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Limits to Exhilarationism: Revisiting Kaldorian Dynamics

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Abstract

It has been argued, in the context of modern economies that there are many margins of compensation that could serve to mask the demand side impact of the deteriorated income distribution. Investment Exhilarationism was seen as one such mechanism that served to mask the impact of deteriorating income distribution. In this paper we revisit the problem of interrelation between the distribution of income and the level of income in an increasing returns regime to understand the limits of an exhilarationist regime from a theoretical point of view. We propose a model where distributive shares are endogenously determined by assuming labour productivity to vary with the level of output/capacity utilization due to economies of scale. With an additional assumption of investment being determined by profit share we model a nice feedback loop between the level of output and distribution of income, i.e. as output increases, labour productivity will increase to bring about a rise in profit share, which, in turn, will increase the level of output through a higher level of investment. Is there a limit to such a cumulative process? Here we address this question purely from a functional distribution point of view without relying on exogenous mechanisms such as ‘ceiling/floor’ capacity utilization ratios.

Key Words: Income distribution, Increasing returns, Investment exhilarationism, Overhead (skilled) labour, Limit cycles

JEL Classification: B22, C62, E12, E32, J31
Introduction
The decade of the Nineties is characterized as the period of ‘Great Expansion’ of the US economy. The impressive empirical facts about the economy explain why this period is hailed as the longest economic expansion in US post-war history. Since the first quarter of 1993, real gross domestic product grew at an average annual rate of 4 per cent. The unemployment rate dropped to 3.9 per cent – the lowest rate in thirty years. The second half of the decade witnessed increased productivity growth. The trend rate of non-farm productivity growth stood at an impressive 2.8 per cent per annum during 1995-1999 as against 1.4 per cent per annum during the period 1972-1995. There has also been a remarkable investment boom in the 90s. Investment spending as a share of GDP rose to 18.4 per cent in 2000 from 13.4 per cent in 1991. It was argued that the rise in the investment share of GDP was mainly due to the higher profit rates and profit share realized in the economy. For instance, the post tax profit rate rose from 4.9 per cent in 1979 to 8.1 per cent in 1999 and profit share rose from 17.7 per cent in 1979 to 20.5 in 1999. However, on the other hand, wage share fell from 82.3 per cent in 1979 to 79.5 per cent in 1999. An interesting point to be noted here is that within the wage share component there has been a shift away from production and nonsupervisory workers to managerial workers. But when the economy slowed down in the second half of 2000 the demand side effects of the deteriorated income distribution were not seen as a major problem. On the contrary, it was argued that “...yet, despite the clear worsening of income distribution, the last two business cycles have seen the U.S. economy still generate substantial increases in aggregate demand...the reason the demand effects of deteriorated income distribution have not yet shown up is because modern financialized economies possess many margins of compensation, and

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1 See Hall, R. E et. al. (2003)
2 See the Department of Commerce’s report on Digital Economy (2000). This kicked off a lively debate about the contribution of IT capital to the acceleration of labour productivity growth. There are various studies accounting for about 40 per cent to 73.3 per cent. See Jentzsch (2001).
3 Even after the downward adjustments in stock prices during 2000, the value of corporate stocks has nearly trebled in the decade. These developments have sparked a lively debate whether or not the U.S. economy had evolved into a “New Economy”. See Jentzsch (2001) for a review of the debate.
4 See Mishel et al. (2000, p.9).
5 See Palley (2002). The real average hourly earnings of production and nonsupervisory workers grew at an annual rate of 2.25 per cent between 1947 and 1973, but fell at an annual rate of 0.12 between 1973 and 1999. The deterioration of U.S. income distribution is a phenomenon that has been proceeding steadily for the last 20 years, and it is well documented in Mishel et al (2000).
these margins can operate for length periods of time before they are exhausted” (Palley, 2002).

Investment exhilarationism was seen as one such mechanism that served to mask the impact of deteriorating income distribution. In such a regime, rising profitability spurs an increase in investment that more than compensates any reduction in consumption spending attributable to worsened income distribution. However, when the economy entered into a recession in 2001, the limit to such a process was articulated in terms of the presence of excess capacity in the manufacturing industry

“the worsening of income distribution, be it the result of a shift to profits or a shift within the wage distribution to upper income groups, results in a situation in which there is insufficient aggregate demand to absorb the additional capacity created through new investment” (Palley, op.cit).

