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# **Optimal Design of an Immigration Points System**

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## **Abstract**

There is growing interest in the United States and elsewhere in the use of a points-based system for selecting immigrants on the basis of their observed human capital. This paper explores the design of an optimal skills-based immigrant selection system based on two basic elements: a predicted-earnings threshold for determining whom to accept and reject, and a human-capital-based earnings regression for making error-minimizing predictions of immigrant success in the host labor market. We first show how to design a points system based on what are assumed to be the optimal predicted-earnings threshold and the optimal prediction regression. We next develop a method for identifying the optimal threshold given the prediction regression. The method produces a “selection frontier” that describes the options facing policy makers. The frontier shows the tradeoff between the average quality of admitted immigrants and the number of immigrants admitted. The frontier shifts out with improved accuracy in predicting earnings as well as with increases in the variation and average quality of the applicant pool. Finally, we show how the policy maker chooses the optimal selection system given the selection frontier.

## 1.0 Introduction

In recent years, a number of industrialized countries have restructured their immigrant selection systems to better target more skilled workers. Australia has increased the share of permanent immigrants selected on the basis of skills, and has fine-tuned its points system to “select for success” based on measures such as mandatory English language testing and rigorous screening of qualifications (Hawthorne, 2005). Canada revamped its points system in 2002 in the face of deteriorating immigrant labor market performance, making it more focused on human capital based indicators of long-term success in the labor market. New Zealand has increasingly focused on attracting highly skilled workers through its points system, with emphasis on current New Zealand employment or firm job offers since 2004. Planning is well advanced in the United Kingdom to introduce a permanent points-based system to replace the pilot system that has been in place since 2002 (U.K. Home Office, 2006).

Elsewhere, skills-based immigration reform is being actively considered, even if the actual reforms have so far been tentative. After extensive debate, Germany adopted a more skill-focused system at the beginning of 2005, but plans to introduce a fully-fledged points system went down in a narrow legislative defeat. The governments of France and Ireland have announced they are actively considering the adoption of points-based systems. In the United States, the immigration reform debate has recently been dominated by question of how to deal with the large flow and stock of illegal migrants. However, the U.S. did significantly expand the availability of temporary H-1B visas for the highly skilled in the late 1990s and early 2000s.<sup>1</sup> And although the bursting of the high tech bubble (and post-September 11, 2001 security concerns) undermined the constituency for renewing the expanded cap after it expired in 2003, fears of an under-supply of skilled workers is leading to calls to reduce restrictions on the

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<sup>1</sup> The cap was expanded from 65,000 to 195,000 visas per year.

recruitment of foreign talent.<sup>2</sup> In May of 2007, the United States Senate began debate on a comprehensive immigration reform bill that includes a points-based system for selecting skilled immigrants. Although this legislation ultimately failed, this policy proposal represents a significant shift from the current emphasis on family reunification as the basis for selecting a large majority of permanent immigrants.

With this level of policy interest, it is surprising that the question of the optimal design of a skill-focused immigrant selection system has not received more attention. This paper explores the optimal design question based on a very simple idea: a selection system can be devised based on a human capital-based earnings regression for predicting how potential immigrants will “perform” in the domestic labor market and a chosen predicted-earnings threshold for deciding whom to accept and reject. We show that these two elements are sufficient to determine the optimal point allocations for various bundles of human capital characteristics. The resulting framework also provides a systematic way of evaluating existing points systems and proposed reforms to those systems. Of course, the idea of giving points for observed human capital characteristics is the essential feature of existing systems. While the allocations of points in the Canadian and Australian systems, for example, are clearly informed by the findings from the human capital literature, the allocation process appears to lack a firm analytical foundation. We hope this paper will provide that foundation.

We hasten to add that having high predicted earnings is a rather narrow basis for selecting immigrants. Most countries also place importance on reunifying families and protecting people fleeing persecution or humanitarian catastrophes. Even from a narrow economic perspective of those already present in the host country, a better measure of immigrant value is the “surplus” that the country gains from the immigrants. This surplus can be defined as the value the country receives less what they must pay to the immigrants. Simple models show that it is not necessarily the most highly skilled immigrants

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<sup>2</sup> See, for example, National Science Board (2004) and Florida (2005).

that generate the greatest surplus.<sup>3</sup> However, the relevance of human capital is likely to increase when we allow for fiscal effects, knowledge spillovers, or the value of specialized skills. Augmenting the relative supply of skilled workers should also reduce overall earnings inequality, so that skilled recruitment can be desirable on both efficiency and equity grounds.<sup>4</sup> But whatever the merits of focusing narrowly on skills, it is the case that a number of countries are striving to select more skilled and higher earning immigrant pools. It is thus worthwhile to look for a more systematic approach to designing a skills-based selection system.

We develop our method for identifying the optimal immigrant selection system as follows. In Section 3.2, we begin with a one-period horizon and an assumption that admitted immigrants would find employment to show the basic method for designing a points system for a given predicted-earnings threshold and a given prediction regression. We initially focus on linear points systems due to their ease of interpretability. We then show how the basic (linear) set-up can be extended to allow for more realistic multi-year horizons, immigrant assimilation, and an earnings threshold measured in terms of the present discounted value of the immigrant's predicted earnings stream. Finally, we discuss the possibility of non-employment and the implications of relaxing the requirement that the points system is linear.

The next two sections focus on the task of identifying the optimal threshold, which we model as a constrained optimization problem. In Section 3.3 we derive the selection frontier, which shows the tradeoff between immigrant quality and quantity and thus captures the constraint facing the policy maker. Each point on the frontier is shown to map to a unique predicted-earnings threshold. To identify the

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<sup>3</sup> See, for example, Borjas (1995).

<sup>4</sup> George Borjas (1999, p. 19) makes the following case for focusing on immigrant skills:

If nothing else, decades of social science research have established an irrefutable link between human capital—a person's endowment of ability and acquired skills—and a wide array of social and economic outcomes, ranging from earnings potential to criminal activity, work effort to drug abuse, and from family stability to life expectancy. In view of this strong link, it is not surprising that the United States cares about whether the immigrant population is composed of skilled or unskilled workers. The skill composition of the immigrant population—and *how the skills compare to those of natives*—determine the social and economic consequences of immigration for the country. [Emphasis in the original.]

frontier, we model the admitted pool as a “selected sample,” in the well defined sense that the pool is an incidentally truncated subset of the applicant pool. The truncation takes place on the basis of a comparison of predicted earnings and the predicted-earnings threshold. We show that the location of the frontier depends on the mean and variance of log earnings in the (lognormally distributed) applicant pool, and also the variance of the prediction error for the log earnings regression. In other words, the policy maker is constrained by the nature of the applicant pool and also their ability to predict the labor market success of given applicants. In Section 3.4, we add an illustrative specification of policy-maker preferences. We assume that the policy maker values immigrant human capital but faces a convex adjustment cost of adjusting to immigration. The optimal threshold is identified as the policy maker’s most preferred point on the selection frontier. Section 3.5 with a brief preview of our companion paper applying the framework to a points system for Canada based on the Longitudinal Immigrant Database (IMDB).

## 2.0 Basic Points System Design

In this section, we describe how a points system can be designed using a human capital-based earnings regression for predicting applicant “success” (i.e. predicted earnings,  $\hat{Y}_i$ ) in the host-country labor market combined with a designated predicted earnings threshold,  $\hat{Y}^*$ .<sup>5</sup> As noted in the introduction, the key idea is that applicants with predicted earnings above the threshold are accepted; all others rejected. We initially restrict attention to linear points systems, which in turn restricts us to using additively separable functional forms for the earnings regression. (A linear system gives the total points by simply adding up the points per unit of each human capital characteristic.) The earnings regression is

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<sup>5</sup> It is worth noting that there are other bases for establishing a points system. Immigrants could be selected via points systems based on linear discriminate analysis, logistic regression, or multi-criteria optimization amongst other methods. We selected our method because of its simplicity, ability to model a continuous measure of success and the direct interpretability of results with respect to a relevant policy variable.

additively separable if there is some monotonically increasing transformation that yields a right-hand side that is linear in the variables.

