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# On the Axiomatic Approach to Freedom as Opportunity: A General Characterization Result

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## Abstract

The aim of this paper is to provide a different, somewhat more general, approach to the axiomatic ranking of opportunity sets in terms of freedom of choice. The opportunity ranking is defined axiomatically relative to some standard of freedom or theory of freedom. The paper then provides a general characterization result of the opportunity ranking in terms of the family of freedom standards.

**Keywords:** axiomatic ranking, freedom of choice, opportunity sets

**JEL Classification:** D71

## Introduction

Recently a growing number of researchers have suggested a great variety of axioms trying to capture different intuitions of the intrinsic value of freedom, see Bossert et al. (1994); Gravel (1994); Jones and Sugden (1982); Klemisch-Ahlert (1993) Pattanaik and Xu (1990, 1996); Puppe (1996); Sen (1985, 1991, 1993); Sugden (1996) and Suppes (1987) among others. Some authors have proposed a set of axioms which are completely independent of an underlying preference relation among basic alternatives, see Klemisch-Ahlert (1993); Pattanaik and Xu (1990); Suppes (1987) among others. On the other hand Sen (1991, 1993) and, subsequently, Arrow (1995); Nehring and Puppe (1996); Puppe (1996) and Sugden (1996) have all strongly advocated the incorporation of preference as an essential ingredient in the evaluation of freedom of choice. This paper suggests a different, somewhat more general, approach to the axiomatic analysis of opportunity ranking in terms of freedom of choice.

The central tenet of the proposed model is an assumption that in order to make comparisons of opportunity sets in terms of freedom, individuals would require a certain standard of freedom or a theory of freedom which would reflect what is essential from individuals' point of view for evaluating opportunity sets. Of course, individual preferences might be an essential part of a freedom standard. However, we believe that, in general, a theory of freedom should include much more than individual preferences only. Although we do not specify precisely what is included in a freedom standard, we do impose a minimal set of requirements on it as well as on the opportunity ranking. In this respect we follow Nehring and Puppe (1996) and Puppe (1996) who believe that, in view of the conceptual complexity and elusiveness of the concept of "freedom of choice", we should start with a minimal set of requirements. What is important to emphasize is that we impose that minimal set of requirements on the opportunity ranking always relative to some standard of freedom or some theory of freedom. In fact, our basic set of axioms for freedom of choice corresponds closely to that of Puppe (1996) However, instead of his central axiom for freedom of choice (Axiom F) which roughly states that every opportunity set contains at least one basic alternative essential for freedom, we utilize a somewhat different axiom on freedom standards (Axiom S) which simply stipulates that those opportunity sets that are included in a freedom standard offer, at least, some degree of freedom. Actually, Axiom S allows us to say somewhat more, namely, that those opportunity sets that belong to a standard or a theory of freedom offer strictly more freedom than those sets that do not.

The plan of this paper is as follows. In Section 2, we introduce some basic notation and axioms. In Section 3, we establish a general characterization result of opportunity ranking in terms of standards of freedom. Specifically, we show that the opportunity ranking can be defined set-theoretically from the family of freedom standards as follows: A offers at least as much freedom as B if and only if A belongs to every freedom standard to which B does. Finally, Section 4 provides a brief assessment of the results.

## 2. Basic definitions and axioms

Let X be a finite, non-empty, set of basic alternatives and let P(X) denote the set of all subsets of X. The elements of P(X) will be denoted by A, B, C, …, and are referred to as opportunity sets. Notice that we include the empty set $\emptyset$ in P(X). In fact, in our framework $\emptyset$ represents an opportunity set which offers zero degree of freedom. In order to compare opportunity sets in terms of freedom, we need some kind of a freedom standard, or theory, which reflects what is essential for freedom from the agent's point of view. Let $\Delta$ be a non-empty subset of P(X) which represents such a standard. Naturally we want to exclude empty set from $\Delta$, so we assume that $\emptyset \notin \Delta$. We also assume that $\Delta$ is closed under supersets.