This argument runs into two problems: Firstly, the argument that excess capacity scales down the level of investment by lowering the profit rate seems to imply\(^6\), from a macrodynamic point of view, that there exits some ‘ceiling’ and ‘floor’ levels of the capacity utilization ratio that acts as upper and lower turning points of the trade cycle respectively. But such an argument lacks a theoretical reasoning on how the ceiling and floor level of capacity utilization are defined in the first place.

Secondly, the presence of excess capacity due to an insufficient aggregate demand in the exhilarationist regime stands contrary to the very definition of such a regime in so far as investment, which has an independent role of generating aggregate demand and responds more than saving to changes in profit share, should offset any reduction in the level of aggregate demand due to a fall in consumption. This highlights our problematic: the interrelation between the distribution of income and the level of income and the limit of exhilarationist regime. In this paper we propose a model where distributive shares are endogenously determined by assuming labour productivity to vary with the level of output/capacity utilization due to economies of scale. With an additional assumption of investment being determined by the profit share we model a feedback loop between the level of output and distribution of income, i.e. as output increases, labour productivity will increase to bring about a rise in profit share, which in turn will increase the level of

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\(^6\) This argument is not new in the literature as it has been well articulated by Joseph Steindl four decades ago in Steindl (1952). See section two on a brief review of Steindl’s model (p. 8-9)
output through a higher level of investment. Is there a limit to such a cumulative process? Here we address this question purely from a functional distribution point of view without relying on exogenous mechanisms such as ‘ceiling/floor’ capacity utilization ratios. And the aim is to show how the formation of effective demand might still operate as a constraint in this increasing returns model, where the distribution of income and the level of output influence each other in a self-reinforcing manner. In the following section we review the literature on how and to what extent this question of interrelation between income distribution and effective demand is addressed by two distinct intellectual traditions originating in Keynesian economics, on the one hand, and in Kaleckian economics, on the other. It is argued that this question of how income distribution affects the level of effective demand and, in turn, how the formation of effective demand itself influences distribution of income, is not answered either by Keynes, Kalecki or their successors. In section III, with the hindsight of empirical evidence from the U.S. economy, we develop a model to understand the possible limits of exhilaration, and follow it with results and discussion in section IV.

Section II.A: Problem of Interrelation - Kalecki and his successors

The origin and, perhaps, the clearest exposition of this problem are to be found in Kalecki’s writing. Kalecki, in his theory of distribution of income postulates a precise relationship between the degree of monopoly and the level of output. The central idea of Kalecki’s theory of income distribution is the mark-up price model, where the average (weighted) price \( \bar{p} \) for an industry is calculated as mark-up (k) over the average (weighted) of unit prime costs \( \bar{u} \) in the industry.

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7 There is a large literature on the empirical estimation of returns to scale in U.S. economy following the two classic papers of Domowitz et al. (1988) and Hall (1988), which report substantial increasing returns in the manufacturing sector. In this literature there is a wide range of returns-to-scale estimates depending on the type of data, level of aggregation and estimation methods. For instance, Basu and Fernald (1997) provide new evidence on the deviations from constant returns and perfect competition. Though their typical industry estimates appear to have significant decreasing returns, their total manufacturing and the total private economy shows apparent increasing returns. They attribute this difference to aggregation bias and heterogeneity effects. In the same vein, Caballero and Lyons (1992) observed the difference in estimates of returns to scale at different levels of aggregation but interpreted this as evidence of productivity spillovers across industries. There is another group of studies that argue that pro-cyclical productivity and increasing returns result from cyclical variations in the intensity of input use, resulting, for example, from labour hoarding (see Shapiro 1993, Blis and Cho 1994, Burnside et al. 1995).

8 The order of discussion is to accord priority to Kalecki, ahead of Keynes, in exploring the issue of interrelation between income distribution and its level within the framework of effective demand. See Feiwell (1975) for a detailed discussion on Kalecki’s discussions with Keynes’s associates.
Kalecki incorporates this mark-up price equation into his theory of distribution by noting that the ratio of aggregate proceeds of an industry to the aggregate prime costs of the industry is the mark-up $k$.

