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<td>Author(s)</td>
<td>O'Connell, Joan</td>
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<tr>
<td>Publication Date</td>
<td>1999</td>
</tr>
<tr>
<td>Publisher</td>
<td>National University of Ireland, Galway</td>
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<tr>
<td>Item record</td>
<td><a href="http://hdl.handle.net/10379/1165">http://hdl.handle.net/10379/1165</a></td>
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Cambridge Distribution in a World Economy

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Working Paper No. 42 November 1999

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Abstract

The paper outlines a two-country Cambridge model of growth and distribution. The condition for the Cambridge equation to apply to the world economy is outlined. When this is satisfied, a dual theorem holds in one of the two countries, and the country with the greater aggregate savings ratio is in current account surplus. The original Cambridge model was formulated as a means of equating the warranted and natural growth rates of Harrod (1939) and Domar (1946) for the case of a closed economy. Thus, the world version is a method of satisfying Harrod's requirement that his model be capable of extension so as to include foreign trade.

Keywords: Distribution; Growth; Balance-of-Payments

JEL Classification: F43; E25
1. **Introduction**

The Cambridge theory of distribution was formulated in the context of a two-class model of exogenous economic growth. Pasinetti's Cambridge equation (1962/1974, p.128), giving the rate of profit as the natural rate of growth, divided by the propensity to save of the capitalist class, comprises the central result of the model. A notable feature of this result is that the rate of profit is independent of the propensity to save of the model's second class, the workers. The Cambridge theorem was swiftly challenged. Pasinetti himself (p.106) pointed out that his result would hold only under certain specified conditions. As is well known, Meade (1966) and Samuelson and Modigliani (1966a, 1966b) demonstrated that when these conditions were not fulfilled, the Cambridge theorem would be replaced by another, dual theorem, equating the ratio of output to capital with the natural rate of growth, divided by the workers' propensity to save. More recently, Fleck and Domenghino (1987) outlined a different sort of objection to the Cambridge theory. Pointing out that Pasinetti's analysis had been conducted under the assumed absence of government activity and foreign trade, their paper suggested that the Cambridge theorem would no longer apply if the model were extended so as to include these sectors.

Beginning with the work of Steedman (1972), a series of papers has dealt with the implications of government activity for the Cambridge theorem. Fleck and Domenghino's objection in relation to foreign trade is taken up in the present paper. The origins of the Cambridge theory lie in the search for a means whereby the warranted and natural growth rates of the Harrod-Domar model (1939, 1946) would be equalized. Thus, the Cambridge theory provided an alternative to Solow's (1956) neoclassical model of economic growth for a closed economy, which proposed a variable capital-output ratio for this purpose. In the original formulation of his theory, Harrod had commented: "To complete the picture, foreign trade must be taken into account" (1939, p.28). The world comprises trading nations, and his model should be capable of accommodating this fact. A two-
country version of the Solow model was outlined by Ruffin (1979) and others. The present paper presents an international Harrod-Domar model based on the Cambridge theory.

Two countries, country 1 and country 2, make up the trading world. The global economy is characterised by full employment, and by economic growth at an exogenously given natural rate. Capital is assumed to be perfectly mobile internationally, giving a uniform rate of profit across the two countries. Under these conditions, a Pasinetti theorem for the international economy is outlined in section 1, and income in each country separately is shown to grow at the same rate as world output. Also, in section 1 it is shown that, when the Cambridge theorem applies throughout the trading world, a dual theorem equating the two ratios, that of income to wealth and that of the growth rate to the savings rate, applies in one of the two countries. The condition required for these conclusions to hold is outlined in section II.

For the case of a single, small open economy, Harrod (1948) and, later, Metcalfe and Steedman (1979) suggested that a problem of excess saving at full employment could be resolved by the export of capital, and vice versa. The balance of payments between countries 1 and 2 is outlined in section III below, and the country with the higher steady-state savings ratio is shown to be in current surplus. Metcalfe and Steedman's analysis required the country's net foreign asset position to alter at the same rate as national income in the steady state. This condition is also considered in section III. The analysis is summarized in section IV.

The content of the paper is linked to the question of the international distribution of property. Substituting optimising, representative consumers for the Cambridge savings relations yields the awkward outcome that the most patient country eventually acquires the whole world's capital stock, a result rejected by e.g., Barro and Sala-i-Martin (1995, chapter 3). In this version of an international economy, all wages are consumed, leaving
profits as the sole source of savings (Bertola, 1993, 1994). Not surprisingly, a two-
country Cambridge model without workers' savings also implies that the more patient
The following analysis includes workers' savings and, thus, this conclusion is not reached.

