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Uncertainty in *A Treatise on Probability* and the *General Theory*

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Abstract

This paper is concerned with the relationship between Keynes's theory of probability and his later work on macroeconomics. It suggests that while the General Theory deals again with the subject of the Treatise on Probability, information and uncertainty, Keynes's treatment of this matter in the later work was influenced by Knight, rather than his own earlier views.

Keywords: uncertainty, risk, probability, information

JEL Classification: B31
1 Introduction


The arguments outlined here are partly, though not entirely, contrary to the continuity thesis. While agreeing that the subject matter of part of the later work falls into the same general area as that of the earlier, it is suggested that the logical theory of probability was as inappropriate to aspects of the *General Theory* (CW, VII) as it was appropriate for the purposes which attracted Keynes to the problem of probability in the first place some three decades earlier. However, the view, sometimes expressed, that Keynes had in mind any precise form of the subjective theory of probability in the *General Theory* is not accepted. Rather, suggested links (e.g., Davidson, 1972) between the work of Knight (1971), on the one hand, and the *General Theory* (CW, VII) and the *Quarterly Journal of Economics* article (CW, XIV, 1973), on the other, are reconsidered and extended. In addition, a possible connexion between the *Quarterly Journal* article and the work of Venn (1888), so vehemently rejected in the *Treatise* (CW, VIII), is suggested.

It is, perhaps, odd to look to Keynes for rigid consistency over twenty or thirty years. Harrod gives as the 'commonly' held view of him 'that he was an in­veterate vacillator' (1966, p.302). Keynes clearly saw no virtue in continuity or consistency, and the juxtaposition of distinct, or even contradictory positions is often found in his writings. He was well aware of this aspect of his work, and defended it on the grounds that changing real world circumstances called for a flexible approach to problems (e.g., CW, XX, 1981, p.502). Indeed, had Keynes had continuity as a goal, he would never have written the *General Theory* (CW, VII).

In section 2, probability, certainty and uncertainty, as these terms arise in the *Treatise* (CW, VIII), are briefly considered. Section 3 notes Keynes's shift with respect to relation between probability and certainty and, therefore, uncertainty,
in the *General Theory* (CW, VII). Also, here, it is pointed out that, in particular, the diversity of opinion among speculators, which yields the downwards sloping liquidity-preference schedule, is incompatible with objective probabilities of future interest rates. The argument is summarized and some conclusions are drawn in the final section.

2 A Treatise on Probability

In *Principia Ethica* (1903) G.E. Moore stressed the insurmountable difficulties of evaluating, even probabilistically, all the future consequences of actions. Moore's conclusion that, therefore, well-tried rules and duties should be adhered to, was unacceptable to Keynes and his friends who, contrary to Moore, insisted on the right to reject general rules, and to judge each individual action for themselves (CW, X, p. 446).

The origins of Keynes's interest in probability are to be found in his need to provide a rational basis for this sort of judgment (see Bateman, 1988, O'Donnell, 1989, p.12, pp. 81-83 and Skidelsky, 1992, p.58). Keynes attributed the frequency theory of probability to Moore. This theory, which had been developed some decades earlier importantly by Venn (1888), though also by others, was of no use for the youthful Keynes's purposes, for in the frequency theory, probabilities of one-off events are not defined. Keynes' early reaction to Venn's work was almost emotional, as shown by the following quotation, taken from O'Donnell (1989, p.14), 'I am very hostile to Venn, a hostility almost amounting to animus.' Probability, Keynes stated in the *Treatise* (CW, VIII), was not statistical frequency as suggested by Venn and his school. Rather, he argued, quoting Bishop Butler, probability was the guide to life, and the importance of probability derived from the judgment that it was 'rational to be guided by it in action' (p.356). According to Keynes, the only acceptable theory was the logical theory, already developed to some extent by the Cambridge logician, W.E. Johnson, whose own work, however, though acknowledged by Keynes in the preface to the *Treatise*, was not published for a further eleven years. Thus, in the *Treatise* 'probability' was presented as an objective logical relation, in general, non-demonstrative, between two sets of propositions, one of which provides the evidence or premisses of an argument, and the second, the conclusions (pp. 4, 9). 'Probability' also meant degrees of rational belief in conclusions (pp.12). The crucial aspect of the logical theory for Keynes was that it allowed him to claim that account could be taken of probability relations between any pair of sets of propositions (p.133). However, he admitted that in particular cases, these might not exist, or when they did, they might not be comparable or even known (p.33). Probabilities as outlined by the frequency theory were
admissible in so far as they coincided with those of the logical theory (pp. 113, 312, 468).