*(i) One-period horizon and certain employment*

To focus on the essential elements, we start with a very simple earnings regression, certain employment, and a one-period horizon for time spent in the host-country labor market. There are just two human capital indicators for a potential immigrant,  $i$ : years of schooling ( $S_i$ ) and years of experience at landing ( $E_i$ ), and the additive separable earnings regression is assumed to take the familiar log-linear (or semi-log) form,

$$(1) \quad \ln Y_i = y_i = \beta_0 + \beta_1 S_i + \beta_2 E_i + u_i \quad u_i \sim n(0, \sigma_u^2).$$

The equation is estimated by OLS, yielding an equation for what we assume is the best-linear-unbiased predictor of earnings for potential immigrant  $i$ ,

$$(2) \quad \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 S_i + \hat{\beta}_2 E_i.$$

Equation (2) is an equation for predicted log earnings ( $\hat{y}_i = \ln \hat{Y}_i$ ). However, what we need is the log of predicted earnings ( $\ln \hat{Y}_i$ ). A consistent estimate of the log of predicted earnings is calculated by adding an adjustment factor  $\sigma_u^2/2$  to the predicted log earnings<sup>6</sup> to obtain the approximation,

$$(3) \quad \ln \hat{Y}_i \approx \frac{\sigma_u^2}{2} + \hat{y}_i = \frac{\sigma_u^2}{2} + \hat{\beta}_0 + \hat{\beta}_1 S_i + \hat{\beta}_2 E_i.$$

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<sup>6</sup> See, for example, Goldberger (1968) and for a discussion on this adjustment factor.

We now slightly rearrange the equation (and henceforth ignore the approximation) to obtain,

$$(3') \quad \ln \hat{Y}_i - \frac{\sigma_u^2}{2} - \hat{\beta}_0 = \hat{\beta}_1 S_i + \hat{\beta}_2 E_i.$$

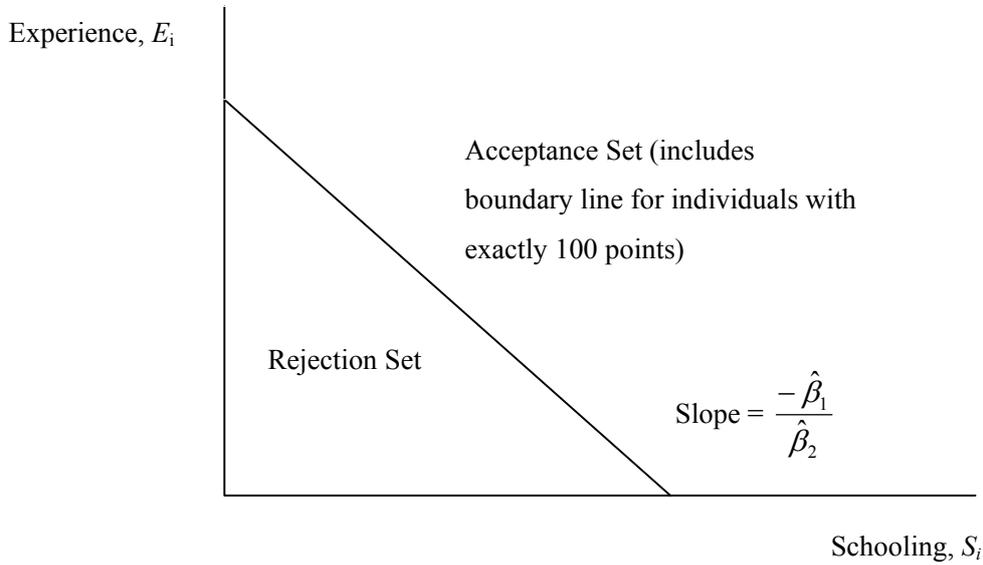
Finally, we rescale so that the left-hand side of (3') is equal to an arbitrarily chosen 100 points when predicted earnings exactly equal the predicted-earnings threshold,  $\hat{Y}_i = Y^*$ .

$$(4) \quad \frac{100\bar{\beta}}{\bar{\beta}} = \left( \frac{100\hat{\beta}_1}{\bar{\beta}} \right) S_i + \left( \frac{100\hat{\beta}_2}{\bar{\beta}} \right) E_i,$$

$$\text{where} \quad \bar{\beta} = \ln Y^* - \frac{\sigma_u^2}{2} - \hat{\beta}_0.$$

In this form, the coefficients on the schooling and experience variables give the number of points that should be granted per unit of schooling and experience respectively. Applicants who score 100 points or more (or equivalently have predicted earnings greater than the threshold,  $Y^*$ ) are accepted; applicants who score less than 100 points are rejected. The combinations of schooling and experience that result in exactly 100 points are shown by the boundary line in Figure 1. The relative value of an additional year of schooling in terms of years of experience is given by the slope of the boundary line. The figure also shows which bundles of human capital characteristics lead to acceptance and which lead to rejection.

**Figure 1**

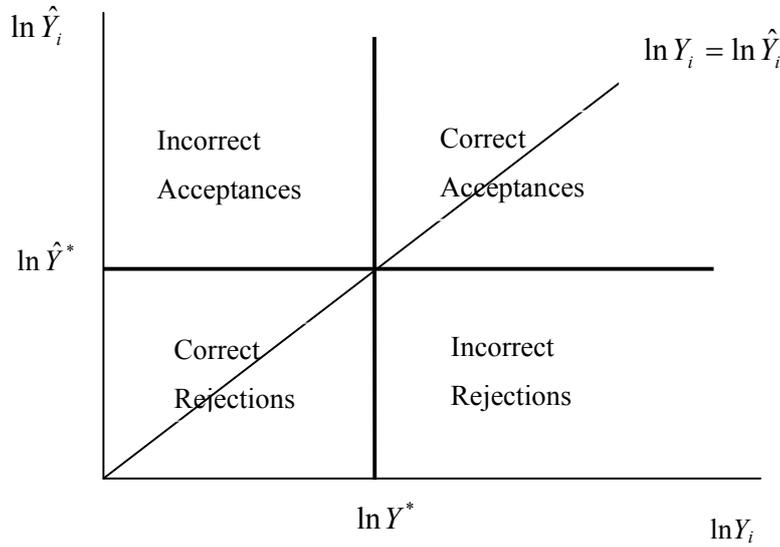


Clearly, the success of any such points system depends on the predictive success of the earnings regression. As is well known, prediction errors can result from biased estimators of the coefficients, sampling error in the estimated coefficients, measurement error in the explanatory variables, and random disturbances in actual earnings. Figure 2 graphically shows how prediction errors will lead to “mistakes” in the immigrant screening process.<sup>7</sup>

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<sup>7</sup> In Sections 3.3 and 3.4 we make the simplifying assumptions that the true earnings regression is known and the human capital variables that the immigration authorities observe are measured without error. Thus the first three sources of error mentioned above are conveniently not present, and we focus on the variance of the disturbance term (which under these simplifying assumptions is equal to the variance of the prediction error) as the key determinant of the success of the screening process for any given applicant pool and predicted earnings threshold. We return to the issue of specifying and estimating the prediction regression given multiple sources of error in Section 3.5.