If \[ \text{Agg. Prime Costs} = \text{Agg. Wage Cost (W)} + \text{Agg. Material Cost (M)} \]
then \[ \text{Agg. Proceeds} = k (W + M) \] (II.A.2)
subtracting \((W+M)\) from both sides, we have

\[ \text{Overheads + Profits} = (k-1) (W+M) \] (II.A.3)

The relative share of wages in value added is

\[ w = \frac{1}{1 + (k-1)(1+j)} \] (II.A.4)

where \[ j = \frac{M}{W} \]

Kalecki explains the effect of distribution on economic activity by postulating an increase in the given degree of monopoly. From equation (II.A.4), an increase in the degree of monopoly will reduce the share of wages in the value added. Conversely, the relative share of profits must increase in response to the higher degree of monopoly. However, this need not imply that the total profits also increase because the level of investment and consumption expenditures determines the total profits. To quote Kalecki,

“The level of income or product will decline to a point at which the higher relative share of profits yields the same absolute level of profits”
(Kalecki, 1954, p.253)

Investment, therefore, determines the level of total profits, which, in turn, determines the level of total output to a proportion that depends upon the given degree of monopoly. This is the core of Kalecki’s theory of distribution. He does not have a theory of what determines the mark-up. As mentioned earlier, for him the degree of monopoly is
determined by a set of institutional factors, and is given exogenously to the system. An increase in the given degree of monopoly reduces output to the extent where this reduction offsets the rise in the share of profits leaving the level of total profits, which are determined by capitalists’ expenditures, unaltered. It is clear from the above analysis that an increase in the degree of monopoly does not alter the distribution of income between capitalists and workers. Kalecki’s theory is often termed as the ‘Monopoly Theory of Distribution’. However, if one delves deeper into his model one sees that the degree of monopoly does not influence the distribution of income in so far as total profit is unaltered for a given level of investment!

Viewed this way, interestingly, there is a common intersection between Kalecki and Classical economists from the point of view of the interrelation between income distribution and level of income. For instance, Marx argued that at a given level of total surplus (or total income) a change in distribution of income occurs through changes in the rate of exploitation affecting relative surplus value. In Kalecki’s model the problem is inverted, i.e. at a given degree of monopoly determining mark-up, changes in distribution occur through changes in the level of output. So, in so far as the question of interrelation is concerned, the common intersection between Kalecki and Classical economists is that the latter group fixes the level of income in determining the distribution between classes. Kalecki, on the other hand, fixes the distribution more or less exogenously, by taking the degree of monopoly as given, to determine the level of income.

The post-war period witnessed a spurt of models inspired by the Kaleckian formulation. These tried to solve the problem of the interrelation between income distribution and the level of output. One set of authors tried to solve this problem by considering the mark-up as a function of the elasticity of demand, trade union power, advertising etc. In other words they tried to solve the problem of interrelation by providing an additional theory of

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10 For instance, from our point of view the central problem with Ricardo’s model lies in the dynamics of the wage fund vis-à-vis the process of accumulation. On the one hand, at a given real wage the size of the available wage fund determines both the amount of labour that can be employed and the margin of cultivation, which in turn, determines the rent, and the wage bill, with profit as a residual. On the other hand, the change in the size of the wage fund is governed entirely by the profit accruing to the capitalists, i.e. a part of profit (saving) is reinvested as the wage fund for the next period. Hence, the size of the wage fund and the margin of cultivation, which are simultaneously given in an exogenous manner, together specify the level of output (total surplus). See Bhaduri and Harris (1987).
the determination of the mark-up. In this class of models the adverse effect of distribution, due to higher profit share or mark-up, on the level of output is shown mainly through a reduction in consumption demand. However, the other component of aggregate demand in a closed economy, namely, investment, plays no role in their analysis. In essence, it is the logic of under-consumptionist that rules in these models.