For Venn, frequencies may be certain or nearly certain, while the outcomes of individual events are uncertain, whether or not these events are members of series on which frequencies are based (1888, e.g., pp.3, 122). To Johnson, 'certitude' characterizes the conclusions of demonstrative inference and is opposed to 'incertitude or probability' (1932, pp.4–5). In A Treatise on Probability (CW, VIII), 'certain' propositions include those that express knowledge directly acquired as well as those that make claims of reason. All the accessible probabilities of the Treatise (CW, VIII) belong to both of these types of statement, for they are represented as directly or intuitively known claims of reason (e.g., pp. 9, 15, 17). Premises of arguments are certain or assumed to be certain for, either they are hypothetical, or they are propositions asserting probability-relations or other knowledge directly acquired (pp.4, 17, 133). Further, conclusions that are of unitary probability are those that can be inferred from the relevant premises and are, therefore, certain. Conclusions which contradict their premises are of zero probability. These are negatively certain (p.16).

What is not certain is uncertain. Conclusions that are 'doubtful' or 'only probable' (p.5), that is, conclusions of probabilities between zero and unity, are uncertain. Also uncertain in the Treatise is the 'vague' or 'incompleted' knowledge that is so inadequate that probabilities either remain unknown or do not exist (e.g., pp. 14, 17). Keynes, however, wished to exclude vague knowledge from the Treatise (pp.17–18) because, as he stated, he did not know how to deal with it. To the extent that he succeeded in this, all the uncertainties in the book are probabilities (see Arrow, 1971, p.16).

Uncertain propositions include the very special class of cases to which the principle of indifference can be properly applied and where, therefore, exact numerical probabilities can be estimated. Examples of such situations are provided by games of chance and, also, though more tentatively, those social and physical statistics where the relevant conditions approximate those obtaining in games of chance (CW, VIII, pp. 458–459, 468). While the probabilities of the Treatise are generally non-numerical, for practical application, some kind of numerical approximation is required (p.33). The resulting conflict in the book between the special circumstances required to generate precise numerical probabilities, and the suggestion there that probability is the guide to life, is partly resolved within the work itself. Keynes argues that, though exact numerical probabilities are only rarely available, the range of probabilities capable of inexact numerical expression is not so limited including, as they do, such probabilities as can be placed between numerical limits (pp.176–180). Resolution, however, is not always possible (p.32).
While, therefore, in the *Treatise*, propositions that are 'only probable' are always uncertain, some such propositions are more uncertain than others. Situations in which probabilities are so imprecise and unquantifiable that 'it will be rational to allow caprice to determine us and to waste no time on the debate' (p.32) are arguably more uncertain than those in which probabilities serve as guides to action. In addition, Keynes considers the weight of arguments or amounts of evidence, but seems finally perplexed about to its significance. In the book, weight is both assigned and denied practical relevance (pp. 345 and 83, respectively).