Let $\geq$ be a binary relation defined over P(X). For all A, B $\in$ P(X), A $\geq$ B will be interpreted as "A offers at least as much freedom as B" relative to $\Delta$. The asymmetric and symmetric part of $\geq$ will be denoted by $>$ and $\sim$, respectively.

We want to impose a minimal set of requirements on $\geq$. First of all, because we are interested in comparisons of opportunity sets in terms of freedom from the point of view of an agent who is rational, we impose a transitivity requirement on $\geq$. Formally,

**Axiom T** (<u>Transitivity</u>). For all A, B, C $\in$ P(X), A $\geq$ B and B $\geq$ C $\Rightarrow$ A $\geq$ C.

Axiom T figures prominently in every paper on the axiomatic approach to freedom as opportunity.

The next axiom is also fairly uncontroversial and is utilized by virtually every researcher in the field, for example, see Arrow (1995); Bossert et al (1994); Gravel (1994); Klemisch-Ahlert (1993); Nehring and Puppe (1996); Puppe (1996); Sen (1991); Suppes (1987). Actually, its origin could be traced to the literature on flexibility under uncertainty about future tastes introduced by Koopmans (1964), see also Kreps (1979). It simply says that if B is a subset of A, then A offers at least as much freedom as B. Formally,

**Axiom M** (<u>Monotonicity</u>). For all A, B $\in$ P(X), B $\subseteq$ A $\Rightarrow$ A $\geq$ B.

Notice that Axiom M implies reflexivity of $\geq$.

The next axiom is an analogue of Puppe's freedom condition in this framework, see Puppe (1996). It stipulates that those opportunity sets that are in $\Delta$ offer some degree of freedom, to wit, they offer strictly more freedom than $\varnothing$. Formally,

**Axiom S** (<u>Standard of Freedom</u>). For all A $\in$ P(X), A $\in$ $\Delta$ $\Leftrightarrow$ not ($\varnothing \geq$ A).

In fact, Axiom S also stipulates that those opportunity sets that are not in $\Delta$ offer strictly less freedom than those that are in $\Delta$. The following lemma establishes that, in presence of transitivity and monotonicity, Axiom S is equivalent to the combination of the following two requirements:

$$\text{For all } A, B \in P(X),\ A \in \Delta \text{ and } B \notin \Delta \Rightarrow A > B. \tag{1}$$

$$\text{For all } A, B \in P(X),\ A \notin \Delta \text{ and } B \notin \Delta \Rightarrow A \sim B. \tag{2}$$

**Lemma 1.** Let $\geq$ be a binary relation on P(X) which satisfies Axioms T and M. Then Axiom S $\Leftrightarrow$ (1) and (2).

**Proof.** Clearly (1) and (2) imply Axiom S. To show the converse, suppose $A \in \Delta$ and $B \notin \Delta$. Then by Axiom S, we have not ($\varnothing \geq A$) and $\varnothing \geq B$. But, we also have, by Axiom M, $A \geq \varnothing$. Therefore, $A > \varnothing$. By Axiom T, then $A > B$. So, we have established that Axiom S implies (1). To show that Axiom S implies (2), assume $A \notin \Delta$ and $B \notin \Delta$. By Axiom S, we have $\varnothing \geq A$ and $\varnothing \geq B$. By Axiom M, we also can derive $A \geq \varnothing$ and $B \geq \varnothing$. Therefore, by Axiom T, we have $A \geq B$ and $B \geq A$, i.e. $A \sim B$. Q. E. D.

We want to point out some obvious differences between Puppe's Axiom F and our Axiom S. Axiom F maintains that every opportunity set offers some degree of freedom, while Axiom S implies that only opportunity sets belonging to $\Delta$ offer some degree of freedom. Next, Axiom F compares every set only with some of its subsets. Obviously, Axiom S does not impose this restriction. The following example clearly illustrates the difference between these two axioms. Imagine a state where politics is dominated by extremist political parties. We assume that there are ten extreme left-wing parties and, also, ten extreme right-wing parties. However, there are only two centrist (moderate) parties (say, $c_1$ and $c_2$). Our agent's theory of freedom is expressed by the motto “extremism is no virtue”, that is, his or her freedom of choice can only be represented by $c_1$ or $c_2$ (and also by their supersets). Suppose $A = \{c_1, c_2\}$ and B consists of three extreme left-wing parties and, also, three extreme right-wing parties. Then Axiom S implies that $A > B$ while, according to Axiom F, A and B are incomparable (incidentally, according to Pattanaik and Xu’s (1990) characterization of $\geq$, $B > A$).