Steindl (1952) provides a theory of investment where the degree of utilization of capacity plays a vital role in determining the level of investment. He tries to overcome the disadvantages of exclusive under-consumptionist logic in explaining the interrelation between the distribution of income and the level of output. However, in his argument, lower capacity utilization puts a drag on the level of investment primarily because of the structure of the manufacturing industry. In his model he considers two types of industries. One is competitive in nature where the profit margin is flexible and excess capacity is driven out by the competitive price-cutting by firms. The other industry is monopolistic in nature where cost differentials exist between firms; consequently the profit margin is less flexible than in the competitive industries. On this industrial structure, Steindl builds up his case for stagnation in the economy as a whole by arguing that the structure of manufacturing, especially US manufacturing in the period under consideration, was evolving more and more towards a monopolistic form. So, the fall in the level of output in his analysis is due to the presence of surplus capacity, brought about by the tendency towards concentration, which depresses investment at a constant profit margin. In other words, since the profit margin is maintained by the evolving structure of industries, the fall in output is not directly due to any adverse distributional effect. Hence, Steindl’s model, getting out of the under-consumptionist mould by bringing investment into consideration and emphasizing the evolving nature of the industrial structure as an essential feature of the analytical model, analyzed partly the problem of interrelation between distribution of income and the level of output. However, Steindl’s model closely allied to Kalecki’s in so far as the industrial structure and its evolution, analogous to

12 In an economy without any economic activity by government and closed to foreign trade, private final expenditure on consumption and on investment are the two main components of aggregate demand. The under-consumptionist logic, for expanding the aggregate output, emphasizes the importance of stimulating high private consumption through a policy of high (real) wages. See Bhaduri and Margin (1993)
13 Steindl (1979) argues that the surplus capacity exits, in his model, due to insufficient aggregate demand.
Kalecki’s degree of monopoly, is largely extraneous to the macroeconomic analysis of aggregate demand. It affects aggregate demand through its impact on the level of investment, whereas in Kalecki’s model it is the degree of monopoly determined exogenously that affects the level of aggregate demand through consumption.

II.B: Problem of Interrelation – Keynesians

Almost obverse to Kalecki’s theory of distribution are Keynesian theories of distribution initiated by Kaldor (1955-56) and later extended by several economists of Keynesian persuasion. Kaldor, referring to Keynes’s Treatise on Money, calls his theory a Keynesian theory of distribution since, “it can be shown to be an application of a specifically Keynesian apparatus of thought” (1956, p.94). In other words, it relies on the same theory of effective demand operating through the multiplier mechanism to derive a theory of distribution between profits and wages. At the same time, his model is radically different from Kalecki’s and other models worked out within the Kaleckian framework, Kaldor relies on a flexible price-wage mechanism to explain the distribution process instead of relying on the exogenously determined degree of monopoly, which specifies an inflexible price-wage configuration. Kaldor’s model deserves discussion in some detail as one of the most important contributions in the class of Keynesian models characterized by endogenous distribution. Kaldor works with Kalecki’s concept of cost-determined prices rather than Keynes’ approach to the problem in the General Theory, i.e.,

\[ p = m.b.w \]  

(II.B.1)

where \( m \) is the percentage mark-up.

He deviates from Kalecki’s definition of the degree of monopoly being determined by institutional factors and assumes the mark-up to be a variable. Now instead of giving a specific functional form for the profit mark-up, Kaldor closes the system by setting the

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14 Keyne in his General Theory takes the distribution of national income as given. See (GT, p. 245).

15 Keynes in his Treatise envisages an economy to pass through three stages during the process of expansion. In Stage one, there is a rise in prices (of capital goods or of consumer goods) without any change in output or in employment. In Stage two, the real activity happens i.e., expansion of employment and output and in Stage three both prices and wages rise. But the peculiarity of the treatment in the Treatise is the extreme concentration on what is called as Stage one. In Hicks’ words “it is stage one alone that is closely analyzed and it is stage one alone to which the ‘Fundamental equations’ essentially refer” (Hicks, 1961, p.192). At the background of Kaldor’s (1955-56) article lie these fundamental equations of the Treatise.

16 Keynes in the General Theory, with the assumption of diminishing returns and perfect competition, works with a price equation such as \( p = \sqrt[f(L)]{} \) (marginal cost).
level of output at the level of appropriate either to the capacity of existing plant and
equipment or full-employment of available labour force, i.e. \( Y = Y_f \). This returns Kaldor’s
model to the Classical separation between the level of output and distribution of income.