3 The General Theory

One of the unsettled questions arising out of Keynes’s work relates to whether he continued to adhere to the logical theory of probability after the publication of the *Treatise* (CW, VIII), or whether he abandoned it in favour of either the frequency theory or Ramsey’s subjective theory. The controversy is summarized by O’Donnell (1989, p.140; 1991, pp. 24–26). It is a debate fueled in part by lack of evidence. O’Donnell himself (1989, p.141) finds no mention of Ramsey’s theory in Keynes’s later writings. Keynes makes no explicit reference to the frequency theory after 1926 (Winslow, 1989, p. 1177), or to objective logical relations after 1931 (Bateman, 1990, p.74). That the *Quarterly Journal* article (CW, XIV) includes implicit references to objective, numerical probabilities is, however, hardly controvertible.

Also indisputable is that the relation between probability and uncertainty in the *General Theory* (CW, VII) is not the same as in the *Treatise* (CW, VIII). Uncertainty arises in the *General Theory* and the *Quarterly Journal* article (CW, XIV) mainly in relation to the outcomes of investment decisions, real and financial. Keynes now states that in the classical economic theory, '[t]he calculus of probability ... was supposed to be capable of reducing uncertainty to the same calculable status as that of certainty' (CW, XIV, pp. 212–213). The certainty of the *Treatise* (CW, VIII) being thus altered, uncertainty must be adjusted as well. This is acknowledged by Keynes in the following well-known extract from the *Quarterly Journal* article (CW, XIV, pp. 113–114), which makes clear that where there are objective, numerical and unchanging probabilities, there is now no uncertainty.

> By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain ... The sense in which I
am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention ... About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know.

While shades of the Treatise (CW, VIII) are discernible here, the more obvious similarities are with Venn's distinction between games of chance and vital statistics (1888, pp. 143-17) and, more importantly, Knight's separation of risk from uncertainty (1971). Venn pointed out that while vital statistics may vary over time, the outcomes of games of chance do not. In his analysis of profit, contained in Risk, Uncertainty and Profit, originally, like the Treatise (CW, VIII), published in 1921, Knight stated that: a "measurable uncertainty, or "risk" proper ... is so far different from an unmeasurable one that it is not in effect an uncertainty at all. We shall accordingly restrict the term "uncertainty" to cases of the non-quantitative type' (1971, p.20). According to Knight, measurability obtains when all possible outcomes, together with the probabilities of occurrence, can be determined precisely (p.198). Relevant probabilities may be either a priori, logical probabilities, in effect, the precise numerical probabilities of Keynes's Treatise (CW, VIII), or frequencies, though in business practice, the latter are the more likely to arise (pp. 214–215). Unlike uncertainty, risk can be converted to certainty through insurance (e.g., p.46). This distinction between risk and uncertainty had been made earlier, but the importance of the distinction had not been recognized (p.44).

However, the hint in the Quarterly Journal article (CW, XIV) that uncertainty is a matter of degree invokes uncertainty in the Treatise (CW, VIII) rather than Knight's account. Moreover, Keynes's outline in the Treatise (CW, VIII) of probabilities incapable of even inexact numerical formulation, has given rise to suggestions of more substantial links between uncertainty in the General Theory (CW, VII) and the Treatise (CW, VIII). Thus, uncertainty in the General Theory (CW, VII) is associated by Lawson (1985, p.914) with the numerically indeterminate and non-comparable probabilities of the Treatise (CW, VIII). O'Donnell, on the other hand (1989, p.260; 1991, pp. 30–31), interprets uncertainty in the General Theory (CW, VII) in terms of the vague knowledge and unknown probabilities of the Treatise (CW, VIII). O'Donnell (1991, p.28) also converts 'weight' in the Treatise (CW, VIII) into 'confidence' in the General Theory (CW, VII).

As Bateman (1990, p.77) points out, reference to probabilities is scattered throughout the General Theory (CW, VII). It will now be argued that these cannot, or cannot consistently, be the objective probabilities of the Treatise (CW, VIII). Keynes's specific economic objection to the classical assumption of
a calculable future is that it leads to a mistaken theory of the rate of interest (CW, XIV, p.122). However, his own theory leaves no room for any ascertainable objective probabilities, incalculable or quantitative.

