z range of TD source field
ez = 100 / f_max * src.SN * c  # set scan range for Kronecker array
ez_rng = linspace(0, ez, src.SN)
WL_rng = src.WL_rng * nm
CWL = src.CWL * nm
BW = src.BW * nm
E_t_i = fftshift(fft(S_w))
E_t_r = E_t_i / E_t_i.max()  # ideal scanning mirror
Sw1 = src.S(w_rng, w_0, sigma_w * 2 * 2 * scale/10)[0]  # order 1
Sw2 = src.S(w_rng, w_0, sigma_w * 2 * 3 * scale/10)[0]
E_t_r1 = fftshift(fft(Sw1))  # virtual scanning field
E_t_i2 = fftshift(fft(Sw2))
bframe = zeros((1, src.SN))
for z in arange(0, 1500, 10):  # in um
    z1 = z / 1  # 1st order displacement
    z2 = z / 2  # 2nd order displacement
    z3 = z / 3  # 3rd order displacement
    z4 = z / 4  # 4th order displacement
    acd1 = z1 - z2  # 1st inter order displacement
    acd2 = z2 - z4  # 2nd inter order displacement
    rs_kd1, zr = Sample(ns=[1+1e-11], z_widths=[z1*scale]).kronecker_deltas(src, new_z_rng=ez_rng)[0:2]
    rs_ac1, zr = Sample(ns=[1+1e-11], z_widths=[acd1*scale]).kronecker_deltas(src, new_z_rng=ez_rng)[0:2]
    rs_ac2, zr = Sample(ns=[1+1e-11], z_widths=[acd2*scale]).kronecker_deltas(src, new_z_rng=ez_rng)[0:2]
    E_t_sl = convolve(E_t_i, rs_kd1, mode='same')  # sample field
    E_t_ac1 = convolve(E_t_i, rs_ac1, mode='same')
    E_t_ac2 = convolve(E_t_i, rs_ac2, mode='same')
    E_t_sl1 = convolve(E_t_r, E_t_sl, mode='same')  # scanning
    E_t_scl = convolve(E_t_r1, E_t_ac1, mode='same')  # scanning 1st inter order
    E_t_sac1 = convolve(E_t_r1, E_t_ac1, mode='same')  # scanning 1st inter order
    E_t_sac2 = convolve(E_t_r1, E_t_ac2, mode='same')  # scanning 2st inter order
    E_t_d = E_t_scl + E_t_sac1 + E_t_sac2
    a_line = log10(abs((E_t_d)**2))
bframe = concatenate((bframe, [a_line]))
```

Listing 2.10: Computation of the MR-OCT calibration B-frame generating artifacts by spectral mixing.

```python
um = 1e6
nm = 1e9
c = speed_of_light

# Simulate for Tomlins data
src = Source(center_wavelength=800e-9, bandwidth=50e-9)
w_rng = src.w_rng
w_0 = src.w_0
sigma_w = src.sigma_w

S_w, w, w_0, s_w = src.S(w_rng, w_0*2, sigma_w*2)
WL_rng = src.WL_rng * nm
CWL = src.CWL * nm * 2
BW = src.BW * nm

E_s = S_w  # sample mirror
bframe = zeros((1,src.SN))
lines = []
ax_lims = None
Sw1 = src.S(w_rng, w_0 * 2 * 1, sigma_w * 4)[0]
Sw2 = src.S(w_rng, w_0 * 2 * 2, sigma_w * 5)[0]