With \( Y = Y_f \), the model boils down to the following set of equations:

\[
pY_f = \pi + w.b.Y_f \\
p\bar{I} = s_\pi \pi + s_w . w.p.Y_f \\
p = m\bar{b}.w
\]

Solving for mark-up, we have

\[
m = \frac{(s_\pi - s_w)}{(s_\pi - \bar{I}/Y_f)} \quad (\text{II.B.2})
\]

The share of profits then becomes, given \( p = m\bar{b}.w \)

\[
h = \frac{\pi/p}{Y_f} = \left[ \frac{\bar{I}/Y_f}{s_\pi - s_w} \right] \quad (\text{II.B.3})
\]

From (II.B.2) and (II.B.3) we see that, as the level of investment increases, with the given
output at full-employment level, the share of profits increases by the multiplier times.
The multiplier, in this case, is the difference between the saving propensities out of profit
and wage income respectively. This argument can be seen by rearranging equation (B.3) as

i.e., \( \bar{I} = [(s_\pi - s_w).h + s_w]Y_f \) \quad (\text{II.B.4})

In Kaldor’s model, as investment rises, the mark-up rises depressing the real wage owing
to the price equation (II.B.1). Since the level of output is at the full-employment level, the
whole adjustment takes place in terms of redistribution of income from wages to profits
(by lowering the real wage). This redistribution is captured by the saving propensities,
which show the additional saving per unit of income redistributed from wages to profits

\[
[s_\pi - s_w] = (1 - s_w) - (1 - s_\pi) \\
= c_y - c_\pi
\]

where, \( c_y \) and \( c_w \) are per unit of consumption out of wages and profits respectively.
Rearranging (II.B.3) we can also see

\[ S = [s_x h + s_w (1 - h)] Y_f \]  

(II.B.5)

From equations (II.B.4) and (II.B.5) it is clear that an increase in the level of investment, at the full employment level, generates its matching level of saving through a redistribution of income between wages and profits, at the full-employment level. This is the reason why Kaldor’s model and subsequent models in this vein have been labeled as Keynesian theories since savings is assumed to adjust passively to an increase in the level of investment. Nevertheless, these models also differ fundamentally from the Keynesian scheme, in so far as changes in the distribution, rather than level of income, ensure the equality between saving and investment.

In contrast to Kaldor’s model, from the point of view of saving/investment equality, Kalecki suggests that a higher level of investment has to bring about its matching level of saving through a higher level of output at a given degree of monopoly. This can be seen from equation (II.B.4),

\[ \bar{I} = [(s_x - s_w) \bar{h} + s_w] Y \]

i.e. profit share (h) is given and output (Y) varies to bring about the saving/investment equality. In short, in Kalecki’s theory, changes in the level of income (output) at a given degree of monopoly ensure this equality. In contrast, this reappears in a different guise in Kaldor’s model, i.e. changes in the distribution (h) through changes in the real wage, at a given level of output (Y=\(Y_f\)) ensures this equality.

From the point of view of our problematic, the interrelation between the distribution of income and its level, both these intellectual traditions provide systems which are so to say, ‘one equation short’. In other words, it is the distribution of income that becomes the exogenous variable in the case of Kalecki and the post-war models inspired by his theoretical approach, whereas, it is the level of total output that becomes an exogenous variable in Kaldor’s and subsequent Keynesian models. In what follows we shall analyze this interrelation in a model where distributive shares are endogenously determined by assuming labour productivity to vary with the level of output/capacity utilization due to economies of scale.
Section III: The Model

National Income in money terms is

\[ p.Y = \pi + W \]  

(III.1)

Here we divide total labour (L) in to two parts

\[ L = N + M \]  

(III.2)

where N is the number of operatives who vary with the level of output i.e.,

\[ N = \beta Y \]  

where Y is the actual output.

M is the number of non-operatives who vary with the level of potential output i.e.

\[ M = \alpha Y^* \]  

where \( Y^* \) is the potential output.\(^{17}\)

With these definitions (III.2) becomes

\[ L = \beta Y + \alpha Y^* \]

\[ \frac{L}{Y} = \beta + \alpha \frac{Y^*}{Y} \]

Dividing through Y we get,

\[ \frac{1}{x} = \beta + \alpha \frac{z}{Y^*} \]  

or

\[ z = \frac{Y}{Y^*} \]

\[ x = \frac{Y}{\alpha + \beta z} \]  

(III.3)

where \( x \) is the labour productivity.