Notice that according to Axiom F, B must offer some degree of freedom, while Axiom S implies that B offers none from the agent’s point of view. In this regard, Axiom S is immune to the criticism that is advanced toward Axiom F, namely that one could easily imagine cases where an opportunity set contains only terrible and dreadful alternatives between all of which an agent is indifferent (see Puppe (1986, p. 181).

Also, notice that $\Delta$ simply represents some fixed family of sets. We do not impose any specific restrictions on $\Delta$ except assuming that it is somewhat “large”. Next we will show that an algebraic construction of a filter specialized to $\geq$ can be interpreted as a somewhat refined standard or theory of freedom (see Lemma 2 below).

## 3. Characterization results

Our ultimate goal is to prove that an opportunity ranking can be defined set-theoretically from freedom standards (or filters). To do this, we introduce an algebraic construction of a filter specialized to $\geq$ (for more details on specialized filters, see Rasiowa (1974). A non-empty family $\Sigma$ of sets is a filter relative to $\geq$ if it satisfies the following conditions:

(f1) $\varnothing \notin \Sigma$;

(f2) $B \in \Sigma$ and $A \geq B \Rightarrow A \in \Sigma$;

Let **F** ($\geq$) be the class of all filters. We will also refer to the elements of this class as freedom standards relative to a binary relation $\geq$ which satisfies Axioms T, M and S.

**Lemma 2.** The class **F**($\geq$) of all filters (or freedom standards) with respect to $\geq$ has the following properties:

(i) If $\Xi$ is a non-empty subclass of **F**($\geq$), then $\cap\, \Xi \in$ **F**($\geq$) and $\cup\, \Xi \in$ **F**($\geq$).

(ii) If $\Gamma \in$ **F** ($\geq$), then $\Gamma \subseteq \Delta$.

(iii) $\Delta \in$ **F**($\geq$).

**Proof.** (i) Let $\Xi \subseteq$ **F**($\geq$). Take $\cap\, \Xi$. Clearly $\varnothing \notin \cap\, \Xi$. Suppose that $B \in \cap\, \Xi$ and $A \geq B$. Then $B \in \Phi$, for all $\Phi$ in $\Xi$. Since $\Phi$ is a filter, we have $A \in \Phi$ for all $\Phi$ in $\Xi$, i. e. $A \in \cap\, \Xi$. Similarly, we can prove that $\cup\, \Xi \in$ **F**($\geq$).

(ii) To the contrary, suppose $A \in \Gamma$ but $A \notin \Delta$. Since $\varnothing \notin \Delta$, we have $A \sim \varnothing$, by Lemma 1. But then $\varnothing \in \Gamma$ and $\varnothing \notin \Gamma$, a contradiction. Hence $\Gamma \subseteq \Delta$.

(iii) By our assumption $\varnothing \notin \Delta$ . Suppose $B \in \Delta$ and $A \geq B$. By Axiom S, we can derive not ($\varnothing \geq B$). By Axiom T then, we have not ($\varnothing \geq A$). Again applying Axiom S, we can conclude that $A \in \Delta$. Q. E. D.

Let $\Sigma$ be any non-empty family of sets and let $[\Sigma]$ denote the smallest filter containing $\Sigma$. $[\Sigma]$ is *called the filter generated by* $\Sigma$ and can be constructed as the intersection of all filters containing $\Sigma$.