for z in arange(0,80,0.4):
    H1, zr = Sample(ns=[1.01], z_widths=[0.0+z]).generate_SRMPM(src,[])
    H2, zr = Sample(ns=[1.01], z_widths=[0.0+z*2]).generate_SRMPM(src,[])
    E_w_s = Sw1*H1 + Sw2*H2
    aline_n = log(abs(fftshift(abs(real(0.1*S_w + E_w_s)))))
    bframe = concatenate((bframe,[aline_n]))
```

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CHAPTER 2. TIME-DOMAIN, FOURIER-DOMAIN, AND MULTIPLE-REFERENCE OPTICAL COHERENCE TOMOGRAPHY

2.8 Signal simulation of the digitizer buffer

2.8.1 Introduction

The computation of the higher order interference signals was simulated as they would occur within the signal buffer of a digitizer. The relation of the Doppler and beat frequency for TD-OCT and MR-OCT are reviewed in the following sections and a method to calibrate the image reconstruction is discussed.

The allocation of the phase linear order signals in the digitizer buffer was measured and plotted. Finally, a discussion about the signal modulation due to the sinusoidal motion of a real scanning mirror is added.

2.8.2 TD-OCT Doppler effect of scanning mirror

For the discussion of the Doppler effect we assume a single frequency for simplicity. If a light source has a center wavelength $\lambda_0 = 1330 \text{ nm}$ then it is possible to calculate the expected frequency based on the speed of light $c=299 792 458.0 \text{ m s}^{-1}$ and the refractive index $n$. Assuming air as medium ($n = 1$) then depending on the refractive index $n$, the frequency $f$ required to be detected would be $c/\lambda = 225 407 863 157 894.75 \text{ s}^{-1}$ [Hz], or in more readable units about 225 THz.

$$\frac{c}{n(\lambda)} = \lambda f$$  \hspace{1cm} (2.67)

For conventional solid-state detectors most often used for OCT systems, the detection speed or upper cut-off frequency is mainly determined by their junction capacitance and the statistics of carrier liberation. Therefore, the heterodyne properties of the interferometric system for OCT is essential to convert the high frequency of the light source down to a suitable lower frequency that also preserves the amplitude and phase information [104]. The heterodyne principle is based on the mixing of a local oscillator with similar frequency relative to the high-frequency signal by multiplying of two sinusoidal wave-forms such that

$$\sin(2\pi f_1 t) \sin(2\pi f_2 t) = \frac{1}{2} \cos(2\pi (f_1 - f_2) t) - \frac{1}{2} \cos(2\pi (f_1 + f_2) t)$$  \hspace{1cm} (2.68)

which results in two new waveforms with one containing a much lower frequency based on the difference $f_1 - f_2$. 

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The heterodyne method using the MI is explained in section 2.3 and the multiplication of two electromagnetic fields $2\text{Re}\{E_R E_S^*\}$ contains the mixed frequency components as described in Eq. 2.68.

To generate a similar frequency for mixing the MI uses the source light by splitting it into two components using a beam-splitter and shifts the frequency in one of the arms by moving the mirror by some speed $v_M$. The source frequency $f_0$ is down converted to $f_D$ by the use of the Doppler effect [105]

$$f_D = f_0 \frac{(c + v)}{(c - v)}. \quad (2.69)$$

Because the wavefront between the BS has to travel towards the mirror and back again $\Delta t = 2z_R/c$ the apparent frequency of the light source would due to constancy of the speed of light $c$ calculate to $f_0 = \lambda_0/2$. Consequently, the Doppler frequency of the reflected wave is $f_1 = 2f_0$ and

$$f_{D1} = 2v_M \frac{f_0}{c - v_M}. \quad (2.70)$$

If the speed of the mirror $v_M$ is small relative to the speed of light $c$ then Eq. 2.70 becomes

$$f_{D1} \approx 2v_M \frac{f_0}{c}; \quad (c \gg v_M) \quad (2.71)$$

and with the relation $\lambda_0 = c/f_0$

$$f_D \approx \frac{2v_M}{\lambda_0}. \quad (2.72)$$

Assuming for TD-OCT a sinusoidal mirror motion with an scanning amplitude of $A=50\, \mu m$ at frequency of $f_s=1\,\text{kHz}$ the maximum mirror speed is $\dot{v}_M = A2\pi f_s = 0.3\,\text{[m/s]}$ and a fractional error $\dot{v}_M/c = 1 \times 10^{-9}\,\text{m}$ would have no relevance compared to the PSF of the light source with $z_{\text{FWHM}} = 15\,\mu m$ based on a bandwidth of about $BW = 50\,\mu m$. Consequently Eq. 2.72 is usually written in OCT literature with an equal relation

$$f_D = \frac{2\dot{v}_M}{\lambda_0}. \quad (2.73)$$
However, Eq. 2.71 has also established the relation of the frequency of the light source $f_0$ and the much lower and detectable frequency $f_D$ based on the mirror scanning velocity. If the light travels within a medium with refractive index $n$ then the Doppler frequency calculates to

$$f_D = 2v_M \frac{f_0}{c/n} = 2 \frac{\Delta l_M}{\Delta t_{\text{scan}}} \frac{f_0}{c/n}. \quad (2.74)$$

Two additional parameters connecting the optical signal to the digitizing timing $\Delta t_{\text{scan}}$ and actual scanned axial range $2\Delta l_M$ which we have available from a real acquisition system and can also relate the according time delay of the total scan per A-line as

$$\tau_{\text{scan}} = \Delta t_{\text{scan}} = \frac{2\Delta l_M}{f_D} \frac{f_0}{c/n} = \frac{2\Delta l_M}{v_{g,p}} \frac{f_0}{f_D} = \tau_{g,p} \frac{f_0}{f_D} \quad (2.75)$$

which was already introduced with Eq. 2.17 assuming refractive index $n$ is unity, the group and phase velocities in air are the same, and equal to the speed of light. Eq. 2.75 means only that the rather low acquisition time vs. the propagation time of light have a direct relation and we can use $\tau_{\text{scan}}$ and $\tau_{g,p}$ interchangeably which may be of interest to calculate and plot certain signal properties.

### 2.8.3 MR-OCT Doppler effect of scanning mirror

For MR-OCT, the application of the Doppler effect is equally valid as it has been shown for TD-OCT (see section 2.8.2). Considering the schematic in Figure 2.28 the difference to conventional TD-OCT is that a PM is inserted in front of the SRM with an arbitrary distance $D$.

The PM generates multiple partial reflections (Figure 2.29) or wave-fronts which are governed by the same fundamental principles as for TD-OCT.

Considering a second reflected wavefront $m_o = 2$ travels twice the distance between the PM and the SRM $D$ causing a time-delay $\Delta t_D = 2D/c$ (see section 2.8.2) then due to $c = \text{const}$ and $f_2 = 2c/\lambda_1 = 4c/\lambda_0$ the resulting Doppler frequency is twice the first Doppler frequency. If one would proceed to calculate the Doppler frequency of the third order of reflection $m_o = 3$ one would obtain $f_3 = 6c/\lambda_0$ meaning three times the first order Doppler frequency and for $m_o = 1, 2, \ldots$ it can be calculated as
Figure 2.28: Michelson interferometer with a partial mirror (PM) in front of the reference mirror (SRM) at a distance $D$.

Figure 2.29: Geometrical deduction of the increased path delay of higher orders of reflections due to the partial mirror (PM) drawn for three reflections. The reflection $m_o=1$ is the first reflection directly from the scanning mirror (SRM). Two more reflections ($m_o=2,3$) due to the PM are shown assuming ($m_o=1,2,\ldots,\infty$) for infinite more reflections will occur. The angle $\alpha$ is close to zero for a well aligned system. Each higher order of reflection ($m_o>1$) has a path delay $m_o \cdot D$ due to the PM-SRM-spacing $D$. The virtual mirrors are drawn for visualizing the reflection on the PM. If the SRM moves a distance $\Delta z$ then spacing changes.

$$f_D(m_o) = 2m_o v_M/\lambda_0. \quad (2.76)$$

Similarly, the Gaussian width of the time-domain signal for each higher order of reflection is changing as well considering that all frequencies in the spectrum will change due to the constancy of the speed of light. From section 2.5 the group time-delay was introduced as $\tau_g = \frac{k'(\omega_0)2\Delta z}{2\Delta z/v_g}$ which, if using the PM-SRM-spacing $D$ and substituting $2\Delta z = 2D$ and $\omega_m = 2\pi f_m = 2\pi m_o f_0$, the group time-delay depending on the PM-SRM-spacing becomes
\[ \Delta \tau_g = \frac{dk}{d(\omega_m)} 2D = \frac{1}{m_o} \frac{dk}{d(\omega)} 2D. \] (2.77)

That means that if each frequency part \( d\omega \) in the spectrum is increased based on equation Eq. 2.76 and substituted with \( d(m_o \omega) \) then the group time-delay reduces. Because the FWHM of the Gaussian envelope is obtained by integration over the coherence function \( \gamma \) which can be expressed as \( G(\omega - \omega_0) \exp[-(\omega - \omega_0)^2 \Delta \tau_g] \) [58] (and see section 2.5) it will equally reduce by \( m_o \)

\[ 2\Delta z_{\text{FWHM}} = c \int_0^\infty |\gamma(\tau_g/m_o)|^2 d\tau_g = c/m_o \int_0^\infty |\gamma(\tau_g)|^2 d\tau_g. \] (2.78)

meaning an apparent reduction of the FWHM of the PSF. As was already mentioned the reduction of the FWHM is caused by the frequency shift due to the Doppler effect which is visible as an interference signal with higher frequency in the digitizer buffer.

The standard deviation related to the FWHM for each higher order signal is then calculated as \( \sigma(m_o)^2 = m_o \sigma_\omega^2 \). The conventional path-delay caused by the spacing of the PM and SRM \( D \) will further displace the center of each scanned region at a depth \( m_o D \).

With the scan time \( \tau_{\text{scan}} \) and the number of samples the addition of noise can simulate the impact of the detection bandwidth on the sensitivity.

### 2.8.4 Sample mirror stepping: calibration line

The method of sample mirror-stepping is to collect multiple A-lines for regularly distributed mirror depth positions. If all A-lines are assembled into an B-frame such that the spacing is imaged with the same step-spacing as for the mirror-steps a diagonal line will be created, called the calibration line.

This is similar to a collection of PSFs to evaluate the intensity roll-off vs. sample mirror position in air. Besides the intensity roll-off the calibration line allows calibration of the processing framework to accurately translate the mirror depth positions but also may reveal other imaging artifacts.

The creation of a calibration line is achieved by attaching the SRM to a translational stage and synchronize acquisition with the stage motion to acquire one A-line per z-position. Consequently,
a real stepping distance $\Delta z_{SRM}$ should be reproduced by the digital processing one-to-one as $\Delta z_{SRM}^r \equiv \Delta z_{SRM}$.

For MR-OCT, a full A-line is generated by reprocessing a selected number of higher order signals and transforming them into a common spatial dimension (see section 4.4.2). Any step shift of the sample mirror can be expressed as a time shift of the maximum of the Gaussian envelope in the digitizer buffer. The stepped size vs. the time interval that a Gaussian envelope would shift in the digitizer buffer can be expressed as $\Delta z(m_o)/(m_o o_{SRM}) = \Delta t$ with $\Delta t$ for the linear acquisition time for one A-line. Because $m_o$ is a quotient the effect for each higher order will have a reduced time shift for the same step size of the sample mirror motion. If multiple points of the sample mirror motion vs. time are plotted for each order of reflection, then each order should follow a straight line with a different slope $1/m_o$. The plot in Figure 2.31 provides a visualization of the sample mirror motion vs. sample points of the digitizer buffer where each sample point can be directly assigned to a time point.

In Figure 2.30 a typical collection of calibration lines in conjunction with the intensity roll-off of the PSFs is shown as it was often used to evaluate the optical and processing characteristics of MR-OCT. Compared to other examples, in Figure 2.30a and Figure 2.30b a widening of the PSF towards deeper scan regions is visible which is increasingly observed if the scan speed is increased.

### 2.8.5 Allocation of the signal in the digitizer buffer

The signal processing for MR-OCT can use the beat frequencies of the interference of the reference reflections that are anticipated for analysis or imaging. The interference components can be separated by digital band-pass filtering based on the beat frequencies. In general, no other information is required to assemble a full A-line after filtering, assuming the linear relationship between the order of reflection $m_o$ vs. the change of the FWHM of the PSF and the imaging range.

The Gaussian width of the higher order signals is reduced increasingly by $1/order$ due to the Doppler effect (see section 2.8.2) which means they occupy an extended axial depth range, and the physical imaging range is recovered by upsampling each higher order by a factor $m_o$. The actual position of each order in space is determined by the spacing between the PM and the SRM $D$ which is usually not known with sufficient accuracy. However, the processing can be calibrated using the calibration line (Figure 2.30) that allows determining the position of each order with pixel accuracy. Any little deviation of the spatial position of a signal order within
(a) Calibration line recorded for 300 steps with a stepping distance of 50 µm.

(b) Gray scaled calibration of the same data with adjusted threshold to reject noise floor. The vertical lines indicate the measurement points for the collection of PSF from Figure 2.30c.

(c) Collection of PSFs vs. depth. The gain is digitally increased for larger depths to improve visibility of noise artifacts which creates a somewhat raising noise floor.

Figure 2.30: Example of three different aspects of the calibration line interpretation.
Figure 2.31: The plot shows multiple sample mirror positions and the corresponding position in the digitizer buffer for up to seven orders of reflections. Depending on the sample rate a time for each sample can be assigned. The measured positions (black dots) correspond well to the computed positions (red dots). The blue depiction to the right shows how the sample mirror position generates multiple signals in the digitizer buffer.

The digital image buffer becomes immediately visible due to disruption of the calibration line. It was determined that the theoretical displacement of each order by D is not entirely sufficient to achieve a smooth line and a small correction is required for accurate alignment. The source of the deviation from D was not yet thoroughly investigated, but it is believed to be related to the time delay of each order due to the speed of light that displaces each signal order in the digitizer buffer by some additional temporal delay.

If each separate higher order signal is plotted according to the sample mirror z-position vs. the buffer time, or in this case buffer samples, it is possible to investigate the different slopes of motion and the allocation of each order in time relative to the sample points (see Figure 2.31).

In Figure 2.32 a schematic view of the displacement of the higher order interference signals in the digitizer buffer is given. Note that in Figure 2.32a only one single sample mirror reflection is shown, but all five scanning reference orders capture the same reflection. Steps Figure 2.32b and Figure 2.32c show schematically the intermediate steps of the change of displacement and the change from spatial-domain to the time-domain.

The total scanning range z(M_o) or imaging depth based on the number of processed orders M_o can then be calculated [4] with
Figure 2.32: This schematic visualizes the virtual conversion from space to time and the apparent location of a single mirror interface during digitizing. In (a) five scanning ranges are shown covering one mirror reflection in air. In (b) the spatial displacement is removed because the scanning of each order starts at the beginning of each order. In (c) spatial gain of the scanning is removed because each scanning range must arrive at the same time due to the speed of light and the end of each scanning range must be equal. Step (c) is showing all scanning ranges merged into one timeline. It can be noted that the higher orders of reflections move closer together (d) which corresponds to the digitized signal.

\[ z(M_0) = \frac{z}{2} + D(M_0 - 1) + \frac{M_0}{2}z. \]  

(2.79)

The simulated buffer signal in Figure 2.33 shows five interference signals of one sample mirror position which was possible by selecting a very small simulated spacing between the PM and the SRM of \( D = 20 \mu m \).

The small spacing \( D \) was chosen for demonstration purposes and would shorten the actual achievable imaging depth (see Eq. 2.79). The intensity for each order for the simulated signal was calculated with

\[ I(t, N) = I(t)(1 - R_{PM})^2(R_{PM})^{(m_0 - 1)}, \]  

(2.80)

as discussed in section 3.1.

A more detailed description of the processing steps are given in section 4 which also shows the allocation of overlaps or multiple scanned regions, which can be either adjoined or summed (see section 4.4).
2.8.6 Carrier signal

In practice the SRM is moving with a sinusoidal position and velocity vs. time and the beat frequency of the interference is modulated. A position vs. time plot in Figure 2.34 shows the linear and nonlinear profile of motion.

The expected Doppler frequency relative to the sinusoidal motion is then using the equations of a simple harmonic oscillator such that the scanning mirror position $z_{SRM}(t)$ vs. time is

$$z_{SRM}(t) = \left(\Delta z_{SRM}/2\right) \cos(2\pi f_{SRM}t_S + \phi)$$  \hspace{1cm} (2.81)

the velocity of the SRM vs. time is then

$$\dot{z}_{SRM}(t) = v_{SRM}(t) = \frac{-\Delta z_{SRM}}{2} 2\pi f_{SRM} \sin(2\pi f_{SRM}t_S + \pi/2)$$ \hspace{1cm} (2.82)

with $v_{SRM}(t)$ is the velocity of the SRM vs. time. The initial phase offset $2\pi$ is arbitrarily defined to match $z_{SRM}(t)$ in (Figure 2.34). Then the maximum velocity is achieved at $\Delta z_{SRM}/2$ or $t_S/2$ in which case $2\pi f_{SRM} t_S = \pi$ then the term above becomes

$$v_{SRM_{\text{max}}} = \frac{\Delta z}{2} 2\pi f_{SRM}.$$ \hspace{1cm} (2.83)
Figure 2.34: Scanning regimes of the scanning mirror with scanning range $z_{SRM}$ vs. scanning time $t_S$. Solid line is an ideal linear motion of the scanning mirror and the dashed line is the real sinusoidal motion due to the mass-spring system following nearly an ideal harmonic-oscillation. The sections $A$ and $B$ are also called “forward” or “reverse”, however each part is considered one full scan region.

Considering that for a linear velocity of the SRM scanning a distance $\Delta z_{SRM}$ over a time $t_S$ the scanning frequency is

$$\frac{\Delta z}{\Delta t_{SRM_{max}}} = \Delta z \pi f_{SRM}$$  \hspace{1cm} (2.84)

$$\frac{1}{\Delta t_{SRM_{max}}} = \pi f_{SRM}$$  \hspace{1cm} (2.85)

then the frequency ratio between linear moving SRM and a sinusoidal motion is

$$f_{SRM_{max}} = \pi f_{SRM}$$  \hspace{1cm} (2.86)

$$\frac{f_{SRM_{max}}}{f_{SRM}} = \pi.$$  \hspace{1cm} (2.87)

For example if the measured scanning rate for real system was $f_{SRM}=152$ Hz and the scanning range was $\Delta z = 90 \mu m$ then the Doppler frequency $f_{max}$ was measured with 61 kHz and the linear Doppler frequency is expected to be about $f_{lin} = \frac{f_{max}}{\pi} = 19.4$ kHz.

The carrier signal in conjunction with the scanning time can be written as

$$g(t) = \cos(m \omega \Delta t_S + \phi)$$  \hspace{1cm} (2.88)

and substituting the harmonic motion (carrier signal) gives a frequency or phase modulated motion.
Figure 2.35: A comparison of the interference with linear phase (a) vs. a nonlinear phase (b) for a monochromatic light source. In (b) the modulation signal (mod phase) is overlaid. The related nonlinear carrier phase with the differential of the carrier phase (carrier diff) is shown in (c). The FFT of the nonlinear or chirped carrier signal (dashed) and the linear carrier signal (solid) is shown in (d).

\[ y(t) = \cos \left( \omega_c \Delta t + \frac{\omega_c}{\omega_M} \sin(\omega_M \Delta t - \pi) \right). \]  

(2.89)

Figure 2.35 demonstrates the impact of the sinusoidal velocity profile of the SRM on an interference signal with linear phase characteristic (top plot) and the resulting nonlinear interference signal with nonlinear phase (second plot).

Because the modulation of the SRM is close to that of simple harmonic motion in conjunction with a spring mounted mirror and electromagnetic actuation, the phase nonlinearity can be digitally removed by remapping the sample bins based on the carrier phase (see Figure 2.35). Other methods to remove the phase modulation due to the scanning can be achieved by directly measuring the modulation and changing the sample width of the digitizer in real time [106].
3.1 Performance review of multiple reference vs. time domain optical coherence tomography

3.1.1 Introduction

A detailed characterization of noise sources in multiple reference optical coherence tomography (MR-OCT) compared to time-domain OCT (TD-OCT) is presented. The noise characteristics were modeled based on the TD-OCT noise model, modified for MR-OCT and confirmed with measurements. The MR-OCT sensitivity characteristics are significantly affected by the reflection from the partial mirror, which also introduces a natural attenuation in the reference arm, partially matching the reflectivity intensity profile of human tissue. At optimal balance between sample and reference arm and using balanced detection, the peak sensitivity was measured to be 95 dB, which is close to simple Fourier-domain systems. The results provide a better understanding of the application range for MR-OCT and higher order effects observed, suggesting a non-trivial noise model for MR-OCT.

The axial imaging depth $z_{MRO}$ can be calculated based on the relation $z_{MRO(M_o)} = \frac{\Delta z}{2}(M_o + 1) + D(M_o - 1)$ using a spacing $D$ between the partial mirror (PM) and scanning reference mirror (SRM), and scanning range $\Delta z$ (Figure 3.1) [107], [108]. Even by processing only ten optical reflections ($M_o = 10$ orders), multiple reference OCT (MR-OCT) enhances the axial imaging depth from an otherwise shallow scanning range of 95 µm to 1870 µm in air using $D = 150$ µm.
Figure 3.1: The path delay for each order within the sample is shown in (a). The delay increases by $D(m_o - 1)$ and as indicated, showing the increasing depths of the scanning layers. The scanning range ($\Delta z$) in (b) is optically increased in the sample (a) which increases the imaging depth indicated by increasing length of the solid horizontal lines by a factor $m_o$; the order of reflection, and first overlap ol. Image (b) shows the generation of reflections and fractional powers with the PM-SRM combination in the reference arm. The PM has a reflectivity $R_{PM}$. The incidence wavefront from the BS with a power $P_r$ is reflected on the PM generating an optical DC with power $P_r R_{PM}$. After the wavefront has passed through the PM the power of each further reflection is calculated as $P_r (1 - R_{PM})^2 R_{PM}^{(m_o-1)}$ ($m_o = 1, 2, \ldots, M_o$).

In this section the step-wise reduction of the powers due to the splitting ratio of the PM is compared to the reflectance profile of human skin and a tentative noise and sensitivity model is provided. This is compared with measurements on a prototype MR-OCT system with a single photodiode and a balanced photo-detection system. The noise model helps to determine the limitations and the application range of MR-OCT.

### 3.1.2 MR-OCT theory

Most of the fundamental theory of MR-OCT is covered by the theory of TD-OCT [56]–[58], [60]. The formation of the interference is the same principle and only the new aspects which are related to MR-OCT are discussed in this thesis. The relation $I = \rho P$ is used to retrieve the photocurrent at the detector, where the detector responsivity is denoted $\rho$. The time-dependent power $P$ of the interference for TD-OCT is then described as

$$P = P_r + P_s + 2\sqrt{P_r P_s G(\Delta z)} \cos(\omega t + \phi)$$

(3.1)

where $P_r$ and $P_s$ are the powers in the reference and sample arm. Assuming a light-source with Gaussian spectral characteristics, the function $G(\Delta z)$ is then the Gaussian envelope of the interference spectrum along the pathlength difference between reference and sample arm $\Delta z$, the angular frequency $\omega = 2\pi f$, and $\phi$ the initial phase of the frequency content of the
interference [56], [58]. The axial resolution for TD-OCT is commonly defined as the 'round trip' coherence length of the light source:

\[ l_c = \frac{2 \ln 2 \bar{\lambda}^2}{\Delta \lambda} \]  

(3.2)

where \( \bar{\lambda} \) is the mean wavelength of the light source and \( \Delta \lambda \) the spectral bandwidth at the full width half maximum (FWHM) of an assumed Gaussian-shaped spectrum. It should be noted that the round-trip coherence length \( l_c \) is half the coherence length of the light source [56], [109]. The superluminescent light emitting diode (SLED) used in this work (DL-CS3152A) has a central wavelength of 1300 nm and a spectral bandwidth of 30 nm, which by Eq. 3.2, gives a best expected axial resolution of 25 \( \mu \)m.

The Gaussian envelope for TD-OCT is given by

\[ G(\Delta l) = \exp \left[ - \left( \frac{2 \sqrt{\ln 2} \Delta l}{l_c} \right)^2 \right]. \]  

(3.3)

For MR-OCT, Eq. 3.3 needs to be extended to accommodate the higher orders of reflections with increasing scanning range. Consequently, we obtain multiple Gaussian envelopes depending on the order \( m_o \) described by

\[ G(\Delta z, m_o) = \exp \left[ - \left( \frac{2 \sqrt{\ln 2} m_o \Delta z}{l_c} \right)^2 \right]. \]  

(3.4)

By digital processing using a calibration line (see section 4.3.4) Eq. 3.4 does not require to including the time delay for each order due to the spacing between the PM and the SRM or the delay due to the thickness of the PM which cannot be measured accurately. It is sufficient to band-pass filter the sum signal of interference signals using a center frequency according to the Doppler frequency \( f_D \) for each order with

\[ f_D(m_o) = \frac{2 m_o v_{SRM}}{\lambda_0}, \]  

(3.5)

where \( v_{SRM} \) is the linear scanning velocity of the SRM and \( \lambda_0 \) is the center wavelength of the light source. The bandwidth should be at least the bandwidth corresponding to the spectral width of the light source but possibly needs to be somewhat larger to account for dispersion effects.
The depth location for each order is found by merging each filtered order into a digital array that describes the final A-line using the calibration line to determine the required shift in depth for accurate reconstruction.

We can describe the apparent scanning range during one acquisition cycle with

\[ \Delta l(m_0)_{\text{digitized}} = (m_0 v_{\text{SRM}}) \Delta t \] (3.6)

indicating the capture of increasing scanning ranges due to increasing SRM velocity into one buffer length and acquisition time \( \Delta t \). That means during observation of the acquired signal with an oscilloscope, the actual increase of the scanning range represents itself as squeezed signals, e.g. the Gaussian FWHM of a mirror signal would reduce to FWHM/\( m_0 \). Consequently, after band-pass filtering, higher orders need to be upsampled by the factor \( m_0 \) to reconstruct the true scanning range.

For the noise discussion we want to evaluate the digitized signal and therefore use Eq. 3.6 to describe the Gaussian shape of the MR-OCT interference signal

\[ P_{\text{MRO}} = P_{\text{PM}} + \sum_{m_0 \to \infty} \{ P_r(m_0) + P_s(m_0) + 2 \sqrt{P_r(m_0)P_s(m_0)} G(\Delta z(m_0)_{\text{digitized}}) \cos(m_0 \omega t) \}. \] (3.7)

Note that the initial phase term \( \phi \) has been dropped here since the absolute phase does not contain any information for this discussion, although the absolute phase will accumulate \( m_0 \) more times after reflection from the SRM. It would only be of interest for evaluating the phase difference, which is the same as in TD-OCT.

The power \( P_r(m_0) \) depends on the cumulative reflectivity and can be calculated by summing all orders up to \( m_0 \) (Figure 3.1). To obtain the reflectivity of reflection \( m_0 \), the equation

\[ R_r(m_0) = R_{\text{PM}} + (1 - R_{\text{PM}})^2 \sum_{m_0=1}^{M_0} R_{\text{PM}}^{(m_0-1)} \] (3.8)

is used. Eq. 3.8 confirms that the total reflectivity is unity if all theoretical orders are summed \( m_0 \to \infty \). The properties of the sequence \( R_r(m_0) \) Eq. 3.8 for reflectivities \( m_0 = 1, 2, \ldots, M_0 \) is geometric [110] and the sum term can be rewritten as
such that for $R_{PM} < 1.0$ and $m_o \rightarrow \infty$ the sum $S$ becomes

$$S(m_o \rightarrow \infty) = \frac{1}{1 - R_{PM}}.$$  

Substituting Eq. 3.11 into Eq. 3.8 gives

$$R_r(m_o \rightarrow \infty) = R_{PM} + (1 - R_{PM})^2 \frac{1}{1 - R_{PM}} = 1$$  

and the result of unity means that the sum of all reflections from the SRM and the PM describes an ideal mirror again. The power characteristics can now be graphically visualized as in Figure 3.2 (solid bold line). Although, not part of this investigation Figure 3.2 also shows the characteristics of different splitting ratios of the PM and the power ranges that can be covered vs. depth. Figure 3.4 shows the power characteristics (Figure 3.4).

To some degree, it is possible with MR-OCT to match the power roll-off in the reference arm, relative to the roll-off in a sample (Figure 3.5).

A simplified one-layer tissue model based on Beer’s law can be used to predict the power vs. depth [112], [113]

$$I_c(z) = I_L(1 - R_{sp})e^{-\mu z^2}$$  

where $I_c(z)$ is the fluence rate at depth $z$, $I_L$ is the fluence of the incidence beam, $R_{sp}$ the specular reflection coefficient (Fresnel reflection), and $\mu$ is the attenuation coefficient ($\mu = \mu_a + \mu_s$) based on the absorption coefficient $\mu_a$ and scattering coefficient $\mu_s$. Although the visibility of the OCT signal depends on the refractive index change, the tissue model used here assumes that the attenuation of some ideal reflecting interface is fully described by the value of $\mu$, which accounts for the apparent reflectivity after the light has traveled through the tissue twice by the distance $2z$. The model is hence only valid to calculate the visibility of one reflecting layer at a time, which is fully sufficient to compare the signal vs. depth characteristics with that of
Figure 3.2: The reference arm power of MR-OCT was calculated based on the SLED power of 13 mW with a PM splitting ratio of R/T=80/20 (solid bold line) and OD 1.0 attenuation. The gray area represents the possible powers that can be achieved through the selection of other splitting ratios. The boundaries are plotted for ratios R/T=20/80 and 95/5 (dotted and dot-dash respectively). Two guides for assumed sensitivity levels of 100 dB [56] and 90 dB [111] are shown.

Figure 3.3: Multiple point spread functions from a mirror interface are plotted to obtain the intensity profile measured on the maximum peak of the Gaussian shape. The step-like structure is apparent in the peak amplitudes of each Gaussian (which are plotted more clearly in Figure 3.4). Because only ten orders were processed, the peaks of the last order level off.
MR-OCT (Figure 3.4) which is obtained from the maximum peak values from multiple point spread functions from a mirror at different depth positions (Figure 3.3).

Figure 3.5 shows different signal power roll-offs for different measured $\mu$ values of skin tissue. The selected roll-off characteristics are shown as a bold black line. As was already mentioned, in relation to Figure 3.4 the slope appears to have a large mismatch, which necessitates the ability to adjust the attenuation for higher orders of reflections (see gray range around the bold line indicating reference arm attenuation from OD 0.0 to OD 4.0). As it is of more interest to capture signals from deeper tissue regions it is better to sacrifice a matching slope and gain the ability to adjust the reference arm power for higher orders of reflections, which is best achieved with a splitting ratio of 0.8/0.2. The mismatch of powers for lower orders of reflections may reduce sensitivity for skin layers at shallower depths. But as the power of reflected photons from those depths is larger anyway sacrificing somewhat in sensitivity for those regions appears to be acceptable for imaging. Based on this conceptual understanding, further investigation of different splitting ratios can be discussed, which may become relevant in conjunction with different types of tissue with different attenuation coefficients. The depth range was calculated based on fourteen orders of reflection ($m_o = 14$) and a PM-to-SRM spacing of 100$\mu$m, resulting in a total imaging depth of 1.4 mm. It is observed that for a splitting ratio of the PM with 0.2/0.8 that the imaging depth is limited by the SNR, although the roll-off would be a better
Figure 3.5: The power roll-off of dermal structures vs. depth for μs values according to various literature resources, calculated with an incident power of 7.5 mW. Sensitivity guides are shown for TD-OCT and FD-OCT [56], [111]. The gray area indicates the range, in which, the MR-OCT reference arm power can be controlled with various attenuations from OD 0.0 to 4.0, and is labeled OD range.

match compared to the tissue roll-off (see Figure 3.5). The selected splitting ratio of 0.8/0.2 generates a step-wise power reduction in the reference arm that is, although much less compared to the power reduction slope from the tissue samples, sufficient to match a range of data points of power values by tuning the reference arm with an OD filter (shaded area of Figure 3.5) for optimal sensitivity. Even with a slope mismatch, one can increase the reference arm attenuation to best match it to the power values coming from deeper region of a scattering tissue, while sacrificing some sensitivity of layers close to the surface.

3.1.3 Performance of TD-OCT vs. MR-OCT

The peak amplitude of the interference signal for TD-OCT [56], [111], [124]-[126] is $I_{\text{peak,SD}} = \rho^2 \sqrt{P_r P_s}$, where $\rho$ is the detector responsivity, $P_r$ and $P_s$ are the reference and sample arm powers, respectively. The responsivity is $\rho = \eta q / h \nu_0$ where $\eta$ is the quantum efficiency of the detector, $q_e$ is the electron charge, $h$ is Planck’s constant, and $\nu_0$ is the center frequency of the light source.

The noise components for TD-OCT and MR-OCT are tabulated in Table 3.1 [111]. For the MR-OCT terms of excess and receiver noise additional scaling terms $\alpha$, $\beta$, and $\nu$ are applied to
account for higher order effects. The parameter $\alpha$ describes the impact of the reflectivity $R_r$, $\beta$ describes the impact of the order of reflection and $\nu$ describes a general noise factor.

The power of the superluminescent diode (SLED) is $P_0$, the degree of polarization is $\Pi$, and the effective optical linewidth [56] is $\Delta \nu_{\text{eff}} = 1.5 \Delta \nu$ with a full width at half maximum (FWHM) of the Gaussian spectrum $\Delta \nu$. The detector is mostly characterized by its noise equivalent current $NEC$ and the detector bandwidth $B$. The splitting ratio of the beam splitter of the Michelson interferometer is described [111] by $\gamma_r + \gamma_s = 1$, whereas for this investigation $\gamma_r = \gamma_s = 0.5$ was used. Furthermore, the reflectivity in the reference and sample arm is expressed as $R_r = R_s = 1$, whereas for MR-OCT the reflectivity in the reference arm is $R_{PM} + R_r(m_o = \infty) = R_r = 1$, and $R_r(m_o)$ corresponds to the reflectivity for a particular reflection of order $m_o$ (see section 3.1.2 MR-OCT theory). The model also denotes $R_r(m_o)$ to indicate that only a fraction of the power depending on the refractive index change contributes to the formation of interference if the waves from the sample and the reference arm are coherent.

The value for $R_r(m_o)$ cannot be measured directly and is encoded in the spectral power of the interference signal, and it depends on the sample structure. As opposed to TD-OCT the shot and excess noise ($i^2_{\text{shot}}, i^2_{\text{excess}}$) for MR-OCT depend additionally on DC power from the front reflection of the PM with reflectivity $R_{PM} = 0.8$. The receiver noise $i^2_{\text{receiver}}$ is independent of the reference arm power for TD-OCT. For MR-OCT, the receiver noise becomes dependent on the sum of partial reflections from each other order. That means only one partial reflection at a time is contributing to interference while in this instance all other reflections are merely optical DC. This DC level changes depending on the splitting ratio of the PM and the attenuation in the reference arm resulting in a more complex contribution of the receiver noise. However, the measurements have shown that possibly other higher order effects dictate the receiver noise characteristics, which were modeled using scaling factors, such as $\nu$, $\alpha$, and $\beta$. The scaling
factor $\upsilon$ describes a general noise factor, which may compensate any other losses of power or gain not accounted for otherwise. The factor $\alpha$ describes a linear gain or attenuation factor per order and the factor $\beta$ describes the nonlinear impact per order. With all the noise components in place the peak SNR with a single detector (SD) is given as [56]

$$\text{SNR}(m_o)_{\text{SD}} = \frac{4 \rho^2 P_r(m_o) P_s(m_o)}{\langle i(m_o)^2_{\text{sh}} \rangle + \langle i(m_o)^2_{\text{ex}} \rangle + \langle i(m_o)^2_{\text{re}} \rangle},$$

while for balanced detection mode the SNR is twice the SNR of single detection mode [124]. The doubling of the SNR is plausible considering the increase of the signal is squared and the noise increases only by a factor of two.

The sensitivity is defined as the ratio of the reflectivity of a perfect mirror ($R = 1$) to the minimum detectable sample reflectivity ($R_{s,\text{min}}$) i.e. $\Sigma = 1/R_{s,\text{min}}$. Since the sample reflectivity is very much weaker than the reflectivity of the reference mirror (i.e. $R_s \ll R_r$), the sensitivity can be written as

$$\Sigma(m_o)_{\text{SD}} = \frac{4 \rho^2 P_r(m_o)}{\langle i(m_o)^2_{\text{sh}} \rangle + \langle i(m_o)^2_{\text{ex}} \rangle + \langle i(m_o)^2_{\text{re}} \rangle}.$$

To measure the sensitivity the sample arm power could be attenuated until the interference signal level matches the noise level which would correspond to the condition of SNR=1. However, the complete attenuation of the sample arm power would require constant adjustment of the reference arm power. Furthermore, it becomes more and more difficult to determine the residual signal intensity accurately at low signal levels. Therefore, it is more reliable to measure the sensitivity with a constant sample arm attenuation that assures sufficient signal visibility. The attenuation provided by an OD2 neutral density filter, which provides a round trip attenuation of 40 dB is an often used value and is used here to be able to compare measurements to values stated in the literature [56], [111]. Consequently, the sensitivity of the system is $\text{SNR}_{\text{attenuated}} + 40$ dB.

### 3.1.4 Optical setup and measurement of sensitivity

#### System parameters

For the noise characterization, two different setups were prepared to measure sensitivity with a single detector (Figure 3.6a) and with balanced detection (Figure 3.6b).

The detectors used were Thorlabs DET10C in conjunction with a trans-impedance ampli-
Figure 3.6: Two configurations of MR-OCT were used; namely (a) single detection MR-OCT and (b) balanced detection MR-OCT. The components used included SLED: superluminescent diode, OF: optical fiber, CM: collimator lens, BS: non-polarizing beam splitter, PBS: polarizing BS, CP: compensator plate, TM: turning mirror, DL: detector lens, D: detector, SL: sample arm lens, RL: reference arm lens, SRM: reference axially scanning mirror, PM: partial mirror, (S/R)AA: sample / reference arm attenuator, POL: polarizer, HWP: half wave plate, QWP: quarter wave plate.

The quantization noise of the selected digitizer (Picoscope 4824) is determined by the bit width (Q = 12 bit). Using the relation for the quantization SNR (QSNR = 20 log_{10}(2^Q)) the digitizer would have limited the imaging SNR to 72 dB. With a first order Doppler frequency which was measured on the frequency spectrum with about 60 kHz and a digitizer sampling frequency of 20 MHz, the first three orders were oversampled by more than one hundred times, which...
Figure 3.7: Digitizer QSNR limitation vs. oversampling factor vs. order of reflection. Due to oversampling the signal SNR can improve somewhat due to averaging effects according to $bit = \log(O)/\log(4)$ where $O$ is the oversampling factor [127]. The position of nine orders of reflections are indicated for the balanced detection limit (12 bit (Picoscope,BD)).

Theoretically improves the achievable imaging SNR to more than 90 dB for the first five orders of reflection (Figure 3.7). This is consistent with the results shown for the sensitivity characteristics.

Figure 3.7 confirms that, for systems with expected SNR values above 100 dB, a digitizer with suitable bit-width is essential. It is perhaps not usually of any concern to rely on some advantages by oversampling but it may be interesting to note that for spectral domain OCT (SD-OCT) the oversampling factor is determined by the number of pixels of the line camera vs. frequency components. Any deeper layer will generate a spectral content with higher frequency and advantages gradually diminish. The Nyquist limit for SD-OCT is given as $N_{px}/2$ [111] based on the number of detector pixels $N_{px}$.

Although, we can assume that the signal is naturally bandwidth limited due to losses during its chirped representation, we estimate the bandwidth for the noise and sensitivity model to be 60 kHz based on the first order Doppler frequency assuming each order has the full bandwidth between the center frequencies.

Sensitivity characteristics for a single detector

Figure 3.8 shows the expected sensitivity characteristics for a single detector (DET10C) in
TD-OCT. To model the sensitivity for TD-OCT the equations for MR-OCT in Table 3.1 were used and the parameters were adjusted such that they zero out all the additional components. The measured sensitivity for a single detector in TD-OCT follows the model reasonably well, using the fitting parameters $\beta_1 = 0$ ($\alpha_1$ and $\alpha_{11}$ zeroed out), $v_1 = 1.5 \times 10^{-2}$, $\alpha_2 = 1.0$, $\beta_2 = 0$, and $v_2 = 0.0$. The factor $v_1$ required some adjustments to account for some power loss to match the expected receiver noise.

Figure 3.9 shows the sensitivity characteristics for a single detector MR-OCT system and three orders of reflections ($m_o = 2, 6, \text{ and } 10$). The first order is not included in the measurements due to the difficulties in measuring the SNR for $m_o = 1$. In any case, the first order is not of significant interest for imaging due to its shallow scanning range and close position to the sample surface or even above. The model parameters used were $\alpha_1 = -1.0$, $\alpha_{11} = 1.8$, $\beta_1 = 8 \times 10^{-3}$, $v_1 = 1.8 \times 10^{-2}$, $\alpha_2 = 1.4$, $\beta_2 = 0.6$, and $v_2 = 4.0$.

**Sensitivity characteristics for a dual balanced detector**

Figure 3.10 shows the sensitivity characteristics of a dual, balanced detector (New Focus 2117) MR-OCT mode. The model parameters used were $\alpha_1 = -1.6$, $\alpha_{11} = 0.7$, $\beta_1 = 3 \times 10^{-2}$, $v_1 = 5 \times 10^{-1}$, $\alpha_2 = 0.8$, $\beta_2 = 0.5$, and $v_2 = 3 \times 10^{-2}$. The model does not include effects of the efficiency of the balanced detector, which can reduce the gain advantages as soon as the detectors have slightly different DC offsets. It appears that at reference arm reflectivities less
Figure 3.9: The measured and modeled sensitivity characteristics for a single detector (DET10C) MR-OCT signal.

Figure 3.10: The measured and modeled sensitivity characteristics for a balanced detector (NF2117) and MR-OCT mode. Please note that the model was weighted towards signal points with larger sensitivity values and does not well account for the characteristics of values close to 80 dB.
than 0.1 that the receiver noise characteristics experience a reduction in slope. The maximum
sensitivity of about $R_r = 0.85$ appears to be surpassed for the second order and reaches values
close to 100 dB. However, the sensitivity of higher order of reflections is more critical for depth
visibility which does not reproduce such sensitivity improvements relative to the maximum peak,
meaning that the higher values shown for the second order may more likely relate to increased
variance of the measured sensitivity which could be caused due to increasing dominant beat
noise [128].

### 3.1.5 Discussion

Table 3.2 compares the model parameters used that allow evaluation of contributions of higher
order effects for which fundamental principles have not been accounted. The parameters $\alpha_1$

<table>
<thead>
<tr>
<th>$v_1$</th>
<th>$\alpha_1$</th>
<th>$\alpha_{11}$</th>
<th>$\beta_1$</th>
<th>$v_2$</th>
<th>$\alpha_2$</th>
<th>$\beta_2$</th>
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<tr>
<td>DET10C, TD</td>
<td>0.015</td>
<td>NA</td>
<td>NA</td>
<td>0.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>DET10C, MR</td>
<td>0.018</td>
<td>-1.0</td>
<td>1.8</td>
<td>0.008</td>
<td>4.0</td>
<td>1.4</td>
</tr>
<tr>
<td>NF2117, MR</td>
<td>0.5</td>
<td>-1.6</td>
<td>0.7</td>
<td>0.03</td>
<td>0.03</td>
<td>0.8</td>
</tr>
</tbody>
</table>

and $\alpha_2$ control the exponential power of the reflectivity for a particular order for the receiver
and excess noise respectively, and $\alpha_{11}$ controls the exponential power of the linear gain of the
reflectivity for the receiver noise per order. The effect of the $\alpha$-parameters is visually represented
in the increased spacing of the sensitivity levels vs order. Another parameter $\beta_1$ and $\beta_2$ controls
the slope of the receiver and excess noise.

For TD-OCT (DET10C, TD) the parameters have no impact or a simple unity relationship
confirming the predicted model characteristics of sensitivity. One correction for TD-OCT was
required for $v_1$, which controls the impact of the receiver noise linearly. The deviation is most
likely related to the difficulties in correctly predicting the noise contribution of the combination
of detector and amplifier, and depends on bandwidth, gain settings, and the beam power.
Nevertheless, even the balanced detector (NF2117) required some reduction of its noise as
well. The $v_2$ parameter controls a linear correction of the excess noise, which increases for the
MR-OCT mode (DET10C, MR). This increase of excess noise could be caused by self-beating
of the interference signals from adjacent orders. Interestingly the effect of excess noise with
balanced detection (NF2117, BD) is reduced which makes sense if the beat noise can be rejected.
3.1.6 Conclusion

The sensitivity characteristics of TD-OCT were confirmed and shown to agree with the model as demonstrated in the literature [56], [111]. A second model for MR-OCT is proposed and the characteristics of the reference arm power vs. the sample power of different tissue types of human skin were compared. The modeled characteristics showed that tuning the spacing between the partial and scanning reference mirror can provide a variable power balance, which is not available in other OCT systems. Comparing the MR-OCT sensitivity characteristics to the measurements showed that the sensitivity reduces faster vs. the change of the reference arm reflectivity from the optimal balanced condition. The faster sensitivity roll-off may reduce the dynamic range.

Furthermore it was shown that by using a balanced detection method, the sensitivity values for the second order of up to 95 dB or better and for the tenth order more than 85 dB can be achieved. The excess noise reduced for the balanced detection by about two orders of magnitude and the receiver noise by about one order. It was also shown that, for MR-OCT, oversampling could improve the SNR related to the quantization limit and in particular for higher orders it may be significant to choose a digitizer with a larger bit-width.

Other aspects of MR-OCT is the capability to tune or modulate the PM vs. SRM spacing and the potential use of the time-delay of the propagation of each order of reflection at the speed of light. One can compare the spatiotemporal characteristics of overlapping orders that originate from multiple scanned regions within the sample, especially if the PM-SRM spacing is reduced. Considering that each order is offset by about $D = 100 \mu m$ depending on the adjusted spacing between the PM and SRM, the time delay $\Delta t$ between each order would be $\Delta t = D/c = 3.34 \times 10^{-13} s$ or 0.334 fs. This would allow an instantaneous resolution of the displacement of a reflecting surface due to oscillation with a frequency of up to $1.50$ THz assuming Nyquist limit ($1/(2 \times 0.334 \times 10^{-13} s) = 1.50 \times 10^{-12} \text{ Hz}$) if the spatial amplitude can be captured by at least two overlapping orders of interference. The displacement is merely detected by the phase shift between two adjacent orders and the phase sensitivity can be used to further resolve smaller amplitudes or displacements due to lower frequencies. But also increasing the PM-SRM spacing and the scan range while lowering the scanning frequency would allow to detect lower frequencies whereas increasing overlapping orders would be equivalent to sampling points in time.


3.2 Optical ray tracing of MR-OCT

3.2.1 Introduction

The generation of the interference was simulated with Zemax for the MI and MR-OCT.

Please take note that the lens arrangement differs from the conventional arrangement as such that only one lens (L1) in front of the light source was used as opposed to two lenses each for the reference and sample arm (see Figure 3.11). The single lens arrangement was chosen as it is not only easier to simulate but would allow reducing material costs for a manufactured system. Beyond that, a single lens assures automatically that the reference and sample arms have path-length mismatch due to manufacturing differences between two separate lenses even with the same parameters.

![Figure 3.11: Schematic of the optical system simulated: (S) light source, (L1,2,3) lenses, (BS) beam splitter, (PM) partial mirror, (SRM) scanning reference mirror, (D) detector, (CP) compensation plate, (SM) sample mirror.](image)

The simulation of MR-OCT with lenses of 5 mm diameter shows that it is suitable to build a compact system using low-cost components. The propagation of light in the interferometric system was investigated by optical ray-tracing with Zemax (Zemax Europe Ltd.) for lens diameters of 5, 12.7 and 25.4 mm for all lenses. The visibility of interference on the detector
plane is simulated and discussed based on the wavefront error related to the lens diameter and higher order of reflections due to a partial mirror.

To limit the complexity of the optical construction the partial mirror was not simulated directly and instead reproduced by the displacement of the scanning reference mirror (SRM) for each order of reflection. The displacement of the SRM was arranged in steps with a width equal to the distance between a real PM and the SRM (Figure 3.1 and Figure 2.8). Five different orders of reflections are discussed and compared in Table 3.7.

The described optical construction of the simulated optical system uses a single doublet lens on the input aperture, which allows to use a lens with a larger focal length, reducing angular divergence. The overall construction is described in Table 3.3 which shows an extract of the lens editor of Zemax and could be used to reproduce the model.

### 3.2.2 Zemax construction

Zemax allows two different simulation modes to construct a system; the sequential-component mode (SC) and the non-sequential component (NSC) mode. The interferometer was constructed in SC mode to allow the use of additional analytic functions, which were at the time with the available Zemax version and license not available in NSC mode, such as the evaluation of the point spread function and ray tracing the polarization states. Otherwise the NSC mode would be preferable as it would have accounted for other aspects such as scattering and internal reflections. To get an initial characterization of the interferometer with a few multiple reflections the SC mode was sufficient.

### 3.2.3 Construction method SC mode

To simulate an interferometer with Zemax in SC mode the two arms of the interferometer are simulated as two separate configurations. The general simulation process with Zemax is along the z-axis only in SC mode. To “break” the z-axis-only limitation a so called coordinate break must be used. As each component can only interact once in SC mode, any simulated reflected ray passing through an interface for a second time, requires a duplicate of the interface at the same position.

The definition of all surfaces is managed in the lens editor in Zemax. Table 3.3 is a technical representation of the content of the lens editor for an optical system with 25.4 mm lens diameters. The table reflects the repeated application of optical components for the prismatic part of
the beam-splitter (P1). Zemax manages two different sets of rays by defining two different configurations, which are shown in Table 3.4.

For the initial test of the simulation the surface $#11$ is set to $100\,\mu\text{m}$ for both configurations. This means for a real optical set-up that the SM-to-CP distance is the depth where the beam is focused into the sample, assuming the sample is placed onto the CP. The SRM-to-PM determines the distance to a higher order reflection. A schematic plot of the construction is shown in Figure 3.12a and a 3D representation in Figure 3.12b.

![2D representation.](image)

![3D representation.](image)

Figure 3.12: Comparing the 2D layout with the 3D representation allows to evaluate the position of the components for the purpose of construction.

The general system performance was evaluated by inspection of the interferogram (Figure 3.13a) and the lateral point spread function (PSF). The Fast Fourier transform (FFT) PSF is shown in Figure 3.13b.
Figure 3.13: General results of interferometric operation of a 25.4 mm system with 3 mm beam diameter.

(a) The interferogram was used to confirm that the simulation generates interference. According to the Zemax manual the interference is limited to a normalized intensity.

(b) The PSF at the detector plane for optimized mirror positions.

According to the description in the Zemax manual the Huygens PSF is calculated in the image plane whereas the FFT PSF is calculated in the pupil space. This is supposed to make the Huygens PSF more robust if coordinate shifts occur and can be used in SC and NSC mode. The FFT PSF is computationally faster but assumes that the pupil aberrations are minimal, among other conditions to achieve valid results. For evaluation of very small differences between wavefronts, the Huygens PSF did provide better sensitivity and was further used for evaluation of the differences on the detector plane and mirrors.

For the optical simulation of the lens diameter with 25.4 mm, achromatic doublet lenses of focal length 100 mm and 30 mm (Thorlabs AC254-100-C and AC254-030-C) and a standard 50:50 beam-splitter cube (Thorlabs BS018) were selected. The partial mirror and related compensation plate were simulated with glass windows (Thorlabs WG11050, glass type N-BK7). Instead of a partial mirror, a glass window is used for simulation. The focal lengths were chosen to correspond to a real system, allowing the assembly in order to be performed on an optical bench with 20 mm distance between lenses and 1 to 5 mm distance between the doublet pair to focus light onto the detector. All components were simulated with anti-reflection (AR) coating, and the reflecting interfaces of the BS prism were defined for 50% transmission.

For the system with 12.7 mm lens diameter, doublet lenses of focal length 50 mm and 25 mm (AC127-050-C and AC127-025-C) and a beam-splitter cube of length 20 mm (BS012) were selected for simulation. The partial mirror and corresponding compensation plate were simulated with a Thorlabs equivalent windows (WG10530-C).

The miniature system with 5 mm lens diameters was simulated with lenses of focal length
of 15 mm and 7.5 mm (AC050-015-C and AC050-008-C) and with a beam-splitter of 5 mm length (BS009). The required glass windows (N-BK7) were scaled to 5 mm diameter and 1 mm thickness.

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Table 3.3: Lens data and position data for optical components for construction of the interferometer in Zemax.

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Table 3.4: Table of configurations to simulate both arms of the interferometer in Zemax.

### 3.2.4 Simulation

Zemax can simulate multiple discrete wavelengths at a time, and the effect for each wavelength simulated separately but does not allow to simulate a continuous emission spectrum such as that of an SLED. It appeared to be not possible at the time with the used version of Zemax to
include the simulation of interference effects in SC mode, which would have allowed to simulate the interference of two beams originating from a beam splitter.

For the simulation with 25.4 and 12.7 mm lens diameters the intensity profiles were assumed to be Gaussian with an entrance beam-diameter of 3 mm and the distance of the entrance pupil was assumed to be 50 mm in front of the first optical component. For the simulation with 5 mm diameter optical components the beam-parameters were varied and also simulated with 1 mm entrance diameter.

The detector plane (D) was simulated to have a diameter of 0.3 mm based on a Newport 2053 detector.

Zemax allows the definition of a merit function to optimize selected system parameters automatically by attempted reduction of the value of the merit function. To calculate the optimal distance for the scanning reference mirror (SRM) and the sample mirror (SM) a merit function was defined to minimize the Gaussian beam-radius on both mirrors, while also keeping the beam-spot size close to the Airy disk diameter. The distance between the SRM and the PM was set to 100 µm and the distance between the CP and the SM was set to 500 µm which would in practice focus into the sample at a depth of 500 µm.

Multiple reflections are simulated separately. In practice, the effect of multiple reflections means an increase in beam-diameter due to beam divergence and a reduction in power due to each reflection at the partial mirror (PM). The divergence can be calculated by a simple geometrical method and it is shown in section 3.2.6 that the divergence is small if the distance between the PM and the SM is small compared to the divergence angle of the beam.

The model was aligned for all systems by setting the distance between CP and the mirrors to 100 µm. This allows the evaluation of the symmetry of the system. The optimized spot size was generally less than the Airy disk diameter and the wave front error and the Huygens PSF are compared in Figure 3.14. The Huygens PSF is sensitive to path-length differences and the variations in relative intensity for the 5 mm system may be caused by minute differences of the placement of the mirrors. The wavefront error for the 5 mm system has increased to an RMS of about 0.05 and a peak-to-valley difference of about 0.16.
Figure 3.14: Comparison of three models simulated for optical components of 25.4 mm, 12.7 mm and 5 mm lens diameter vs. Huygens PSF (a) and wavefront error (b). Zemax shows the wavefront distortions normalized and exaggerated. The peak-to-valley and RMS values are a measure of relative distortion strength.

Figure 3.15: The simulation of the wavefront error improves for a 5 mm system by reducing the beam diameter from 2 to 1 mm.
symmetric and circular, and the spot width is less than the Airy disc diameter. Figure 3.15 shows that the RMS is reduced to 0.0007 waves for a beam diameter of 1 mm.

Zemax allows the estimation of the effects by varying the value of a large set of parameters (geometry, material, positional, etc.) in parallel. The calculation of those parameters is called “tolerancing” and uses a Monte Carlo method to vary the parameters randomly. Although tolerancing was not the primary aim of this investigation a simulation was prepared including both configurations and merit function. Zemax creates a list of components causing the largest effects under a section called “Worst offenders”. According to the list of “Worst offenders” the first lens (L1) has the strongest effect if the radius, refractive index and the thickness is changing, and the beam-splitter causes effects if it is tilted in x or y.

3.2.5 Simulation and Optimization Results

The Airy disk describes the theoretical minimum spot size possible that can be achieved by a diffraction limited optical system. The diffraction limit is given as $\sin \theta = \frac{1.22 \lambda}{d}$ where $d$ is the lens aperture, $\lambda$ the wavelength of the light used, and $\sin \theta$ the solid angle. For small solid angles the approximation $y/D = \tan \theta \approx \sin \theta \approx \theta$ is valid describing the radius $y$ of the Airy at a distance $D$ from the input aperture. For OCT systems, the visibility of the interference depends not only on the lateral resolution but also on the efficiency to superimpose the wavefronts from the sample and the reference arm. The Airy disk describes the best possible lateral resolution and Table 3.5 shows different spot sizes for different beam diameters and focal lengths $L_n/L_3$. To achieve maximum depth of field the Rayleigh range of a lens requires to use a small beam diameter vs. a large lens diameter. Consequently, the visibility of interference also depends on the depth of the reflected light from the sample and must be optimized by the lateral spot size and axial beam waist. Other aspects that affect the visibility of interference is the injected power per unit volume into the sample, which depends on the beam profile and the scattering probability of the sample material, which again is to some degree determined by the lens and beam parameters.

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<th>5.0</th>
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Table 3.5: Change of Airy disk diameter on the image plane (detector) depending on the beam diameter and the lens parameters (diameter and combination of focal lengths for lens L3 and all other lenses $L_n$).
Although, a reduced Airy disk means a better lateral resolution the Rayleigh range and the Gaussian beam-width reduce as well (Table 3.6). The Rayleigh range describes roughly the depth of field for an optical system which is also important for OCT to achieve optimal resolution along the imaging depth. On the other hand the Gaussian beam waist describes the diameter where most of the beam energy or intensity is allocated. A reduction of intensity would mean a reduced number of reflected photons are available to produce interference and reduces the detectability of structures in a sample.

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<tr>
<th>Input parameter</th>
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Table 3.6: Paraxial Gaussian Beam parameters calculated by Zemax depending on beam diameter and lens parameters (diameter and combination of focal lengths for lens L3 and all other lenses Ln).

Table 3.6 shows that a beam-waist radius and the Rayleigh-range increase if the beam diameter is reduced from 3 to 1 mm. The relative intensity on the detector for different lens diameters remains approximately constant and the simulation shows changes of a maximum of 2% (Table 3.6).

It appears to be plausible that with constant beam-diameter the intensity should stay constant. The reduction of the thickness of components should reduce absorption, however scatter effects and dispersion may have a stronger effect and increasingly attenuate the beam intensity.

### 3.2.6 Multiple reflections

The beam is focused on the SRM (see section 2.8.3) and the spot size will increase depending on how many times the beam will be reflected between the PM and the SRM that are spaced at a distance $D$. Each new reflection (order) can be simulated by placing an additional mirror at a distance (integer multiple $D$) away from the PM.

The simulation of multiple reflections with Zemax requires the placement of a new surface for each reflection and the introduction of a negative thickness of lenses and distances to assure that the beam propagation continues. However, the additional information gained are only the increased spot size due to the angular divergence depending on the incidence angle of the input ray (Figure 2.8).
Any additional reflection can be easily investigated with Zemax by increasing the distance by a value $D$.

After the 5 mm optical system was optimized for the position of all components the distance from lens L3 to the detector plane was 5.2402 mm (beam-width 1 mm).

For displacements of the detector of up to $\pm 200 \mu m$ the geometrical calculated divergence of rays matched the Airy disk diameter (Figure 3.16).

![Figure 3.16: The spot size diagrams show the simulation of the displacement of the detector plane. The detector plane displacement creates the same effect as a mirror reflection. Consequently, all spots can be related to one order of reflection. For a range over 400 µm the spot size is equal or smaller than the Airy disk suggesting an optimal focus for four orders.](image)

The aberration of the wavefront due to the detector displacement is shown for a range of $\pm 80 \mu m$ (Figure 3.17). Zemax shows the aberration metrics as peak-to-valley values with 0.1347 of the wavelength, which appears to be acceptable for sufficient visibility of interference, as it is less than half the wavelength.

![Figure 3.17: Considering a scanning range from $-80 \mu m$ to $80 \mu m$ the wavefront error in RMS changes from 0.04 to 0.001 times of a wave cycle. The wavefront error of one wave cycle would mean extinction and no visible interference. Therefore, the measured values suggest a minimal or negligible effect and optimal visibility of interference is to be expected.](image)

The simulation shows a slight asymmetry of the wavefront error values, which shows up also in the wavefront error characteristics (Figure 3.18).