Since in the Treatise (CW, VIII), rational beliefs are justified by objective logical relations, only diverse preferences can account for heterogeneous actions. In the General Theory (CW, VII), business is guided predominantly by the profit motive, though lack of a calculable future may result in decisions based on whim or sentiment (pp. 161–163). In his treatment of the money market, Keynes briefly considers 'the simplest case, where everyone is similar and similarly placed', but rejects this possibility 'by reason partly of differences in environment and the reason for which money is held and partly of differences in knowledge and interpretation' (pp. 198–199).

In accounting for liquidity preference elsewhere in the book, Keynes describes the speculative demand for money as 'the object of securing profit from knowing better than the market what the future will bring forth' (p.170). In general, therefore, speculators differ among themselves regarding the future of the interest rate. Indeed, the usual downwards-sloping liquidity preference schedule depends on such diverse opinions (Tobin, 1958). In the General Theory (CW, VII), effective monetary policy, in turn, depends on such a downwards-sloping curve:

...opinion about the future of the rate of interest may be so unanimous that a small change in present rates may cause a mass movement into cash. It is interesting that the stability of the system and its sensitiveness to changes in the quantity of money should be so dependent on the existence of a variety of opinion about what is uncertain. Best of all that we should know the future. But if not, then, if we are to control the activity of the economic system by changing the quantity of money, it is important that opinions should differ.

(p.172)

That a continuous downwards-sloping liquidity-preference curve exists in practice is proven by the success of open market operations (pp. 196-197).

Therefore, the 'existing probabilities' of future interest rates, referred to in Keynes's discussion of the demand for money (CW, VII, p.169) cannot be logical relations, whether known or ascertainable (see also Davidson, 1991). Further, Ramsey's view of probability theory as 'a set of numbers ... obeying the calculus of probabilities' (1990, p.96) is equally inapplicable to the uncertainty of the General Theory (CW, VII). 'Confidence' also is treated by Ramsey (1990.
pp. 71, 91, 94) in more precise terms than in the General Theory (CW, VII, pp. 148-149). The true uncertainty arising because of the absence of quantitative probabilities, referred to in the foregoing passage from the Quarterly Journal article (CW, XIV), could be compared with that ascribed to individual events by Venn (1888). However, Venn's uncertainty is due to the absence of probabilities.

Ramsey did not think of all beliefs as potentially measurable (1990, pp. 68, 79-80). Indeed, he preceded Keynes in outlawing 'excessive scholasticism' (CW, X, p.341). While in his backwards glance at his earlier life in My Early Beliefs (CW, X), Keynes does not mention Ramsey by name, he almost quotes himself quoting him in his reference to 'Moore's method, according to which you could hope to make essentially vague notions clear by using precise language' (pp.440, 343). Ramsey's influence on Keynes continues to be debated (Bateman, 1987, 1990; Moggridge, pp. 364-366; O'Donnell, 1989, pp. 139-148; Skidelsky, pp. 67-73). That it existed can hardly be doubted. What is suggested here, however, is that the influence of Knight extended beyond Keynes's Quarterly Journal (CW, XIV) account of the connexion between probability and certainty, and that Keynes's description of behaviour under uncertainty in the General Theory (CW, VII) most closely resembles the earlier description given in Risk, Uncertainty and Profit (1971).