2. Pasinetti and Dual Theorems

In the Cambridge model, economic agents are divided into capitalists who own capital but
do not work and workers who work and generally, also, own capital. Each of these two
broad classes may be further broken down into separate groups, distinguished from one
another by different propensities to save. In the world economy, workers and capitalists
naturally decompose into two distinct groups, one for each country. For this reason, a
trading world with perfect capital mobility is, to a degree, analogous with the closed
economies of Vaughan (1971), Pasinetti (1974), and Samuelson and Modigliani (1966a)
in which each of the two more comprehensive categories, capitalists and workers, may
break down into further distinct subsets.

In the following analysis, \( r \) represents the uniform rate of profit. The world economy is
assumed to grow at the exogenously given, constant proportional rate, \( n \). The
assumptions with regard to savings behaviour comprise suitably modified versions of
those already familiar from Pasinetti's analysis of a closed economy. Workers' savings
ratios in countries 1 and 2 are denoted by \( s_{w1} \) and \( s_{w2} \), respectively. Corresponding
savings propensities of the two countries' capitalists are given by \( s_{c1} \) and \( s_{c2} \). All four
savings ratios are assumed to be positive and, in addition, \( s_{c1} > s_{c2} \), \( s_{c1} > s_{w1} \) and \( s_{c1} > s_{w2} \). Where \( W_i \) is wages and \( P_{wi} \) profits of the workers of country \( i \) \((i=1,2)\),
workers' savings in \( i \) are represented by \( S_{wi} = s_{wi}(W_i + P_{wi}) \). \( P_{wi} \) is equal to \( rQ_{wi} \) in
which \( Q_{wi} \) stands for the capital owned by the workers of country \( i \). Capitalists
savings in \( \text{i, } S_{ci} = s_{ci}^r Q_{ci} \) here \( Q_{ci} \) denotes the property of country \( \text{i's capitalists, and } rQ_{ci} = P_{ci} \) the profits accruing to them.

In the steady state of a closed economy, with a single group of capitalists, the condition that capitalists' capital must grow at the given natural rate provides the basis for the Cambridge equation. By contrast, in the world economy, with two distinct group of capitalists, the equations

\[
\begin{align*}
sc1r - n & = 0 \\
sc2r - n & = 0
\end{align*}
\]

cannot both be satisfied simultaneously. Because \( sc1 > sc2 \) capitalist wealth in country 1 grows faster than in country 2. Adapting the argument of Pasinetti (1974, pp. 141-2) for a closed economy with many groups of capitalists, the steady-state rate of profit is now given by equation (1). It is

\[
r = \frac{n}{sc1}
\]

This is the Cambridge equation for the world economy. It shows that the rate of profit is equal to the natural rate of growth, divided by the higher of the two capitalists' propensities to save. At this rate of profit, wealth of country 1's capitalists grows at the rate \( n \), and wealth of country 2's capitalists at the rate \( nsc2/s_{c1} < n \). Since wealth-owners are identified by their property, country 2's capitalists gradually become negligible. In equilibrium growth the world economy is effectively populated by three types of agent, the two groups of workers, and the capitalists of country 1.

In the steady state, the property of each class grows at the rate \( n \). Equilibrium growth in the world economy, therefore, implies that
\[
\frac{S_{c1}}{Q_{c1}} = \frac{S_{w1}}{Q_{w1}} = \frac{S_{w2}}{Q_{w2}}
\] (3)

Profits received by each class are proportional to amounts of property owned so that

\[
\frac{P_{c1}}{Q_{c1}} = \frac{P_{w1}}{Q_{w1}} = \frac{P_{w2}}{Q_{w2}}
\] (4)

Dividing (3) by the corresponding equations (4) gives

\[
\frac{S_{c1}}{P_{c1}} = \frac{S_{w1}}{P_{w1}} = \frac{S_{w2}}{P_{w2}}
\] (5)

In a Pasinetti equilibrium, national income in country 1, \(Y_1\), may be appropriately presented as

\[
Y_1 = P_{c1} + W_1 + P_{w1}
\] (6)

This equation, equation (5) and the savings relations \(S_{c1} = s_{c1}P_{c1}\) and \(S_{w1} = s_{w1}(W_1 + P_{w1})\), together yield

\[
Y_1 = \frac{s_{w1}P_{c1} + s_{c1}P_{w1}}{s_{w1}}
\] (7)

Equations (2) and (4) indicate that \(P_{c1}\) and \(P_{w1}\) both grow at the rate \(n\). Therefore, income in country 1 also grows at the same rate.

In country 2, steady-state national income \(Y_2\) is
\[ Y_2 = W_2 + P_{w2} \]  

(8)

which, when combined with \( S_{w2} = s_{w2}(W_2 + P_{w2}) = nQ_{w2} \), yields

\[ s_{w2}Y_2 = nQ_{w2} \]  

(9)

\( Y_2 \) has to grow at the same constant rate as \( Q_{w2} \), that is, also at the rate \( n \). By equation (9)

\[ \frac{Y_2}{Q_{w2}} = \frac{n}{s_{w2}} \]  

(9')

Equation (9') is the open-economy version of the Meade-Samuelson-Modigliani dual theorem of the output-to-capital ratio. It represents the ratio of income to wealth in country 2 as equal to the rate of growth \( n \) divided by country 2's aggregate propensity to save which, in the absence of domestic capitalists, is given by \( s_{w2} \). When the Cambridge theorem determines the world rate of profit, a dual theorem holds in one of the two countries.