**Figure 2**



(ii) *Allowing for a multi-period horizon*

An obvious limitation of this simple model is that immigrants will typically be present in the host-country labor market for longer than a single year. This will force the policy maker to consider a threshold for the present discounted value of the predicted earnings stream rather than a single year's predicted earnings. Predicted earnings for later years will be affected by how the immigrants earnings evolve with time spent in the host-country labor market. The simplest possible modification of our basic case is to add a years-since-migration variable,  $t_{it}$ , to our basic framework. The revised earnings regression is,

$$(5) \quad \ln Y_{it} = \beta_0 + \beta_1 S_i + \beta_2 E_i + \beta_3 t_{it} + u_{it} \quad u_{it} \sim n(0, \sigma_u^2).$$

Again using OLS to obtain the best-linear-unbiased predictor of earnings and the approximation used above, we can write the log of predicted earnings as,

$$(6) \quad \ln \hat{Y}_{it} \approx \hat{y}_{it} + \frac{\sigma_{\hat{y}}^2}{2} = \hat{\beta}_0 + \hat{\beta}_1 S_i + \hat{\beta}_2 E_i + \hat{\beta}_3 t_i + \frac{\sigma_{\hat{y}}^2}{2}.$$

Assuming a time horizon of  $T_i$  years and a discount rate of  $\delta$ , we use (6) to write the present discounted value of earnings as,

$$(7) \quad \begin{aligned} \hat{Z}_i &= \int_0^{T_i} e^{-\delta t_i} \hat{Y}_{it} dt_i \\ &= e^{\frac{\sigma_{\hat{y}}^2}{2} + \hat{\beta}_0 + \hat{\beta}_1 S_i + \hat{\beta}_2 E_i} \int_0^{T_i} e^{(\hat{\beta}_3 - \delta)t_i} dt_i \\ &= e^{\frac{\sigma_{\hat{y}}^2}{2} + \hat{\beta}_0 + \hat{\beta}_1 S_i + \hat{\beta}_2 E_i} \left( \frac{1}{\hat{\beta}_3 - \delta} \right) \left( e^{(\hat{\beta}_3 - \delta)T_i} - 1 \right) \end{aligned}$$

For now, we can preserve linearity in the points system if we approximate the last term in parentheses by  $e^{(\hat{\beta}_3 - \delta)T_i}$ . (That is, we simply ignore the fact that 1 is subtracted from this term, which should be a reasonable approximation for longer horizons)<sup>8</sup>. Using this approximation and taking logs yields,

$$\ln \hat{Z}_i \approx \frac{\sigma_{\hat{y}}^2}{2} + \hat{\beta}_0 + \hat{\beta}_1 S_i + \hat{\beta}_2 E_i - \ln(\hat{\beta}_3 - \delta) + (\hat{\beta}_3 - \delta)T_i.$$

We further assume that the immigrant will work until age  $\bar{A}$ , so that  $T_i = \bar{A} - A_i$ , where  $A_i$  is age at arrival. Now letting our threshold for the present discounted value of the predicted earnings stream equal  $Z^*$  (and again imposing a points cut off of 100 and ignoring the approximations), we can write our key points equation as,

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<sup>8</sup> Removing this approximation leads to a system where points are allocated linearly to human capital characteristics with a non-linear penalty for age.

$$\frac{100\tilde{\beta}}{\tilde{\beta}} = \left( \frac{100\hat{\beta}_1}{\tilde{\beta}} \right) S_i + \left( \frac{100\hat{\beta}_2}{\tilde{\beta}} \right) E_i - \left( \frac{100\hat{\beta}_3 - \delta}{\tilde{\beta}} \right) A_i,$$

(8)

where  $\tilde{\beta} = \ln Z^* - \frac{\sigma_i^2}{2} - \hat{\beta}_0 + \ln(\hat{\beta}_3 - \delta) - (\hat{\beta}_3 - \delta)\bar{A}$ .

Once again, the terms in parentheses on the right-hand-side provide the points per unit of the human capital characteristic. The key difference between (8) and (4) is that (8) allows for a points penalty based on the age of the applicant (reflecting the fact that older applicants will have fewer years of earnings in the host-country labor market).

*(iii) Allowing for non-employment*

Up to this point, we have assumed all immigrants are employed in the host economy and have focused on developing a log-linear model for predicted earnings conditional on that employment. However, immigrant non-employment is likely to be a significant concern for various reasons—lengthy job search in an unfamiliar labor market, non-recognition of immigrant credentials, poorly transferable skills, etc. If we continue to use predicted earnings as the basis for selection, the obvious extension to our model is to treat predicted (unconditional) earnings ( $\hat{Y}_{it}^u$ ) as the product of the predicted probability of employment in year t ( $\hat{J}_{it}$ ) and predicted earnings conditional on employment ( $\hat{Y}_{it}$ ). Taking logs, we have an amended predicted earnings equation:  $\ln \hat{Y}_{it}^u = \ln \hat{J}_{it} + \ln \hat{Y}_{it}$ . Based on the vast literature on empirical earnings functions, we have argued that a log-linear specification is defensible for modeling the determinants of  $\ln \hat{Y}_{it}$ . This is what allowed us develop a user-friendly linear points system. But additive separability is much harder to defend for the probability of employment. If, for example, we let  $J_{it} = e^{\alpha_0 + \alpha_1 S_i + \alpha_2 E_i}$ , then taking logs of both sides clearly gives us the necessary linear right-hand side:

$\ln J_{it} = \alpha_0 + \alpha_1 S_i + \alpha_2 E_i$ . But a log-linear specification is unlikely to be defensible for modeling the probability of employment. One obvious limitation is that the probability of employment can exceed unity. Of course, more standard models of this probability, such as probit or logit, do not produce the necessary linearity after the log transformation. This forces us to balance the costs of restrictive functional forms against the simplicity of linearity.<sup>9</sup>

Taking stock, our approach thus far has been to assume a given (optimal) earnings threshold and an (optimal) predication regression, and then to derive the optimal points system. The next two sections focus on how the selection frontier and optimal threshold are determined.

### **3.0 Derivation of the Selection Frontier**

The optimal threshold is determined as the result of a constrained optimization problem, where the constraint – what we term the selection frontier – is the tradeoff between the mean earnings of the admitted pool and number of immigrants admitted. Each point on the frontier maps to a unique predicted-earnings threshold. In this section, we derive the selection frontier and explore the factors that affect its position. In the next section, we examine how the optimal point on the frontier is chosen.

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<sup>9</sup> Note also that even though the use of the log-linear functional form is standard for earnings functions (conditional on employment), our requirement of additive separability rules out interaction and higher order terms (e.g. experience squared) as additional explanatory variables. Given the ubiquity of such terms in estimated earnings regressions, such exclusions are likely to be quite restrictive. Thus policy makers may wish to abandon the simplicity of a linear points system even without the complication of the possibility of non-employment. This raises the broader question of how important it is that the points system is linear. Clearly, a linear system is user friendly, as potential applicants can easily understand how their points total is arrived at, and also what they would need to do to increase their score. On the other hand, even highly non-linear systems can be made reasonably user friendly through the use of an on-line points calculator. The potential applicant could enter a set of characteristics, and the calculator would reveal the points score based on the predicted earnings for someone with those characteristics. The decision about adopting a non-linear system would involve a tradeoff between lost simplicity and improved predictions. The systematic approach to points-system design we develop in this paper has the virtue of allowing designers to explore how predictions are improved when non-linearities are allowed (see McHale and Rogers, 2009, for an empirical application for Canada).