Hence, with our assumption of labour productivity being an increasing function of capacity utilization, the profit share (h) is no longer exogenously given, but varies with the level of output/capacity utilization (z).

\[ i.e., \quad h = 1 - \frac{w}{p} \frac{\alpha + \beta z}{z}; \quad \frac{dh}{dz} > 0 \]  

(III.4)

\(^{17}\) There are different models in this framework that generate the Kaldorian dynamics. For instance, Velupillai (1982) was able to generate such dynamics when workers not only save but also invest. Here we are able to generate the Kaldorian dynamics without assuming saving propensities for these two types of labour. The mere division of total labour in to operatives and non-operatives, who generate scale economies, is sufficient to generate the Kaldorian dynamics in a much broader context.
Equation (III.4) can be written as

\[ h = 1 - \frac{w_p}{x(z)} \]

Or

\[ h = 1 - \frac{w}{x(z)} \cdot \frac{p}{w} \]  

(III.5)

Thus, in this case of increasing returns due to the fixed overhead wage bill of the non-operative labour, as capacity utilization \((z)\) increases labour productivity increases to bring about a fall in the unit variable cost \(w_{x(z)}\) which, in turn, would increase the share of profits \((h)\)\(^{18}\). This result concurs with the widely observed empirical fact of procyclical behaviour of profit share emphasized by Okun.\(^{19}\) However, note that, unless we explicitly postulate how money wage rate \((w)\) changes and its consequence for unit prime cost \(w_{x(z)}\) understanding the direction of change in profit share, under increasing returns regime, remains incomplete.

Here we assume the money wage rate to be a function of capacity utilization because the bargaining power of the workers increases with the tightness of the labour market which in turn depends on the level of capacity utilization. This is postulated to be a simple linear relation,\(^{20}\)

\[ w = v + \theta z, \quad v, \theta > 0 \]  

(III.6)

With labour productivity being an increasing function of capacity utilization (see eqn.III.3), the unit variable cost (UC) now becomes a more complex function of capacity utilization

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\(^{18}\) See Okun (1981, p. 15-18) for scale economies owing to overhead labour.

\(^{19}\) See Okun (1981 p. 16 and p.227) for a discussion on this issue.

\(^{20}\) Here we are defining a wage curve, with nominal wage as a function of capacity utilization.
i.e. \[ UC = \frac{w(z)}{x(z)} \]

\[ = \frac{(y + \theta z)(\alpha + \beta z)}{z} \quad (\text{III.7}) \]

From the above equation (III.7), it can be seen that unit variable cost changes as capacity utilization changes according to,

\[ \frac{d(UC)}{dz} = \frac{\theta \beta z^2 - y \alpha}{z^2} \]

Therefore, \[ \frac{d(UC)}{dz} > 0 \] depending upon whether \[ z > \sqrt{\frac{y \alpha}{\theta \beta}} \] \quad (III.8)\(^\text{21}\)

Finally, the price equation is given by,

\[ p = m.b.w \quad (\text{III.9}) \]

where \( m \) is given mark-up, \( b \) is the labour-output coefficient and \( w \) is the money wage rate and we define the following price adjustment equation\(^{22}\):

\[ \dot{p} = \lambda \left[ m \frac{w(z)}{x(z)} - p \right] \quad \lambda > 0 \quad (\text{III.10}) \]

Note that higher (lower) value of \( \lambda \) entails faster (slower) adjustment in price in response to unit variable cost.