Table 3.7: Five orders of reflection with increasing depth from 100 µm up to 500 µm. ($l_R$ distance PM-to-ScM, $l_S$ distance CP-to-SpM, $\Delta F_D$ change of focal point of optimal detector position, RMS wavefront error, $I_H$ relative intensity of Huygens PSF)

<table>
<thead>
<tr>
<th>$l_R$ (mm)</th>
<th>$l_S$ (mm)</th>
<th>$\Delta F_D$ (mm)</th>
<th>RMS (waves)</th>
<th>$I_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.200</td>
<td>0.205</td>
<td>0.055</td>
<td>0.025</td>
<td>0.9</td>
</tr>
<tr>
<td>0.300</td>
<td>0.3015</td>
<td>0.11</td>
<td>0.050</td>
<td>0.85*</td>
</tr>
<tr>
<td>0.400</td>
<td>0.402</td>
<td>0.16</td>
<td>0.074</td>
<td>0.62*</td>
</tr>
<tr>
<td>0.500</td>
<td>0.501</td>
<td>0.22</td>
<td>0.099</td>
<td>0.52</td>
</tr>
</tbody>
</table>

* possibly low accuracy

The simulation values for different orders are shown in Table 3.7. The increased virtual distance for a higher order reflection was simulated by placing the reference mirror at the distance $z_R$ corresponding to the PM-to-SRM distance. As the relative intensity of the Huygens PSF ($I_H$) also reacts on the path-length difference, the distance $z_S$ (CP-to-SM distance) was adjusted, to find the maximum of $I_H$ of a path-length match. Consequently, the values for $z_S$ are slightly different, but this also means that the values for $I_H$ are at best estimates, and the jump between 0.300 and 0.400 mm may be spurious. The increased path-length realizes an image-spot at a deeper layer, which shifts the focus for such a higher order on the detector plane by a distance $\Delta F_D$ and the detector is slightly out of focus. Also the wavefront error RMS increases, but remains below 0.1 wavelengths and sufficient visibility of interference is to be expected.

3.3 The impact of relative intensity noise on the signal in multiple reference optical coherence tomography

3.3.1 Background

The direct integration of a superluminescent light-emitting diode (SLED) is a preferable solution to reduce the form-factor of an MR-OCT system. Such direct integration exposes the light source
to environmental conditions that can increase fluctuations in heat dissipation and vibrations and affect the noise characteristics of the output spectrum. This work describes the impact of relative intensity noise (RIN) on the quality of the interference signal of MR-OCT related to a variety of environmental conditions, such as temperature.

### 3.3.2 Introduction

The relative intensity noise (RIN) of laser diodes and superluminescent diodes (SLED) is the fluctuation of power output relative to their average power. It is known for particular devices that the spectral power density of the RIN can peak at frequencies below the gigahertz range. RIN can have multiple sources that depend on the constructional details of the laser cavity, the carrier injection method, and mode interactions of the light emitting system [129]. For the modern SLEDs used in optical coherence tomography systems and telecommunication the values for RIN may range from -130 to -140 dB or even less [130]. Due to optical feedback and fluctuations of the injection current and temperature, the RIN of an SLED light sources may increase and become dominant in reducing the SNR of the OCT imaging system. The SNR in OCT systems is described by the maximum signal of the interference $S$ for a reflector in the sample arm of the Michelson Interferometer (MI) and the noise $N$. The SNR is then the fractional relation of $S$ over $N$. It is desirable to reduce the effort to stabilize the parameters of an SLED to achieve low-cost systems, such as MR-OCT [3] technology.

### 3.3.3 Theory

The measurement and significance of RIN in OCT and telecommunication is thoroughly described by Shin, Sharma, Tu, et al., Hashemi [130], [131] among others. Both authors use a spectrum analyzer (SA) and a low noise photodetector to analyze the source spectrum directly. The general expression to calculate RIN based on the average spectral power square $(\Delta P)^2$ and the average DC current squared $(P_{avg})^2$ is

$$\text{RIN} = \frac{(\Delta P)^2}{(P_{avg})^2}.$$  \hspace{1cm} (3.16)

The impact of the RIN on the SNR is described by Derickson [132] as

$$\frac{S}{N} = \frac{m^2}{2B \times \text{RIN}}.$$  \hspace{1cm} (3.17)
with \( m \) for the optical modulation index, and \( B \) the noise bandwidth. It should be noted that the spectrum analyzer shows the measured power of the detector directly as \((\Delta P^2)\), in which case any \( dB \) calculations assume a factor \([132]\) of 10.

Other noise components were subtracted as much as possible using the relationship

\[
P_n = P_{\text{laser}} + P_{\text{shot}} + P_{\text{thermal}}
\]

in which \( P_n \) describes the total power measured on the SA and \( P_{\text{laser}} \) is the source noise, \( P_{\text{shot}} \) is the shot noise component, and \( P_{\text{thermal}} \) is the thermal noise component.

### 3.3.4 Setup and methods

![Setup diagram](image)

Figure 3.19: The setup to measure RIN includes the mount for the SLED, current driver, and temperature controller. After the TO-can a collimation lens was positioned and aligned. The collimated beam was coupled into a fiber collimator and attenuated with an fiber attenuator to avoid saturation of the photodetector (New Focus 1554-B). The detector was connected to an electrical spectrum analyzer (ESA, RBW = 3 MHz) and the average DC level was monitored with an digital multimeter.¹

Figure 3.19 shows a setup similar to that from Shin, Sharma, Tu, et al. [130] that was used to determine the RIN for two different SLEDs, from Exalos and Denselight. The Exalos SLED was specified with a center wavelength of about \( \lambda_c = 1310 \text{ nm} \) and a bandwidth of about 68 nm (10 dB from peak). The Denselight SLED was used as a reference light source and specified with a bandwidth of about 130 nm. The center wavelength for the Denselight SLED does not strictly apply as the spectrum was not exactly Gaussian, and the “center of the mass” of the spectral power spectrum was about 1300 nm.

The setup was intended to capture the change of RIN for different injection currents and different temperatures. All RIN measurements where performed without a dedicated optical isolator. The
feedback was minimized by using anti-reflective coating on lenses and tapered optical fiber. As the experiment was mainly concerned with obtaining a baseline estimation about the impact of RIN in a Michelson Interferometer that is using a partial mirror, a larger resolution bandwidth was used.

3.3.5 Results

The emission spectrum of the Exalos SLED for a frequency range from 10 kHz to 5 GHz is shown in Figure 3.21 after subtracting detector and ESA noise. There is no significant peak-like characteristics to observe, which may have multiple reasons, such as the large resolution bandwidth, or the peak frequency is above 5 GHz. Some artifacts which may be caused by unaccounted noise sources can be observed below 0.5 GHz, showing a drop of the spectral power. The SLED was measured in open loop mode and drop of the spectral power for 200 mA compared to 150 mA, may be caused due to unstable operation.

The large errors indicated in the plots in Figure 3.22 do not allow to make assumption for direct
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Figure 3.21: Captured spectrum of the Exalos TO-can SLED for different injection currents.

Figure 3.22: The RIN values for the Exalos TO-can SLED in comparison with the Denselight SLED.

Figure 3.23: Relative SNR values for different order of reflections of the Exalos SLED.
correlation with the SNR values shown in plots Figure 3.23. However, the tendency that can be observed in Figure 3.22a can be an indication that the decrease of the RIN values do improve the SNR, as can be observed in plot Figure 3.23a. Those results would be sufficient for further investigation of the SNR directly in the MI setup. Any direct correlation of RIN and SNR vs. temperature could not be observed. However, it appears to be consistent that better relative SNR values (closer to one) appear at lower temperatures (see Figure 3.23b). The deterioration of the relative SNR appears to be stronger for higher order of reflections, which indicates that with higher temperatures the sensitivity for deeper scan areas will reduce faster.

3.3.6 Discussion

A setup to evaluate the relative change of SNR in an MI that incorporated a partial mirror in correlation with measured RIN values of a SLED light source was presented. The relative SNR was measured for different orders of reflections, temperatures, and injection currents. The results show a tendency of increased (improved) relative SNR for higher injection currents. No correlation was found between RIN and SNR values for the selected temperature range. However, better relative SNR is consistently observed for lower temperatures. All measurements were performed without dedicated optical isolator suggesting that the optical feedback can be controlled sufficiently with a reduced set of optical components.

3.4 Simultaneous en-face scanning with MR-OCT

3.4.1 Introduction

Due to the inherently flexible way to arbitrarily increase the distance between the PM and the SRM it was possible to describe and validate a layered scanning method using the MR-OCT technique. The layered scanning method allows acquiring multiple scans at different depth simultaneously. The technique is validated by imaging a reference sample and a fingerprint in-vivo. The principle of scanning multiple selected layers is shown by imaging a partial fingerprint with $200 \times 200 \times 200$ voxels of $3 \times 3 \times 0.5$ mm size and obtaining an arbitrary number of layers merely by digital processing. A mirror scan frequency of 1 kHz and an A-line rate of 2 kHz was achieved that allowed the acquisition time of 20 s for one volume. The results show the feasibility of the application of layered scanning MR-OCT (lsMR-OCT) that uses a partial mirror in the reference arm of the Michelson interferometer. The reduced scan range required for layer scanning allows even higher scan rates which are limited only by the voice coil design and the mass-spring system, e.g., mirror mass, spring constant, and damping.
This layered scanning method allows the simultaneous formation of *en-face* imaging at regular depth intervals with minimal modification of an MR-OCT system. The advantage of lsMR-OCT is the availability of all interference signals from all depth layers in one sum-signal on one photodetector as opposed to other methods that require multiple detectors [47], [133]. The benefits of lsMR-OCT over MR-OCT are A-line rates of currently 2 kHz and the ability to adjust the spacing between the selected *en-face* layers on demand which has not been shown to be possible in other OCT systems. Such a selected layer imaging method allows obtaining *en-face* images that have an exact position within the sample.

The sensitivity of the lsMR-OCT system is sufficient to extract biometric markers, such as the fingerprint ridges, minutia, and sub-dermal features, such as sweat ducts, which can be used for the improved detection of fingerprint presentation attacks [41].

The lsMR-OCT method was compared with images taken from a commercial swept-source OCT (OCS1300SS, Thorlabs). The images were scanned with equal lateral scan parameters (200x200 A-lines) and demonstrate the validity of the lsMR-OCT method at the same finger tip.

### 3.4.2 Materials and setup

The optical layout of the investigated MR-OCT system to test the lsMR-OCT method is shown in Figure 3.24. The partial mirror (PM) can be attached to a piezo-positioner for easy re-configuration of the system for different scanning protocols requiring different spacings between SRM and PM. To understand the effort required to assemble a low-cost scanning system a self-made system made of a voice coil was constructed. The scanning mirror (SRM) was then attached on top of the leaf spring assembled using a conventional cage lens mount from Thorlabs (Figure 3.25). Due to the low-grade construction, it was expected that the pointing stability of the SRM was not perfect and caused some dispersion effects for the higher order reflections. On the other hand, it can be assumed that it is possible to improve on the pointing of the SRM with minimal effort using better tools and engineering methods. The focal length of lens L1 was determined by the position of the PM. To achieve sufficient lateral resolution it was necessary that the focal length of L2 be less than L1. For the axial scanning system a mirror with a diameter of 5 mm was mounted on a bar spring (Figure 3.25) and actuated by a voice coil. The resonance of the mass-spring system was determined to be 1 kHz and a scan range of the mirror was measured to be 12 µm. Usually, for ordinary TD-OCT, such a small scan range would be of little use while lsMR-OCT uses multiple reflections from the PM to obtain multiple imaging layers to a depth of approximately 800 µm. For other applications, if necessary, this maximum depth can easily be increased. The scanning mirror (SRM) in conjunction with the partial mirror
Figure 3.24: The optical configuration of the lsMR-OCT system. The low coherent light source (SLED, DL-CS3207A) is a super-luminescent diode and fiber coupled over a collimator (CM). The reference arm consists of the scanning mirror (SRM), partial mirror (PM), a 75 mm achromatic lens doublet (L2), and attenuator (A) together with the beam splitter (BS1). The sample arm consists of a sample (e.g. in-vivo finger or mirror), a 30 mm lens doublet (L1), the xy-galvo for lateral scanning, a turning mirror (TM), a detector lens (L3) and the detector (D1, New Focus 2053).

Figure 3.25: Voice coil mounted on a bar-spring made of brass (3 × 0.3 × 40)mm. The resulting resonance frequency was close to 1 kHz.

(PM) does in theory generate an infinite number of path length delays due to the light being reflected multiple times on the PM. Those multiple orders of reflections would mean an infinite number of interference signals for infinite depth layers. The actual number of visible interference signals is limited by the system SNR, the scattering properties of the sample material, and the chosen splitting ratio of the PM. During digital processing, the interference signals are separated by digital filtering and can be further selected depending on the application.

The interference signal for each scan range or depth layer will have a distinct Doppler frequency \( f_D \) according to \( f_D(m_o) = m_o \times f_0 \) where \( m_o \) is the order of reflection, and \( f_0 = 2\bar{v}_M/\lambda_0 \) is
the fundamental interference frequency from the first direct reflection from the SRM with an average linear mirror velocity $\bar{v}_M$ and a center wavelength of the light source $\lambda_0$.

The interference signals are separated by digital filtering and after envelope detection assembled into one A-line buffer. The total reconstructed scan range SR can be calculated \cite{134} with

$$SR = (\Delta z_M/2) \cdot (m_o + 1) + D \cdot (m_o - 1)$$

using the scan range $\Delta z_M$ and the spacing $D$ between the PM and the rest position of the SRM, and a preselected number of orders of reflections $m_o$ to be processed.

The reconstructed signal response for a mirror in the sample arm at different depth positions is shown in Figure 3.26. The light source used was a DL-CS3207A from Denselight with a center wavelength $\lambda_0 = 1310 \text{ nm}$ and a bandwidth of $\Delta \lambda = 65 \text{ nm}$ which gives a full width at half maximum (FWHM) of the PSF of $l_c = 13.5 \mu\text{m}$. The measured FWHM of the PSF was 20 $\mu\text{m}$, which is somewhat larger than the theoretical value but may be attributed to dispersion effects due to the mismatched lenses L1 and L2. The actual PSF at a depth close to the top surface (lower orders) is limited by the scan range of the scanning mirror which is 12 $\mu\text{m}$ (Figure 3.26). The thickness of the scanned layer increases for each order ($D \cdot m_o$). Furthermore, the FWHM of the PSF increases slightly for higher orders due to dispersion effects.

![Profile Mirror signals at 35µm spacing](image)

Figure 3.26: Signal response (PSF) for three different spacings between PM and SRM and each order of reflection. (a) SRM-PM spacing 95 $\mu\text{m}$, (b) SRM-PM spacing 65 $\mu\text{m}$, (c) SRM-PM spacing.

The SNR roll-off for the different spacings is shown in Figure 3.27. The SNR reduces with increasing depth and higher orders, but if a sufficient number of orders are closely spaced (Figure 3.27) the increasing overlap of adjacent orders allows to reduce the roll-off somewhat (see reduction of roll-off in Figure 3.27 for 37 $\mu\text{m}$ spacing at range 400 $\mu\text{m}$ to 500 $\mu\text{m}$). In comparison, a reduced number of orders with larger spacing will provide better SNR at deeper regions.

The theoretical signal for TD-OCT \cite{58} on the detector is given by
Figure 3.27: The plot shows the intensity roll-off for all PM-SRM spacings. Each data point corresponds to one order of reflection. A larger spacing has less number of orders and less roll-off. The SNR of higher orders of reflections is increasingly reduced due to the effect of the splitting ratio of the partial mirror (PM). A partial reduction of the SNR roll-off occurs at 400µm to 500µm when adjacent orders overlap (37µm).

\[ I = I_R + I_S + 2\sqrt{I_R I_S} \cdot G(z) \cdot \cos(\omega t + \phi). \]  

(3.19)

The parameters in Eq. 3.19 are the intensity from the reference and sample arm \( I_R \) and \( I_S \), the Gaussian shape \( G(z) \) with a maximum of the Gaussian response at depth \( z \) with a frequency content \( \cos(\omega t + \phi) \) and angular frequency \( \omega t \) and phase \( \phi \).

For MR-OCT Eq. 3.19 must be modified to

\[ I = I_{PM} + \sum_{m_o} \left[ I_R(m_o) + I_S(m_o) + 2\sqrt{I_R(m_o)I_S(m_o)} \cdot G(m_o) \cdot \cos(\omega(m_o)t + \phi(m_o)) \right]. \]  

(3.20)

The modified equation (Eq. 3.20) includes the total residual reflection from the PM, \( I_{PM} \) and the summing of all generated interference signals. Mathematically an infinite number of frequencies of the interference signals can be calculated whereas in practice the useful interference frequencies are limited due to the optical losses on the PM, scattering in the sample, and other losses in the system.

It is not obvious that multiple reflections contribute with multiple powers originating from the reference arm \( I_R(m_o) \), governed by the transmission \( T \) of the PM which only interfere with a fractional power from the sample \( I_S(m_o) \). That means, that \( I_S(m_o) \) describes the fraction of light that does create interference due to being in coherence with the fractional beam-power \( I_R(m_o) \)
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at depth $z_S$. The reference arm power for each higher order of reflection can be calculated by $T^2(1 - T)^{m_o - 1}$ with $T$ for the transmission ratio of the PM.

The signal characteristics for each higher order of reflection exhibit an increasing reduction of the width of the Gaussian envelope due to the increased scan range “squeezed” into the buffer of the digitizer of constant length and time [3]. This squeezing effect of the Gaussian shape is based on the relative change of the time delay vs. the coherence length and the resulting Gaussian characteristics.

The coherence length $l_c$ can be related to a time delay $\tau$ with Eq. 3.21 [135]. Therefore summing over time elements $d\tau$ of the coherence function $G(\tau(m_o))$ with Gaussian characteristics

$$l_c = c \int_0^\infty |G(\tau(m_o))|^2 \, d\tau,$$

(3.21)

allows to retrieve the coherence length based on the speed of light $c$ (See also section B.3). If $\tau(m_o) = (n_S z_S - z_M(m_o))/c$ is the time delay introduced by the path-length difference between the sample reflector position $n_S z$ and the scan position of the scanning mirror $z_M$ with the refractive index of the sample $n_S$ and the speed of light $c$ then, Eq. 3.21 allows the Gaussian term to be retrieved by considering that the coherence length vs. the change of the time delay $\Delta \tau(m_o) = \tau(m_o) - \tau(m_o)$ which effectively depends on the scan length $\Delta z_M(m_o)$. The scan distance $\Delta z_M$ will occur increased in each mirror reflection due to the Doppler effect and the light spectrum is shifted to shorter wavelengths effectively narrowing the Gaussian shape. The increased mirror scan range can be expressed as $\Delta z_M(m_o) = m_o \cdot \Delta z_M$ or expressed as a time delay difference $\Delta \tau(m_o) = (n_S z_S - \Delta z_M \cdot m_o)$. Consequently, each higher order interference will have a distinct higher Doppler frequency with reduced Gaussian width related to the ordinal number of reflection $m_o$. To recover the actual corresponding axial scan range after digitizing, each order of reflection is corrected by up-sampling the buffer containing the filtered signal by a factor of $m_o$. Furthermore, the position of the Gaussian signal needs to be corrected within the sample buffer by calculating the depth position $z(m_o)$ based on the center of the scan range $\Delta z_M/2$ using $z(m_o) = \Delta z_M/2 + D \cdot (m_o - 1)$. That means each order of reflection delivers an interference pattern at increasing depth intervals and each higher order increases in scan range. The increase of the scan range is a key feature of MR-OCT to allow larger imaging depths with a moderate or small scan range of the scanning mirror. The increasing length of the scan regions causes increased overlaps between adjacent scan ranges, which could be useful for particular signal analysis tasks. The position of the PM can easily be changed if attached to a piezo positioning system and the scan range depends only on the driving voltage of the voice coil.
of the SRM, which would allow easy switching between layer-mode imaging and conventional B-frame imaging.

For the investigated MR-OCT system it was possible to detect up to 30 orders of reflections using a broadband mirror in the sample arm, while up to 20 orders have been found to be useful for imaging, which may vary depending on the adjusted spacing between PM and SRM (see Figure 3.27). The number of orders providing interference is usually less due to dependence on the scattering and absorption properties of the sample. It was shown that it is possible to image up to 1 mm depth into a human finger tip \textit{in-vivo} [134].

\subsection*{3.4.3 Results}

To demonstrate the imaging capability of scanning selected layers a phantom was constructed using a microscope slide, on which a calibration grid is engraved (Thorlabs R1L3S3P, Figure 3.28). For the anticipated application to scan fingerprints it is of interest to show the reproduction of images below a glass window that is compatible with FTIR fingerprint scanners [136]. The phantom, therefore, was prepared with 5 layers of clear scotch tape and a final bottom paper layer to evaluate the sensitivity of structures below a slab of glass.

The tape layers in Figure 3.28 are numbered from 1 to 5. Although, the B-frame visibility (Figure 3.28g) is limited for lsMR-OCT the anticipated imaging mode for \textit{en-face} shows impressive sensitivity comparable to a commercial OCT system (OCS1300-SS) from Thorlabs. Particular material structures (Figure 3.28b,e), such as circular patterns in the tape layers not visible otherwise were detected equally well with the lsMR-OCT system.

A human fingertip was scanned \textit{in-vivo} and the images of the fingerprint structures compared between the same swept-source system and lsMR-OCT (see Figure 3.29 and Figure 3.30). All images have a resolution of $200 \times 200 \times 200$ pixel and $3 \times 3$ mm scan width. The images for the lsMR-OCT are shown for a PM-SRM spacing of $37 \mu m$.

Furthermore, it can be shown that the lsMR-OCT method is a viable tool to identify a partial fingerprint based on the pattern of the sweat ducts (Figure 3.31).

Figure 3.31 shows a cut-out of two regions of which one was scanned with the OCS1300-SS and the second with lsMR-OCT. The scan for the lsMR-OCT was much smaller and used to correlate the position within the larger OCS1300-SS scan.
Figure 3.28: The phantom was composed of a calibration grid slide (R1L3S3P) from Thorlabs and five layers of tape below the surface including a paper layer at the bottom with printed letters (all images 200 × 200 pixel and 3 × 3 mm). The images a), b), and c) show en-face structures from different layers taken with the Telesto II, and images f), g), h) show corresponding en-face images taken with lsMR-OCT. The images g) and i) show a B-frame taken with a Telesto II g) and taken with lsMR-OCT i). A corresponding camera image of the printed letters is shown in h). The top structure a) and d) show the grid and air bubbles in the glass-glue interface. The second layer b) and e) show structures in the tape material, and c) and f) show the bottom layer with printed letters “er”.

3.4.4 Discussion

A new technique, lsMR-OCT, for simultaneous formation of en-face imaging at various depths has been proposed, which in comparison to other similar proposed systems [47], [133] can be constructed with a reduced set of components and provides additional imaging modalities not available otherwise. The lsMR-OCT system was investigated for fingerprint identification applications using a light source with a wavelength of 1310 nm and about 5 mW sample power. The calculated spot diameter was 13 µm and the scan velocity of the spot over a 5 mm distance at 1000 Hz scan rate and 200 A-lines was 25 mm s⁻¹ which provides sufficient maximum permissible exposure (MPE) for skin [137].

The image quality obtained is sufficient to identify the surface ridge structure and compare it with
(a) Fingerprint structure finger-glass interface.  
(b) Sweat duct visibility.  
(c) Subdermal boundary.

Figure 3.29: *En-face* images of a fingertip below glass-window taken with an SS-OCT system (OCS1300-SS) at three different depths.

(a) Fingerprint structure finger-glass interface.  
(b) Sweat duct visibility (normalized, 0.2% pixel saturation).  
(c) Subdermal boundary (histogram equalized and normalized).

Figure 3.30: *En-face* images of a fingertip below glass-window taken with lsMR-OCT at three different depths.

Figure 3.31: Correlating a partial fingerprint between the OCS1300-SS (cyan) scan and a lsMR-OCT scan (black).
the sub-dermal structure including the outline of the locations of the sweat ducts. If the relative geometric position of the sweat ducts is included as a biometric feature another identification layer would be available using lsMR-OCT. Consequently, three biometric markers, such as ridge structure, matching subdermal boundary, and sweat duct pattern would be available to improve biometric identification systems against presentation and spoofing attacks. The system was tested with a residual fingerprint on the glass surface in which case no sweat ducts were visible and the subdermal boundary was missing. The proposed lsMR-OCT combines simple technologies into a novel application system that can help to improve current identification systems with a minimal effort and cost. Existing biometric databases can be used with lsMR-OCT while the enhanced detection of additional biomarkers is an inherent feature of the technology. Upgrading databases with the added biomarkers can even further push the safety level of identification and protection of a wide area of attacks related to fingerprint identification.

Beyond the fingerprint detection applications, the MR-OCT and lsMR-OCT lends itself to production of simple constructed systems at high volumes that can find application in remote monitoring or prescreening of biomarkers which does not require medical grade speed or sensitivity. Some unique features due to the multiple reflections may also be combined with other OCT technologies to create new acquisition and detection mechanisms that have not been explored yet. Further research is envisioned to detect the flow of blood in capillaries of human skin, using different wavelengths of light to detect features of the human eye, and investigating applications for distributed sensing on thin layer materials [138] as primary and secondary markers.
CHAPTER 4

Aspects of MR-OCT signal and image processing

4.1 Introduction

The signal and image processing for MR-OCT requires additional considerations related to filtering and separation of multiple scanning layers and the potential additional information that is not required or not available using other OCT methods. For conventional TD-OCT, it is usually recommendable to limit the bandwidth of the interference signal to match that of the digitizer to avoid noise artifacts. For MR-OCT, the full bandwidth was digitized as the frequency for each interference signal that can cover a range reaching from the fundamental beat frequency based on the scan frequency up to the detectable order of interference. As an example, with a scan frequency of 152 Hz and a scan range of 90 µm the beat-frequency is

\[ f_B = \frac{2v_M}{\lambda_0} = 2.42\pi f_M/\lambda_0 = 2(45 \times 10^{-6} \text{ m})2\pi(152 \text{ s}^{-1})/(1330 \times 10^{-9} \text{ m}) = 64 \text{ kHz}. \]

Consequently, if up to 20 orders of interference are to be processed then the maximum frequency to be digitized will be \(20 \times 64\text{kHz} = 1300\text{kHz}\) or \(1.3\text{MHz}\). However, the actual sampling rate was chosen five to ten times more than the highest expected frequency to assure sufficient sample points for filtering a noisy signal. A sampling rate of 10 MHz was used to digitize the signal but higher sampling frequencies were also used to investigate signal processing methods to use the additional amount of data to analyze the noise content. Because MR-OCT contains multiple frequency bands for each higher order interference, a band-pass filter method must be used to separate each order before performing image processing. In this chapter a particular FFT filter method is investigated which shows better results to suppress artifacts and side-channel suppression.
CHAPTER 4. ASPECTS OF MR-OCT SIGNAL AND IMAGE PROCESSING

compared to conventional elliptic or Chebyshev type filters. Furthermore, the generation of the calibration line is discussed that provides a visual guide for evaluating the change of PSF from a sample mirror at regularly spaced positions. The calibration line is a different method compared to plotting a series of PSF into a 2D plot to evaluate the roll-off of the intensity vs. depth. In general, if each PSF is converted into an intensity projection serving as a single A-line a B-frame of many PSFs can be assembled providing the calibration line.

4.2 Voice Coil parameters

4.2.1 Frequency characteristic and damping

The investigation of the voice coil (VC) based delay line was motivated by the simplicity of a CD/DVD pick-up system (PUH) and to understand advantages and limitations.

The PUH construction demonstrates a miniature optical bench including light source, detectors, lenses, and a beam splitter among others in conjunction with an integrated electromagnetic actuated lens positioning system. It was easy to attach a mirror to the extracted VC and use a sinusoidal current to achieve axial scanning.

Such an oscillatory system is best described as a damped spring-mass system with a driven harmonic motion.

The motion of the mirror $y(t)$ follows a forced harmonic oscillation or driven oscillation [139]

$$m \ddot{y} + c \dot{y} + ky = F(t) = F_0 \cos(\omega_f t + \varphi_f).$$  (4.1)

The driving force $F(t)$ then is in equilibrium with the sum of the forces due to acceleration $\ddot{y}$ of the mass $m$ (mirror, mount, etc.), the force due to damping $c$ and velocity of the mass $\dot{y}$, and the force due to the spring constant $k$ and displacement $y$. The driving force is described by the circular driving frequency $\omega_f$ at time $t$ and its instantaneous phase $\varphi_f$.

The forced oscillation is preferably underdamped to allow oscillation with sufficiently high scanning frequencies for axial scanning of the optical delay in the reference arm, which can reach a maximum at resonance.

The solution [139] of Eq. 4.1 for $y(t)$ is given as
\begin{equation}
\begin{aligned}
y(t) &= A_h e^{-c^2 m t} \sin(\omega' t + \varphi_h) + A \cos(\omega_d t + \varphi_d) . \\
&= \text{transient} \quad \text{steady state}
\end{aligned}
\tag{4.2}
\end{equation}

Eq. 4.2 can be simplified for a steady state describing the oscillation for time going towards infinity \((t \to \infty)\). Due to the vanishing exponential term Eq. 4.2 can be rewritten as

\begin{equation}
\begin{aligned}
y(t) &= \sin(\omega t + \varphi) = \sin(\omega_d t + \varphi_d') \\
&= A sin(\omega t + \varphi) = A sin(\omega_d t + \varphi_d')
\end{aligned}
\tag{4.3}
\end{equation}

However, the transient term converges usually quickly to zero due to the exponential term \(e^{-c^2 m t}\). If the mirror mass \(m\) is relatively small even with a small damping \(c\) the steady state is reached after a short time \(t\) which is usually in the range of a few seconds. After steady state is dominant the the oscillation frequency is determined by the driving frequency. It is important to have some knowledge about the oscillation characteristics of the scanning mirror to avoid needing feedback to monitor mirror position vs. time.

Other parameters of interest may be to estimate the maximum amplitude based on the driving frequency \(\omega_d\) with

\begin{equation}
A = \frac{F_0/m}{\sqrt{[\omega_0^2 - \omega_d^2]^2 + \frac{c^2 m \omega_d^2}{m}}}.
\tag{4.4}
\end{equation}

The mass \(m\) is perhaps the easier part to measure, while the driving force \(F_0\) depends on the construction of the VC, which includes the strength of the magnet, coil geometry, number of turns of wire, and the electrical current. Moreover the damping constant or ratio \(c\) will not be well known and must be estimated by other means, but may depend on air pressure, temperature, and humidity. The resonance frequency \(\omega_0\) can be calculated with

\begin{equation}
\omega_0 = \sqrt{\frac{k}{m}}.
\tag{4.5}
\end{equation}

whereas the spring constant \(k\) must be obtained from supplier’s specifications or must be determined experimentally. If one is able to measure the phase shift between the driving signal and the response of the scanning mirror one could extract the spring constant from

\begin{equation}
\varphi_d' = \tan^{-1} \left[ \frac{c \omega_d}{k - m \omega_d^2} \right] = \varphi_d.
\tag{4.6}
\end{equation}
4.2.2 Maximum amplitude of scanning mirror

The maximum amplitude of the scanning mirror can be approximated using a simple mass-spring system in which case it is assumed that the mirror mass behaves like a point mass and damping effects are negligible. The energy stored in a mass-spring system is described with Eq. 4.7 based on the kinetic energy (KE) of the mass and potential energy (PE) stored in the spring with a spring constant \( k \). In theory, it is possible to operate such a mass-spring system at resonance to maximize the scanning range. However, at resonance the acting forces on the mounts and the spring are large and anisotropic material properties quickly become significant causing reduced control of the repeatability of position and spatial stability of the mass. The spatial stability is directly linked to the ability to control the normal vector of the mirror plane or pointing stability.

\[
E = KE + PE = \frac{1}{2} kA^2
\]  

(4.7)

This equation (Eq. 4.7) shows the potential limits of the maximum achievable amplitude. For instance if it is desired to double the amplitude then at least four times the initial energy is required.

Considering that the power over time is proportional to the electrical energy, the amount of maximum current can be predicted, by further including the diameter of the wire in the coil, the strength of the magnetic field based on the coil area, and how much the magnetic core is interacting with the electromagnetic field of the coil. Subsequently, it also becomes clear, that the maximum energy that can be transferred into the mass (mirror and mount) is limited by the maximum current the wire can transport and the distance of the magnetic core relative to the coil. Although, for particular constructions the voice coil is designed to provide a magnetic core over the length of the motion required \( y(t) \), with increased scan range the acceleration \( \vec{a} \) will increase by

\[
\vec{a} = -\omega^2 A \sin(\omega t + \varphi) = -\omega^2 y(t).
\]  

(4.8)

Including the relation of the force \( F \) vs. acceleration and amplitude

\[
\vec{F} = m \cdot \vec{a} = kA
\]  

(4.9)
it can be predicted that the amplitude will be limited due to the force available.

In conclusion, the frequency and the scan range of a VC are mainly determined by its design specifications. If the VC can be driven in resonance the amplitude of the oscillation becomes maximal. However, if operating the VC at resonance more care needs to be taken to assure sufficient pointing stability of the SRM, meaning that the normal of the mirror plane remains axially aligned over the full scan range.

4.3 FFT band-pass filter with Gaussian window

4.3.1 Theory of MR-OCT frequency bands

Based on the theory in section 2.7.3 the scanning range physically increases as $\Delta z_{m_o} = m_o \Delta z$ where $\Delta z$ corresponds to the conventional scanning range of a TD-OCT. The increased scanning range requires an increase of the virtual scanning velocity as $v_{m_o} = m_o v$ which further causes higher order interference frequencies $\nu_{m_o} = m_o \nu$ (see Figure 4.1).

The detector effectively detects the sum of all interference signals which are further digitally processed to reconstruct an A-line for each scan of the SRM. The digital processing includes the removal of DC, phase linearization, windowing, band-pass filtering, the Hilbert transform, spatial correction, and A-line reconstruction (see Figure 4.11).

The removal of the DC is required to get an accurate representation of the Gaussian envelope from the interference signal, while phase linearization is required to remove frequency dispersion and efficient separation of the different interference signals based on their beat frequency. To avoid artifacts due to the signal cut-off on the buffer boundaries of the signal a Tukey window is applied before filtering. For band-pass filtering different filter types can be used to separate the different interference signals. The elliptic filter was chosen due to its excellent transition rate from stop to pass-band, while the Chebyshev type filter is known for their low ripple characteristic. However, as it was theorized the mismatch of the filter window might cause artifacts that may be difficult to remove completely. Therefore filtering the signal using an FFT operation in conjunction with a Gaussian window in the frequency domain is investigated.

Depending on the desired number of orders of interference signals to be processed the number of frequency bands of the band-pass filter can be chosen, whereas only up to twenty orders have been shown to be detectable. Subsequently, each order is then stored in a separate buffer segment (Figure 4.12 and Figure 4.13). Because each higher order represents an increased scanning range detected in the same time frame it must be spatially corrected which is simply
achieved by upsampling the related buffer segments. After spatial correction, the buffer segments are assembled into one A-line by either adjoining or summing (see Figure 4.13 in section 4.4).

Because the interference frequency based on a linear mirror speed is \( \nu = 2v_{\text{SRM}}/\lambda \) [56], the frequency orders for MR-OCT can be calculated with \( \nu_{m_o} = 2m_o v_{\text{SRM}}/\lambda \), with \( v_{\text{SRM}} = \{v_0, v_1, \ldots, v_{M_o}\} \), and \( M_o \) the last order to be processed.

![Figure 4.1](image) Principle of MR-OCT showing the relation of velocity of the virtual scanning mirrors \( v \) and scanning ranges \( \Delta z \). The components are the Michelson interferometer (MI) with light source (SLED), photo diode (PD), beam splitter (BS), partial mirror (PM), and scanning mirror (SRM), blank.

### 4.3.2 Theory of FFT filter with Gaussian window

The signal parameters are dictated by the wavelength \( \lambda_0 \) and bandwidth \( \Delta \lambda \) of the light source and the average linear scan speed \( v_{\text{ref}} \) of the scanning mirror in the reference arm. Consequently the coherence length \( l_c = 2ln(2)\lambda_0^2/(\pi\Delta \lambda) \) and the Doppler shifted apparent interference frequency \( f_D = 2v_{\text{ref}}/\lambda_0 \) provide the information of center frequency and bandwidth of the Gaussian shaped interference signal [64] on the PD in the MI. The signal is preprocessed by removing the DC and linearizing the phase before filtering to separate the different frequency components. In this case, the frequency components were in a range from 32 kHz up to 448 kHz with a bandwidth of about 1 kHz. In theory, each frequency component carries the Gaussian characteristics of the spectrum of the light source and matching the filter characteristic to the spectrum should result be better suited for an optimal filter performance. The parameters for the filters, such as pass-bandwidth, stop-bandwidth, and filter types relative to the SNR are compared and explained in section 4.3.5.
The ripple attenuation was kept constant at 40 dB, and only the infinite impulse response type filter is considered, as it has reduced constraints compared to finite impulse response type filter. A second-order section type was selected to be able to design the filter for the stringent frequency versus sample-conditions at the low-frequency range.

The FFT filter, on the other hand, is only described by the shape of the window in the frequency domain. The window shape was chosen to match the Gaussian characteristic of the signal. Because the window is Gaussian, it should not produce any ripples in the stop or pass-band, and multiplication with the signal should ideally suppress all adjacent frequency terms to zero. Depending on the machine precision, the Gaussian may not always reach zero values, but it is to be expected that the suppression is still better compared to other filter types. To compare the FFT filter with the two other filters described the bandwidth of the Gaussian shape was measured at levels of -3 dB and -40 dB.

In simple mathematical terms the FFT filter is described by the application of the FFT $\mathcal{F}$ on the signal $S$ multiplied by the Gaussian window $G$ and the inverse FFT $\mathcal{F}^{-1}$ (Eq. 4.10)

$$S_f = \mathcal{F}^{-1}G\mathcal{F}(S),$$  \hspace{1cm} (4.10)

with $S_f$ as the band-pass filtered signal.

The Gaussian shape was aligned with the center frequency, and the width was calculated relative to the signal bandwidth. To calculate the SNR values the mean of the median of all interference signals was taken. Although the noise is preferably measured with the RMS, the median did allow to measure a noise level including the PSF. The estimated difference between the two methods was measured to be negligible (RMS = 30.87, median = 31.01; arbitrary units) or below the expected error of the noise estimation.

We can expect the best filter performance if the filter bandwidth is optimally matched to the signal of interest. The bandwidth of the signal is related to the Gaussian spectrum of the light source [140] and was used as a reference parameter where the highest SNR is expected. If the filter bandwidth is too narrow the loss of signal will reduce the SNR whereas a too large filter bandwidth will increase the noise content and reduce the SNR as well.
4.3.3 Characterization of the MR-OCT signal

To characterize the digital processing (section 4.4.2), such as filtering and merging of the different interference signals into an A-line, a reflection from a mirror is recorded for different depth positions $z_{m_o}$ (Figure 4.2). A single A-line can be described as an array of samples $X_\xi = \{\zeta_0, \zeta_1, \ldots, \zeta_\eta\}$ creating an image matrix $\chi(\xi, \eta)$. Consequently, we get one A-line $X_\xi$ for each position $z_{m_o}$ and expect after processing to find a maximum of the Gaussian response at $\zeta_\eta$ correlating to the position $z_{m_o}$ of the mirror (diagonal line Figure 4.2).

![Figure 4.2: Characterization of processing by acquiring multiple scans at regular spaced sample mirror distances $z_{m_o}$. The position of the maximum of the Gaussian response occurs at the sample $\zeta_{m_o}$ within the A-line array $\chi$. Plotting the image matrix of all A-lines $\chi(\xi, \eta)$ a diagonal line can be reconstructed with bright pixels at $\chi(\xi = m_o, \eta = z_{m_o})$. While the diagonally reconstructed line itself shows the accuracy of the processing to combine the different A-line sections from the different orders of reflection, the log-scaled 2D images of the recombined A-lines can reveal artifacts introduced during digital processing and allow to some extent to evaluate the acquisition quality (see Figure 4.4).]

4.3.4 Band-pass filtering and calibration line reconstruction

The processing steps of band-pass filtering described above (section 4.3.1) can be visualized with Figure 4.3.

Figure 4.3 shows the steps for the band-pass filter process for an elliptic filter Figure 4.3a and a Gaussian filter Figure 4.3b. In this case, the phase response was of no immediate interest and is not shown. Both, the elliptic filter and the Gaussian filter response are represented in the frequency domain together with the signal spectrum for a single reflection (O2) for a second-order signal. The elliptic filter (Figure 4.3a) required different bandwidths and reduction of the transition rate to avoid ringing artifacts (Figure 4.5). The Gaussian FFT filter (Figure 4.3b) required less design consideration and did produce less artifacts. The phantom signal is directly
(a) Elliptic filter. The time-domain plot (1) has linear scale and the frequency plots (2) and (3) have log scale.

(b) Gaussian filter. The time-domain plot (1) has linear scale and the frequency plots (2) and (3) have log scale.

Figure 4.3: The steps of the band-pass filter process for a single reflection. The input signal is windowed (1), the frequency response of the band-pass filter and the signal spectrum (2), and the filtered signal (3). The signal frequency (O2) is a second order interference frequency and a phantom frequency (P) is also visible.

originating from the time-domain signal and occupies the position of the fourth frequency band which makes it difficult to suppress the phantom signal. However, as those signals are difficult to be reproduced it is to expect that they will not be visible in scattering samples.

The related calibration lines are shown in Figure 4.4. Many artifacts are still present for the elliptic filter while for the Gaussian FFT filter most of the artifacts have been removed. It should be noted that the artifacts for the elliptic filter can be further reduced by adjusting its parameters. However, the sensitivity to slight changes from the input signal would require a constant readjustment which was not the case for the Gaussian FFT filter. The signal order (O2) is indicated together with the phantom signal (P) which propagates through all interference orders representing a phantom mirror signal at a wrong position. Further artifacts at the very left are not continuous but present in all higher orders, and it is believed to originate from the interaction of two adjacent orders.

A corresponding collection of point spread functions related to the calibration line shows the increasing ringing artifacts for the elliptic filter (Figure 4.5).
Figure 4.4: MR-OCT calibration line for elliptic filter (a) and Gaussian FFT filter (b). The strong reflection from the sample mirror also caused a large amount of broadband noise which was not fully rejected by the band-pass filter and occurs at the edges of each order as repeating dots. The dots are not visible for weakly scattering reflectors. The digital gain was somewhat increased for deeper regions increasing the noise level visible as a yellow tint.

Figure 4.5: MR-OCT intensity roll-off vs. z-position of a sample mirror in air for an elliptic filter (a) and a Gaussian FFT filter (b). The intensity was normalized to increase the visibility of the noise at deeper regions.
4.3.5 Results and conclusion

To be able to compare multiple different filter types the spectral bandwidth parameters were normalized against the first order bandwidth. That means the first order coherence length in the spatial domain is about 15 µm and if converted into the frequency domain the bandwidth is about 1 kHz. Consequently, a pass bandwidth of 4 would mean about 4 kHz. The best SNR values achieved that also had minimal artifacts were for the Chebyshev 2 85.5 dB, for the elliptic 86.8 dB, and for the Gaussian FFT filter 87.8 dB (Figure 4.6). The characteristics of a test signal with the least artifacts are shown in Figure 4.8. The optimal SNR values are clustered around the pass bandwidth between 2 to 4 times, and a stop bandwidth between 10 to 20 times the signal bandwidth.

![SNR vs stop/pass bandwidth](image)

Figure 4.6: Variation of SNR values versus pass and stop bandwidth in normalized units relative to the signal bandwidth.

In Figure 4.7 and Figure 4.8 the filter results are compared using 400 A-lines for the calibration lines of an sample mirror in air moved at regular intervals of Δz = 5 µm. The results in Figure 4.7 are based on bandwidth settings arbitrarily maximizing the intensity and results in Figure 4.8 use more optimized filter parameters. Nevertheless, even with worst parameter settings, the Gaussian FFT filter (Figure 4.7a) performs better than the elliptic (Figure 4.7b) and the Chebyshev filter (Figure 4.7c). It should also be noted that in Figure 4.7 some artifacts may be mistaken as phantom signals. However, this would not correlate with the distinct occurrence
of a real phantom signal (compare Figure 4.4) and on the other hand, the filter artifacts only occur after filtering was performed and the phantom signal is visible even before filtering. At maximum SNR the Gaussian filter exhibits a substantial broadening of the PSF, while the elliptic and the Chebyshev filter show strong leaking of signals visible as ripples.

Figure 4.7: Calibration line using 400 A-lines showing filter leaking and broadening of the PSF due to suboptimal filter parameters.

Figure 4.8: Calibration line using 400 A-lines showing optimal characteristics. The elliptic and the Chebyshev filter show somewhat more residual leaking artifacts.

The calibration line shows that the image contrast-to-noise ratio (CNR) would be somewhat better using a Chebyshev 2 filter compared to an elliptic filter. However, the Gaussian FFT filter shows a further improvement due to the reduction in side lobes that are still visible as a red shadow for the elliptic and the Chebyshev filter (see Figure 4.8).

If the Gaussian filter is optimally matched to the signal it shows a reduced roll-off (Gauss, no artifacts, Figure 4.9).

Consequently, using a Gaussian window in the frequency domain performs better compared to an elliptic and a Chebyshev type 2 filter. The Gaussian filter showed reduced artifacts and was at least as good or better regarding SNR. The better performance can be attributed to the
Figure 4.9: Roll-off vs. depth compared between the investigated filter types. The spurious increase of intensity after 1.50 mm is caused due to an added gain towards signals for deeper regions to allow better investigation of noise artifacts in those areas. The increase is less visible in Figure 4.8 but present.

<table>
<thead>
<tr>
<th>Filter type</th>
<th>best SNR (dB)</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>elliptic</td>
<td>86.8</td>
<td>leaking artifacts</td>
</tr>
<tr>
<td>elliptic</td>
<td>84.5</td>
<td>minor side lobe</td>
</tr>
<tr>
<td>Chebyshev2</td>
<td>85.5</td>
<td>leaking artifacts</td>
</tr>
<tr>
<td>Chebyshev2</td>
<td>83.0</td>
<td>minor side lobe (less vs. elliptic)</td>
</tr>
<tr>
<td>Gaussian FFT</td>
<td>88.3</td>
<td>widened PSF</td>
</tr>
<tr>
<td>Gaussian FFT</td>
<td>87.8</td>
<td>no artifacts</td>
</tr>
</tbody>
</table>

Table 4.1: The maximum SNR values compared for the investigated filter types.

better matching of the window to the spectral shape of the signal. A disadvantage is that the signal must be available offline to be able to perform the FFT.

It should also be mentioned that no electronic band-pass filters were investigated as it is much more challenging to build such filters and allow sufficient flexibility to change their parameters for research purpose. In general, digital filters are much easier to modify as long as the optimal filter parameters are not known. Furthermore, an analog electronic filter-bank must take care of sufficient shielding to avoid cross talk of the signal and may introduce losses due to parasitic capacitors and inductivities, and can add noise themselves. On the other hand, a well-designed filter-bank would reduce substantially the demand on the digital processing power that can be used for image reconstruction and processing.
4.4 Signal simulation and signal processing for MR-OCT

This section explores the signal generation and simulation from the data processing point of view which does not require the full theory as described in section 2.7.3. Although, it is possible to describe the position of each interference order within the digitizer buffer listing A.3-5 in section A.3 this information is not strictly required to reconstruct an A-line. The theoretical aspects in this chapter have been discussed in detail in section 2.7.3 and here only the essential parameters required for signal processing and image reconstruction are described. A flow chart describes the operational principle of the signal processing for MR-OCT and a visual schematic of how an A-line is assembled to aid the understanding of the image reconstruction.

4.4.1 Introduction

In currently demonstrated MR-OCT systems [61] the PM is designed to reflect 80 percent of the light back to the SRM, while the remaining 20 percent pass through the PM, back into the interferometric system for imaging. The distance between the SRM and the PM can be adjusted and was demonstrated with about 50 µm. Due to the spring-mass system of the actuator of the SRM the movement will be a simple harmonic motion. The light is reflected back and forth multiple times between the PM and the SRM generating a composite reference signal. Each further reflection (order) corresponds to an increased path length in the reference arm which only interferes with light from the sample arm matching the path length. Consequently, there is a systematic increase in the magnitude of the scan range associated with the corresponding order of reflections from the reference arm.

In Figure 4.10a a schematic of the MR-OCT system is shown. A first order reflection will have a path length of the distance between SRM and the beam splitter (BS) interface. For a second order reflection the path length increases by \( m_o \). Any higher order reflection will have a path length increased by \( m_o z \) due to the additional distance the light has to travel between the PM and the SRM. More details about the origin of the multiple reflections and the higher order signals are discussed in section 2.7.3 Figure 2.8.

4.4.2 MR-OCT signal processing

The displacement and position of multiple signals for a single reflecting layer within the digitizer buffer is a combination of the displacement \( \Delta z \), spatial change due to the increased scanning range and the position of the zero crossing of the scanning mirror (see section 2.8.5).
The processing steps can be visualized in Figure 4.12 in conjunction with the processing flow-chart in Figure 4.11. Besides the conventional DC removal, and phase linearization, the MR-OCT signal must be band-pass filtered before any other processing steps. The flow chart (Figure 4.11) indicates that after band-pass filtering multiple buffer segments with all the higher order interference signals are now separately available. After the envelope detection by Hilbert transformation the signal is further upsampled to achieve the proper pixel spacing relative to the first order. That means each higher order is now corresponding to its optical scan range (Figure 4.12) which is \( m_o \) times longer. Furthermore, the zero position of each virtual scanning mirror is displaced by \( D \) and a portion of the scanning range \( \Delta z \). Although, all the correction steps involve the assumed physical and geometrical beam path parameters some additional correctional shift must be added which was usually in the range of 5 to 100 buffer bins.

The shift was assumed to originate from the light passing through the PM and was estimated based on a PM PR1-1550-80-0525 (CVI) with a thickness of 6.35 mm and a refractive index for fused silica at 1330 nm with a value of 1.4466 [141]. The spacing between the PM and the SRM typically used was \( D = 125 \) µm. Then the time delay can be estimated based on the equation for the refractive index \( n = \frac{c}{v} \) using the speed of light in vacuum \( c \) and the speed of light in a material \( v \) with refractive index \( n \). The velocity \( v \) can be related to the travelled distance \( s \) and the time \( t \) as \( v = \frac{s}{t} \). The distance travelled between the SRM and the PM is \( D \) and through the PM is determined by its thickness \( w \). Then the delay is

\[
t = \frac{D}{c} + \frac{nw}{c} = \frac{125 \times 10^{-6} \text{ m}}{300 \times 10^6 \text{ m/s}} + \frac{1.4466 \times 6.35 \times 10^{-3} \text{ m}}{300 \times 10^6 \text{ m/s}} = 3.11 \times 10^{-11} \text{ s}
\]

If this is compared...
to the signal as it is recorded using the digitizing parameters of 120000 sample points, a 20 MHz sample rate, and a signal shift by 50 buffer bins, then the time delay calculates as $t = \frac{50}{120} \times 10^3 / 20 \times 10^6 \, \text{s}^{-1} = 2.1 \times 10^{-11} \, \text{s}$. The exact parameters of the PM were not known as it was supplied by a third party and we may expect some uncertainty related to the specifications used. Therefore the obtained time delays of $3.11 \times 10^{-11}$ s and $2.1 \times 10^{-11}$ s appear to support the assumption that the refractive index of the PM is contributing to the shift of the interference signal.

If all the spatial corrections have been performed the buffer segments are merged into a single A-line (Figure 4.12(a) and Figure 4.13).

The number of depth ranges to be processed can be controlled simply by selecting the number of filter slots. Up to twenty orders of reflections have been shown to produce detectable signals which may, however, also depend on the scattering and absorption characteristics of the sample. Early attempts to test feasibility on scattering samples such as a human finger or skin regions have shown that up to ten layers for imaging are useful [134].
Figure 4.11: Signal processing flow for MR-OCT includes band pass filter with a center frequency \( f_1 \) for each order of reflection \( m_o \) with \( M_o \) for the number of processed orders. The image matrix processing with the function \( g \) performs a correction for the wavenumber \( k \) for each A-line signal \( y \) and assembles a full B-frame \( I \) to display.
Figure 4.12: MR-OCT orders of reflections. An A-line is composed of multiple A-line segments. During acquisition a structure can be scanned multiple times due to the overlap of the scan range $s_{\text{scan}}$. The time-domain signal is the sum of all scan ranges, and the same structure appears displaced in time.

Figure 4.13: Schematic image matrix processing with four orders of reflections color coded as "signal patches". A signal patch is a data array of fixed sample length containing the OCT signal of the corresponding order of reflection. The graph indicates the processing steps to correct for length and placement of the signal patches and finally merging them into an A-line. The spacing between the scanning mirror and the partial mirror is typically larger than the maximum amplitude of the scanning mirror to avoid contact with the partial mirror during scanning. Subsequently the first order patch is shorter than the distance to the second order and a space between the signal patches occurs.
4.4.3 Method

Most of the fundamental theoretical aspects are covered in section 2.7.3 and only the relevant equations are given here.

Considering only the cross-correlation term and ignoring the DC and autocorrelation term the detector current $I$ can be written as:

$$I(k) = \frac{\rho}{2} \left[ S(k) \sum_{m_o=1}^{M_o} \sqrt{R_R R_{S_m_o}} \cos[2k(z_R - z_{S_{m_o}})] \right]. \quad (4.11)$$

The detector current is the sum over all reflecting elements $R$ at a distance $z$ from the beam splitter multiplied by the power spectral dependence of the light source $S(k)$.

The light source is assumed to have a Gaussian-shaped spectrum

$$S(k) = \frac{1}{\Delta k \sqrt{\pi}} \exp\left(-\left[\frac{(k - k_0)}{\Delta k}\right]^2\right) \quad (4.12)$$

with $\Delta k$ corresponding to the bandwidth of the light source,

$$\Delta k = \frac{\pi}{\sqrt{\ln(2)}} \frac{\Delta \lambda}{\lambda^2}. \quad (4.13)$$

Eq. 4.13 can be reshaped by using the full width half maximum (FWHM) of a Gaussian to obtain the coherence length $l_c$:

$$l_c = \frac{2 \sqrt{\ln 2}}{\Delta k} = \frac{2 \ln(2)}{\pi} \frac{\lambda_0^2}{\Delta \lambda}. \quad (4.14)$$

The oscillatory term $\tilde{J}$ of the detector signal for a single reflecting layer is

$$\tilde{J} = \cos[2k(z_R - z_S)] = \cos[2k \Delta z]. \quad (4.15)$$

For only one reflecting layer the summing operation can be omitted which is useful to discuss only the sum of multiple reflections generated by the partial mirror.
The reference mirror is assumed to be moving with a velocity \( v_{SRM} \) and changing the distance \( \Delta z \) which results in the Doppler frequency \[ f_D = 2v_{SRM} \frac{\lambda}{1}. \] (4.16)

The movement of the reference mirror is often stated as linear, which simplifies the mathematical treatment. In this investigation the reference mirror was attached to a voice coil extracted from a CD/DVD pickup system, which will closely follow a harmonic motion due to the mass-spring system. The harmonic motion of the reference scanning mirror is

\[
v_{SRM} = -A\omega_r \cos(\omega_r t + \phi_r),
\]

with \( A \) for the amplitude or the scan range, \( \omega_r \) for the frequency and \( \phi_r \) the phase.

With \( k = \frac{2\pi}{\lambda} \) and the amplitude normalized, the oscillatory term can be written as:

\[
\tilde{J}(t) = \cos[2\pi f_D t].
\] (4.17)

Substituting \( v = \frac{\Delta z}{\Delta t} \) and \( \Delta z \) with \( \Delta z m_o \) in Eq. 4.16 gives the beat frequency \[ f_D(m_o) = 2\frac{\Delta z m_o}{\lambda} = 2v_{Rm} \frac{m_o}{\lambda}; \quad m_o = 1, 2, \ldots, M_o \] (4.18)

where \( \Delta z(m_o) \) is the path delay, \( m_o \) the ordinal number of a higher order of reflection, and \( M_o \) with the maximum number of orders to be processed.

Rewriting the Eq. 4.15 incorporating \( m_o \) path delays:

\[
\tilde{J}(\Delta t) = \sum_{m_o=1}^{M_o} \cos(2\pi f_D(m_o)\Delta t)
\] (4.19)

a sum of multiple frequency components for a single sample reflector can be computed. The different frequency of each component is used to separate the signals by filtering, obtaining a series of signals.

The Gaussian component determines the peak position at \( k_0 \) of the envelope for each frequency component related to wavelength and bandwidth of the light source. For the purpose of synthesizing an idealized signal any dispersion effects are neglected, and the Gaussian envelope related to a reflecting layer can be defined as:
Using Eq. 4.20 a valid signal can be synthesized for the first order reflection (Figure 4.14). For higher order reflections the center position of the Gaussian envelope shifts according to Eq. 4.21. As the scan length increases the FWHM of the Gaussian envelope will reduce in the digitizer buffer by a factor \( m_o \)

\[
S(z, m_o) = \exp \left( - \left[ \frac{(z - (z_0 + \Delta z(m_o - 1)))}{l_c/m_o} \right]^2 \right). 
\]  
(4.21)

The signal depends also on the scan mirror velocity \( v_{SRM} \) which will distort signal. The distortion is synthesized using a look-up function \( K \) which displaces the spatial data accordingly (see also section 5.4.1). Synthesizing a signal can now be performed by combining Eq. 4.20 and 4.21 and creating the sum of all \( m_o \) reflections in Eq. 4.22:

\[
I(t, z) = \sum_{m_o=1}^{M_o} K(v_{SRM}, S(z, m_o) \cdot \tilde{J}(t, m_o)).
\]  
(4.22)

For a SLED from Denselight Semiconductors (DL-CS3207a) a wavelength of \( \lambda_0 = 1310 \) nm and a bandwidth of \( \Delta \lambda = 56 \) nm it was possible to use the mathematical relation Eq. 4.15 to estimate the actual scan range (amplitude of the harmonic motion) of the SRM.

Figure 4.14: Calibrated Gaussian envelope (solid) matching a real signal (dashed). It was possible to extract the scan width by fitting the Gaussian envelope based on the wavelength of the light source with about 70 \( \mu \)m.
4.4.4 Results

For test purposes a CD/DVD pick-up coil was extracted from a decommissioned DVD-drive to excite the scan mirror with a frequency of 400 Hz. The voice coil was driven by a computer generated sinusoidal voltage via a preamplifier. The frequency stability of the voice coil appears to be sufficient for imaging [134].

![Simulated and real signal](image1)

(a) Gaussian response for the 3rd and 4th order of a mirror placed in the sample arm of the MR-OCT.

![FFT response real signal](image2)

(b) Comparison of FFT between simulated and measured data.

Figure 4.15: The results show the measured and simulated data.

The strong reflection from a sample mirror generates a clear Gaussian response, as shown in Figure 4.15a. The signal synthesis did reproduce a similar characteristic compared to the acquired signal from the voice coil actuated system, confirming the validity of the current model. This very simple model is useful for quick evaluation of a filter response but can also be used by adding calibration constants to perform fitting to a noisy signal.