Under our points system, prospective immigrants are accepted or rejected on the basis of points which reflect their expected earnings. Those who are rejected do not produce earnings in the receiving country and therefore do not contribute to the mean earnings of the immigrant pool. Thus the selection frontier is developed as a variant of incidental truncation of immigrant earnings. In this case, the earnings variable is truncated based on a forecast of earnings, not its actual value. In deriving the selection frontier, we assume that actual (post-entry) earnings are lognormally distributed across the applicant pool. In addition, we assume that the earnings prediction errors that result from the earnings regression are lognormally distributed. Together these assumptions imply that actual earnings and predicted earnings have a joint lognormal distribution with a correlation coefficient equal to the square root of the coefficient of determination ( $R^2$ ) from the earnings regression.<sup>10</sup>

The first step in deriving the frontier is to determine the mean earnings of the admitted pool for any given earnings threshold. To do this, we treat the admitted pool as resulting from the incidental truncation of the applicant pool, where the applicant is admitted if their predicted earnings,  $\hat{Y}$ , is greater than a policy-determined earnings threshold of  $Y^*$  (where  $i$  subscripts are now dropped for notional convenience). The mean earnings of the admitted pool with the general probability density function (PDF)  $f(\cdot)$  is given by,

$$(9) \quad E[Y | \hat{Y} \geq Y^*] = E[Y | \hat{y} \geq y^*] = \int_{y^*}^{\infty} E[Y | \hat{y} = x] \frac{f(x)}{1 - P(x \leq y^*)} dx.$$

We apply this general specification to our model by substituting in the conditional estimate of earnings based on the estimator of log earnings that was introduced in the previous section:

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<sup>10</sup> Using notation described in the main text the relationship between log earnings, predicted log earnings and the residual is:  $y_i = \hat{y}_i + \hat{u}_i$  since  $V(y) = V(\hat{y}) + V(\hat{u})$ , it follows that  $\hat{y} \sim N(\mu_y, \sigma_y^2 - \sigma_{\hat{u}}^2)$ .

$E[Y | \hat{y} = x] \approx e^{\sigma_u^2/2} e^x$ . We also substitute the normal PDF, for the general density function  $f(\cdot)$  to yield the conditional expectation:<sup>11</sup>

$$(10) \quad E[Y | \hat{y} \geq y^*] = \frac{1}{P[\hat{Y} \geq Y^*]} \int_{y^*}^{\infty} \frac{1}{\sigma_{\hat{y}} \sqrt{2\pi}} e^{\sigma_u^2/2} e^{-\frac{(x-\mu_{\hat{y}})^2}{2\sigma_{\hat{y}}^2}} e^x dx.$$

Collecting like terms and completing the square in the exponents yields:

$$(11) \quad E[Y | \hat{y} \geq y^*] = \frac{e^{\sigma_u^2/2} e^{(-\mu_{\hat{y}}^2 + (\sigma_{\hat{y}}^2 + \mu_{\hat{y}})^2)/2\sigma_{\hat{y}}^2}}{1 - \Phi(z(y^*))} \int_{\xi}^{\infty} \frac{1}{\sigma_{\hat{y}} \sqrt{2\pi}} e^{-[(v-\mu_{\hat{y}}/\sigma_{\hat{y}})^2/2]} dv.$$

Integrating and substituting in the standard normal transformation,  $z_{\hat{y}}(k) = \left(\frac{k - \mu_{\hat{y}}}{\sigma_{\hat{y}}}\right)$  and the fact that

$e^{\frac{\sigma_{\hat{y}}^2}{2}} = e^{\frac{\sigma_u^2}{2}} e^{\frac{\sigma_{\hat{y}}^2}{2}}$  yields:<sup>12</sup>

$$(12) \quad E[Y | \hat{y} \geq y^*] = e^{\sigma_{\hat{y}}^2/2} e^{\mu_{\hat{y}}} \left[ \frac{1 - \Phi(z_{\hat{y}}(y^* - \sigma_{\hat{y}}^2))}{1 - \Phi(z_{\hat{y}}(y^*))} \right].$$

For the final step we use the fact that  $z_{\hat{y}}(y^*) - \sigma_{\hat{y}} = z_{\hat{y}}(y^* - \sigma_{\hat{y}}^2)$  to obtain:

$$(13) \quad E[Y | \hat{y} \geq y^*] = \bar{Y} \left[ \frac{1 - \Phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})}{1 - \Phi(z_{\hat{y}}(y^*))} \right],$$

<sup>11</sup> To establish notation, we will frequently use  $\phi(\cdot)$  to denote the standard normal PDF and  $\Phi(\cdot)$  to denote the standard normal cumulative distribution (CDF).

<sup>12</sup> See Appendix B for additional details on steps from (10) to (13).

where  $\bar{Y}$  is the mean earnings of the applicant pool. With this parsimonious equation for the mean earnings of the admitted pool, we can determine how the “quality” of the admitted pool changes with the predicted-earnings threshold.

*Proposition 1: The mean earnings of the admitted pool is increasing in the predicted-earnings threshold.*<sup>13</sup>

Proof:

Many of our proofs will follow the form of calculating a partial derivative of the conditional expectation function and then characterizing its sign. In this case, we will characterize the partial derivative of the mean earnings with respect to the cutoff earnings:

$$(14) \quad \frac{\partial E[Y | \hat{y} \geq y^*]}{\partial y^*} = \bar{Y} \left[ \frac{-\phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})(1 - \Phi(z_{\hat{y}}(y^*))) + \phi(z_{\hat{y}}(y^*))(1 - \Phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}}))}{\sigma_{\hat{y}}(1 - \Phi(z_{\hat{y}}(y^*)))^2} \right].$$

Since the mean earnings of the candidate pool,  $\bar{Y}$ , and the denominator of the fraction are both positive, the sign depends on the numerator of the quotient. The mean earnings of the admitted pool will increase in the earnings threshold if and only if:

$$(15) \quad \phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})(1 - \Phi(z_{\hat{y}}(y^*))) < \phi(z_{\hat{y}}(y^*))(1 - \Phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})).$$

Since  $\phi(\cdot)$  is the standard normal PDF and  $\Phi(\cdot)$  is the standard normal CDF, this condition can be expressed in terms of normal hazard rate functions:

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<sup>13</sup> See Appendix A for additional details on the proof.

$$(16) \quad \frac{\phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})}{1 - \Phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})} < \frac{\phi(z_{\hat{y}}(y^*))}{1 - \Phi(z_{\hat{y}}(y^*))}.$$

The fact that  $\sigma_{\hat{y}} > 0$  and the normal hazard rate is an increasing function completes the proof.  $\square$

The second step in deriving the selection frontier is to determine the relationship between the percentage of immigrants admitted and the predicted earnings threshold. We define the share of candidates admitted as  $p = N/T$ , where  $N$  is the number admitted and  $T$  is the total pool size.

Candidates are admitted when their predicted earnings exceeds the predicted-earnings threshold,  $Y^*$ . Given the lognormal distribution of earnings in the applicant pool, the expected share of applicants admitted for any given threshold in log earnings is that share of applicants whose incomes exceed the threshold:

$$(17) \quad p = 1 - \Phi(z_{\hat{y}}(y^*)).$$

*Proposition 2: The share of the applicant pool that is admitted is decreasing in the predicted-earnings threshold.*

Proof: This follows directly from the monotonicity of the  $z_{\hat{y}}(\cdot)$  and the fact that  $\Phi(\cdot)$  is a CDF.  $\square$

We are now in a position to derive the selection frontier. Since  $z_{\hat{y}}(\cdot)$  and  $\Phi(\cdot)$  are positive monotonic functions, we can invert them and make a minor rearrangement of (17) to obtain:

$$(18) \quad y^* = z_{\hat{y}}^{-1}(\Phi^{-1}(1 - p)).$$

This establishes a one-to-one relationship between a cutoff threshold and share of the applicant pool that is admitted. We next subtract  $\sigma_{\hat{y}}^2$  from both sides and transform both sides by  $z_{\hat{y}}(\cdot)$  and use the fact that  $z_{\hat{y}}(y^*) - \sigma_{\hat{y}} = z_{\hat{y}}(y^* - \sigma_{\hat{y}})$  to obtain:

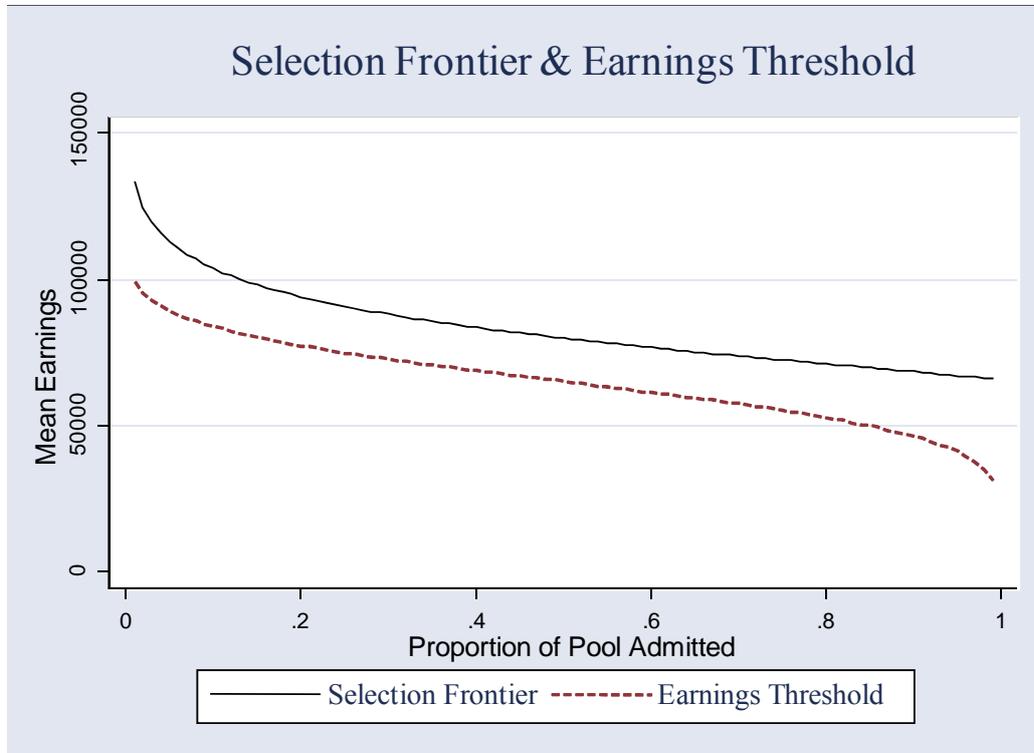
$$(19) \quad z_{\hat{y}}(y^*) - \sigma_{\hat{y}} = z_{\hat{y}}(z_{\hat{y}}^{-1}(\Phi^{-1}(1-p)) - \sigma_{\hat{y}}).$$

We substitute (19) into (13) and work through the  $z_{\hat{y}}(\cdot)$  and  $z_{\hat{y}}^{-1}(\cdot)$  transformations to obtain an equation for the selection frontier.

$$(20) \quad E[Y | p] = \frac{\bar{Y}}{p}(1 - \Phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}})).$$

The selection frontier and the associated predicted-earnings thresholds are shown in Figure 3. For illustrative purposes, the mean earnings of the applicant pool,  $\bar{Y}$  has been set to \$65,000. In addition,  $\sigma_y^2$  and  $\sigma_u^2$  are set to 0.4 and 0.32 respectively which imply that the  $R^2$  from the earnings regression is equal to 0.20 – a number that is consistent with earnings regressions.

**Figure 3**



The selection frontier shows how the “quality” of the admitted pool declines as a larger share of the applicant pool is admitted, where it is assumed that the best possible method of predicting earnings is being utilized. As expected, as we move towards admitting all applicants, the mean earnings of the admitted pool converges to the mean earnings of the applicant pool. The lower curve shows the relationship between the predicted earnings threshold and the proportion of the pool admitted. Together these curves illustrate how a predicted earnings threshold would be established. If the country chooses the point on the selection frontier where 25% of the pool will be admitted, yielding mean earnings of \$90,500 for successful candidates, the implied earnings threshold would be \$75,000.

We next confirm that the selection frontier is strictly downward sloping, so that the admission of a larger share of the applicant pool (which is achieved by lowering the earnings threshold) is associated with a decline in the mean earnings of the admitted pool.

*Proposition 3: The mean earnings of the admitted pool decreases with the share of applicant pool admitted.*<sup>14</sup>

Proof:

This time we take the partial derivative of the selection frontier function with respect to price and show that it is always negative.

$$(21) \quad \frac{\partial E[Y | p]}{\partial p} = -\frac{\bar{Y}}{p} \left( \frac{(1 - \Phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}}))}{p} + \frac{\phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}})}{\phi(\Phi(1-p))} \right)$$

This partial derivative will be negative if:

$$(22) \quad \frac{\phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}})}{1 - \Phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}})} < \frac{\phi(\Phi^{-1}(1-p))}{p}$$

Let  $x = \Phi^{-1}(1-p)$ , rearranging we get  $p = 1 - \Phi(x)$ . Substituting these into (22) allows us to express the constraint in terms of hazard rate functions:

$$(23) \quad \frac{\phi(x - \sigma_{\hat{y}})}{1 - \Phi(x - \sigma_{\hat{y}})} < \frac{\phi(x)}{1 - \Phi(x)}$$

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<sup>14</sup> See Appendix A for additional details on the proof.

As before, the fact that the normal hazard rate is an increasing function completes the proof.  $\square$

The selection frontier determines the quality-quantity tradeoff available to policy makers. We next examine the factors that determine the position of the frontier. Given our lognormality assumptions, the position of the frontier is determined by just three parameters: the mean earnings of the applicant pool ( $\bar{Y}$ ), the variance of earnings in the applicant pool ( $\sigma_y^2$ ), and the variance of the prediction error ( $\sigma_u^2$ ).

*Proposition 4: The selection frontier is shifted upwards by an increase in the mean earnings of the applicant pool.*

Proof:

Here we show that the partial derivative of the selection frontier function with respect to mean earnings of the candidate pool is positive.

$$(24) \quad \frac{\partial E[Y | p]}{\partial \bar{Y}} = \frac{1}{p} (1 - \Phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}})).$$

Because  $p$  is positive and the  $\Phi(\cdot)$  function is bounded above by 1, the value of this partial derivative is positive for all values of  $p$ . Thus an increase in  $\bar{Y}$  results in higher earnings for accepted candidates at each value of  $p$  and is therefore associated with an upward shift of the selection frontier.  $\square$

*Proposition 5: The selection frontier is rotated upwards by an increase in the variance of earnings in the applicant pool and rotates downwards by an increase in the variance of the prediction error.*

Proof:

Since neither  $\sigma_y^2$  nor  $\sigma_u^2$  appear in the formula for the selection frontier, we must substitute them in and then determine how the function responds to changes. Recall the formula for the selection frontier given by equation (20):

$$(20) \quad E[Y | p] = \frac{\bar{Y}}{p} (1 - \Phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}})).$$

Substituting in the identity  $\sigma_{\hat{y}} = (\sigma_y^2 - \sigma_u^2)^{1/2}$  we get:

$$(25) \quad E[Y | p] = \frac{\bar{Y}}{p} (1 - \Phi(\Phi^{-1}(1-p) - (\sigma_y^2 - \sigma_u^2)^{1/2})).$$