**Lemma 3.** $[\Sigma] = \{A : A \geq B \text{ for } B \text{ in } \Sigma\}$.

**Proof.** Let $\Gamma = \{A : A \geq C \text{ for } C \text{ in } \Sigma\}$ We establish that $\Gamma = [\Sigma]$. First, we prove that $[\Sigma] \subseteq \Gamma$. To do this, we have to show that $\Gamma$ is a filter containing $\Sigma$. Suppose $A \in \Sigma$. By Axiom M, we have $A \geq A$. Hence $A \in \Gamma$, by the definition of $\Gamma$. Therefore $\Sigma \subseteq \Gamma$. Clearly, $\varnothing \notin \Gamma$.

Suppose now that $B \in \Gamma$ and $A \geq B$. Then we have $B \geq C$ for $C$ in $\Sigma$. By Axiom T, we derive $A \geq C$. Therefore, $A \in \Gamma$.

To prove that $\Phi \subseteq [\Sigma]$, let $\Phi \in \mathbf{F}(\geq)$ such that $\Sigma \subseteq \Phi$. Suppose $A \geq C$ for $C$ in $\Sigma$. Then, since $\Phi$ is a filter, we have $A \in \Phi$. Hence $\Gamma \subseteq [\Sigma]$. Q. E. D.

**Lemma 4.** (i) $\Sigma \subseteq [\Sigma]$;

(ii) $\Sigma \subseteq \Gamma \Rightarrow [\Sigma] \subseteq [\Gamma]$;

(iii) $[\Sigma] = [[\Sigma]]$;

(iv) $[\Sigma] = \cup \{[\Gamma] : \Gamma \subseteq \Sigma\}$.

Furthermore, $\mathbf{F}(\geq) = \{\Sigma \subseteq P(X) : [\Sigma] = \Sigma\}$.

**Proof.** To prove (i), suppose that $A \in \Sigma$. Since $[\Sigma] = \cap \{\Phi : \Sigma \subseteq \Phi\}$, $A \in \Phi$ for all $\Phi$ such that $\Sigma \subseteq \Phi$ and, hence, $A \in \cap \{\Phi : \Sigma \subseteq \Phi\}$. Therefore, $A \in [\Sigma]$. (ii) and (iii) can be handled similarly. To prove (iv), assume first that $A \in [\Sigma]$. Then $A \in \Phi$ for all $\Phi$ such that $\Sigma \subseteq \Phi$. Since $\Gamma \subseteq \Sigma$, $A \in \Phi$ for all $\Phi$ such that $\Gamma \subseteq \Phi$, that is, $A \in \cap$

$\{\Phi : \Gamma \subseteq \Phi\}$, i. e. $A \in [\Gamma]$. Hence $A \in \cup\{[\Gamma] : \Gamma \subseteq \Sigma\}$. Conversely, suppose $A \in \cup \{[\Gamma]: \Gamma \subseteq \Sigma\}$. Then $A \in [\Gamma]$ for some $[\Gamma]$ such that $\Gamma \subseteq \Sigma$. By Lemma 3, we have $[\Gamma] = \{A : A \geq B \text{ for } B \text{ in } \Gamma\}$. But then we also have $A \geq B$ for $B \in \Sigma$. Therefore, $A \in [\Sigma]$. Q. E. D.

**Proposition 1.** $A \geq B \Leftrightarrow$ for all $\Phi \in \mathbf{F}(\geq)$, if $B \in \Phi$, then $A \in \Phi$.

**Proof.** ($\Rightarrow$). Suppose $A \geq B$ and let $\Phi$ be any filter from $\mathbf{F}(\geq)$ such that $B \in \Phi$. Then $A \in \Phi$.

($\Leftarrow$). To prove the converse, suppose that not ($A \geq B$). Take $\Psi = \{C : C \geq B\}$. We prove that $\Psi$ is a filter such that $A \notin \Psi$ and $B \in \Psi$. By Axiom M, we have $B \geq B$ and hence $B \in \Psi$ while $A \notin \Psi$ by the assumption. Now suppose $C \in \Psi$ and $D \geq C$. Then we have $C \geq B$ and, by Axiom T, can derive $D \geq B$. Therefore, $D \in \Psi$. Q. E. D.