The simplified signal simulation is suitable for testing aspects of digital filtering or other aspects of the MR-OCT signal and image processing. Another option is to use the generated signal to be fitted over a noisy signal to improve the SNR.
Noise sources, MR-OCT tuning procedure, hardware configuration, and image results

5.1 Noise sources

**Shot noise** also called Poisson noise due to the statistical Poisson characteristics. The shot noise originates from the random character of liberating a charge carrier due to an incident photon. If the number of events are sufficiently large, which it is typically for light incident on photodiodes or detectors to measure intensity or power, the distribution characteristic is very similar to a Gaussian distribution. If the standard deviation of the shot noise is equal to the square root of the average numbers of events $N$, the SNR can be expressed as $SNR = \frac{N}{\sqrt{N}} = \sqrt{N}$. OCT systems are called “shot noise limited” if the shot noise is dominant, which does not mean that the shot noise itself is accessible [56] as such. The sensitivity of the system is optimized if the shot noise becomes dominant by balancing the powers of the arms in the MI and the residual noise is dominated by the shot noise. The frequency spectrum of shot noise is considered white or broad band $i_n = \sqrt{2qI_{dc}}$ [142], with $i_n$ noise current, $q$ electron charge, and $I_{dc}$ a constant current applied.

**Poisson noise** (see Shot noise).

**Johnson noise** is generated by thermal agitation of charge carriers, which occurs also if no voltage is applied [143], [144]. The Johnson noise may become dominant over the Shot noise and in some IR photodiodes or detector circuits particular cooling options are provided to
minimize the noise. The noise has a power depending on temperature \( P = k_B T \Delta f \), with \( k_B \) Boltzmann’s constant, temperature \( T \) in Kelvin, and \( \Delta f \) the bandwidth in Hertz.

**Flicker noise** originates from fluctuations of resistive systems [145] and is considered to be negligible [111] considering signals above 10 kHz [56]. Consequently, the flicker noise is also called “pink” noise or \( 1/f \) noise due to the spectral power reducing towards higher frequencies.

**Excess intensity noise** [56] is also called excess photon noise [128] is the noise generated due to self-beating of broad-band light waves. The excess noise is responsible for the line-width of monochromatic light sources and is negligible for low-coherence radiation according to Hodara [128]. Consequently, this noise may have some significance for SS-OCT light sources, but also for telecommunication applications [131]. Some measurements for SLEDs suggest values between \(-101 \text{ dBm}\) to \(-115 \text{ dBm}\) for incident powers of 30 µW [130].

**Relative intensity noise** (see excess intensity noise)

**Receiver noise** \( \sigma_R \) is the noise contribution from the detection system. For a photodiode it can be estimated based on \( \sigma_R^2 = NEC^2 B \) with the noise equivalent current \( (NEC) \) and the bandwidth \( B \) and for CCD line cameras it is estimated by the dark and read noise [111], [124].

### 5.2 Signal and image parameters

**Signal-to-noise ratio (SNR)** is the ratio of signal \( S \) to noise \( N \) calculated as \( \text{SNR} = S/N \).

Historically it can be stated as a log-scaled value with \( \text{SNR}_{dB} = 10 \log_{10}(S/N) \) based on the units of the input values in Watts. Consequently, if the input values occur in Volts the calculation changes to \( \text{SNR}_{dB} = 10 \log_{10}(v_S^2/v_N^2) = 20 \log_{10}(v_S/v_N) \). The bel unit was initially introduced as the transmission unit (TU) [146] and later renamed in honor of Alexander Graham Bell as a measure of power ratios for transmission lines. Although the unit is an accepted standard [147] it is not an SI unit, and the only reason to use it would be to refer to quantities related to electro-technical journals or other historical documents. For scientific purposes, it is therefore of good practice to state the bare \( S \) and \( N \) values along with the SI units and if dB values are required the related conversion should be made explicit with an equation.

To allow for a broader audience to deduce the input values it may be sufficient to determine the light sensing component as photodiode or charge coupled device (CCD camera). A
photodiode produces a photocurrent proportional to the energy of a photon based on the responsiveness and quantum efficiency. The detection voltage is then only determined due to the input impedance of the preamplifier or current driver. On the other hand, the CCD voltage is proportional to the power of the incident light due to the photon counting characteristics, whereas the number of generated photons depends equally on the detector responsivity.

Some caution should be exercised not to use the SNR as a sole measure to evaluate the performance of a system [58, p.175]. As a typical standard approach in the OCT community, SNR in conjunction with an ideal reflector in the sample arm is used to evaluate the initial system performance, which may not necessarily reflect the actual final imaging performance and only gives a limited value to compare different OCT technologies. The reasons are related to the signal to noise bandwidth that is entering the acquisition and processing framework and needs to be included for further analysis besides the bare SNR values [64], [124], [148]. For example, an ideal reflector has no absorption or scattering, and measurements of materials with a variety of different refractive indexes, different turbidity, and surface roughness can be provided depending on the anticipated imaging goals [149].

Finally, the SNR should be distinct to sensitivity, even though the SNR provides a value of the best sensitivity possible.

**Dynamic range (DR)** Beyond the detector it is the ratio of the maximum to the smallest signal level step size that a system can acquire, transfer, or process [150], [151] and is related to the contrast a reconstructed image will be able to be displayed with. The dynamic of a system is consequently a measure of the detection of a change of signal level and is mostly relevant in conjunction with a digitizer. However, as the SNR of a photodiode will limit also the DR and both properties are sometimes stated interchangeably for detectors or sample materials. For CCD devices typically no access to the analogue signal is available which is limited by the full-well capacity and the read-noise, and usually is further reduced by the A/D converter. [56], [58], [111], [152]. CMOS devices with “smart pixel” technology may have a better DR compared to CCD devices [56]. In other words the DR is the number of grey levels that can be acquired or reconstructed. Consequently a reduction of the DR will reduce visibility of deeper structures by either merging them into the noise floor or into an adjacent grey level [148]. Transparent samples therefore would require a lower DR while turbid samples need a large DR to be able to separate grey levels of weak reflecting structures. The DR of display devices would need to be included into this
discussion considering budget computer screens provide typical values that are around 100:1 [153] while newer devices may go up to 10000:1 or more. This would include a look at the bit-depth of the attached graphics system of the computing device which can go from 8 bit to 32 bit for more modern graphics cards which also requires the monitor to be able to display that much. Most authors will show images in log-scaled grey-levels to shift low intensities into the visible range of display devices, which is by the way not the reason of using the dB scale because a wider variety of algorithm exists to provide much better compensation of the display’s limited DR. This means, in conclusion, that it makes sense to store images with the acquired DR to be able to extract arbitrary grey level ranges for later inspection independently from the display device.

**Sensitivity (S)** is the strength of a response \( R \) to a stimulus \( I \) defined as \( S = \frac{\Delta R}{\Delta I} \). Assuming the stimulus and response are noiseless values then after adding noise the minimal stimulus required must be at least the same as the noise level, which means that the SNR of the stimulus would have the value one (\( \frac{I}{N} = 1 \)) [56]. The latter expression relates therefore to the minimal stimulus to the lowest noise level.

**Measuring sensitivity** is described for a FD-OCT system by measuring the signal of the sample mirror attenuated using a neutral density filter (ND) with attenuation OD-2 in the sample arm, while the interference of the mirror signal was maximized beforehand by adjusting the reference arm attenuation [111]. Due to the light passing through the OD-2 ND filter twice (double pass) the sample arm attenuation is 40 dB. The sum of the residual signal after DFT and 40 dB is then an estimate of the sensitivity \( S \) of the system calculated as

\[
S_{dB} = \text{SNR}_{dB} + \text{ND}_{dB}.
\] (5.1)

The idea is based on the values of a measured signal change from zero to a level \( X \) vs. a change of reflectivity from zero to some attenuated reflectivity (\( A \cdot R_S \)) with attenuation \( A \) and ideal reflectivity in the sample \( R_S = 1 \). Then the sensitivity can be written as

\[
S = \frac{0 - X}{0 - A \cdot R_S} = \frac{0 - X}{0 - A}.
\] (5.2)

If the signal \( X \) will have some noise \( N \) then we can write the sensitivity in terms of the SNR of the signal \( X \)
\[ S = \frac{-X/N}{-A} = \frac{X/N}{A}. \]  \hspace{1cm} (5.3)

Converting the values to dB scales again gives then

\[ 10 \log_{10}(S) = 10 \log_{10}\left(\frac{X/N}{A}\right) = SNR_X - 10 \log_{10}(A). \]  \hspace{1cm} (5.4)

The round-trip attenuation of 40 dB is close to the CCD 8 bit A/D conversion that would correspond to a DR of about 48 dB, which supports the difference between values for DR and SNR [148].

However, it should be added that other sources in the literature suggest to adjust the source power to \( P_0 = 175 \mu W \) and the initial sample power to \( P_S = 60 \mu W \), and then measures the sensitivity for a few different reference arm reflectivities [111]. However, also the values of the fractional powers should be included in the measurements that exit the interferometer which is essential if the OCT system is operated in unbalanced mode. In some cases, it may make sense to use unbalanced configuration although if feasible, the balanced mode is preferred due to an optimal rejection of vibrational-induced noise, reduced back-reflection to the source, and best SNR [154]. Sometimes OCT systems are declared to be shot-noise limited which is too vague for comparison. It would be much better to provide a sensitivity vs. reference attenuation curve [111] that shows how close a system reaches the shot-noise limit relative to the reference arm attenuation.

### 5.3 MR-OCT tuning procedure and hardware configuration

#### 5.3.1 Introduction

Throughout the research multiple different MR-OCT configurations were used to evaluate different aspects of signal and imaging parameters.

Two main configurations were used that differ mainly in the scan speed and the scanning hardware.

The fast scan system was equipped with an 2D galvo scanner and was typically operated with a scan rate between 600 Hz and 1000 Hz using a selection of different custom voice coil types.
This system was used to study the layer-scanning method with MR-OCT at a scan rate of 1000 Hz (see section 3.4).

The slow scan system was placed on a translational stage such that scanning was achieved by moving the whole system and a scan rate of 152 Hz. The slow scan method was of interest to investigate applications that will use a miniature scan head with the Michelson interferometer and all other optical components integrated. It was then also used to measure the sensitivity characteristic (see section 3.1) for multiple separate orders of reflections in conjunction with different detection schemes such as single detector mode and balanced detection mode.

The detectors used were a PDB210C (Thorlabs), a DET10C (Thorlabs), and a NF2117 (Newport). The PDB210C did not provide sufficient bandwidth for frequencies of higher order frequency signals which increased heavily the noise in balanced mode. So the PDB210C was only used in single channel mode to gather data in comparison to the DET10C detector. The DET10C detector was used in conjunction with a preamplifier (DHCPA-100) from Femto which offered a wide range of bandwidth vs. gain settings. For MR-OCT it appears to be relevant to chose a particular gain and bandwidth to achieve optimal sensitivity for a particular higher order interference which does not necessarily assure highest sensitivity for the first order interference (see section 5.8).

The NF2117 detector was used to compare the sensitivity advantage between balanced and single channel detection (see section 3.1).

5.3.2 Materials and setup used

Slow-scan system

The slow-scan system was especially designed to study methods of low-cost and limited resource settings (Figure 5.1).

Consequently, the scan rate was set to 152 Hz A-line rate and scanning was performed by moving the stage. Although, the low-cost approach was always an underlying concept to incorporate into the investigation a lot of hardware components are still expensive and bulky. Single detector and balanced detectors were used to compare the different detection modes with each other. The currently used optomechanical design is very flexible to quickly reconfigure the system for use of different components.

Figure 5.1 shows an MR-OCT configuration equipped with an balanced detection scheme using a detector NF2117 from New Focus. The voice coil used in this configuration was a commercial
system with relatively large dimensions (NCM02-17-035-2F) to avoid problems with the axial scan stability so that the attached scanning mirror can achieve optimal pointing precision and was driven with a custom waveform generator and amplifier.

Other major components not visible in the image are the SLED type from Exalos EXS210041-02 that was controlled by a custom current driver and temperature stabilized by an ILX Lightwave TEC (LDT5412). The data acquisition was performed with a Picoscope 4824 from Pico Technology.

**Fast-scan system**

The fast-scanning system was equipped with a 2D galvo scanner that allowed to perform imaging of 3D OCT volumes and was used to evaluate the concept of an layered-scanning MR-OCT (section 3.4). This system was also equipped with an ATS 9440 from Alazar Technologies Inc. to achieve sufficient data transmission for the expected data volume for 3D volume scans. The system is also uses an SLED type from Exalos, EXS210041-02, as a light source and can be configured with different detectors such as the DET10C from Thorlabs and different voice coils.

Typically the scanning system was a custom voice coil made from an pick-up head extracted from an CD/DVD-ROM drive. However, the scanning frequency was limited to about 600 Hz before the scan range became too short for scanning. To increase the scanning frequency a custom resonant scanner was constructed using a voice coil from BEI kimco (Figure 5.2).
The voice coil was then directly attached to the rear of the spring and the scanning mirror was attached at the front. In section 3.4 it was shown that even with minimal engineering effort the scanning system can achieve reasonable scanning results to detect sweat duct patterns for fingerprint identification.

5.4 Raw signal measurements

The acquisition hardware was tailored towards reduced costs of equipment and a digitizer matching more the scan speed of the reference mirror was selected. The Picoscope 4824 from Picotech was used (price 1955 EUR) providing a memory size of 256 MB, 80 MS/s at a bandwidth of 20 MHz. Despite the availability of a USB 3.0 port, the acquisition PC was only able to perform with USB 2.0 speed. The digitizer was configured for two channel mode, one trigger channel and one data channel, which provided a buffer size of 128 mega samples (MS). At a scan rate of 152 Hz and a selected sample rate of 20 MS/s the sample length per A-line is 131579 samples. Consequently, at 128 MS buffer available about 973 A-lines could be recorded at high speed before transferred over USB 2.0 onto the computer. Assuming a USB 2.0 speed of about 480 Mbits/s and data bit width of 12 bit, the transfer for 973 A-lines would take 128 MS/(480 Mbits/s/12 bit)=3.2 s. The actual transfer time will be somewhat larger due
Figure 5.3: A typical oscilloscope screen recording the MR-OCT signal with the Picoscope showing at the top the scanning mirror actuation (red), the rectangular trigger window (brown), and the interference signal (blue) of one order. The acquisition time was adjusted for test purposes to accommodate three scanning cycles (three forward and reverse scans, blue). The bottom plot shows the FFT of the interference signal and the dispersion of frequency components towards low frequencies due to the nonlinear phase.

to communication overhead on the USB bus, and a maximum of 600 A-lines was sufficient for most imaging tasks. The slow transfer speed matched well with the repositioning time of the scanning system before each next B-frame. A raw signal measured with the Picoscope oscilloscope software is shown in Figure 5.3 and Figure 5.4.

5.4.1 Phase detection

The phase detection was performed by summing multiple raw interference signals and extracting the phase after Hilbert transformation (see Figure 5.5).

A nonlinear phase causes dispersion of frequencies in the frequency domain (Figure 5.4).

The method of summing the raw signal allows to evaluate the relative phase variations and is therefore some indication of the phase stability. Although, the characterization of the phase stability is limited due to the time interval between each interference which was determined by the motion of the translational stage.

The background of Figure 5.5 shows the extraction of the motion characteristic of the scanning mirror based on the phase of the interference signal. While the upper left plot shows only a single sample mirror position, the upper right plot shows the sum of about ten interference signals from ten sample mirror positions generating a more or less continuous waveform. The Hilbert function (lower left) of the signal sum (upper right) provides the envelope which is, in this case, an indicator of the interference phase stability. If the phase shifts precisely by one
Figure 5.4: Compared to Figure 5.3 the position of the sample mirror was moved closer to the zero crossing of the scanning mirror. The scanning mirror velocity is most linear if it passes through its zero position and the interference frequency can be determined from the FFT plot (bottom). The interference frequency of the first order reflection was 0.06 MHz with an estimated SNR of 64 dB.

Half cycle extinction occurs and the envelope amplitude becomes zero. From the envelope signal the unwrapped phase of the interference can be extracted (lower right) which corresponds to the position vs. time characteristic of the scanning mirror showing the expected sinusoidal half cycle.

Zooming into the plot from the lower right phase characteristic (Figure 5.5) is shown in Figure 5.6. The phase characteristic can be fitted with a sinusoidal half-cycle to very high accuracy confirming the nearly ideal simple harmonic motion of the voice coil’s mass-spring system.

The important aspect is that a further refined method would allow to remove the nonlinearity of the phase merely by signal processing and does not require any additional optical components to monitor the scanning mirror motion.

In practice small deviations need to be considered due to air friction and non-isotropic material constants in the spring.

5.4.2 Evaluation of phase linearity

The phase of the interference signal is linearized using a prerecorded signal from a sample mirror by applying a pixel remapping [106]. The pixel remapping method was used due its simple and fast way to reconstruct the interference signal. Another approach is to first combine all nonlinear A-lines into a B-frame image matrix and perform a 1D inverse barrel distortion [4],

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Figure 5.5: The extraction of the scanning mirror motion profile by summing of multiple interference signals. Single interference at step position 17 (top left). Sum of interference signals with about 20 mirror step positions (top right). The envelope of the sum is detected with the Hilbert function (lower left) and the unwrapped phase of the envelope corresponds to the motion profile of the scanning mirror (lower right).

Figure 5.6: Fitting a sinusoidal function over the nonlinear signal phase.

[155], [156]. However, for large sizes of A-lines it may be difficult to fit all data into a work memory.

Similar to section 5.4.1 the linear interference signal can be investigated based on different metrics to evaluate the quality of the MR-OCT configuration (Figure 5.7a).

The subplots in Figure 5.7a represent the sum of multiple sample mirror positions of the first order signal (top left) allowing again to extract the phase (bottom right). Additionally, the FFT of the sum signal (top right) reveals the frequency and noise structure. Summing all FFT signals gives a representation of all available frequencies vs. order of reflections (bottom left).
(a) Overview of control plots to evaluate multiple signals from 300 sample mirror positions in depth. Sum of all first order signals (top left), the FFT on the sum of all first order signals (top right), all available center frequencies (bottom left), and the linear phase (bottom right). The sum of all center frequencies also shows the signal power vs. noise power. Another acquisition (see 5.7b) shows a better (reduced) noise floor.

(b) A zoomed in version compared to 5.7a bottom left. This sample mirror acquisition shows a better SNR for higher orders.

Figure 5.7: The plots are demonstrating the measurement during different processing steps, particularly at the step of measuring the fundamental signal parameters such as center frequencies and phase linearity. Furthermore, the plots allow to evaluate the broadening of the spectra for each higher order of reflection and an estimation of available orders for processing.

This allows to measure the location of their center frequency, spectral power, and line width. In Figure 5.7a ten orders of reflections are marked with vertical markers.
5.5 Frequency analysis of artifact signals

5.5.1 Introduction

To evaluate the origin of the artifact signals the filter configuration of the processing of the digital signal is discussed. The processing chain is reviewed beginning from the raw signal, the phase linearization, and the final band-pass filter process. In all cases, digital filtering can be performed by applying a Gaussian window for any particular frequency band in the frequency domain which avoids a lot of artifacts that can otherwise not easily be avoided with other filter types. The elliptic filter is often considered to have a superior transition characteristic, minimal pass-band ripple, and optimal side-band suppression. Other aspects are the windowing of the signal before filtering to reduce the broadband noise generated by the cut-off of the signal on the buffer boundaries. However, it can be shown that even with a rectangular window only the broadband noise will increase and at best reduces the SNR of the signal, but will not be able to generate distinct artifacts. Finally, it is shown that signal saturation did not occur, and any signal saturation generates multiple high-frequency bands with regular frequency spacing in the frequency domain.

5.5.2 Analysis of single detector DET10C

A frequency analysis was performed to analyze the source of the artifacts of additional mirror signals using the DET10C detector.

For this purpose a signal with nonlinear phase and a linearized signal were compared (see Figure 5.8, 5.9, and 5.10). Figure 5.8 is demonstrating that the raw interference signal is not visibly distorted in the time-domain (upper plots). The spectrograms show the frequency distribution of the time-domain signals vs. acquisition time. The nonlinear phase components cause most of the frequency dispersion at the beginning and end of the acquisition window corresponding to the slow-down of the scanning mirror close to the turnaround point. The signals after linearizing their phase are shown in Figure 5.9 and Figure 5.10.

Both figures can be used to compare the frequency location of a second order artifact with a signal. In Figure 5.9 the first artifact occupies a frequency close to 240 kHz between the time from 0 to 1.25 ms. In Figure 5.10 the signal for the fourth order can be found at the same location from 240 kHz between the time from 0 to 0.75 ms. The band-pass filter can therefore not reject the parasitic signals and those will occur in the calibration image Figure 5.12.

Figure 4.3 shows how the filter windows are allocated in the frequency domain and how the
(a) Representation of a second and third order interference signal with nonlinear phase from a single sample mirror at position 130 \( \mu m \).

(b) Representation of a fourth, fifth, and sixth order interference signal with nonlinear phase from a single sample mirror at position 295 \( \mu m \).

Figure 5.8: Figures (a) and (b) show a set of different orders of signals together with a spectrogram. The spectrogram allows analyzing the frequency composition vs. the acquisition time. The nonlinear phase of the signals represents itself as a slight slope of the frequency in (a). For (b) the nonlinearity is apparent due to the lower frequency for the highest order signal (third right). Furthermore, additional (parasitic) frequency components are visible which are believed to originate from beating between different orders of signals. It should be noted that the signals have not been preprocessed at this point and show the raw digitized signal.

Figure 5.9: The raw second and third order before (a) and after linearizing the phase (b). Compare with nonlinear phase in Figure 5.8a. The FFT (c) reveals the frequency location of the parasitic components vs. the signal components. The spectrogram (d) shows the frequency components as straight lines. The frequency lines become somewhat fuzzy at the endpoints due to the reduction of the power of the frequency components related to the Gaussian envelope. The parasitic components remain visible and occupy higher order frequency bands. I.e., the first parasitic component here does occupy precisely the frequency band of the fourth order (see Figure 5.10).
separated signals appear after processing. The parasitic signal was marked in the frequency domain (Figure 4.3, upper right) at about 240 kHz and occupies the frequency band of the fourth order. Experimentally different window types and filter types were tested, but none rejected the artifact signal. Consequently, the signal artifacts will propagate through the remaining image processing steps and are visible in the calibration line occurring as a second sample mirror motion (Figure 5.11a).

In Figure 5.11 an attempt to investigate the reproducibility with the same detector shows that a second artifact signal can become visible which would corroborate the theoretical considerations that all higher order frequencies can potentially create beat frequencies.

It should be noted in Figure 5.11 that the artifact signals are not always reproducible. It is currently hypothesized that the visibility depends strongly on the parallelism between the SRM and the PM. Potentially, a slight misalignment could avoid the artifacts by sacrificing a minimal level of the interference visibility. Notably, the artifacts were only visible with the DET10C detector that has a large photosensitive area. The large detector area may also contribute to the visibility of artifacts considering that the spot size is different compared to the scanning mirror spot.

A dense plot Figure 5.12 of multiple PSFs also shows the that the artifacts are exactly located at the frequency bands of signals such that there is no obvious way to eliminate them.
(a) Recording 1 and processing of calibration line for a sample mirror at 300 different z-positions up to a distance \( z = 1.4 \text{ mm} \). Artifact signal (A) and other aliasing artifacts (C).

(b) Recording 2 and processing of calibration line for a sample mirror at 300 different z-positions up to a distance \( z = 1.4 \text{ mm} \). Artifact signal (A) and second artifact signal (B). Aliasing artifacts (C).

Figure 5.11: Two different recordings (a) and (b) were performed with the same detector. The artifact signal (A) occurred while the artifact signal (B) is only visible in the second recording attempt.

Figure 5.12: Multiple plotted PSFs to evaluate the intensity roll-off vs. z-position of the sample mirror in air.

Also, it is to be expected that for weak scattering samples the visibility of artifacts is negligible (Figure 5.13).

So far all shown calibration lines were obtained with a DET10C detector in conjunction with a DHPCA-100 preamplifier and the artifacts for other detectors are not occurring or too weak to
be detectable. Further research could investigate if balanced detection can either reproduce the artifacts or is able to suppress them.

5.5.3 Analysis of digital filtering

Although, the filter response by applying a Gaussian window in the frequency domain shows a very distinct signal that propagates into another frequency band with the relationship $f_{\text{leak}} = 2f_m$, we can evaluate how other filter types are able to suppress the artifact signal.

A similar investigation was already performed in section 4.3.2 that could show that band-pass filtering using a Gaussian window in the frequency domain is better able to remove ringing and side lobes of the mirror signal. As it was already mentioned the filter optimization is not able to improve suppression of the artifact signals.

Different filter types usually perform better for particular signal and noise characteristics. The elliptic and the Chebychev type filter are usually optimal for most general purpose signals. Their transition characteristics, stop-band attenuation, and pass-band ripple are usually sufficient to suppress noise efficiently. The comparison of the filter performance of an elliptic filter for different window parameters on the signal before filtering is shown in Figure 5.14.
The change of the parameter of the Tukey window suggests that the window type has minimal
effect on the line washout and no effect on the suppression of the phantom lines.

(a) Tukey window $\alpha = 1.0$
(b) Tukey window $\alpha = 0.0$
(c) Tukey window $\alpha = 0.5$

Figure 5.14: Processing of calibration line using an elliptic filter configured to have a pass-band
loss of 0.001 dB, a stop-band attenuation of 120 dB, and use the pole-zero parameters to convert
to an second order sections form for three different settings on the raw signal using a Tukey
window.

It should also be mentioned that for MR-OCT signals the second order section (SOS) form of
the filter were used to be able to filter the low order signals for the relatively low sample rates.
Another option could have been to zero pad the signal and shift it each time into a frequency
band that can be calculated more reliably, but this option was not further evaluated yet. The
examination of an elliptic filter in Figure 5.15 does not show any obvious improvements and no
better suppression of the artifact signal either.

Figure 5.15: Filter response of an elliptic filter using the same detector DET10C for a second
order interference (a) and a zoomed view (b). The filter bandwidth for each center frequency is
slightly different for each higher order to account for the frequency dispersion of higher order
signals. The first order filter bandwidth is somewhat asymmetric to avoid most of the low
frequency content.

In Figure 5.15 the response of the elliptic filter shows that no particular ripple is above the
noise threshold compared to the raw interference signal (blue, 5.15b). However, compared to
the true signal at about 120 kHz the parasitic phantom signal at about 240 kHz remains visible and is not or cannot be rejected.

### 5.5.4 Conclusion

So far the artifact signals are unlikely to originate from the digital processing steps as they occur in general in the raw signal after digitizing. The phantom signals are believed to originate from beating between the frequencies of different higher order interference frequencies. Although, the well-separated frequency bands do not support well this theory as it is difficult to see how any interaction could occur. Another theory is that if the detector area is large enough, a cross-talk between the higher order signals could happen that leads to the generation of beat frequencies.

So far the repeatability of the artifact signal remains part of future research that requires a particular setup that includes the evaluation of intermediate electronic components and the position of optical components.

Nevertheless, although the phantom signals are not desired for conventional OCT imaging, they may point to other underlying physics that may be of interest to be investigated further.

### 5.6 Effect of windowing on the nonlinear signal

The raw interference spectrum of a first order reflection was windowed to evaluate which window can produce the best SNR and peak shape. Although, the peak shape is dispersed the criteria was to obtain minimal ripple and no peak widening.

A visual index of the peak shapes vs. window type is given in the list Figure 5.16.

The rectangular window includes a lot of high frequency terms originating from the instantaneous amplitude change at the beginning and end of the time-domain signal. On the other hand the Flat-top window shows a significant reduction in signal power. Comparing the window types according to a minimal high frequency content and maximum signal power the Hamming and the Blackmann window appear to provide best results.
5.7 Roll-off control

The roll-off can be partially controlled by moving the reference lens at a small distance vs. the SRM. However, the measured effect were minute yet significant enough to be reported for further evaluation with a dedicated MR-OCT system. Not only is the roll-off characteristic important for imaging it may also be sensitive to the position of optical components and may carry other information about the system status and the propagating light.

Please take note that the roll-off is plotted vs. the depth position (Figure 5.17) for each contributing order of reflection. Usually the intensity is maximized by observing the 1st order of reflection and adjusting the position of the reference lens. In Figure 5.17 the adjustment was also performed by positioning first the sample lens at the position of the order 5 and 10 and respectively maximizing the intensity for each order by adjusting the reference arm lens again.

It can be theorized that the roll-off should change depending on the position of the reference arm lens and the maximum intensity should be available for the adjusted order.
5.8 Baseline sensitivity

This section evaluates the variance to expect between multiple sensitivity measurements including the effect of different gain settings of the preamplifier (DHPCA-100) and power settings of the light source. The gain settings were preselected for minimal noise. Four different settings with different bandwidths HBW = 3.5 MHz and LBW = 1.8 MHz were compared.

![Graphs showing roll-off characteristics vs. depth for three different positions of the lens in the reference arm.](image)

Figure 5.17: Roll-off characteristics vs. depth for three different positions of the lens in the reference arm.

![Graphs showing roll-off characteristics vs. depth for three different positions of the lens in the reference arm.](image)
CHAPTER 5. NOISE SOURCES, MR-OCT TUNING PROCEDURE, HARDWARE CONFIGURATION, AND IMAGE RESULTS

OD_r = 0.86, gain = 10^5, LBW, SLED = 10 mW

OD_r = 0.97, gain = 10^6, LBW, SLED = 5 mW

OD_r = 1.11, gain = 10^6, LBW, SLED = 10 mW

OD_r = 0.50, gain = 10^5, HBW, SLED = 5 mW

OD_r = 0.67, gain = 10^5, HBW, SLED = 10 mW

OD_r = 0.81, gain = 10^6, HBW, SLED = 5 mW

OD_r = 0.78, gain = 10^6, HBW, SLED = 10 mW

OD_r = 0.75, gain = 10^7, HBW, SLED = 5 mW
Table 5.1: Sensitivity measurements including the evaluation of the roll-off. All measurements are performed with OD2.0 in the sample arm.

The results of the measurements in Table 5.1 are compared in Figure 5.18.

Figure 5.18: Visual comparison of sensitivities measured vs. gain and source power for order $m_o = 1$ and $m_o = 10$. The gain parameters (see Table 5.2) are 'H' for a high bandwidth of 3.5 MHz and 'L' for 1.8 MHz, whereas the digit describes the gain exponent $10^n$. The gain settings for maximum sensitivity of order $m_o = 1$ (a) and (b) are different for gain settings to achieve maximum sensitivity for order $m_o = 10$ (c) and (d).

It is apparent that the different configurations cannot be related one-to-one from a first order signal and a tenth order signal. As expected the sensitivities in Figure 5.18 for the first order ($m_o = 1$) are largest for the highest SLED powers and highest selected gain settings $H7 = 10^7$ HBW (a) and $L6 = 10^6$ LBW (b). To achieve the best sensitivity for higher orders ($m_o = 10$) then again the largest SLED powers are prevalent but the amplifier gain must be reduced $H6 = 10^6$ HBW (c) and $L5 = 10^5$ LBW (d).

The differences demonstrate that the detector in conjunction with the amplifier settings need to be included into the considerations if maximal sensitivity of higher orders of reflections are required. The effect is possibly related due to nonlinear noise response of the amplifier in conjunction with the detector noise. The first order signal appear to benefit from a higher bandwidth at high gain settings while the higher order signal (here the 10th order) benefit from
a reduced bandwidth. But also if the noise structure in the corresponding plot Table 5.1 (w) and (x) is investigated it appears that the noise floor increases much quicker and the intensity roll-off is nonlinearly falling off much faster for higher orders (larger z-positions). In contrast for higher orders the reduced gain appears to flatten out the noise floor higher orders and the intensity roll-off becomes more linear (Table 5.1 (s) and (t)). The same characteristic can be observed for Table 5.1 (g) and (h) showing a flat noise floor and linear roll-off. For MR-OCT the total DC power on the detector is a combination of all higher orders of reflections not in coherence or not generating any interference.

<table>
<thead>
<tr>
<th></th>
<th>S1, $m_o = 1$</th>
<th>S2, $m_o = 1$</th>
<th>D, $m_o = 1$</th>
<th>S1, $m_o = 10$</th>
<th>S2, $m_o = 10$</th>
<th>D, $m_o = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OD$_r$=0.81H6,10 mW</td>
<td>80.6</td>
<td>80.8</td>
<td>0.2</td>
<td>64.6</td>
<td>65.5</td>
<td>0.9</td>
</tr>
<tr>
<td>OD$_r$=0.67H6.5 mW</td>
<td>73.3</td>
<td>74.2</td>
<td>0.9</td>
<td>58.0</td>
<td>59.2</td>
<td>1.2</td>
</tr>
<tr>
<td>OD$_r$=0.81L5,10 mW</td>
<td>80.6</td>
<td>82.8</td>
<td>2.2</td>
<td>65.3</td>
<td>64.8</td>
<td>0.5</td>
</tr>
<tr>
<td>OD$_r$=0.72L5.5 mW</td>
<td>75.8</td>
<td>76.0</td>
<td>0.2</td>
<td>60.5</td>
<td>58.0</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 5.2: Estimation of variance of sensitivity measurements based on two values of Sensitivity (S1, S2) and the difference (D) between the two values. The variance was estimated for $m_o = 1$ and $m_o = 10$, powers of $P = 5 \text{ mW}$ and $P = 10 \text{ mW}$, at $H6 = 10^6$ high bandwidth, and $L5 = 10^5$ low bandwidth. The values OD$_r$ is the according reference arm attenuation for the most optimal sensitivity.

Table 5.2 gives some estimate about the variance of the sensitivity measurements for different gain settings, injected power from the light source and orders $m_o = 1$ and $m_o = 10$. It should be noted that the sensitivities were determined measured at two different days. The measurement of the sensitivity depends on factors such as the source power, the gain of the preamplifier, detector temperature, and in the case of MR-OCT of the stability of the parallelity between the PM and the SRM. It is to expect that therefore the variance of the sensitivities will be somewhat more substantial than the determined values. The worst case of the sensitivity variance was 2.5 dB which corresponds to about 4%.

5.9 Imaging results

The images in this section demonstrate the capabilities of MR-OCT for conventional OCT imaging of semi-transparent and scattering specimens. In Figure 5.19 it was investigated if the reduction of power would allow a better adjustment of the reference arm power to further improve sensitivity. However, from section 3.1 it was concluded that each partial reflection is attenuated due to the PM splitting ratio and the reference arm attenuation may only minimally improve visibility due to the strong overall optical DC originating from all orders. Nevertheless, the images of the scotch reel suggest an imaging depth of up to 1 mm in a scattering medium.
The Figure 5.20 shows the change of contrast vs. A-line densities. To avoid any overhead due to averaging it was of interest to see how different number of A-lines vs. a constant scan length change the visibility of scattering structures. Although not conclusive it appears that undersampling the B-frame scan with a reduced number of A-lines can provide somewhat better results without further postprocessing. Postprocessing to average A-lines of an oversampled B-frame was not further followed up in this case. Layers up to 750 µm are visible, and some structures up to 1 mm appear to be present.

Imaging a weak scattering sample allows to evaluate the axial dispersion and widening of the PSF in the sample Figure 5.21 and confirms well that MR-OCT is capable of imaging beyond 1 mm depth.
CHAPTER 5. NOISE SOURCES, MR-OCT TUNING PROCEDURE, HARDWARE CONFIGURATION, AND IMAGE RESULTS

The images of an onion skin compared to an image taken with the OCS1300-SS (Figure 5.22) demonstrate good visibility of structures within a depth range of about 750 µm.

Unfortunately, the scanning system for the research setup was not anticipated for in-vivo imaging, and it was not possible to obtain more images for living samples. The system could be easily equipped with a scanning galvo mirror allowing for rapid lateral scanning in two dimensions. The axial scan rate of the SRM would needed to be maximized. It was shown that a 1 kilohertz scan rate can be achieved (see section 3.4) and with improved engineering methods perhaps even larger scan rates are possible.

Nevertheless, even with the slow scanning system a B-frame of a fingertip was imaged showing typical structures of the epidermis and the top of the papillary dermal layer (Figure 5.23).

The images show in conclusion that no obvious artifacts are present due to the expected beating

Figure 5.21: Image of a stack of pop-note (Scotch) using the DET10C.

Figure 5.22: Onion skin imaged with MR-OCT (DET10C) (a) and OCS1300-SS (b). The position and the lateral scan range is not matching.
between different interference frequencies. The effect of frequency beating is too weak in scattering media and no artifacts will be visible. However, secondary effects could cause a reduction in resolution and sensitivity. The image quality of onion skin is comparable to the quality of an OCS1300SS system, and the tape images show the reproduction of conventional structures. For the research setup, a low-cost digitizer oscilloscope was connected over USB 3.0 which was matching the scan speed of the translational stage. Such a slow scan configuration is possibly more relevant for a purpose-built application but imposes unwanted limitations to investigate a wider variety of samples and other research tasks.
6.1 Frequency dependency vs. path delay

A significant aspect of MR-OCT is the description of the origin of the increasing frequency of the interference signal for each higher order of reflection. The traditional analysis was related to the Doppler effect and the TD-OCT principle [4]. The increase of frequency meant that the higher orders of reflections represent a scanning mirror with increased scan range scanning within the same time interval as the physical scanning mirror effectively having a scan velocity of \( v_M(m_o) = m_o v_M \). However, due to the constancy of the speed of light, the path delay has the same effect of increased frequency which relates to the effect of light traveling with increasing distance between the scanning mirror and partial mirror physically. The increase of frequency corresponding to the displacement of the virtual scanning mirror with increasing distance is the same principle of the frequency encoding of sample layers of FD/SD-OCT. That was the background of the research question 1 to obtain scientific evidence if the principle of FD/SD-OCT can describe the path delay in the reference arm of MR-OCT. Although, for MR-OCT this only valid for the displacement of the zero-crossing of the virtual scanning mirror and time-domain scanning is still required to acquire the position of the layers in the sample. Perhaps equally relevant is that the higher order interference signals are not entirely independent and beating between the signals is possible under certain conditions. Although, most aspects can cause some limiting factors for conventional OCT imaging goals the minute delay between
each higher order interference signal can have applications beyond mere imaging and potentially being used to detect rapid motion in samples comparing the overlapping regions.

6.2 Wavefront distortion

A lens in the reference arm must be used as otherwise the spatial imperfections in the partial mirror and the scanning reference mirror, increase the noise of the interference signal. Although, the reference lens alleviates the spatial noise and sensitivities of up to 90 dB for the first order interference were achievable it was not well understood yet how the wavefront of higher order reflections is affected. Because due to the multiple reflections the multiple beam spots occur beyond the focal point and the spot size increases. Using Zemax as an optical simulation framework (see section 3.2), it was possible to obtain more accurate information about the wavefront deformation on the detector plane. The simulation predicted a minimal wavefront distortion for higher order wavefronts.

6.3 Simultaneous scanning of thin layers

The simultaneous scanning of multiple scanning layers is not an entirely new concept and was demonstrated with different degree of complexity and demand for hardware [133], [157], [158]. The lsMR-OCT method, on the other hand, demonstrated scanning of multiple layers by merely reducing the scanning range and increasing the spacing between the partial mirror and the scanning reference mirror. That would allow planning for a low-cost eye screening application by projecting the more sensitive lower order scan ranges into the retinal layer while the remaining part of the eye is not scanned. This straightforward method underlines once more the versatility of the MR-OCT method and the potential of future low-cost applications.

6.4 Filtering using a Gaussian window in the frequency domain

In section 4.3.2 an FFT filter method is proposed that shows improved artifact suppression compared to conventional filters. Related to research question 4, if a Gaussian window in the frequency domain can improve the signal characteristics, it was hypothesized that the Gaussian-shaped interference signal having equally a Gaussian spectral shape could benefit by using a Gaussian filter response. Filtering a signal using the FFT transformation is a well know strategy with the limitation that the signal has to be stored in memory. However, if the signal
can be processed in blocks, it is shown that using a Gaussian window for each frequency band produces fewer artifacts and the stopband attenuation is nearly arbitrarily infinity depending on the machine precision if the Gaussian is reduced to values of zero.

### 6.5 Performance of higher order interference signals

Another important research question was the way how the performance of each higher order is changing, related to research question 5. The established method to evaluate the sensitivity performance of OCT systems measures the residual SNR of an attenuated sample mirror reflection by varying the attenuation of the reference arm power. The variation of the reference arm power allows finding the most sensitive configuration of the Michelson interferometer to detect the attenuated light from the sample arm and typically shows some maximum response at some reference arm attenuation. Because an MR-OCT system generates multiple higher order interference signals, each signal should have hypothetically a slightly different sensitivity peak vs. the reference arm attenuation. The reasoning behind this theory is that each higher order interference has increasingly attenuated reference power due to the splitting ratio of the partial mirror and every single order is riding on top of the remaining DC from the average from each other partial reflection. The experiment performed in this work shows only a minimal change of the sensitivity peak, and it may be negligible for any practical purpose. More importantly, however, it showed that the overall sensitivity characteristic follows at least a second-order function, meaning roll-off is nonlinear. The results showed as expected that a balanced detection scheme brings the sensitivity closer to the shot noise limit. However, interestingly it was not possible to bring the system into saturation even with zero attenuation in the reference arm. The natural attenuation due to the splitting ratio of the partial mirror causes that each higher order signal has only a minimal power available for interference. On the other hand, the reduced measurement repeatability for high reference arm powers suggesting that the increasing dominance of the excess noise contributes to low-frequency drifts of the sensitivity. The low-frequency drift is relevant for balanced MR-OCT because the sensitivity has no distinct peak and it is easy to pick some higher reference arm power that coincidentally provides some higher sensitivity that can be up to 10 dB more. However, it is assumed that those higher sensitivities are spurious and are not continuously available during imaging. Performing the sensitivity measurements with a mirror interface can cause that higher order signals start increasingly to affect each other and additional beat signals can fade in and out. Improving the understanding of the interactions between the higher order signals could be a subject for further research that
6.6 Relevance of MR-OCT

In general, this work considered aspects of the MR-OCT method that have not been described in the literature before. The overall sentiment to classify MR-OCT as TD-OCT is not sufficient to explain the observed artifacts described in this work. In consequence, the theory of MR-OCT is more complicated and needs to include the spectral properties of the higher order interference signals. Other aspects such as the overlapping scan ranges that have a natural delay based on the speed of light having traveled a few micrometers more each time could open the path to novel OCT measurement systems, detecting high-frequency oscillations in scattering media. So far, the performance of MR-OCT will not challenge the performance of FD/SD-OCT and SS-OCT systems but will contribute with additional features that go beyond the mere OCT imaging.

The most important contribution of MR-OCT is undoubtedly to open up a second pathway compared to optically integrated OCT systems to achieve a reduction in costs and enable production in large quantities. Furthermore, although TD-OCT is known not to achieve the SNR levels compared to SD-OCT this thesis provided evidence to confirm that this also applies to MR-OCT. Considering in anticipation of a low-cost OCT system a particular scanning regime reducing the data volume, it was shown that indeed MR-OCT is a much simpler way to achieve scanning of multiple en-face layers compared to other methods [133]. Moreover, MR-OCT can easily be reconfigured for different layer thicknesses and spacings by merely changing the distance between the partial mirror and the scanning reference mirror as well as the scanning range and detects all depth layers quasi-parallel and instantaneously. The frequency encoding of the depth position of the scanning layers into discrete interference frequencies is a unique property of MR-OCT and not available in any other OCT method known so far except one allows the comparison to SD-OCT and the continuously encoded depth layers as a frequency spectrum. If the scanned layers are allowed to overlap such that some depth regions are scanned multiple times, the multiple interference signals can be used for noise reduction or to compare the spectral content of the signals.
6.7 Outlook and future work

The research provided in this thesis did predictably not show that MR-OCT can surpass any FD/SD-OCT or SS-OCT with akinetic light-sources. However, additional features of MR-OCT such as the simple reconfiguration of the reference arm allowing to increase the scanned depth region among others are unique and not available in any other OCT imaging method. The simplicity of the optomechanical setup of MR-OCT is undoubtedly challenged by its scanning system to achieve high axial scanning rates, but this should not distract from the fact that crude measurements and imaging tasks are well in reach. Therefore it would be interesting to see engineering work that can use purpose built scanning systems, a purpose-built and machined small optical bench to demonstrate a portable and low-cost OCT system. Some early attempts to demonstrate the miniaturizations of MR-OCT show a promising overall concept that can be easily further reduced in size [159]. On the other hand, it would be of interest to understand how the multiple interference signals interact with each other and if the development of new detection methods is possible.
A.1 Comparison of programming languages for scientific analysis and application development

A.1.1 Background

For the implementation of an algorithm for MR-OCT on available computing platforms it was of interest to investigate how three main classes of programming and development languages perform in comparison based on the correlation-mapping algorithm. Besides measuring the performance, the flexibility of a language to quickly move from the research system to an end consumer prototype is briefly discussed as well.

All three are high-level languages that target ease of use, are used for scientific research, and are relatively easy to be implemented for a wide range of different hardware platforms. However, the selected languages are not strictly representative, and especially Java programs cannot be assumed to run on a desktop and Android environments. Objective-C was not included as it is mostly limited to one environment and targeting application development. Matlab is often the language of choice for scientific research and analysis. Although it allows connecting a smartphone to a remote Matlab engine, it requires good experience to be able to port and test an algorithm that can run on a mobile platform directly. Python was included as it equally like Matlab provides a host of scientific tools but also has access to libraries that indeed allow running code directly on Android systems even if those are not native applications. Therefore, the selection was limited to Java, Matlab, and Python and tested with the correlation mapping
algorithm (cmOCT). The cmOCT algorithm is used for post-processing OCT signals to measure the spatiotemporal variations in intensity to distinguish dynamic from static scatterers.

The implementation with Matlab and Python took least of the time, and Java required most time to assure correct use of the class structures and syntactic aspects. Running time with Java was fastest which is to be expected as the code is efficiently precompiled for the Java virtual machine (JVM). The JVM allows caching of previous runs which can cause that initial measurements are moderate in speed and follow-on measurements tend to achieve an implausible increase in speed, making it difficult to obtain accurate measurements. Matlab provided an overall good performance not requiring to optimize the code while the Python performance was initially somewhat less compared to Matlab. However, with some simple changes using NumPy arrays and ctypes, it was possible to close in on the overall Matlab performance. Some further considerations for Python would be the availability of many different pathways to convert the code to C or call binary libraries from commercial scientific and mathematical libraries or legacy programs.

A.1.2 Introduction

The performance of an algorithm depends on the hardware platform it was designed for and the tool it was designed with. Computing systems available nowadays are powerful enough to compensate for penalties due to naive design patterns and even smartphone devices move into the focus for medical data processing [160], [161].

Programming environments such as Java target the hardware with particularly optimized software virtual machines (the JVM) to achieve highest performance. Java is of particular interest due its multi-platform capabilities by providing run-time engines for a wide variety of hardware platforms, such as smartphones [162]. Java [163] was designed with the goal to run “everywhere”, which is somewhat true, as it is possible to design a run-time engine for any new hardware platform if required but not fully applicable for Android environments. Matlab is a commercial system for high-level algorithmic design used in science and engineering, and promises sufficient performance on desktop systems and provides a large choice of add-on modules that can be purchased. Python is an open-source general purpose “gluing” [164], high-level scripting and programming language that intends to connect algorithmic tools from a wide choice of libraries and other languages available. It does not provide performance on its own but rather harvests the performance of native libraries that can be conveniently accessed using modules.

Other high-level languages not included but might be of interest are Mathematica (http://www.wolfram.com/mathematica/), Julia (www.julialang.org) and Octave (www.gnu.org/software/octave). Mathematica was not considered as it is similar to Matlab but with more emphasis on symbolic mathematical computation. Octave is a clone of the core language of
Matlab and provides an open source alternative to running Matlab-code, but it does not provide all the toolboxes and does not perform as well. Julia is advertised as a high-speed and high-level programming environment with a syntax similar to Python but is not known to be portable to mobile platforms. Due to Python’s capability to call Julia functions and the capability to import a wide variety of other programming environments, Python was considered for further discussion in this review. For performance evaluation of Python the NumPy [165] module was used. NumPy was selected, as it is the primary standard module for mathematical computation and matrix manipulation.

A.1.3 Methods of Comparison

For comparing the efficiency of programming environments, the plain processing speed was measured, and the used hardware configuration stated. The correlation-mapping (CM) algorithm was reimplemented [166] for each programming environment and then run for different data volumes. A simplified description of the CM algorithm is shown in the schematic (Figure A.1) with two image arrays and the movement of a kernel matrix over the images. The calculation of the correlation value for each position of the kernel matrix is then representing a difference that may indicate a flow or other dynamical processes in samples.

\[
corr = \sum \sum \frac{[I_A - \bar{I}_A][I_B - \bar{I}_B]}{\sqrt{[I_A - \bar{I}_A]^2[I_B - \bar{I}_B]^2}}
\]

Figure A.1: Simplified graphical representation of the correlation-mapping algorithm.
The code for the program was not optimized, and it is possible to achieve different performance results with more efficient algorithm or including parallel distribution of tasks.

### A.1.4 Speed Measurements

The speed measurements were performed on a Windows 7 64 bit environment (i7-2600, 3.4 GHz, 8 GB RAM) and a Linux environment with Mint 17.1 (kernel 3.16, i7-4710HQ, 3.4 GHz, 16 GB RAM) as shown in Figure A.2. For the algorithmic programming and data array handling with Python the mathematical toolbox NumPy was used, as indicated in the figure.

![Processing Speed vs Data Volume](image)

**Figure A.2:** Performance comparison for non-optimized code.

The results show the problem if relying solely on the evaluation of algorithmic performance in a monolithic programming environment. Perhaps the speed characteristics for Java are most dramatic, as it is not apparent how the underlying run-time engine can compensate for inefficient code structures. The Java implementation indeed shows an impressive performance on the particular hardware and software environment but may skew the expectations of performance on mobile platforms. Considering that Android is working with the Linux kernel and Dalvik as Java run-time environment the expectations of the speed for different algorithms can vary substantially [167]. Lin, Lin, Dow, *et al.* reports an average difference of performance of 34 percent between algorithms and coding in Java or native code.

Matlab performs well on both machines, although it cannot challenge the Java performance. The reasons for the slower speed are most likely found in the additional convenience functions to simplify the programming and coding of array structures.

The penalties using only list structures with Python are pronounced but can be quickly alleviated
using array structures from the NumPy module. Another example is the use of ctypes to set the variable types to C-code equivalents and let the module compile parts of our code into C. In particular, no actual C-code was touched, and all programming remained in the Python domain, and the processing speed could be increased matching nearly that of the Matlab processing speed. More significant may be, that with Python a broad set of modules support the export to Windows, Mac and Linux platforms, or the direct call of native function from system libraries. A similar approach can be taken with Matlab and the generation of C-code with the MEX compiler toolbox can accelerate algorithms in development. However, at the time of writing this document, no methods were known to generate code for mobile platforms directly.

A.1.5 Modules for Visualization and Research

Matlab provides 3D presentation capabilities including a separate program called Viewer3D on MatlabCentral. While with Java one would have the potential to program such a tool oneself, and ImageJ (www.imagej.net) is an excellent example of Java’s GUI and image processing capabilities. Although, ImageJ is capable to interpret Python-script as well, available modules for Python allow us to extend the functionality for visualizing scientific data within the native environment. For instance, to show isometric 3D volume data from a set of image-slices the MayaVi [168] module is a nice example for the multitude of options with Python (Figure A.3).

A more difficult and debatable subject is about the coding efficiency, meaning how much code needs to be written for a given problem. For Python, it is reported that it requires two to three times less code and about 50 percent less time to solve a given problem compared to Java and C/C++ [169]. The strength of Matlab is that it is easy to learn and to use [170] while enhanced programming techniques are in favor of simplicity somewhat suppressed. A
survey of top forty-eight US universities concluded with the recommendation to use Python “as a preferred language” [171].

A.1.6 Conclusion

In conclusion, it was shown that the measurement of raw speed on one hardware platform might not be sufficient to predict the performance of an algorithm if ported towards another hardware. The speed results with Java illustrate the risk of overestimating the performance, which was excellent on a Windows desktop computer but showed a considerable reduction of performance on a Linux system. Considering mobile platforms running on Android, which is using a Linux kernel variant, more detailed studies need to be included before an algorithm can be validated.

The ability to test algorithms on different environments and hardware platforms may require a significant amount of software engineering expertise. Although Matlab provides tools to generate C-code to accelerate Matlab-code, such code is not easily portable to other hardware platforms.

Python is an environment to “glue” different software libraries together, either available as binary or source code. Using Python’s NumPy package to do matrix calculations provides an excellent initial performance and is excellent for algorithmic design. For increased performance, one can select one of the modules available from a continuous growing online database. Python also provides modules for generating code for Mac, Linux and Windows, which makes it a competitive system in a fast paced environment marked with quickly changing hardware requirements. Also, the coding efficiency will help to develop more rapidly robust algorithms [172].

“Python has now entered a phase where it’s clearly a valid choice for high-level scientific code development, and its use is rapidly growing.” Pérez, Granger, and Hunter (2011)
A.2 Comments on the scripting with Python

The used language to demonstrate different selected signal properties was Python. However, for the sake of clarity and readability for readers not familiar with the Python syntax most of the syntactic optimization and simplifications were not used to keep the code as closely to plain text as possible.

Generation of list of numbers is achieved by according function calls as shown in the listing below.