Since  $\Phi(\cdot)$  and  $\Phi^{-1}(\cdot)$  are increasing functions, the selection frontier moves up with  $\sigma_y^2$  and down with  $\sigma_u^2$ . Equivalently, the mean earnings of admitted candidates falls with an increases in prediction error and rises with an increase in the variance of the candidate pool earnings.  $\square$

*Proposition 6: Holding forecast accuracy ( $R^2$ ) constant, an increase in the variance of earnings in the applicant pool rotates the selection frontier upwards.*

Proof:

Since  $R^2 = \frac{\sigma_{\hat{y}}^2}{\sigma_y^2}$  it follows that  $\sigma_{\hat{y}}^2 = R^2 \sigma_y^2$  and an increase in the variance of the earnings in the applicant

pool is matched by a corresponding increase of the prediction variance. Substituting this result into equation (20):

$$(26) \quad E[Y | p] = \frac{\bar{Y}}{p} (1 - \Phi(\Phi^{-1}(1-p) - R\sigma_y)),$$

which is increasing in  $\sigma_y$ .  $\square$

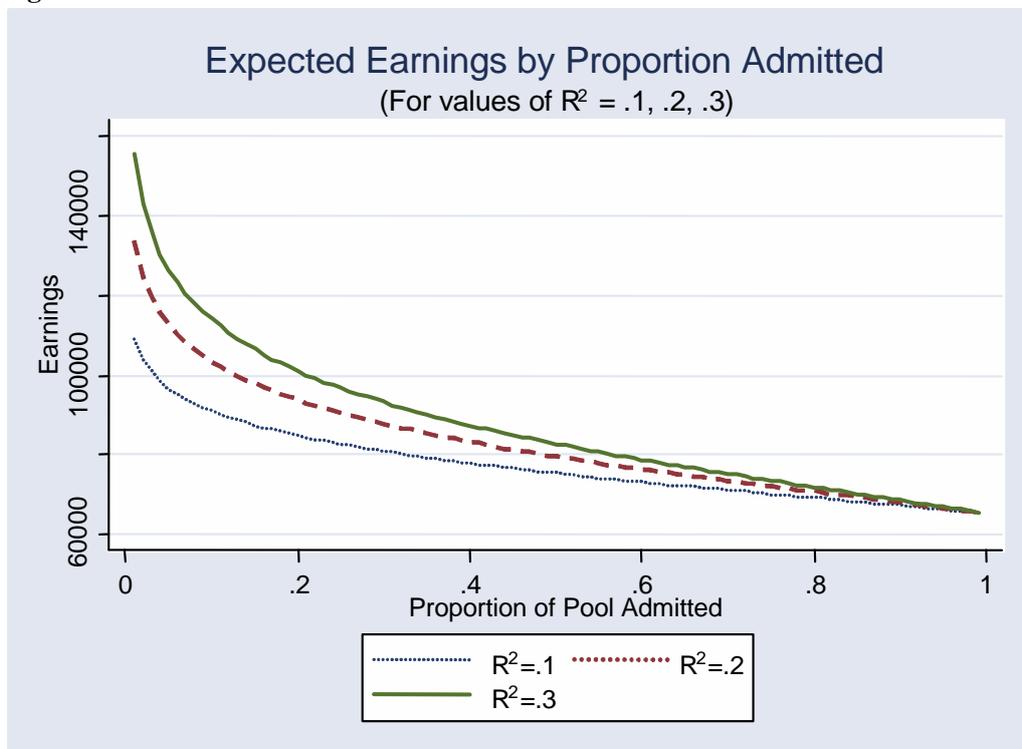
*Proposition 7: Increasing forecast accuracy ( $R^2$ ) holding the variance of earnings in the applicant pool constant rotates the selection frontier upwards.*

Proof:

This follows directly from proposition 6 by interpreting equation (20') as a function of  $R^2$  with a constant  $\sigma_y$ .  $\square$

Figure 4 shows the selection frontier for different values of the  $R^2$  from the earnings regression. A reduction in the variance of the prediction error will raise the  $R^2$  and shift the selection frontier upwards. As shown in the graph, the mean earnings of the admitted pool of immigrants increases more in response to improved forecasts when a small proportion of the applicant pool is admitted. In other words, the benefits of better prediction increase as the system becomes more selective.

**Figure 4**



#### 4.0 Choice of the Predicted-Earnings Threshold: An Illustrative Example

Our focus to this point has been the determination of the selection frontier facing policy makers. We now turn briefly to the question of how policy makers should choose which point on the frontier to operate on; that is, how they choose the predicted-earnings threshold. The chosen point will depend on the willingness of policy makers to trade off immigrant “quality” for “quantity,” which in turn will depend on the details of how skilled immigration affects the economy and politics of recruiting foreign workers. Rather than attempt to provide a detailed model of the determinants of policy-maker preferences, we limit ourselves here to an illustrative example of the tradeoffs that are likely to be involved. The essence of the example is that policy makers like immigrant human capital but face a convex cost in adjusting to immigration.

Letting  $q$  denote the average quality (as measured by the mean earnings) of the admitted pool and  $N$  the total number of immigrants admitted, we can write the total human capital of the admitted pool as  $q \times N$ . We assume that policy makers place a value of  $a$  on a unit of human capital (which we take to be measured by a dollar of earnings). There is also a cost to immigration that is a convex function of the number of immigrants. To model this, we will use  $(b/2)N^2$ . Total policy maker utility is thus,

$$(27) \quad U = aqN - \frac{b}{2}N^2.$$

The resulting policy-maker indifference curves in quality-quantity space will be U-shaped in quantity-quality space (see Figure 5). This implies that over a certain range a fall in quality can be compensated by an increase in quantity; however, once the level of immigration reaches a certain level,

further immigration must be compensated by an increase in quality. More specifically, the slope of an indifference curve through a given point is given by,

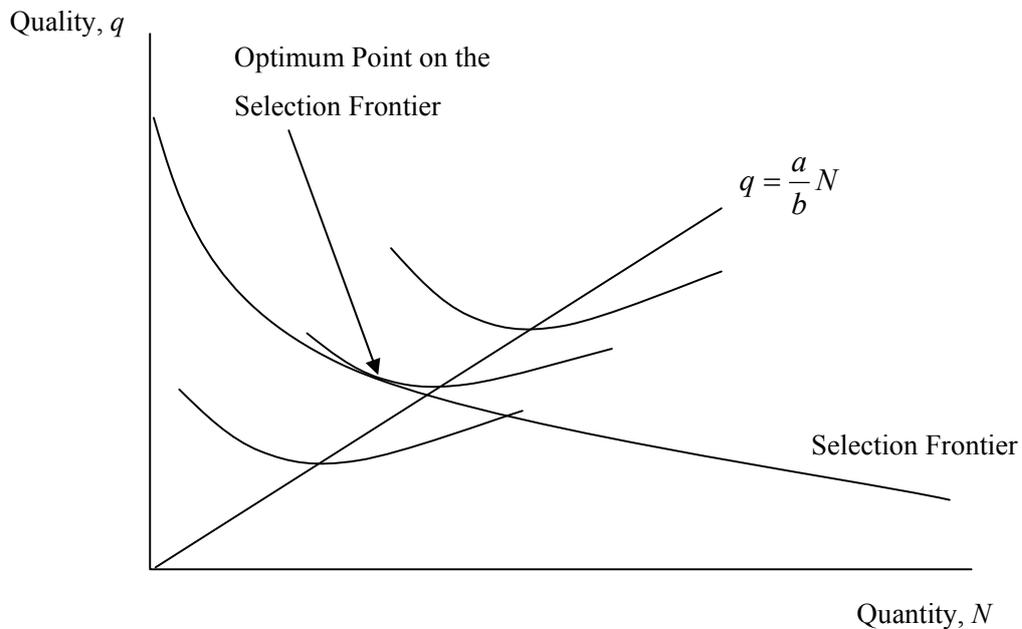
$$(28) \quad \frac{dq}{dN} = \frac{bN - aq}{aN}.$$

An indifference curve will reach its minimum point when the numerator is equal to zero. Thus the set of minimum points as we move to higher and higher indifference curves will rise along the linear schedule,

$$(29) \quad q = \frac{b}{a} N.$$

Figure 5 illustrates how the policy maker chooses the predicted-earnings threshold by reaching the highest feasible indifference given the selection frontier.