**Corollary .** $A \geq B \Leftrightarrow [A] \subseteq [B]$.

Let **F** be a family of freedom standards satisfying conditions (i) -(iii) of Lemma 2. We call $\mathbf{F}(\geq)$ the family of freedom standards determined by $\geq$, where $\mathbf{F}(\geq)$ is the class of all filters relative to $\geq$. By Lemma 2, $F(\geq)$ is indeed a family of freedom standards, and Proposition 1 guarantees that $\geq$ can be defined in terms of $F(\geq)$. Next, we establish that, for any family of freedom standards **F**, the opportunity ranking $\geq$ defined by:

$$(*)\ A \geq B \Leftrightarrow \text{for all } \Phi \in \mathbf{F}, \text{ if } B \in \Phi, \text{ then } A \in \Phi$$

is, indeed, an opportunity ranking. Namely, there is a one-to-one correspondence between opportunity rankings and families of freedom standards such that for any opportunity ranking $\geq$ the corresponding family of freedom standards can be defined set-theoretically from it and vice versa. Moreover, $\mathbf{F} = \mathbf{F}(\geq)$.

**Proposition 2.** Let **F** be any family of freedom standards and $\geq$ the corresponding opportunity ranking on P(X) defined by condition (∗). Then $\geq$ is an opportunity ranking in terms of freedom relative to Δ. Furthermore, $\mathbf{F} = \mathbf{F}(\geq)$.

**Proof.** We have to verify that $\geq$ satisfies Axioms T, M and S. We start with verification of Axiom M. Suppose $B \in \mathbf{\Phi}$ and $B \subseteq A$. Since $\mathbf{\Phi}$ is a freedom standard, we have $A \in \mathbf{\Phi}$. By (∗) then we derive that $A \geq B$.

To verify Axiom S, suppose first that not $(\varnothing \geq A)$. Then, by (∗) and condition (iii) of Lemma 2, we have $\varnothing \notin \Delta$ and $A \in \Delta$. Conversely, assume $\varnothing \geq A$. Again, by (∗) and condition (iii) of Lemma 2, we derive that if $A \in \Delta$, then $\varnothing \in \Delta$. Suppose $A \in \Delta$. Then, we have both $\varnothing \in \Delta$ and $\varnothing \notin \Delta$, a contradiction. Therefore, $A \notin \Delta$. By contraposition then if $A \in \Delta$, then not $(\varnothing \geq A)$.

To verify Axiom T, suppose $A \geq B$ and $B \geq C$. Then, by (∗), we have that if $\in \mathbf{\Phi}$, then $A \in \mathbf{\Phi}$ and, also, if $C \in \mathbf{\Phi}$, then $B \in \mathbf{\Phi}$. Suppose $C \in \mathbf{\Phi}$. Then, we can derive that $A \in \mathbf{\Phi}$. Therefore, by (∗) we have $A \geq C$.

To prove that $\mathbf{F} \subseteq \mathbf{F}(\geq)$, assume $\mathbf{\Phi} \in \mathbf{F}$. We have to show that $\mathbf{\Phi}$ is a filter. By our assumption $\varnothing \notin \mathbf{\Phi}$. Suppose now that $B \in \mathbf{\Phi}$ and $A \geq B$. Using (∗), we derive that if $B \in \mathbf{\Phi}$, then $A \in \mathbf{\Phi}$. Therefore, $A \in \mathbf{\Phi}$, i. e. $\mathbf{\Phi} \in \mathbf{F}(\geq)$.

To prove the converse, assume $\mathbf{\Phi} \in \mathbf{F}(\geq)$. By condition (iv) of Lemma 4, we have $[\mathbf{\Phi}] = \cup \{[\Sigma] : \Sigma \subseteq \mathbf{\Phi}\} = \cup \{[A] : A \in \mathbf{\Phi}\}$. By Lemma 3, we also have

$[A] = \{B : B \geq A\}$. Therefore, using (∗), we can obtain $B \in [A] \Leftrightarrow$ for all $\Gamma$ in **F**, if $A \in \Gamma$, then $B \in \Gamma$, i. e. $[A] = \cap\{\Gamma \in \mathbf{F} : A \in \Gamma\}$. By condition (i) of Lemma 2, we can conclude that $[A] \in \mathbf{F}$ for any A. Clearly $\{[A] : A \in \Phi\}$ is a non-empty class. Therefore, again by condition (i) of Lemma 2, $\cup\{[A] : A \in \Phi\} \in \mathbf{F}$. Hence $\Phi \in \mathbf{F}$. Q. E. D.