```
Listing A.2-1: Generate list of numbers.

range(3) -> [0, 1, 2]
arrange(3.) -> array([0., 1., 2.])
linspace(start=0, stop=3, num=4) -> array([0., 1., 2., 3.])
```

The example in A.2-1 shows some additional helpful features such as named parameters as for the `linspace` functions are being used for clarity and are valid code that can be executed in this manner. The handling of number sequences can have multiple different representations, such as a plain list or vector, array, and matrix. No attempt is being made to explicitly distinguish those data types as it usually possible to recognize from the context. However, if conversion between one or the other container type is required then function such as `list(array) to convert an array to a plain sequence, and array(list) to convert an plain sequence (list) to an array may occur.

A particular helpful feature is the separation of code blocks which is easily achieved by aligning groups of code lines at the same level of indentation as shown in the schematics below.

```
< ... code ... >:
< code line >
< code line >
...
< ... code ... >:
...
}
code block
```

The colon ':' indicates the start of a new code block.

Compared to other scripting languages Python uses objects for almost all entities. That means an object entity provides function calls that can be accessed by simply writing `<entity>..<function_name>`. For example in the listing below (A.2-2) the function `linspace` returns an object (list of numbers) that provides a function `astype` among many other functions, that can be used to convert the list into another data type (here complex).