**Figure 5**



Although the constraint imposed by the selection frontier is non-concave, we show in Appendix C that there exists a unique solution to this constrained-optimization problem. Finally, as demonstrated in Section 3.4, this optimal point on the frontier maps to a unique predicted-earnings threshold (see also Figure 3). The combination of this optimal threshold and the “optimal” prediction regression are sufficient to identify the optimal immigrant selection system.

## **5.0 Concluding Comments**

This paper has explored a simple idea for designing a more rational skills-based immigration selection system: combine a human capital based earnings regression to predict potential immigrants success in the labor market with a threshold predicted earnings level to determine which immigrants to select. Our approach is motivated by the apparent desire by policy makers to select immigrants based on their likely labor market success, which explains the focus on human capital indicators in existing selection systems. However, the findings from human capital research have heretofore been applied to selection system design in a seemingly ad hoc way. Our proposed approach provides an objective basis for the evaluation of existing points systems, reforms to those systems, and the designs of new systems. It also provides a useful link between the burgeoning work on the determinants of immigrant labor market success and the growing interest in the improved design of immigrant selection policies.

In a companion paper (McHale and Rogers, 2009), we implement our design framework for Canada using the Longitudinal Immigrant Database (IMDB). The IMDB is one of the few datasets that combines information on the human capital characteristics of immigrants at arrival with income data derived from post-arrival tax filings, and is uniquely suited to developing our design approach. Most importantly in the context of the present paper, the application shows the feasibility of the design approach. More specifically, it draws attention to weaknesses in the existing Canadian points system in terms of selecting immigrants most likely to be successful in the labor market. These weaknesses include

an overweighting of foreign experience in the points grid and excessively flat educational attainment points gradient.

The New Immigrant Survey will provide an ideal dataset for applying the framework to the design of a selection system for the United States. The NIS will be a multi-cohort, longitudinal survey containing rich data on of immigrant characteristics and outcomes. One of the attractive features of the NIS is that it contains information on the pre-immigration incomes of immigrants. If pre-immigration income is predictive of post-immigration income, it could be included as a source of points (as is currently the case in the new points system in the United Kingdom). Data is now available from the initial survey of the 2003/4 NIS cohort. The 2003/4 cohort was resurveyed for the year 2007, with data expected to be available in late 2010. With the present interest in “merit-based” immigrant selection, an application of our framework based on the panel data should provide valuable input into the immigration reform debate.

## 6.0 Appendices

### 6.1 Appendix A – Details on Proofs

*Proposition 1: The mean earnings of the admitted pool is increasing in the predicted –earnings threshold.*

Starting with the formula for mean earnings conditional on admission:

$$(A1) \quad E[Y | \hat{y} \geq y^*] = \bar{Y} \left[ \frac{1 - \Phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})}{1 - \Phi(z_{\hat{y}}(y^*))} \right],$$

$$(A2) \quad \frac{\partial E[Y | \hat{y} \geq y^*]}{\partial y^*} = \bar{Y} \left[ \frac{-\phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})}{\sigma_{\hat{y}}(1 - \Phi(z_{\hat{y}}(y^*)))} + \frac{\phi(z_{\hat{y}}(y^*))(1 - \Phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}}))}{\sigma_{\hat{y}}(1 - \Phi(z_{\hat{y}}(y^*)))^2} \right],$$

$$(A3) \quad = \bar{Y} \left[ \frac{-\phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})(1 - \Phi(z_{\hat{y}}(y^*))) + \phi(z_{\hat{y}}(y^*))(1 - \Phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}}))}{\sigma_{\hat{y}}(1 - \Phi(z_{\hat{y}}(y^*)))^2} \right].$$

The denominator is positive so the sign depends on the numerator. It will be positive if:

$$(A4) \quad \phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})(1 - \Phi(z_{\hat{y}}(y^*))) < \phi(z_{\hat{y}}(y^*))(1 - \Phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})).$$

As shown in the main text, this can be rearranged as a condition on the normal hazard rate functions as given by equation (16) as used in the main text:

$$(16) \quad \frac{\phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})}{1 - \Phi(z_{\hat{y}}(y^*) - \sigma_{\hat{y}})} < \frac{\phi(z_{\hat{y}}(y^*))}{1 - \Phi(z_{\hat{y}}(y^*))}.$$

*Proposition 3: The mean earnings of the admitted pool is decreasing in the share of applicant pool admitted.*

$$(A5) \quad E[Y | p] = \frac{\bar{Y}}{p} (1 - \Phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}})),$$

$$(A6) \quad \frac{\partial E[Y | p]}{\partial p} = -\frac{\bar{Y}}{p^2} (1 - \Phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}})) - \frac{\bar{Y}}{p} (\phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}}) D_p(\Phi^{-1}(1-p))).$$

By the Inverse Function Theorem:

$$(A7) \quad D_p(\Phi^{-1}(1-p)) = -\frac{1}{\phi(\Phi^{-1}(1-p))}.$$

Substituting (A7) into (A6) and simplifying we get equation (21) in the main text:

$$(21) \quad \frac{\partial E[Y | p]}{\partial p} = -\frac{\bar{Y}}{p} \left( \frac{(1 - \Phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}}))}{p} - \frac{\phi(\Phi^{-1}(1-p) - \sigma_{\hat{y}})}{\phi(\Phi^{-1}(1-p))} \right).$$

## 6.2 Appendix B – Omitted Steps Between Equations (10) and (11)

The exponential component of the integrand in equation (10) is:

$$(B1) \quad = -\left(\frac{x - \mu_{\hat{y}}}{\sigma_{\hat{y}}}\right)^2 / 2 + x,$$

$$(B2) \quad = -\left(\frac{x - (\mu_{\hat{y}} + \sigma_{\hat{y}}^2)}{\sigma_{\hat{y}}}\right)^2 / 2 + \frac{-\mu_{\hat{y}}^2 + (\mu_{\hat{y}} + \sigma_{\hat{y}}^2)^2}{2\sigma_{\hat{y}}^2}.$$

Substituting this back into (10) we get:

$$(B3) \quad = \frac{e^{\sigma_{\hat{y}}^2 / 2}}{1 - \Phi(z_{\hat{y}}(y^*))} \int_{y^*}^{\infty} \frac{1}{\sigma_{\hat{y}} \sqrt{2\pi}} e^{-(x - (\sigma_{\hat{y}}^2 + \mu_{\hat{y}}))^2 / 2\sigma_{\hat{y}}^2} e^{-\mu_{\hat{y}}^2 + (\sigma_{\hat{y}}^2 + \mu_{\hat{y}})^2 / 2\sigma_{\hat{y}}^2} dx.$$

Moving the constant through the integral:

$$(B4) \quad = \frac{e^{\sigma_{\hat{y}}^2 / 2} e^{-\mu_{\hat{y}}^2 + (\sigma_{\hat{y}}^2 + \mu_{\hat{y}})^2 / 2\sigma_{\hat{y}}^2}}{1 - \Phi(z_{\hat{y}}(y^*))} \int_{y^*}^{\infty} \frac{1}{\sigma_{\hat{y}} \sqrt{2\pi}} e^{-(x - (\sigma_{\hat{y}}^2 + \mu_{\hat{y}}))^2 / 2\sigma_{\hat{y}}^2} dx.$$