Notice that we have not imposed a completeness requirement on $\geq$:

$$\text{For all } A, B \in P(X), \text{ either } A \geq B \text{ or } B \geq A$$

In this respect we follow many researchers in the field. For instance, Sen (1993, p. 529) maintains that "comparisons of opportunity-freedom must frequently take the form of incomplete orderings". Klemisch-Ahlert (1993); Pattanaik and Xu (1990, 1996); Puppe (1996) all do not assume completeness. In fact, characterization results above provide a natural explanation for incompleteness of $\geq$ in terms of freedom standards. We can easily imagine the following situation: for any pair of opportunity sets $A, B \in P(X)$, let $A \in \Gamma$ but $B \notin \Gamma$. On the other hand, let $B \in \Sigma$ but $A \notin \Sigma$. Then, neither $A \geq B$ nor $B \geq A$.

The following lemma provides a rather natural requirement in this framework for $\geq$ to be complete.

**Lemma 5.** An opportunity ranking $\geq$ is complete if and only if the corresponding family of freedom standards is a chain.

**Proof.** ($\Rightarrow$). Suppose that $\geq$ is complete but $\mathbf{F}(\geq)$ is not a chain. Then, there are $\Phi$ and $\Gamma$ in $\mathbf{F}(\geq)$ such that neither $\Phi \subseteq \Gamma$ nor $\Gamma \subseteq \Phi$. Hence, there are A, B such that $A \in \Phi$,

$A \notin \Gamma$ and $B \in \Gamma$, $B \notin \Phi$. By the definition of a filter, neither $A \geq B$ nor $B \geq A$ contrary to the completeness condition.

($\Leftarrow$). To prove the converse, suppose that $\mathbf{F}(\geq)$ is a chain. Take any pair of sets A, B. We have either $[A] \subseteq [B]$ or $[B] \subseteq [A]$. By Corollary, we then derive that either $A \geq B$ or $B \geq A$. Q. E. D.

## 4. Concluding remarks

In this paper we have suggested a somewhat different approach to the axiomatic ranking of opportunity sets in terms of freedom, namely, we have relativized the axiomatic ranking of opportunity sets relative to some standard of freedom or to some theory. The central axiom of this model is Axiom S which guarantees that opportunity sets belonging to a freedom standard, offer, at least, some degree of freedom. We then provide a general characterization result of the opportunity ranking in terms of the family of freedom standards. A natural question then arises: Can we extend this characterization result if we impose some additional requirements on the opportunity ranking which would be entirely plausible from a freedom point of view? For example, we can easily extend the characterization result if we impose the following condition on the opportunity ranking:

$$\text{For all } A, B, C \in P(X),\ A \geq C \text{ and } B \geq C \Rightarrow A \cap B \geq C.$$

However, this condition is clearly unacceptable for those who value the intrinsic value of freedom as opportunity (the reader can easily construct a suitable counterexample).

Another natural extension of this work is to examine the dynamics of the standards of freedom. How can rational agents change or update their standards? We believe that the recent work on the logic of theory change might be relevant in providing an answer (for an excellent survey of this field, see Gardenfors (1988).

It is also possible to extend all the proofs of Section 3 to a case where X is an infinite set. Naturally, in this case we have to modify and reformulate some conditions. However, we have decided not to pursue this generalization in this paper.

Finally, van Hees and Wissenburg (forthcoming) have recently criticized preferential approaches to freedom as opportunity. They argue that any preferential approach to freedom must ultimately be based on some moral standard. The present paper might lend some formal ammunition to their argumentation. However, we do not claim that standards must necessarily be moral or ethical. It seems that neither morality nor ethics enters into consideration in numerous comparisons of opportunity sets.

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