Knight and, in the General Theory (CW, VII) Keynes, dealt with a common subject-matter, how profit projections are possible in the light of 'ignorance of the future' ( Knight, 1971, pp. 37, 198; Keynes, CW, VII, pp. 157, 163). Both point to the singular nature of many business decisions (Knight, pp. 226, 231; Keynes, CW, XIV, pp.113-114), and the vagueness of the background against which decisions may have to be reached (Knight, p.211, Keynes, CW, VII, pp. 24, 149). In the absence of objective, reliable information, agents fall back on experience (Knight, pp. 227-228; Keynes, CW, VII, pp. 147-148, 152; CW, XIV, p.114), and form expectations or opinions or forecasts or, if possible, estimates or calculations (Knight, pp. 210, 226, 231; Keynes, CW, VII, pp.147-150, 154,163,169-172). Business decisions are based not only on these estimates or opinions, but also on the confidence with which they are held (Knight, pp. 226-227, 229; Keynes, CW, VII, pp. 147-149). Confidence, however, is a subjective feeling, related to business psychology, and not amenable to formal analysis (Knight, pp. 227, 229; Keynes, p.149). Knight wishes to call both estimates and the confidence vested in them probabilities, probabilities that are, however, subjective, and very different from a priori and statistical probabilities (pp. 225-229).

In this respect, Keynes in the General Theory (CW, VII) is less forthcoming. The imprecision and subjectivity of Knight's estimate-probabilities are, however, indicated by Keynes's description of the state of expectation of an en-
trepreneur both as a 'bundle of vague and ... various possibilities' and as 'several hypothetical expectations held with varying degrees of probability and definiteness' (CW, VII, p.24). The doubts about the probability of future returns, entertained by an entrepreneur or borrower, (p.144) also echo Knight’s estimates of estimates (1971, p.227). It may be noted that such doubtful probabilities do not have their origin in the Treatise (CW, VIII), where all ascertainable probabilities are known with certainty.

4 Conclusions

Perhaps the most surprising aspect of the Treatise (CW, VIII) is that it was ever published. This comment is provoked by Keynes’s disastrous experience on foreign exchange markets in 1920 (Moggridge, 1992, p.349), about a year before the submission for publication of the final manuscript of the Treatise (CW, VIII), experience hardly conducive to belief in the pervasiveness of objective logical relations. As he himself was to remark in the General Theory (CW, VII, pp. 198-99), if everyone were ‘similar and similarly placed’, no transactions would result from ‘a change in circumstances or expectations’. Later, following publication of the Treatise (CW, VIII), the logical theory of probability was rejected by the two dominant figures of Cambridge philosophy at the time, Wittgenstein and Ramsey (Skidelsky,1992, pp. 67-73).

In the Treatise (CW, VIII), Keynes attempts to abstract from vague knowledge, and to consider objective logical relations as existing ‘between any pair of sets of propositions’ (pp. 17, 133). That which is ‘only probable’ is uncertain (p. 5). In the General Theory (CW, VII), by contrast, Keynes wishes to consider investment decisions, though knowledge of the future, over which outcomes accrue, is ‘fluctuating, vague and uncertain’ (CW, XIV, p.113).

In Keynes’s Quarterly Journal article and Knight’s Risk, Uncertainty and Profit (1971), probability, certainty and uncertainty are related in a similar, though not identical, fashion. In the Quarterly Journal article (CW, XIV), Keynes outlines certainty and near certainty in the terms in which Knight defines risk. And while to Keynes in the General Theory (CW, VII), uncertainty remains partly a matter of degree, this difference from Knight’s uncertainty is formal rather than substantial. The account of behaviour under uncertainty, given in the General Theory (CW, VII) is very close to that of Knight.

What is carried over to the General Theory from the Treatise (CW, VIII) is Keynes’s interest in information and uncertainty. While this interest constitutes a basic ingredient of the General Theory (CW, VII), the shift in the treatment
of uncertainty is also essential. The ubiquity of objective logical relations in the Treatise (CW, VIII) together with their serviceability as guides to life comprise a necessary prop to Keynes's belief in an objective though idiosyncratic morality. In the General Theory, on the other hand, logical probabilities would inhibit the development of Keynes's theory of the rate of interest, a determinant of investment expenditure which, in turn, is the 'causa causans ...of the level of output and employment as a whole' (CW, XIV, p. 121)
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