By defining  $\xi = (\hat{y}^* - \sigma_{\hat{y}}^2)$  and using a change of variables to  $v = (x - \sigma_{\hat{y}}^2)$  the second part of this expression becomes:

$$(B5) \quad \int_{y^*}^{\infty} \frac{1}{\sigma_{\hat{y}} \sqrt{2\pi}} e^{-(x - (\sigma_{\hat{y}}^2 + \mu_{\hat{y}}))^2 / 2\sigma_{\hat{y}}^2} dx = \int_{\xi}^{\infty} \frac{1}{\sigma_{\hat{y}} \sqrt{2\pi}} e^{-(v - \mu_{\hat{y}})^2 / 2\sigma_{\hat{y}}^2} dv.$$

The second component is just the definition of the normal PDF so:

$$(B6) \quad \int_{\xi}^{\infty} \frac{1}{\sigma_{\hat{y}} \sqrt{2\pi}} e^{-(v - \mu_{\hat{y}})^2 / 2\sigma_{\hat{y}}^2} dv = 1 - \Phi(\hat{y} - \sigma_{\hat{y}}^2).$$

Substituting this result into (B4) yields:

$$(B7) \quad E[Y | \hat{y} \geq y^*] = e^{\sigma_{\hat{y}}^2 / 2} e^{(-\mu_{\hat{y}}^2 + (\sigma_{\hat{y}}^2 + \mu_{\hat{y}})^2) / 2\sigma_{\hat{y}}^2} \left[ \frac{1 - \Phi(z_{\hat{y}}(y^* - \sigma_{\hat{y}}^2))}{1 - \Phi(z_{\hat{y}}(y^*))} \right].$$

Collecting terms this simplifies to:

$$(B8) \quad E[Y | \hat{y} \geq y^*] = e^{\sigma_a^2/2} e^{\sigma_y^2/2} e^{\mu_y} \left[ \frac{1 - \Phi(z_{\hat{y}}(y^* - \sigma_{\hat{y}}^2))}{1 - \Phi(z_{\hat{y}}(y^*))} \right]$$

Finally, using  $e^{\frac{\sigma_y^2}{2}} = e^{\frac{\sigma_a^2}{2}} e^{\frac{\sigma_{\hat{y}}^2}{2}}$  AND  $z_{\hat{y}}(y^*) - \sigma_{\hat{y}} = z_{\hat{y}}(y^* - \sigma_{\hat{y}}^2)$  we have the formula for the selection frontier as shown in equation (13):

$$(13) \quad E[Y | \hat{y} \geq y^*] = \bar{Y} \left[ \frac{1 - \Phi(z_{\hat{y}}(y^* - \sigma_{\hat{y}}^2))}{1 - \Phi(z_{\hat{y}}(y^*))} \right].$$

### 6.3 Appendix C – Proof of Existence of a Unique Interior Solution

By our assumptions above, the country's objective function is:

$$(C1) \quad U = aqN - \frac{b}{2}N^2.$$

Where  $q$  is the measure of pool quality,  $N$  is the number of immigrants admitted and  $a$  and  $b$  are parameters reflecting taste. The policy frontier which constrains this choice was given by equation (22):

$$(C2) \quad E[Y | p] = \frac{\bar{Y}}{p}(1 - \Phi(\Phi^{-1}(1 - p) - \sigma_{\hat{y}})).$$

We substituting this constraint on the quality measure into (C1) and replacing  $p$  with  $N/T$ . This transforms our constrained optimization problem in two variables into an unconstrained problem in one – variable, the number of candidates admitted. This specification of the problem is given by.

$$(C3) \quad U(N) = a \frac{\bar{Y}}{N/T} (1 - \Phi(\Phi^{-1}(1 - N/T) - \sigma_{\hat{y}})) N - \frac{b}{2} N^2,$$

$$(C4) \quad = a \bar{Y} T (1 - \Phi(\Phi^{-1}(1 - N/T) - \sigma_{\hat{y}})) - \frac{b}{2} N^2.$$

Since  $\Phi^{-1}(x)$  is not defined for  $x = 0,1$  the domain of this function is  $N \in (0, T)$ . We can extend the

domain to  $N \in (0, T]$  by defining  $U(T)$  to be equal to its limiting value i.e.  $U(T) = a \bar{Y} T - \frac{b}{2} T^2$ . To

show that this problem has a unique interior solution, we will show that the function is concave and that the first order condition gives rise to a critical point on  $N \in (0, T]$ <sup>15</sup>.

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<sup>15</sup> We ignore the issue of continuity in the domain as  $T$ , the number of immigrants in the candidate pool will typically be so large as to make  $N/T$  an approximately continuous number.

To show that it is concave, we must show that the second derivative is always negative.

$$(C6) \quad \frac{dU}{dN} = -a\bar{Y}T\phi(\Phi^{-1}(1-N/T)-\sigma_{\hat{y}}))\frac{-1}{T\phi(\Phi^{-1}(1-N/T))}-bN,$$

$$(C7) \quad = a\bar{Y}\frac{\phi(\Phi^{-1}(1-N/T)-\sigma_{\hat{y}}))}{\phi(\Phi^{-1}(1-N/T))}-bN.$$

Substituting in the formula for  $\phi(\cdot)$

$$(C8) \quad = a\bar{Y}\frac{1/\sqrt{2\pi}e^{[-(\Phi^{-1}(1-N/T)-\sigma_{\hat{y}})^2/2]}}{1/\sqrt{2\pi}e^{[-(\Phi^{-1}(1-N/T))^2/2]}}-bN,$$

$$(C9) \quad = a\bar{Y}e^{[(-\Phi^{-1}(1-N/T))^2+2\Phi^{-1}(1-N/T)\sigma_{\hat{y}}-\sigma_{\hat{y}}^2+\Phi^{-1}(1-N/T)^2]/2]}-bN.$$

Cancelling the  $\Phi^{-1}(1-N/T)^2$  terms we get a simplified version of the first derivative:

$$(C10) \quad \frac{dU}{dN} = a\bar{Y}e^{[\Phi^{-1}(1-N/T)\sigma_{\hat{y}}-\sigma_{\hat{y}}^2/2]}-bN.$$

From this, we calculate the second derivative as:

$$(C11) \quad \frac{d^2U}{dN^2} = D_n(\Phi^{-1}(1-N/T)\sigma_{\hat{y}}-\sigma_{\hat{y}}^2/2)a\bar{Y}e^{[\Phi^{-1}(1-N/T)\sigma_{\hat{y}}-\sigma_{\hat{y}}^2/2]}-b,$$

$$(C12) \quad \frac{d^2U}{dN^2} = -\frac{\sigma_{\hat{y}}a\bar{Y}e^{[\Phi^{-1}(1-N/T)\sigma_{\hat{y}}-\sigma_{\hat{y}}^2/2]}}{T\phi(\Phi^{-1}(1-N/T))}-b.$$

Since  $a$ ,  $b$  and  $T$  are positive and  $e^{[\Phi^{-1}(1-N/T)\sigma_y - \sigma_y^2/2]}$  and  $\phi(\Phi^{-1}(1-N/T))$  are always greater than 0, the second derivative is negative. As a result, the function is concave.

We still have to show that there is an interior solution to this function. For that we return to the first order condition. Rearranging the (C10) we obtain the first order condition:

$$(C13) \quad a\bar{Y}e^{[\Phi^{-1}(1-N/T)\sigma_y - \sigma_y^2/2]} = bN.$$

Since the left hand side of this equation is restricted to non-negative numbers  $N$  must be positive for equality to hold. As a result, a positive (though possibly arbitrarily small and non-integer) number of immigrants would be selected.  $\square$

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