```
Listing A.2-2: Use of objects in Python

linspace(0, 3, 4).astype(complex) -> array([[0.+0.j, 1.+0.j, 2.+0.j, 3.+0.j]])
```

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Finally, Python does not require loops to index elements because each list of numbers is an object that returns an iterator. This means nothing else than to get an item in a list object in a step-by-step fashion as shown in A.2-3 which generates a list_of_values and does loop over each item without the need to provide an index.

Listing A.2-3: Python for-loop example.

```python
list_of_values = linspace(0,3,11)
for value in list_of_values:
    print(value, end=', ', flush=True)
>>> 0.0, 0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, 2.7, 3.0,
```

Indexing of elements can still be performed as usual as shown in listing A.2-4. However, it becomes immediately clear that the indexed syntax is less concise and can cause confusion about what is printed.

Listing A.2-4: Python for-loop example using indexing.

```python
list_of_values = linspace(0,3,11)
indexes = range(list_of_values.size)
for i in indexes:
    print(list_of_values[i], end=', ', flush=True)
>>> 0.0, 0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, 2.7, 3.0,
```

Finally most or all of the commands to label and annotate the plots are not shown to avoid cluttering the code with non-relevant information. The use of the `end` and the `flush` arguments for printing allow to arrange the list to be shown in one line as opposed to printing each number on a new line.
A.3 Fundamental scripts

We calculate the signal with explicitly given number of samples to be able to compare with different sample rates provided by some digitizer, using the number of samples $N$, sample rate $SR$, total acquisition or scanning time $\tau_{\text{scan}}$, wavelength $\lambda$, linear scanning reference mirror velocity $v_M$, and circular frequency $w$.

**Listing A.3-1: Monochromatic wave vs. time.**

```python
1  N = 20000  # buffer size
2  SR = 20e6  # sample rate [Hz] [1/s]
3  wavelength = 1330e-9  # [m]
4  v_M = 0.003  # [m/s]
5  tau_scan = linspace(0, N/SR, N).astype(complex)  # [s]
6  f_D = 2 * v_M / wavelength  # [1/s], [Hz]
7  w = 2*pi*f_D
8  coherent_wave = exp(-1j*w*tau_scan)
9  tau_scan_ms = tau_scan * 1e3  # x-axis: time
10 D_L_um = v_M * tau_scan * 1e6  # x-axis: space
11 plot(xaxes=[tau_scan_ms, D_L_um], yaxis=coherent_wave)
```

![Figure A.4: 1D wave relation of space and time generated with Listing A.3-1](image)

Please note that we control the sample rate and number of samples independently of the reference mirror scanning velocity. In practice, it will most often occur that the scanning mirror is oscillating around a zero position of some scan range, for instance $\Delta l_{\text{scan}} = 50 \mu m$, and the time of one sweep of the mirror scanning frequency should be matched with the sample rate and sample range of the digitizer. For simulation purpose we do not have this limitation can move the reference mirror linearly at any velocity. As an example in this code segment in conjunction with the plot below we have chosen the values such that the acquisition time $\tau_{\text{scan}}$ is 0.001 s.
and we can confirm the Doppler frequency based on the used wavelength \( \text{wavelength}=1330 \times 10^{-9} \text{ m} \) and mirror velocity \( v_M = 0.005 \text{ m/s} \) which results in about seven and one half of waves during that time.

A second axis for the spatial dimension (top) was added to estimate the wavelength from the plot directly and confirms that the Doppler frequency relative to the time is twice that of the source frequency.

Please take note about the special plot function that takes two x-axes (\( \text{tau\_scan} \) and \( D_{\text{L\_um}} \)) to demonstrate that we can obtain the mapping between time base and space without recalculating \( \text{coherent\_wave} \). The code contains only one line to calculate the wave with \( \text{tau\_scan} \) and we calculate only a second array for the x-values \( D_{\text{L\_um}} \). It should be noted that the digitizing time base is independent of the spatial space.
As it was already discussed in listing A.3-1 we can calculate the dimensions by mapping the x-axis values or by true re-calculation with the appropriate independent variable. To demonstrate the calculation of $\text{coherent\_wave}$ with the wavenumber $k$ and the scanning mirror moved by a distance $D_L$ we obtain the results directly as spatial values. Equally like we can estimate the wavelength of the source from the plot considering the double pass in the reference arm yielding $\lambda_D = 2\lambda_0$ with $\lambda_0$ the center wavelength of the source.

Figure A.5: 1D wave generated with Listing A.3-2
Listing A.3-3: Interference of broadband light source.

```python
N = 2e6  # buffer size
SR = 20e6  # sample rate [s]
tau_scan = linspace(0, N / SR, N)  # time range [s]
wavelength = 1330e-9  # [m]
wavelengthBW = 60e-9  # [m]
L_FWHM = 2*log(2)/pi * wavelength**2 / wavelengthBW
L_sigma = L_FWHM/2/sqrt(2*log(2))
D_k = pi/sqrt(log(2)) * wavelengthBW/wavelength**2
v_M = 0.05  # [m/s]
D_L = tau_scan * v_M  # spatial range [m]
D_L_um = D_L*1e6
f_D = 2 * v_M / wavelength  # [1/s]
L_0 = 23e-6  # [m]
K = 2*pi / wavelength
I_t = exp(-1j * 2 * K * D_L) * exp(-(D_L-L_0)**2 + (D_k**2))
plot(D_L_um,I_t)
```

This code (A.3-3) describes the computation of a single and ideal reflector in the sample arm for a conventional TD-OCT. The code idealizes the propagation and assumes an refractive index in vacuum or air and assumes no dispersion.

![Interference for Gaussian light source generated with Listing A.3-3](image)

Figure A.6: Interference for Gaussian light source generated with Listing A.3-3
Listing A.3-4: Computation of MR-OCT signal with five order of reflection vs. time.

```python
1  SN = 20000  # buffer size
2  SR = 20e6  # sample rate [1/s]
3  tau_scan = linspace(0, SN / SR, SN)  # time range [s]
4  wavelength = 1330e-9  # [m]
5  wavelengthBW = 60e-9  # [m]
6  L_FWHM = 2*log(2)/pi * wavelength**2 / wavelengthBW  # [m]
7  L_sigma = L_FWHM / sqrt(2*log(2))  # [m]
8  v_M = 0.1  # [m/s]
9  D = 20e-6  # spacing [m]
10 D_T = D / v_M  # spacing [s]
11 N_O = 5  # amount of orders of reflection
12 T_FWHM = L_FWHM / v_M  # [s]
13 T_sigma = L_sigma / v_M  # [s] sigma_w
14 f_D = 2 * v_M / wavelength  # [1/s]
15 t0 = 0.1e-3  # sample location [s]
16 w = 2*pi*f_D
17 I_t = []
18 for N in range(1,N_O+1):
19     I_t.append( exp(-1j * N * w * tau_scan) * exp(-(tau_scan-t0-(N-1)*D_T)**2 /
20                 (2*(T_sigma/N)**2)) )
21 plot(tau_scan*1e3,sum(I_t,axis=0))
```

Figure A.7: Linear MR-OCT signal with five orders generated with Listing A.3-4
Listing A.3-5: Computation of MR-OCT signal with five order of reflection vs. time with attenuation due to PM splitting ratio.

```python
SN = 20000  # buffer size
SR = 20e6  # sample rate [1/s]
tau_scan = linspace(0, SN / SR, SN)  # time range [s]
wavelength = 1330e-9  # [m]
wavelengthBW = 60e-9  # [m]
R_PM = 0.8  # reflectivity fraction of PM
L_FWHM = 2*log(2)/pi * wavelength**2 / wavelengthBW  # [m]
L_sigma = L_FWHM/2/sqrt(2*log(2))  # [m]
v_M = 0.1  # [m/s]
D = 20e-6  # spacing [m]
D_T = D/v_M  # spacing [s]
N_O = 5  # amount of orders of reflection
T_FWHM = L_FWHM / v_M  # [s]
T_sigma = L_sigma / v_M  # sigma_w
f_D = 2*v_M / wavelength  # [1/s]
t0 = 0.1e-3  # sample location [s]
w = 2*pi*f_D
I_t = []
def G(): return exp(-(tau_scan-t0-(N-1)*D_T)**2 / (2*(T_sigma/N)**2))  # envelope
def O(): return exp(-1j * N * w * tau_scan)  # carrier
def T_PM(): return (1-R_PM)**2 * R_PM**(N-1)  # attenuation PM splitting ratio
for N in range(1,N_O+1):
    I_t.append(T_PM() * G() * O())
plot(tau_scan*1e3, sum(I_t,axis=0))
```

Figure A.8: MR-OCT signals with attenuation of higher orders generated with Listing A.3-5
Please note that we use functions such as T_PM() to calculate the intensity roll-off due to the splitting ratio of the PM, O() to calculate the carrier frequency, and G() to calculate the Gaussian envelope.
Listing A.3-6: Computation of MR-OCT signal and applied frequency chirp due to scanning.