3. Condition for Cambridge Theorem

For the closed economy case, Pasinetti (1974) Meade (1966) and Samuelson and Modigliani (1966a, 1966b) all demonstrated that the Cambridge equation is not universally valid, but applies only under certain specified conditions. The analogous problem here relates to the conditions necessary for the open-economy Cambridge equation (2) to hold. If equation (2) does not apply country 1's class structure eventually disappears, leaving country 1's aggregate savings ratio equal to \( s_{w1} \).
The savings assumption $S_{wl} = S_{wl}(W_I + P_{wl})$, and equations (2), (3), (4) and (6) together give

$$Q_{wl} = \frac{s_{wl}(Y_I - nQ_{c1}/s_{cl})}{n}$$

(10)

Where $Q$ is the world's capital stock, we have $Q = Q_{c1} + Q_{wl} + Q_{w2}$. When this definition is combined with equations (9) and (10), we may derive

$$\frac{Q_{c1}}{Q} \cdot \frac{s_{cl}^{s_{cl}}s_{wl}}{s_{cl}} = 1 - \frac{s_{wl}Y_I + s_{w2}Y_2}{nQ}$$

(11)

Since, in the steady state, $Y_1, Y_2$ and $Q$ all grow at the rate $n$, this equation shows that the ratio of the property of country 1's capitalists to world capital $Q_{c1}/Q$ is constant over time. Equation (2) applies, and country 1's class structure survives if $Q_{c1}/Q$ is positive, so that $Q_{c1}$ also grows at the rate $n$.

By equation (11), we have $Q_{c1}/Q > 0$ providing

$$\frac{s_{wl}Y_I + s_{w2}Y_2}{nQ} < 1$$

Letting aggregate wealth of country 1, $Q_{c1} + Q_{wl}$, be denoted by $Q_1$, and using equation (9), this inequality may be re-written as

$$s_{wl} < \frac{nQ_1}{Y_I}$$

or, because of the Cambridge equation (2), as
in which \( P_1 = rQ_1 \), all unearned income of country 1’s residents. In the world economy, this inequality corresponds to Pasinetti’s condition (1974, p.106) for a two-class single country.

National saving in country 1 is equal to \( S_{w1} + S_{c1} \). Combining equations (5) with the savings relation \( S_{c1} = s_{c1}P_{c1} \), this term may be given as \( s_{c1}(P_{c1}+P_{w1}) = s_{c1}P_1 \). Thus, the right-hand-side of inequality (12) represents the steady-state aggregate savings ratio of country 1.

4. The Balance of Payments

Neither conditions (12) nor (9’) reproduce exactly their respective closed-economy counterparts. In each case, the difference between the closed and open economy condition arises from a common source, that is, the accessibility, in the case of open economies, of international capital markets. In trading countries, residents' wealth generally deviates from capital employed domestically, and the profit component of national income includes net income from abroad.

In this section, condition (12) is assumed to hold throughout. The uniform rate of profit is given by the Cambridge equation (2), and the ratio of income to residents' wealth in country 2 by the dual equation (9’). We let \( K_1 \) and \( K_2 \) represent capital employed in country 1 and country 2, respectively. In country 1, domestic production \( Y_{1d} \) may be represented by

\[
s_{w1} < \frac{s_{c1}P_1}{Y_1} \quad (12)
\]
\[ Y_{1d} = Y_1 - r(Q_1 - K_1) \]  \hspace{1cm} (13)

where \( Q_1 - K_1 \) is the net foreign asset position of country 1. Domestic production in country 2 is

\[ Y_{2d} = Y_2 - r(Q_{w2} - K_2) \]  \hspace{1cm} (14)

in which the net foreign asset position of country 2 is written as \( Q_{w2} - K_2 = K_1 - Q_1 \).

In the closed economy Harrod-Domar model, equilibrium growth requires that planned savings equal planned investment. This condition carries over to open economies, with the difference, however, that total planned investment may now be broken down into domestic investment and the current account. Thus, where \( I_1 \) stands for planned domestic investment in country 1 and \( B_1 \) for country 1’s planned current account, we must have

\[ s_{c1}P_1 = I_1 + B_1 \]  \hspace{1cm} (15)

Similarly, in the case of country 2, the steady state requires

\[ s_{w2}Y_2 = I_2 + B_2 \]  \hspace{1cm} (16)

Here \( I_2 \) is planned domestic investment in country 2, and \( B_2 \) is the planned current account of that country.