```python
1 SN = 60000 # buffer size
2 SR = 20e6 # sample rate [1/s]
3 tau_scan = linspace(0, SN / SR, SN) # time range [s]
4 wavelength = 1330e-9 # [m]
5 wavelengthBW = 60e-9 # [m]
6 R_PM = 0.8 # reflectivity fraction of PM
7 L_FWHM = 2*log(2)/pi * wavelength**2 / wavelengthBW # [m]
8 L_sigma = L_FWHM/2/sqrt(2*log(2)) # [m]
9 v_M = 0.03 # [m/s]
10 D = 16e-6 # spacing [m]
11 D_T = D/v_M # spacing [s]
12 N_O = 5 # amount of orders of reflection
13 T_FWHM = L_FWHM / v_M # [s]
14 T_sigma = L_sigma / v_M # [s] sigma_w
15 f_D = 2 * v_M / wavelength # [1/s]
16 v_M_phs = f_D * SN/SR # distortion due to scanning.
17 t0 = 0.6e-3 # sample location [s]
18 w = 2*pi*f_D
19 I_t = []
20 def G(N): return exp(-(tau_scan-t0-((N-1)*(D_T)))**2 / (2*(T_sigma/N)**2)) # envelope
21 def O(N): return exp(-1j*N * (w * tau_scan + v_M_phs)) # carrier
22 def T_PM(N): return (1-R_PM)**2 * R_PM**(N-1) # attenuation PM splitting ratio
23 for N in range(1,N_O):
24     I_t.append(T_PM(N) * G(N) * O(N))
25 plot(tau_scan*1e3,sum(I_t,axis=0))
```

Figure A.9: MR-OCT signal with non-linear phase due to simple harmonic motion of the SRM generated with Listing A.3-6
Listing A.3-7: Kronecker delta array creation and plotting.

```python
air = 1.0
ns = array([air, 1.001, 1.002, 1.003])
z_widths = array([15, 60, 90])
z_widths = z_widths * ns[0:-1]  # correct with ref. index
z_locs = z_widths.cumsum()
total_width = z_widths.sum()
z_rng_max = total_width * 2
z_rng = linspace(0, z_rng_max, SN)  # use sample length of E_i self.SN
getlocidx = interpolate.interp1d([0, z_rng_max], [0, SN])
rs_kd = zeros(SN)  # create empty Kronecker delta array -> all zeros
rjs = array([(np-nn)/(np+nn) for np, nn in zip(ns[0:-1], ns[1:])).squeeze()]
rs_kd[getlocidx(z_locs).astype(int)] = 1 * rjs  # We indicate the Kron. — 
Delta by explicitly using the value 1
```
A.4 Final simulation script

Listing A.4-8: Full simulation script.

```python
from scipy import *
from scipy.constants import speed_of_light
from scipy.fftpack import *
from scipy.signal import *
from scipy.optimize import fsolve
from numpy.random import normal, randn
import numpy as np
import matplotlib
matplotlib.use('Qt5Agg')
from matplotlib.pyplot import *
from matplotlib.path import Path
import warnings
import time
from scipy.interpolate import UnivariateSpline
from scipy import interpolate
import scipy as sy

class SimulationHelpers(object):
    def measure_FWHM(self, x, g):
        '''
        Measure FWHM by fitting a spline on any shape in g.
        Usually the shape should be a Gaussian envelope but other envelopes may
        produce some results as well, though then it is not a FWHM anymore.
        :param x:
        :param g:
        :return:
        '''
        spline = UnivariateSpline(x, g - max(g) / 2, s=0)
        rts = spline.roots()
        r1, r2 = rts.min(), rts.max()
        plt = UnivariateSpline(x, g - 0)(x)
        return abs(r1 - r2), plt, x

    def measure_FWHM_hr(self, x, y):
        '''
        Measure FWHM but make y real: y' = abs(hilbert(real(y))
```
@param x:
@param y:
@return:
'''
env = abs(hilbert(real(y)))
spline = UnivariateSpline(x,env−max(env)/2,s=0)
rts = spline.roots()
r1, r2 = rts.min(), rts.max()
plt = UnivariateSpline(x,env,s=0)(x)
return abs(r1−r2),plt,x

def measure_FWHM_h(self,x,y):
    '''
Measure FWHM by y’ = abs(hilbert(y))
    @param x:
    @param y:
    @return:
    '''
env = abs(hilbert(y))
spline = UnivariateSpline(x,env−max(env)/2,s=0)
rts = spline.roots()
r1, r2 = rts.min(), rts.max()
plt = UnivariateSpline(x,env,s=0)(x)
return abs(r1−r2),plt,x

class Sample(object):
    '''
The Sample object is providing all functions to generate reflecting sample layers.
E.g. initializing a sample by calling
    smp = Sample()
creates by default a three layer structure with widts = [15, 60, 90] and refractive
index = [1.001, 1.002, 1.003].
To create your own structure you would call
    smp = Sample(ns = [1.001,1.1], z_widths = [100, 200])
To plot the response of the sample response function along the sample z-axis you
need
also a light source
    src = Source()
which generates by default a center wavelength of 1330 nm and a bandwidth of 60 nm.
    smp.plot_H_td(src)
    ...
air = 1.0
ns = None
```
micrometer = 1e-6
z_widths = None

def __init__(self, ns=[1.001, 1.002, 1.003], z_widths=[15, 60, 90]):
    
    Initialize a sample layer structure
    @param ns:
    @param z_widths: in micrometer

    self.ns = array([self.air, ∗ns])
    self.z_widths = array(z_widths) ∗ self.micrometer

def __mul__(self, other):
    assert False, 'This is not implemented yet!
    pass

def kronecker_deltas(self, source, new_z_rng=None, verbose=False):
    
    The Kronecker deltas are actually just the r_j array – the reflectivities.
    **Return the Kronecker deltas**

    FFT(Kronecker delta) = int( r_s * exp(2ikz_s) ) = H. |n
    So FFT(Kronecker delta) = H. |n
    so |n
    H = r * exp(2ikz_n) |n
    and |n
    FFT(H) = convolve( r, exp(2ikz_n)). |n
    
    According to Izatt, Choma p52
    r = sum( r * kd(z_S - z_Sn)) where kd(...) is the delta for Kronecker delta.
    E_s = E_i * convolve(r, exp(2i∗k∗z_Sn)) |n
    then |n
    E_s = E_i ∗ H

    :param z_rng: If given this range is used to place the Kronecker deltas into
    :return: r_s_kd, z_rng, z_rng_max
    
    _ = self
    SN = source.SN
    z_widths = _.z_widths * _.ns[0:-1]
    z_locs = z_widths.cumsum()
    # print('z_locs',z_locs)
    if verbose:
        print('kronecker_deltas.ns',_.ns)
        print('kronecker_deltas.z',_.z_widths)
        print('kronecker_deltas.z_locs',z_locs)
        cum_width = z_widths.sum()
        if type(new_z_rng) is list or type(new_z_rng) is ndarray:
            # assert cum_width ∗ 4 <= max(new_z_rng), 'The new_z_rng is too small
            # cum_width=4, new_z_rng=4: format(cum_width∗4,max(new_z_rng))
            z_rng_max = max(new_z_rng)
        else:
```

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```
z_rng_max = cum_width*4
z_rng = linspace(-z_rng_max,z_rng_max,SN)
getlocidx = interpolate.interp1d([-z_rng_max,z_rng_max],[-SN, SN])

rs_kd = zeros(SN)  # create empty Kronecker delta array  ->  all zeros
rjs = array([(np-nn)/(np+nn) for np,nn in zip(_.ns[0:-1],_.ns[1:])]).squeeze()

try:
    rs_kd[getlocidx(-z_locs).astype(int)] = 1 * rjs
except ValueError as ve:
    print('z_locs',z_locs)
    print('z_rng_max',z_rng_max)
    raise(ValueError('z_locs',z_locs,'z_rng_max',z_rng_max))

rs_kd = fftshift(rs_kd)  # reverse to correct orientation.

return rs_kd, z_rng, z_rng_max

def plot_kronecker(self,source,do_plot=True,do_save=False):
    '''
    Plot the Kronecker delta based on positions z_widths and refractive indexes 'ns'.
    See more details below.
    
    :param do_plot:
    :param do_save:
    :return:
    
    **To get the Kronecker deltas call kronecker_deltas**
    
    Principally FFT(Kronecker deltas) == H.
    so
    
    H = r * exp(2ikz_n)  
    and
    
    FFT(H) = convolve( r , exp(2ikz_n)).
    
    According to Izatt, Choma p52
    
    r = sum( r * kd(z_S - z_Sn)) where kd(...) is the delta for Kronecker delta.
    then
    
    E_s = E_i * convolve(r , exp(2i*k*z_Sn))
    
    then
    
    E_s = E_i * H
    
    **
    
    rs_kd,z_rng,z_rng_max = self.kronecker_deltas(source)
    
    if do_plot:
        figure('kronecker deltas',tight_layout=True)
        plot(z_rng+1e6,abs(rs_kd),'-,-',lw=0.5)
        # xlim((0,z_rng_max+1e6))
        xlim((0,200))
        title('Kronecker delta')
        xlabel('z ($\text{\mu m}$)')
    ```
ylabel('field reflectivity $r_j$')

if do_save: savefig('kronecker_deltas.pdf')

def generate_H(self, source):
    '''
    Compute the sample response function $H$ and return.
    **Please take note that FFT($H$) = Kronecker deltas.**
    :return: $H$, $z_{rng}$
    
    The SRF is defined for one layer as (Ch2: eq 1.12) as
    $H = r \cdot \exp(2i * \pi * c * z_s) = r \cdot \exp(2i * \pi * w / c * z_s)$.
    
    According to Izatt, Choma p53
    the sample field is calculated as
    $E_S = E_i \cdot \text{convolve}( r, \exp(2i * \pi * k * z_z) )$
    $E_S = E_i \cdot H$

    This function computes $H$ for multiple layers and in principle means
    to track all interface positions $z$ and the layers with refractive index $n$ between.
    Therefore, a layer boundary at $z[n]$ needs to account for all $n[n-1]$: $n[n-1] = \sum(n[0..n-1] * z[0..n])$ (see code). $n$
    $H_j = r_j \cdot \exp( 2i * \pi * w / c * \sum( n[z] ) )$ for all covering layers $n$, and $z$. $n$
    $H = \sum( H_j )$
    '''
    _ = self
    print('generate_H.ns',_.ns)
    print('generate_H.z',_.z_widths)

    r_j_f = lambda n1,n2: (n1-n2)/(n1+n2)
    src = source
    w_rng = src.w_rng
    SN = src.SN
    c = speed_of_light
    f_max = 2 * w_rng[-1] / 2 / pi  # note here that we use 2 x the _.w_rng due to
    # the double pass.
    ST = 1 / f_max * SN  # s
    z = ST * c  # s * m/s == m
    z_rng = linspace(-z/2,z/2,SN)

    z_widths = _.z_widths * _.ns[0:-1]  # correct with ref. index
    Z_j = z_widths.cumsum()
    rjs = array([(n1-n2)/(n1+n2) for n1,n2 in zip(_.ns[0:-1],_.ns[1:])])
    Hj = []
    for r_j,z_j in zip(rjs,Z_j):
        Hj.append(r_j * exp( 1j * 2 * w_rng / c * z_j))
    H = sum(Hj,axis=0)

    return H, z_rng

def generate_SRM_PM(self,source, spacing=[]):
    '''
Generate multiple reflecting scanning layers. In principle the same as the sample response function, with the addition to allow to set a spacing.

@param source: 
@param spacing: array to zero out values to create a spacing. 
@return: H, z_rng
'''
assert not any(spacing), 'Spacing is not used yet!'

r_j_f = lambda n1,n2: (n1−n2)/(n1+n2)
src = source
w_rng = src.w_rng
SN = src.SN
c = speed_of_light
f_max = 2 * w_rng[−1]/2/pi # note here that we use 2 x the _w_rng due to the double pass.
ST = 1/f_max*SN #s
z = ST * c # s * m/s == m
z_rng = linspace(−z/2,z/2,SN)

z_widths = _z_widths * _ns[0:−1] # correct with ref. index
Z_j = z_widths.cumsum()
rjs = array([(n1−n2)/(n1+n2) for n1,n2 in zip(_ns[0:−1],_ns[1:])])
Hj = []
for r_j,z_j in zip(rjs,Z_j):
    hj.append(r_j * exp( 1j * 2 * w_rng / c * z_j))
H = sum(Hj,axis=0)

return H, z_rng

def plot_H_td(self, source, do_plot=True, do_save=False, tukey_alpha=False):
    
    Generate the H according to Tomlins 2005, eq.11. 
    H(w) is the spectral modulation depending on depth!
    
    more notes below.

:param do_plot:
:param do_save:
:return:

Tomlins states for a TD-OCT
I(d,z) = 1/4 * int( S * (H**2 + 1) + 1/2* int(S * H * exp( PHI(z)) )

Take note that the integrals convert the spectrum into fields and then intensity fractions 
are the results.
H, z_rng = self.generate_H(source)
SN = source.SN
if tukey_alpha is type(number):
    apply_tukey = True
else:
    apply_tukey = False
if apply_tukey:
    # tukey_alpha = 0.9 was tested but the
    print('Apply Tukey window for plot_H.
    tukwin = tukey(len(H),alpha=tukey_alpha,sym=False)
H = tukwin*H
if do_plot:
    figure(num='sample resp fun',tight_layout=True)
    plot(z_rng, abs(fftshift(fft(H)))/SN,'.-',lw=0.5)
    xlim((0,200))
    title('Sample response function FFT(H)')
    xlabel('z ($\mu m$)')
    ylabel('reflectivity $r_j$')
    if do_save:
        savefig('sample_response_function.pdf')

def plot_H_freqDom(self,source,do_plot=True,do_save=True):
    ...
    Please take note that the SRF alone is only of limited use although the FFT can
    be used
    if it consistent with the Kronecker deltas.
    :param do_plot:
    :param do_save:
    :return:
    ...
H, z_rng = self.generate_H(source)
if do_plot:
    figure(num='sample resp fun FD',tight_layout=True)
    subplot(211)
    plot(H )#,'.',ms=1.5)
    title('${\mathcal{H}(\omega)}$')
    subplot(212)
    plot(H )#,'.',ms=1.5)
    xlim((0,200))
    title('${\mathcal{H}(\omega)}$(zoom)')
    if do_save:
        savefig('sample_response_function_freqDom.pdf')
class Source(SimulationHelpers):
    ...
    Source describes the spectral content of the light source.
    ...
    SN = 2 ** 16
c = speed_of_light
range_factor = 50  # How much more relative to the CWL of range should be
generated?
# This has inverse impact on the freq range or spatial range.
# This is somewhat similar to zero padding sufficient sample points.
# Initialize all values to None to make them to exist here.
# Values are set below within __init__.
CWL = None
BW = None  # 50 nm
FWHM_psf = None
sigma = None
WL_rng_max = None
WL_rng = None  # wavelength range to use for
WLR = None
f_0 = None
w_0 = None
FWHM_w = None
sigma_w = None
w_rng_max = None
w_rng = None
SRF = 'SRF'
KRN = 'Kron'
def __init__(self, center_wavelength=1330e-9, bandwidth=60e-9):
    '''Print values only if object is used and initialized.'''
    self.CWL = center_wavelength
    self.BW = bandwidth
    self.FWHM_psf = 2 * log(2) / pi * self.CWL ** 2 / self.BW
    # note here that we use 2 x the w_rng due to the double pass.
    # todo Perform this for frequency here as well.
    self.sigma = self.BW / sqrt(8 * log(2))
    self.WL_rng_max = self.CWL + self.CWL * self.range_factor
    self.WL_rng = linspace(0, self.WL_rng_max, self.SN)  # wavelength range to
    use for
    self.WLR = self.WL_rng
    self.f_0 = self.c / self.CWL
    self.w_0 = 2 * pi * self.c / self.CWL
    self.FWHM_w = 2 * pi * self.c * self.BW / self.CWL ** 2
    # note here that we use 2 x the w_rng due to the double pass.
    self.sigma_w = self.FWHM_w / sqrt(8 * log(2))
    # w_rng = linspace(w_0 - sigma_w*5, w_0 + sigma_w*5, SN)
    self.w_rng_max = self.w_0 + self.w_0 * self.range_factor
    self.w_rng = linspace(0, self.w_rng_max, self.SN)
print('CWL',self.CWL)
print('BW',self.BW)
print('FWHM_z {:.1f} um'.format(self.FWHM_psf * 1e6))
print('f_0 {} THz'.format(self.f_0 * 1e12))
print('FWHM_w {:.1e} rad/s/THz'.format(self.FWHM_w / 2 / pi * 1e-12))
print('Sigma_w {:.1e} rad/s/THz'.format(self.sigma_w))

if 'PSF th' in normalize:
    norm_value = 1/sqrt(2*pi/self.CWL**2)
elif 'probability' in normalize:
    # see http://mathworld.wolfram.com/GaussianFunction.html
    print('Normalize for probability.')
    norm_value = 1/s_w/sqrt(2*pi/2)
elif 'liu' in normalize:
    print('Normalize acc. to Liu p28.')
    norm_value = sqrt(2*pi/s_w**2) # see Liu p28
else:
    print('No normalization.')
    norm_value = 1/sqrt(s_w**2 / pi) 
    # normalize = 1/sqrt(2*pi/s_w**2) # this generates a spectrum exactly 1
    # S = sqrt(2*pi / s_w ** 2) * exp(-(w - w0) ** 2 / (2 * s_w ** 2)) * norm_value
    S = sqrt(2*pi / s_w ** 2) * exp(-(w - w0) ** 2 / (2 * s_w ** 2)) * norm_value
    return S, w, w0, s_w
def get_E_w_i(self):
    
    Return the source field in the spectral domain.
    :param mode: Source.SRF or Source.KRN this is currently required due to FFT
double sided changing the z_rng.
    :return: E_w_i, z_rng
    
    _ = self
    c = speed_of_light
    f_max = 2 * _.w_rng[-1]/2/pi  # note here that we use 2 x the _.w_rng due to
    print("f_max",f_max,'Hz')
    ST = 1/f_max*_.SN  #s
    print("ST",ST,'s')
    z = ST * c  # s * m/s == m
    zC = z/2*1e6
    print("zC",zC)
    print("z",z)  # s * m/s == m
    print("z",z)  # s * m/s == m
    z_rng = linspace(0,z,_.SN)*1e6
    S_w_w0 = self.spectrum(_.w_rng, _.w_0, _.sigma_w*2*sqrt(2))[0]
    E_i = S_w_w0
    return E_i, z_rng


def get_E_i_td(self,sigma_x = 1,w_x =1):
    
    Return the field from the source.
    :return: E_i, z_rng
    
    _ = self
    c = speed_of_light
    f_max = 2 * _.w_rng[-1]/2/pi  # note here that we use 2 x the _.w_rng due to
    print("f_max",f_max,'Hz')
    ST = 1/f_max*_.SN  #s
    print("ST",ST,'s')
    z = ST * c  # s * m/s == m
    zC = z/2
    print("zC",zC+1e6)
    print("z",z)  # s * m/s == m
    print("z",z)  # s * m/s == m
    z_rng = linspace(0,z,_.SN)
    S_w_w0 = self.spectrum(_.w_rng, _.w_0*_.w_x, _.sigma_w*sigma_x )[0]
    E_i_td = fftshift(fft(S_w_w0))
    return E_i_td, z_rng


def plot_E_i_td(self, do_plot=False, do_save=False, do_envelope_hilbert=False, 
do_envelope_absolute=False):
    
    According to Izatt, Choma p52
    
    E_i = s(k,w) * exp(i(kz - wt))
    
    this is also

   195
$E_i = \text{FFT}(S(k,w))$

Please take note that in this case the spatial spacing must be adapted by $2\pi$ or the bandwidth to obtain the right width of the PSF.

Otherwise and currently the $\sigma = \text{FWHM} / \sqrt{2 \pi \cdot 8 \log(2)}$ the $2\pi$ conversion is required to account for the FFT operation.

:param mode: Possibly we should use src.SRF if the KRD is now double sided.
:param do_save: Save a plot.
:return: $E_i$, $z$ range

...
APPENDIX A. PROGRAMMING LANGUAGE PERFORMANCE AND PROGRAM SCRIPTS

if do_envelope_absolute:
    envelope_on_absolute_values()

xlabel('z (um)')
xlim(array([zC−25,zC+25]))
legend()

if do_save:
    savefig('source{:1.0f}nm_rf{:3.0f}_SN{}_space_z.pdf'.format(_.CWL*1e9,_.range_factor*100,_.SN))

return E_i_td,z_rng

def __mul__(self, other):
    assert False, 'This is not implemented yet!'

pass

def plot_Freq(self,do_save=False):
    _ = self
    S_w_w0 = self.spectrum(_.w_rng, _.w_0, _.sigma_w)[0]
    figure(num='frequency',tight_layout=True)
    plot(_.w_rng/2/pi*1e−12, S_w_w0/S_w_w0.max(),'.−',lw=0.5,label='$\lambda_0$={:1.0f} nm, $\Delta\lambda$ = {:1.0f} nm'.format(_.CWL*1e9,_.BW*1e9))
    # stem(_.w_rng/2/pi*1e−12, S_w_w0/S_w_w0.max(),basefmt=' ',label='$\lambda_0$={:1.0f} nm, $\Delta\lambda$ = {:1.0f} nm'.format(_.CWL*1e9,_.BW*1e9))
    sw = 5
    xlim(array([_.w_0−_.sigma_w*sw,_.w_0+_.sigma_w*sw])/2/pi*1e−12)
    grid(True)
    xlabel('Frequency (THz)')
    ylabel('Power vs. frequency (a.u.)')
    title('Plotted with {} sigma frequency width,
$f_{max}$={:1.0f} THz'.format(sw,_.w_rng−1/2/pi*1e−12))
    legend(loc='upper right')
    if do_save:
        savefig('source{:1.0f}nm_rf{:3.0f}_SN{}_freq.pdf'.format(_.CWL*1e9,_.range_factor*100,_.SN))

def plot_WL(self,do_save=False):
    _ = self
    S_WLR_CWL = self.spectrum(_.WLR, _.CWL, _.sigma/2)[0]
    figure(num='wavelength',tight_layout=True)
    plot(_.WLR*1e9, S_WLR_CWL ,'.−',lw=0.5,label='$\lambda_0$={:1.0f} nm, $\Delta\lambda$ = {:1.0f} nm'.format(_.CWL*1e9,_.BW*1e9))
    # stem(WLR*1e9, S_WLR_CWL/S_WLR_CWL.max(),basefmt=' ',label='$\lambda_0$={:1.0f} nm, $\Delta\lambda$ = {:1.0f} nm'.format(CWL*1e9,BW*1e9))
    sw = 5 # show times more than the BW
    xlim(array([_.CWL−_.sigma*sw,_.CWL+_.sigma*sw])*1e9)
    grid(True)
    xlabel('Wavelength (nm)')
    ylabel('Power vs. wavelength (a.u.)')
title('Plotted with {} sigma wavelength width, n {}\lambda_{max}\$={:1.0f} nm'. format(sw,.WLR[-1]*1e9))

legend(loc='upper right')

if do_save:
    savefig('source_{:1.0f}nm_rf{:3.0f}_SN{}.pdf'. format(_.CWL*1e9,self.
    range_factor*100,_.SN))

def plot_circFreq(self,do_save=False):
    _ = self
    S_w = self.spectrum(_.w_rng, _.w_0, _.sigma_w)[0]
    figure(num='circular frequency',tight_layout=True)
    plot(_.w_rng*1e-12, S_w,'.',lw=0.5,label='$\lambda_0$={:1.0f} nm, $\Delta \lambda$ = {:1.0f} nm'. format(_.CWL*1e9,_.BW*1e9))
    # stem(WLR*1e9, S_WLR_CWL/S_WLR_CWL.max(),basefmt=' ',label='$\lambda_0$={:1.0f} nm, $\Delta \lambda$ = {:1.0f} nm'.format(CWL*1e9,BW*1e9))
    sw = 5 # show times more than the BW
    xlim(array([_.w_0-_.sigma_w*sw,_.w_0+_.sigma_w*sw])*1e-12)
    grid(True)
    xlabel('$\omega$ (T rad/s)')
    ylabel('Power vs. $\omega$ (a.u.)')
    title('Plotted with {} sigma wavelength width, range {}$\times$1e12 T rad/s'. format(sw,_.range_factor*100))
    legend(loc='upper right')
    if do_save:
        savefig('source_{:1.0f}nm_rf{:3.0f}_SN{}.pdf'. format(_.CWL*1e9,self.
        range_factor*100,_.SN))

class Simulate_TD(SimulationHelpers):
    def __init__(self):
        rcParams['text.usetex']=True
        rcParams['font.size']=24
        rcParams['lines.linewidth']=0.5

    def run_simu_TD(self):
        um = 1e6
        c = speed_of_light
        # Simulate for Tomlins data
        smp = Sample(ns=[1.3,1.5,1.0],z_widths=[5,15,30])
        src = Source(center_wavelength=800e-9, bandwidth=50e-9)

        E_t_i, ez_rng = src.get_E_i_td(sigma_x=(4*sqrt(2)), w_x=2)
        E_t_i /= E_t_i.max()

        figure(tight_layout=True)
        plot(ez_rng*um, real(E_t_i)),title('Source field')
        rs_kd, kz_rng, z_rng_max = smp.kronecker_deltas(src,new_z_rng=ez_rng)
        H, hz_rng = smp.generate_H(src)
        # correct the SRF frequency powers
        alpha = 1.0
        win = tukey(len(H), alpha=alpha, sym=False)
SN = src.SN
H = win * H

figure(tight_layout=True)
plot(kz_rng*um, abs(rs_kd), label='KRN')
plot(hz_rng*um, abs(fftshift(fft(H))/src.SN, label='SRF')
plot(hz_rng*um, win)
ylabel('Intensity (arb.)')
xlabel('Displacement in air (\textmu m)')
# xlim((0, 80))

E_t_s = convolve(E_t_i, rs_kd, mode='same')

figure(tight_layout=True)
title(r'$E(t)_S$ Kronecker delta')
plot(kz_rng, real(E_t_s))
# position OK
ylabel('Reflectivity (n.u.)')
xlabel('Displacement in air (\textmu m)')
# xlim((0, 80))
# ylim((-40, 40))

# measure one layer PSF
def measure():
  E_t_s[0:36500] = 0
  E_t_s[37800:] = 0
  plot(kz_rng, real(E_t_s))
  psf, plt, x = self.measure_FWHM_hr(kz_rng, E_t_s)
  print('PSF kr', psf) # PSF OK, Frequ. OK
  plot(x, plt, label='psf_kr= {:1.6} \mathrm{\mu m}'.format(psf)), 
  grid(True)
measure()

E_t_i2, ez_rng = src.get_E_i_td(sigma_x=2*sqrt(2), w_x=1)
E_t_i2 /= E_t_i2.max()

# divide by SN to normalize getting reflectivities
E_t_s2 = convolve(E_t_i2, abs(fftshift(fft(H)))/src.SN, mode='same')
figure(tight_layout=True)
title(r'$E(t)_S$ Sample response function')
plot(hz_rng*um, real(E_t_s2))
ylabel('Intensity (arb.)')
xlabel('Displacement in air (\textmu m)')
# xlim((0, 80))
# ylim((-40, 40))

# measure one layer PSF
def measure():
  E_t_s2[0:40000] = 0
  E_t_s2[43000:] = 0
  plot(hz_rng*um, real(E_t_s2))
  psf, plt, x = self.measure_FWHM_hr(hz_rng*um, E_t_s2)
print(’PSF srf’, psf)  # PSF OK, Freq. OK
plot(x, plt, label=’psf_h= {:1.6} $\mathrm{{\mu m}}$’.format(psf)), grid
(True)
measure()
E_w_i3, wz_rng = src.get_E_w_i()
# E_w_i3 /= E_w_i3.max()
E_t_i3 = abs(fftshift(fft(E_w_i3*H)))
figure(tight_layout=True)
title(’FFT\{E(\omega)_S \cdot H(\omega)\}'
plot(hz_rng*um, E_t_i3)
ylabel(’Intensity (arb.)’)
xlabel(’Displacement in air (\textmu m)’)’
xlim((0,100))

def measure():
    figure(tight_layout=True), title(’Measure psf’)
    plot(z_rng[47000:51000],E_t_s[47000:51000])
    psf,plt,x = self.measure_FWHM_h(ez_rng[47000:51000],(E_t_s[47000:51000]))
    print(’PSF’,psf)
    plot(x,plt,label=’psf= {:1.6} $\mathrm{{\mu m}}$’
        format(psf)),grid(True)
legend()
# measure()

class Simulate_SD(SimulationHelpers):
    ‘’’
    This basically means to take the E_s(w) and E_r(w) and compute the
    E_d(w) = E_S(w) + E_R(w).
    Because we have the H(w)*E_i(w) = E_S(w) and
    E_i(w) == E_r(w).
    ‘’’
    def __init__(self):
        rcParams[‘text.usetex’] = True
        rcParams[‘text.latex.preamble’]=[’\usepackage{siunitx}’]
        rcParams[‘font.size’] = 20
        rcParams[‘lines.linewidth’] = 1.0
        rcParams[‘lines.markersize’] = 1.0

    def run_simu_SD(self):
        um = 1e6
        nm = 1e9
        c = speed_of_light
        # Simulate for Tomlins data
        smp = Sample(ns=[1.3, 1.5, 1.0], z_widths=[5, 15, 30])
        src = Source(center_wavelength=800e-9, bandwidth=50e-9)
        w_rng = src.w_rng
        w_0 = src.w_0
        scale_sigma = 4
sigma_w = src.sigma_w * scale_sigma
S_w, w, w_0, s_w = src.spectrum(w_rng, w_0*2, sigma_w)

WL_rng = src.WL_rng * nm
CWL = src.CWL * nm
BW = src.BW * nm

figure(tight_layout=True)
title('Source spectrum $S(\omega)$')
plot(WL_rng, S_w)
xlabel('Wavelength (nm)')
ylabel('Power (arb.)')
xlim((CWL - 5 * BW, CWL + 5 * BW))

H_w, hz_rng = smp.generate_H(src)
E_w_s = S_w * H_w
E_w_r = S_w
E_w_d = E_w_r + E_w_s

k_max = 2*pi/src.WL_rng[-1]
with warnings.catch_warnings(record=True) as w:
    # Accept division by zero.
    # This has only impact on the image having missing data points where it
    # happens.
    warnings.filterwarnings('always')
k_rng = 2*pi/src.WL_rng
k_0 = pi/src.CWL

figure(tight_layout=True)
title('Interferogram')
plot(k_rng/1e3, abs(E_w_d)**2)
view_rng = (k_0/1e3-.25e3,k_0/1e3+.25e3)
xlim(view_rng)
gca().set_xticks([view_rng[0],k_0/1e3,view_rng[1]])
xlabel('k (1/mm)')
ylabel('Power (arb.)')

I_w_d = abs(E_w_d)**2
I_z = abs(fftshift(ifft(I_w_d)))

figure(tight_layout=True)
title('A-line')
plot(hz_rng*um, I_z, label='A-line')
# scale the H_w by sqrt( samples ) to get matching powers
plot(hz_rng*um, abs(fftshift(ifft(real(H_w))))/sqrt(src.SN), label='$r\delta(z)$')

legend()
xlim((0,80)),ylim((-.00001,.00045))
xlabel('z (\SI{}{\micro\meter})')
ylabel('Intenstiy (arb.)')

class Simulate_MRO(SimulationHelpers):
    """
Please be aware that all orders are simulated beginning to scan directly from the PM. That means all orders overlap over the full scan range and no segments are considered here.

To simplify the processing at this stage no order segments were simulated. Otherwise a full A-line reconstruction algorithm must be included if desired to do so.

In this case, within the for loop, the sample creation must be called as many times as orders of reflections are considered and the desired z-range must be adapted.

This does not yet construct overlapping orders for which the Sample() object would need to be modified if required.

Also the simulation can be arbitrarily scaled to any depth or scan width. No particular parameter is yet provided to adjust a physical spatial dimension to the pixel number. This can be improved in future versions.

Currently, any scaling is done directly during plotting for whatever shape required.

```python

```
for z in arange(0, 1500, 10):
    rs_kd1, zr = Sample(ns=[1.04], z_widths=[0.0 + z]).kronecker_deltas(src, new_z_rng=ez_rng)[0:2]
    E_t_s1 = convolve(E_t_i, rs_kd1, mode='same')  # sample field
    E_t_d1 = convolve(E_t_r, E_t_s1, mode='same')  # scanning
    E_t_d1 /= E_t_d1.max()
    a_line = log(abs(E_t_d1))**2
    bframe = concatenate((bframe, [a_line]))

figure(1)
plot(zr, a_line)
figure(gcf().number + 1)
imshow(bframe[:, src.SN // 2:], aspect=220, 
      vmin=-55, vmax=10 * src.SN)
colorbar(ax=gca())

def simulate_MRO_O2(self):
    '''
    Simulate a single calibration line with artifacts summing additional parasitic sample signals.
    @return:
    '''
    um = 1e6
    nm = 1e9
c = speed_of_light
src = Source(center_wavelength=1300e-9, bandwidth=60e-9)
w_rng = src.w_rng
w_0 = src.w_0
sigma_w = src.sigma_w
scale = 30

S_w = src.spectrum(w_rng, w_0, sigma_w * 4 * scale / 10)[0]
# initial z range of TD source field
f_max = 2 * w_rng[-1] / pi
ez = 100 / f_max * src.SN * c  # set scan range
print('ez (um)', ez * 1e6)
ez_rng = linspace(0, ez, src.SN)
WL_rng = src.WL_rng * nm
CWL = src.CWL * nm
BW = src.BW * nm

E_t_i = fftshift(fft(S_w))
E_t_r = E_t_i / E_t_i.max()  # ideal scanning mirror
Sw1 = src.spectrum(w_rng, w_0, sigma_w * 4 * 2 * scale / 10)[0]  # order 1
Sw2 = src.spectrum(w_rng, w_0, sigma_w * 4 * 3 * scale / 10)[0]
E_t_r1 = fftshift(fft(Sw1))  # virtual scanning field
E_t_i2 = fftshift(fft(Sw2))
bframe = zeros((1, src.SN))
```python
i1 = [1,2]
print('i1',i1)
i2 = [2,4]
print('i2',i2)

for z in arange(0,1500,10): # in um
    zn = [z/i for i in range(0,10)]
    acd1 = zn[i1[0]] - zn[i1[1]]
    acd2 = zn[i2[0]] - zn[i2[1]]

    rs_kd1, zr = Sample(ns=[1+1e−01],z_widths=[0.0+zn[1]+scale]).
        kronecker_deltas(src,new_z_rng=ez_rng)[0:2]
    rs_ac1, zr = Sample(ns=[1+1e−11],z_widths=[0.0+acd1+scale]).
        kronecker_deltas(src,new_z_rng=ez_rng)[0:2]
    rs_ac2, zr = Sample(ns=[1+1e−15],z_widths=[0.0+acd2+scale]).
        kronecker_deltas(src,new_z_rng=ez_rng)[0:2]

    E_t_s1 = convolve(E_t_i, rs_kd1, mode='same') # sample field
    E_t_ac1 = convolve(E_t_i, rs_ac1, mode='same')
    E_t_ac2 = convolve(E_t_i, rs_ac2, mode='same')

    E_t_sc1 = convolve(E_t_r, E_t_s1, mode='same') # scanning
    E_t_sac1 = convolve(E_t_r1, E_t_ac1, mode='same') # scanning 1st inter order
    E_t_sac2 = convolve(E_t_r1, E_t_ac2, mode='same') # scanning 2st inter order

    E_t_d = E_t_sc1 + E_t_sac1 + E_t_sac2

    a_line = log10(abs((E_t_d))**2)
    bframe = concatenate((bframe, [a_line]))

figure()
imshow(bframe[0:125,src.SN//2:61700], cmap='CMRmap', aspect=220, vmin=-40, vmax=0)
    colorbar(ax=gca())

# uncomment the desired operation
# Source().plot_E_i_td(do_plot=True)
# Source().plot_WL()
# Source().plot_circFreq()
# Source().plot_Freq()

Simulate_TD().run_simu_TD()

# Simulate_SD().run_simu_SD()

# Simulate_MRO().simulate_MRO_O1()
```
# Simulate_MRO().simulate_MRO_O2()

show()  # keep here to see plots
APPENDIX B

Definitions

B.1 Wiener-Khinchin Theorem

The Wiener Khinchin Theorem is assumed throughout this work to describe the conversion of a Gaussian spectrum from the frequency domain and vice versa [58, p98]. That means an electric field can be expressed mathematically as

\[ E(\omega) = \int_{-\infty}^{\infty} E(t) \exp[i2\pi\omega t] \, dt \]  

(B.1)

where \( E(\omega) \) is the field in the frequency domain and the right side of Eq. B.1 is the sum of frequency components \( E(t) \exp[i2\pi\omega t] \) in the time domain. A complete mathematical description of the theorem is given elsewhere [173], and the concept can be summarized using the relationship between the autocorrelation function and the power spectral density of a wide sense stationary stochastic process. This can be expressed with Eq. B.2 that shows that the autocorrelation function equals the power spectral density of the time-domain function

\[ ACF(E) = |E(\omega)|^2 = \iint E^*(t)E(t+\tau) \exp[i2\pi\omega t] \, dt \, d\tau. \]  

(B.2)

or more concise the Fourier transform of the autocorrelation function is the power spectrum [174]–[176]
APPENDIX B. DEFINITIONS

\[
\text{FFT}\{ACF(E(t))\} = |E(\omega)|^2. \quad (B.3)
\]

The Eq. B.1 can be rewritten for discrete values

\[
E(\omega) = \sum_{t=-\infty}^{\infty} E(t) \exp[i2\pi\omega t] \quad (B.4)
\]

which was used for the numerical computation and the generation of the interference signals based on the power spectrum of the light source. It should be noted that in practice Eq. B.4 will have finite boundaries and does not meet the definition of a wide sense stationary process anymore and is considered to be an approximation. The choice of the boundary conditions vs. accuracy is discussed in section 2.7.4.

B.2 OCT resolution: FWHM of the Gaussian envelope

The definition of the coherence length \(l_c\) are given in section 2.6 and section 2.7.2 without any further explanation. This section provides some more details about the deduction of the coherence function and the coherence length or FWHM of the Gaussian envelope.

In section 2.7.2 it was explained that the detector current \(I_D\) is the squared time average of the sum of the reference and sample fields

\[
I_D = \rho \langle |E_R + E_S|^2 \rangle = \langle |E_D|^2 \rangle \quad (B.5) = \frac{\rho}{2} (\langle E_S E_S^* \rangle + \langle E_R E_R^* \rangle + 2\Re\{\langle E_S E_R^* \rangle\}). \quad (B.6)
\]

The last term in Eq. B.6 is the cross correlation between the fields and assuming that the sample arm is a mirror it is the auto correlation of the source spectrum.

Based on the Wiener-Khinchin theorem it was shown that power spectrum on the detector \(S(\omega)\) can be expressed with the autocorrelation function Eq. B.2 or in the case of using a sample mirror \(E_S \equiv E_R\) then the cross-correlation is also the autocorrelation and

\[
S(\omega) \exp[-i\Delta\Phi(\omega)] = |E_S E_R^*|. \quad (B.7)
\]
The time averaged signal is then the sum of all frequency components $d\omega$ and by anticipating a Gaussian distribution where the normalized integral is $\int_{-\infty}^{\infty} S(\omega) \frac{d\omega}{2\pi} = 1$ the detector current is proportional to the sum of all frequency components [58], [60]

$$I_D \propto \Re \left\{ \int_{-\infty}^{\infty} S(\omega) \exp[-i\Delta \Phi(\omega)] \frac{d\omega}{2\pi} \right\} = \Re \left\{ \int_{-\infty}^{\infty} |E_S E_R^*| \frac{d\omega}{2\pi} \right\}. \quad (B.8)$$

The phase delay $\Delta \Phi(\omega)$ of the high frequency spectrum is the difference between the reference and sample arm $2k\Delta z$. As it was discussed in section 2.5 in a non-dispersive medium the propagation constant changes proportionally to the frequency components $\omega$

$$\Delta \Phi(\omega) = 2k_S(\omega) z_S - 2k_R(\omega) z_R. \quad (B.9)$$

The Taylor expansion allows to separate the dependency of the propagation constant around the center frequency $\omega_0$ related to the phase delay $\Delta \tau_p$ and the changing part $k'$ depending all other frequencies $\omega$ that is related to the group delay $\Delta \tau_g$

$$\Delta \tau_p = k_S(\omega_0) \omega_0 = k(\omega) + k'(\omega_0)(\omega - \omega_0). \quad (B.10)$$

The relating phase delay $\Delta \tau_p$ and phase group delay $\Delta \tau_g$ is

$$\Delta \tau_p = \frac{k(\omega_0)}{\omega_0} 2\Delta z = \frac{2\Delta z}{v_p} \quad (B.11)$$

and

$$\Delta \tau_g = k'(\omega_0) 2\Delta z = \frac{2\Delta z}{v_g}. \quad (B.12)$$

This allows to write Eq. B.8 by separating the components of the $\Delta \Phi(\omega)$ term as

$$I \propto \Re \left\{ \exp[-i\omega_0 \Delta \tau_p] \int_{-\infty}^{\infty} S(\omega) \exp[-i(\omega - \omega_0)\Delta \tau_g] \frac{d(\omega - \omega_0)}{2\pi} \right\}. \quad (B.13)$$

Eq. B.13 has now two parts, one for the center frequency and one for all other frequencies.

Using the description of a Gaussian power spectrum with standard deviation $\Delta k$ and using the normalized Gaussian integral $\int_{-\infty}^{\infty} S(\omega) \frac{d\omega}{2\pi} = 1$, substituting
\[ S(\omega - \omega_0) = \sqrt{\frac{2\pi}{\sigma_\omega^2}} e^{-\frac{(\omega - \omega_0)^2}{2\sigma_\omega^2}} \]  
(B.14)

in Eq. B.13 results in

\[ I \propto \exp[-i\omega_0 \Delta \tau_p] \exp \left[ -\frac{\Delta \tau_g^2}{2\sigma_t^2} \right] . \]  
(B.15)

The standard deviation in the frequency domain \(2\sigma_\omega\) has an inverse relationship to the standard deviation in the time domain \(2\sigma_t = \frac{2}{\sigma_\omega}\).  

(B.16)

The Gaussian part \(\exp \left[ -\frac{\Delta \tau_g^2}{2\sigma_t^2} \right]\) from Eq. B.15 contains the source band width \(\sigma_\omega\) of the light source. The half maximum intensity is then

\[ I_{AC} = \exp \left[ -\frac{(\Delta \tau_g \sigma_\omega)^2}{2} \right] \]  
(B.17)

\[ 1/2 = \exp \left[ -\frac{(\Delta \tau_g \sigma_\omega)^2}{2} \right] . \]  
(B.18)

The time delay is directly related to a spatial distance by \(\Delta \tau_g = 2\Delta z/c\) assuming that \(v_g = c\) which is approximately true in air. Rearranging Eq. B.18 and substituting the time delay we get

\[ -\ln 2 = -\frac{(\Delta \tau_g \sigma_\omega)^2}{2} \]  
(B.19)

\[ 2\ln 2 = (\Delta \tau_g \sigma_\omega)^2 \]  
(B.20)

\[ \sqrt{2\ln 2} = \Delta \tau_g \sigma_\omega \]  
(B.21)

\[ \frac{2\sqrt{2\ln 2}}{\sigma_\omega} = \Delta \tau_g \]  
(B.22)

\[ \frac{2\sqrt{2\ln 2}}{\sigma_\omega} = \frac{2\Delta z}{c} \]  
(B.23)

\[ \sqrt{2\ln 2} \frac{2\Delta z}{c} = \Delta \tau_g \]  
(B.24)

With \(c = \lambda f\), \(\sigma_\omega = 2\pi \sigma_f\), and relating the standard deviation to the wavelength bandwidth.
\[ \frac{\sigma_f}{f} = \frac{\sigma_\lambda}{\lambda} \]  
(B.25)

\[ \sigma_f = f \frac{\sigma_\lambda}{\lambda} \]  
(B.26)

\[ \sigma_\omega = \frac{2\pi f \sigma_\lambda}{\lambda} \]  
(B.27)

\[ \sigma_\omega = \frac{2\pi c \sigma_\lambda}{\lambda \lambda} \]  
(B.28)

\[ \sigma_\omega = \frac{2\pi c \sigma_\lambda}{\lambda^2}. \]  
(B.29)

Substituting Eq. B.29 into Eq. B.24 we get

\[ \frac{\lambda^2 \sqrt{2\ln 2}}{2\pi c \sigma_\lambda} = \frac{\Delta z}{c} \]  
(B.30)

\[ \frac{\lambda^2 \sqrt{2\ln 2}}{2\pi \sigma_\lambda} = \Delta z. \]  
(B.31)

For the FWHM of the Gaussian vs. wavelength we can apply the same method as above considering that we get half of the width \( \lambda_{1/2} \)

\[ \gamma(\lambda) = \exp \left[ \frac{\lambda^2}{2\sigma_\lambda^2} \right] \]  
(B.32)

\[ 1/2 = \exp \left[ \frac{\Delta \lambda_{1/2}^2}{2\sigma_\lambda^2} \right] \]  
(B.33)

\[ -\ln 2 = \frac{\Delta \lambda_{1/2}^2}{2\sigma_\lambda^2} \]  
(B.34)

\[ 2\sigma_\lambda^2 \ln 2 = \Delta \lambda_{1/2}^2 \]  
(B.35)

\[ \sigma_\lambda \sqrt{2\ln 2} = \Delta \lambda. \]  
(B.36)

Substituting Eq. B.36 into Eq. B.31 gives then the resolution at FWHM vs. the wavelength-to-bandwidth ratio.
\[ \Delta z = \frac{\lambda^2 \sqrt{2 \ln 2}}{2\pi \Delta \lambda} \] (B.37)

\[ \Delta z = \frac{\lambda^2 2\sqrt{2 \ln 2 \ln 2}}{2\pi \Delta \lambda} \] (B.38)

\[ \Delta z = \frac{4\ln 2\lambda^2}{2\pi \Delta \lambda} \] (B.39)

\[ \Delta z = \frac{2\ln 2\lambda^2}{\pi \Delta \lambda} \] (B.40)

**B.3 The self coherence function and coherence time**

In section 3.4 the coherence length \( l_c \) is calculated with

\[ l_c(m_o) = c \int_0^\infty |G(\tau(m_o))|^2 d\tau, \] (B.41)

where \( c \) is the speed of light, \( G \) is the coherence function, and \( \tau(m_o) \) is the time delay for each order of interference [135]. The time delay is calculated with

\[ \tau(m_o) = (n_S z_S - z_M(m_o))/c \] (B.42)

where \( n_S \) is the refractive index of the sample or air for a mirror, \( z_S \) the sample layer, and \( z_M(m_o) \) the position of the multiple virtual scanning mirrors for MR-OCT.

The Eq. B.41 for TD-OCT is

\[ l_c = c \int_0^\infty |G(\tau)|^2 d\tau, \] (B.43)

and with \( \frac{\xi}{c} = \tau_c \)

\[ \tau_c = \int_{-\infty}^\infty |G(\tau)|^2 d\tau, \] (B.44)

simply provides the coherence time [177]. The change of the boundaries relates to the assumption that the Gaussian spectrum in practice can have only positive values.
The solution of Gaussian integral (Eq. B.44) is provided [177, p168] using $f$ for frequency

$$\tau_c = \sqrt{\frac{2\ln 2 \cdot 1}{\pi \Delta f}}. \tag{B.45}$$

The same integral is solved in section B.2 for wavelength and with $c = \lambda f$ yields

$$l_c = \Delta z = \frac{2\ln 2\lambda^2}{\pi \Delta \lambda}. \tag{B.46}$$

The interesting aspect of this second approach which is essentially the same as in section B.2 is, that Eq. B.41 underlines the mathematical description of the reduction of the coherence time of the light source due to the Doppler effect, essentially meaning that the wavelength is reduced for each higher order of reflection for MR-OCT.

### B.4 Reference scanning path-delay systems

This review of axial scanning systems for TD-OCT underlines the flexibility of the MR-OCT method to be adapted for a wide variety of scanning methods and the use of a PM is in most cases superior concerning the complexity of construction and the total imaging range. Nevertheless, the listed scanning methods are an overview of the most important concepts reported in the literature.

**Dispersion based scanning delay line** The dispersion based delay line uses the relationship of the frequency and speed of light to create a rapid axial scanning with a galvo mirror, a lens and a diffraction grating [178]–[183]. Such dispersive delay lines are not only able to generate an axial path delay but are also used in some swept-source OCT systems to rapidly changing the frequency of light. The construction of such a delay line for TD-OCT allows great flexibility in design and allows for high scanning speeds and scan ranges of 3 mm at 2000 Hz [178] and 4000 Hz with a resonant galvo scanning mirror [182].

The axial delay is dictated by the focal length of the lens and the number of lines of the diffraction grating. Therefore, the multitude of optical components and their required relative position does not lend itself for miniaturization attempts but also assembling and tuning would make it difficult to produce such delay lines at low costs.

**Piezo actuated axial scanning** Axial scanning can be achieved using a piezoelectric transducer that can change thickness based on the applied voltage. However, the achieved scan
ranges using a piezoelectric element directly are only in the range of a few micrometers, and an attached scanning mirror would provide only a very shallow scanning range.

A fiber-based TD-OCT is described that is using fiber stretching amplifying the effect of the small change of the geometrical dimension of the piezo is reported to achieve a scan rate of about 2000 A-scans per second and an imaging depth of about 600 µm at an SNR of 113 dB [184]. This approach appears to be very robust due to the protected waveguide in the optical fiber. However, it needs to be noted that the fiber bending radius will limit miniaturization attempts. For a fiber with 6 µm core a bending radius smaller than 2.5 cm the loss increases from about 10 to 10000 dB/m. Consequently, an estimated smallest device volume should be larger than (5 × 5 × 5) cm and miniaturization at low production costs appear to be not likely.

On the other hand MR-OCT in conjunction with a piezo stack that can achieve up to 50 µm scan range (PI, E-625.CR) could enhance the imaging depth to about 1 mm depending on the spacing between PM and SRM $D = 90$ µm using only ten orders of reflections ($m_0 = 10$) allowing already for a much better prospect of miniaturization attempts [4].

The challenges of using a piezo actuator remain in the controller electronics and the generation of high-voltages. Because the controller may required calibration for the piezo material to achieve sufficient scanning repeatability which may counteract the low-cost goal. Furthermore, the piezo stack needs to have a minimal volume to achieve a minimal travel range and for the mentioned proof of concept, the used PI piezo stack would require an additional volume of (30 × 30 × 10) mm.

**Amplified resonant piezo actuator** The amplified resonant piezo actuator (APA) would be an option to increase the scan range of a piezo actuator attaching a spring-loaded mass. For example, an APA from Thorlabs (PK2FSF1) does achieve a travel range of 220 µm without load at resonance of 1 kHz.

The travel range would still be limited for a conventional TD-OCT application. The interference using an APA in a MR-OCT system was characterized, and values for SNR and sensitivity were in the same region as for a first attempt using a voice coil from a CD/DVD-ROM drive (see text below) [185].

A problem with pushing for a large mechanical travel ranges is related to the increased mechanical forces. The vibrations do not only increase that impair the stability of the optical system and introduce noise, but also the pointing accuracy of the attached mirror may reduce due to anisotropic material constants becoming more dominant.
**Piezoelectric membrane** A piezoelectric membrane actuator (PMA) is a component for generating sound widely used in low-cost signaling systems such as buzzers but are also reported to be feasible for membranes in micro pumps [186]–[188]. Such PMAs are genuinely low-cost systems and commercially available in a wide selection of shapes and sizes and can be purchased in large numbers. The displacement of such membranes are limited to a few micrometers depending on the diameter and thickness of the membrane and reported with about 10 µm. For ordinary TD-OCT such small scan ranges are of insufficient use, and no implementations for TD-OCT systems are reported. However, in principle, those PMAs could be easily implemented for a layer scanning MR-OCT as it was reported for a voice coil driven system that requires only a small scanning amplitude after all [108]. Furthermore, it can be theorized that with purpose-built membranes the scan range can still be enhanced to some degree [189].

The geometrical shape mainly determines the volume of the PMA and with a thickness of 1 mm or less in conjunction with disk diameters in the range of 20 to 30 mm would undoubtedly support miniaturization at low-costs.

**Voice coil based axial scanning actuator** A voice coil actuator extracted from a pickup head from a decommissioned CD/DVD-ROM drive was used in an early MR-OCT research system demonstrating once more the versatile reconfigurability of the MR-OCT method [4]. Such a pickup head appears to be an obvious choice considering that such components can be produced in large numbers at low costs. The voice coil is required to follow the wobble of the CD or DVD and the rotation per minutes dictates the maximum scanning frequency, while the amplitude of the wobble is the requirement of the working distance of the lenses. For DVD drives stated values for the focus servo are said to require 4 kHz at 4× constant-linear-velocity mode and a lens working distance of 480 µm [190]. However, it is to expect that the focus servo will operate at lower frequencies and requires a reduced tracking distance at normal operating conditions of the drive. For MR-OCT scanning ranges from 50 µm to 150 µm at a scan rate of about 600 Hz were demonstrated. Including both directions of the scanning motion, 1200 A-lines per seconds can be used for imaging. Nevertheless, a disadvantage of a voice coil system is the spring-mass system which is susceptible to the motion of the scanning system meaning it may not be the best choice for portable or hand-held systems. Other problems are the ohmic heating of the coil which can cause drift of the scan range if not compensated.

**Electro-optical modulator** Electro-optical modulators (EOM) are often used in optical systems to modulate the phase at very high frequencies to generate a carrier for different
purposes. The path delay is too short to be used for any useful axial scanning, but the EOM generated carrier allows to stabilize high-speed scanning systems and improve SNR and sensitivity [191].

However, not only is the change of the achievable path-delay small, the EOM cannot be fitted in between the PM and the SRM of a MR-OCT system which would otherwise infeasibly increase the distance between the orders of reflections. Although this is one instance where MR-OCT cannot easily be adapted to, it may still be possible to use the EOM fitted in before the PM to generate a carrier signal.

Fiber-optic thermo-optical delay line An exciting method to miniaturize a scanning delay line is the use of the temperature dependent properties of integrated waveguides on a silicon wafer. A so-called time-domain thermo-optical delay line (TODL) was reported to allow a scanning delay of $200 \mu m$ at a frequency of 10 kHz and $950 \mu m$ at 2 kHz [192]–[194]. Although, no component size was given the length of the active region was given with 10 mm. A similar method is attempted with a so-called MOEMS delay line, but no up-to-date application notes are available [195]. The TODL technology is preferably suitable for fiber-based interferometric systems which would limit the smallest volume of a compact system. Only if the interferometer can be integrated on the same chip the potential of systems with very small form factors is one step closer.

Multilayered scattering reference mirror The use of a scattering multilayered mirror (SMM) is described to provide multiple reference reflections based on the number of stacked mirrors [51]. The SMM could help to increase the imaging depth if the mechanical travel range of a scanner is not sufficient. It was reported that the SMM is composed of several layers of mylar sheets of which each is $6 \mu m$ thick and spaced by silica microspheres with $4.8 \mu m$ in diameter. The reported imaging depth with about $100 \mu m$ is however not much better compared to conventional scanning delay lines.

The challenges to stack multiple mirror interfaces on top of each other will be increased internal reflections that cause either phantom signals and limit the number of reflections with sufficient power, limiting the total imaging depth and reduce axial resolution.

Quasi-stationary time-domain delay line A simulation of a quasi-stationary delay line for TD-OCT was described using planar optics distributing a beam through a liquid crystal window and a staircase mirror [86]. Each step of the staircase mirror acts as a static mirror at an increased path length. Without the liquid crystal window, the planar optics
would distribute a beam on all mirror steps, and multiple reference waves at all path delays would be available.

Such a method appears to pose multiple challenges regarding coherence, as the planar optical system introduces additional wavefront distortions and path length dispersion that may be difficult to control in practice. The complex geometry of the staircase mirror may make production expensive, and the asymmetric geometry may make it challenging to achieve high scan rates if use is intended in a scanning delay line.

Microelectromechanical scanning mirrors Scanning mirrors based on microelectromechanical systems (MEMS) appear to be the most promising technology to miniaturize TD and MR-OCT systems. However, currently available axial scanning mirrors for delay lines provide similar speed and scan range to conventional mechanical delay lines. MEMS technology remains mechanical after all and only offers the advantage to push the boundaries to smallest geometrical sizes possible that remain operational. The smaller size and reduced mass can undoubtedly contribute to increased scanning frequencies. However, if operated in ambient conditions the smaller components encounter air more and more like a liquid medium and frictional forces can cause undesired effects. So far in one investigation, reported a MEMS mirror with a travel range of 500 µm and an axial scanning frequency of about 390 Hz with pointing accuracy $\leq 0.14^\circ$ [196]. Especially for MR-OCT the pointing stability and accuracy is a more critical requirement to assure sufficient visibility of higher orders of interference signals.
Publications and other outputs

Journal papers

2014  Dsouza, Roshan; Subhash, Hrebesh; Neuhaus, Kai; Hogan, Josh; Wilson, Carol; Leahy, Martin, *Dermoscope guided multiple reference optical coherence tomography*, Biomedical Optics Express.

2015  Dsouza, Roshan; Subhash, Hrebesh M.; Neuhaus, Kai; Hogan, Josh; Wilson, Carol; Leahy, Martin, *3D nondestructive testing system with an affordable multiple reference optical-delay-based optical coherence tomography*, Applied Optics.

2016  Dsouza, Roshan; Subhash, Hrebesh; Neuhaus, Kai; Kantamneni, Ramakrishna; McNamara, Paul M.; Hogan, Josh; Wilson, Carol; Leahy, Martin, *Assessment of curing behavior of light-activated dental composites using intensity correlation based multiple reference optical coherence tomography*, Lasers Surg. Med.

2017  Neuhaus, Kai; O’Gorman, Seán; McNamara, Paul M.; Alexandrov, Sergey A.; Hogan, Josh; Wilson, Carol; Leahy, Martin J., *Simultaneous en-face imaging of multiple layers with multiple reference optical coherence tomography*, Journal of Biomedical Optics, 10.1117/1.JBO.22.8.086006.


2017  Neuhaus, Kai; McNamara, Paul M.; Alexandrov, Sergey; O’Gorman, Seán; Hogan, Josh; Wilson, Carol; Leahy, Martin J., *Performance review of multiple reference vs. time domain optical coherence tomography*, IEEE Photonics Journal, 10.1109/JPHOT.2018.2828419.

2018  O’Gorman, Seán; Neuhaus, Kai; Alexandrov, Sergey; Hogan, Josh; Wilson, Carol; McNamara, Paul; Leahy, Martin, *Characterization of an amplified piezoelectric actuator for multiple reference optical coherence tomography*, Applied Optics.
Conference proceedings


APPENDIX C. PUBLICATIONS AND OTHER OUTPUTS

Presentations

2014 Poster: Multiple Reflection OCT (MR-OCT) Miniaturization and Data Processing; BIGSS, Galway, Ireland
2015 Oral: Characterization of Light Distribution and Optimization of Detector Position for Multiple Reference Optical Coherence Tomography; SPIE Photonics West, San Francisco, California, United States
2015 Poster: Signal Simulation and Signal Processing for Multiple Reflection Optical Coherence Tomography; SPIE Photonics West, San Francisco, California, United States
2015 Poster: Microcirculation Detection with Correlation Mapping Processing: A Review with MATLAB, Java and Python; Photonics Ireland, Cork, Ireland
2016 Oral: The Impact of Relative Intensity Noise on the Signal in Multiple Reference Optical Coherence Tomography; SPIE Photonics West, San Francisco, California, United States
2016 Poster: Improved signal quality of filtered signals for multiple reference optical coherence tomography (MR-OCT) using a Gaussian window; Advanced Laser Technologies, Galway, Ireland
2017 Poster: Signal processing for simultaneous en-face imaging of multiple layers with multiple reference optical coherence tomography; Galway, Ireland
2017 Poster: Comparing an FFT filter for multiple reference optical coherence tomography (MR-OCT) with a Chebyshev and an elliptic filter; SPIE Photonics West, San Francisco, California, United States
2017 Oral: Simultaneous en-face imaging of a human finger tip using multiple reference optical coherence tomography; American Association of Anatomists, Galway, Ireland

Awards

2016 SFI Poster Prize; Advanced Laser Technologies Conference

Successful application for funding

2015 SPIE Photonics West Student Chapter Officer Travel Grant
2016 SPIE Travel Scholarship Cycle 2

Structured PhD modules

1. (GS507) Statistical Methods for Research
2. (GS506) Teaching and Learning
3. (GS530) Graduate Research Information Skills
4. (PH506) Principles of Optical Design & Image Formation
5. (PH504) High Performance Computing and Parallel Processing
6. (PH508) Biophotonics and Imaging Summer School (BIGSS) 2014 and 2016


E. Auksorius and A. C. Boccara, “Fingerprint imaging from the inside of a finger with full-field optical coherence tomography”, *Biomedical Optics Express*, vol. 6, no. 11, p. 4465, Nov. 1, 2015, ISSN: 2156-7085, 2156-7085. DOI: 10.1364/BOE.6.004465.


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L. Wu, D. Q. Xiao, J. G. Zhu, P. Yu, Q. Wei, and Y. Zhuang, “Fabrication and properties of lead-free piezoelectric buzzers made from k0.5na0.5nbo3-based ceramics”, in *2009 18th IEEE International Symposium on the Applications of Ferroelectrics*, (Aug. 2009), pp. 1–4. DOI: 10.1109/ISAF.2009.5307587.