In the spirit of Harrod and Domar, domestic investment in each country is assumed to be such as to maintain a constant ratio between capital employed and domestic production. In addition, the accelerator coefficient, \( v \), is assumed to be common to the two countries. Therefore, we have

\[ Y_{1d} = Y_1 - r(Q_1 - K_1) \]  \hspace{1cm} (13)

\[ Y_{2d} = Y_2 - r(Q_{w2} - K_2) \]  \hspace{1cm} (14)
\[ v = \frac{K_1}{Y_{1d}} = \frac{K_2}{Y_{2d}} \]

The usual closed-economy, steady state condition that capital employed in each country grows at the rate \( n \) is imposed. Domestic investment in country 1 is, therefore, given by

\[ I_1 = \frac{dK_1}{dt} = nK_1 = v \frac{dY_{1d}}{dt} \tag{17} \]

and in country 2 by

\[ I_2 = \frac{dK_2}{dt} = nK_2 = v \frac{dY_{2d}}{dt} \tag{18} \]

Since, in any meaningful economic system wages are positive, we have \( I/v > r \), or \( s_{c1} > nv \). On differentiating equation (14) with respect to time, and noting equations (2), (3), (9) and (18), we may derive \( I_2 \) in terms of \( Y_2 \) as follows

\[ I_2 = nvY_2 \frac{s_{c1}-s_{w2}}{s_{c1}-nv} \tag{19} \]

Equations (16) and (19) now together give country 2's current account as a constant fraction of the home national income

\[ \frac{B_2}{Y_2} = \frac{s_{c1}(s_{w2}-nv)}{s_{c1}-nv} \tag{20} \]

Similarly, differentiating equation (13) with respect to time, and using conditions (15) and (17), yields the constant ratio
In the sense of Machlup (1943) and Meade (1951), the overall balance of payments between the two countries is always balanced. Surpluses (deficits) on current account are offset by deficits (surpluses) on autonomous capital flows. We also have the equilibrium condition that

\[ B_1 + B_2 = 0 \]

(22)

which requires that the planned current account imbalance of country 1 offset exactly the planned current imbalance of country 2. Recalling that \( s_{c1} P_1 / Y_1 \) represents the aggregate savings ratio of country 1, and \( s_{w2} \) that of country 2, it becomes clear from equations (20) and (21) that the country in which the overall savings ratio, divided by the accelerator coefficient \( v \), exceeds the rate of growth \( n \) is that which is in current account surplus. Since \( v \) and \( n \) are uniform throughout the trading world, this condition can be put more concisely: the country in current account surplus is that with the greater overall savings ratio. Equation (22) requires saving in the two countries combined to equal world investment. Therefore, this equation represents the extension to the aggregate world economy of the closed-economy Harrod-Domar condition for equilibrium growth.

Finally, Metcalfe and Steedman's steady-state condition for an open economy, that the net foreign asset position should alter at the same rate as product, can be shown to hold for each country. Using equations (13), (15), (17), (21), as well as the definition \( v = K_1 / Y_1 d \), we can derive the following expression for \( B_1 \)

\[
\frac{B_1}{Y_1} = \frac{s_{c1}}{s_{c1} - n} \left( \frac{s_{c1} P_1}{Y_1} - n v \right)
\]

(21)
Manipulating this expression, and adding equation (2), yields $B_1$ in terms of country 1's net foreign asset position $Q_1 - K_1 = K_2 - Q_{w2}$

$$B_1 = n(Q_1 - K_1) \tag{23}$$

But $B_1$ is equal to the incremental change in the net foreign asset position of country 1. Therefore, equation (23) means that Metcalfe and Steedman's condition is satisfied for that country.

Parallel steps can be taken to demonstrate that this condition is satisfied also in the case of country 2. Equations (2), (14), (16), (20) and the definition $v = K_2/Y_{2d}$ can be used together to derive

$$B_2 = n(Q_{w2} - K_2) \tag{24}$$

showing that the net foreign asset position of country 2, $Q_{w2} - K_2 = K_1 - Q_1$ alters at the rate $n$.

5. Conclusions

The original closed-economy growth model of Harrod (1939) and Domar (1946) left unresolved the problem of how to equate the warranted and natural rates of growth at full employment. For the case of a single trading country, Harrod and following him, Metcalfe and Steedman (1979), proposed foreign lending and borrowing as a solution to the difficulty.
For the closed-economy case, Solow (1956) suggested a variable capital-output ratio as a way out of the Harrod-Domar impasse, while Kaldor (1956) and Pasinetti (1962/74) outlined the Cambridge theory of endogenous savings for the same purpose. The analysis of this paper incorporates both that of Metcalfe and Steedman and the Cambridge theory in the context of a two-country trading world.
References


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