\(^{21}\text{Note 1} > z > 0 \text{ implies} \quad ---\)

\(^{22}\text{Alternatively, the accelerationist Phillips curve could be used here as a closure. But here we used an indirect method of endogenizing both nominal wage and the labour-output coefficient in the price equation and specified this price adjustment equation that reflects the cost pressure in the price inflation dynamics. Interestingly, in the recent debate on Wage curve vs Phillips curve, it is argued that the micro-level specification of money wage curve and real wage curve is completely consistent with the aggregate accelerationist Phillips curve in the case of constant markup. See Whelan (1997) for an analytical proof of this argument. However, we hasten to note that this argument was not in consideration in our original model specification. Given the overall objective of the study, our motive in using this price adjustment mechanism in the analysis is a naïve one: we wanted to endogenize the unit variable cost by using a first-approximation nominal wage curve, as a function of capacity utilization and expressing labour-output coefficient, again, as a function of capacity utilization in the price dynamics to see its effect on the distributive dynamics. Though simplistic, these simple assumptions lead to some interesting insights in terms of the magnitude of change in price in relation to unit variable cost and its implications for profit share (see eq. III.20). I am greatful to the referee for this insight. See Blanchflower and Oswald (1995), Card (1995) and Blanchard and Katz (1997) for the debate on Wage curve versus Phillips curve.}\)
When mark-up tends to be fixed, it can be interpreted to mean that it is targeted at a certain level to achieve a target profit share. This can be shown in the price adjustment equation using the definition of wage share (1-h),

\[
\frac{\dot{p}}{p} = \lambda \left[ m(1 - h) - 1 \right]
\]

\[
= \lambda \left[ m - m\cdot h - 1 \right]
\]

\[
= \lambda \left[ (1 - m) - m\cdot h \right]
\]

\[
= m\lambda \left[ \frac{(m - 1)}{m} - h \right]
\]

(III.11)

For any given level of mark-up \( m \) the first component in equation (III.11) gives us the intended profit share. The second component \( h \) is the actual profit share. Changes in this component \( h \) are brought about by adjustment in price in relation to unit variable cost, where the latter varies with capacity utilization, as shown by (III.10). Hence, the absolute change in price is defined in terms of deviations from the actual profit share from the targeted profit share, and \( \frac{\dot{p}}{p} = 0 \) means that the actual profit share \( h \) is equal to the targeted profit share \( (m-1)/m \), which is targeted through the mark-up \( m \) in this case.

With these postulates we are now in a position to outline a formal model capturing the dynamic interrelation between the distribution of income and its level in the course of changing price and unit variable cost under increasing returns and wage bargain by organized labour in accordance with (III.6).

The realized profit share equation is given definitionally from (III.5) as,

\[
\dot{h} = (1 - h) \left[ \frac{\dot{p}}{p} - \left( \hat{w} - \hat{x} \right) \right]
\]

Using equation (III.3), (III.6), (III.10) and (III.11) we have
\[ \dot{h} = c.(1 - h) \left[ \lambda.(m.(1 - h) - 1) + \left( \frac{\alpha \nu - \beta \theta.z^2}{(\alpha + \beta.z)(\nu + \theta.z)} \right) \cdot \frac{1}{z} \right] \] (III.12)

Let \( g(z) = \left( \frac{\alpha \nu - \beta \theta.z^2}{(\alpha + \beta.z)(\nu + \theta.z)} \right) \cdot \frac{1}{z} \) in equation (III.12)

Now consider the rate of change in capacity utilization, which is governed by excess demand in the product market, i.e.

\[ \dot{z} = a \left[ I(.) - S(.) \right] \] (III.13)

Here \( I(.) \) is the investment function and is defined as \( I = l(h,z) \), where \( h \) is the profit share and \( z \) is the capacity utilization ratio.\(^{23}\)

The saving function is defined as

\[ S = s \cdot \pi \], where \( s \) is the propensity to save out of profits (\( \pi \)) and \( 1 \geq s \geq 0 \)

This is further decomposed as

\[ S = s \cdot \frac{\pi}{Y} \cdot \frac{Y^*}{Y} \cdot Y^* \], where \( Y \) is the actual output and \( Y^* \) is the potential output.

Normalizing with respect to \( Y^* \) we have

\[ S = s \cdot h \cdot z \] (III.14)

Therefore, equation (III.13) can be rewritten as

\[ \dot{z} = a \left[ I(h,z) - s.h.z \right] \] (III.15)

\(^{23}\) See Bhaduri and Marglin (1990, p.105) for deriving this functional form. They argue for a formulation of investment as a function of profit share, rather than profit rate, on the ground that this clearly separates the two influences at work whereas the rate of profit reflects the dual influences of profit share and capacity utilization.
Equations (III.12) and (III.15) characterize our coupled dynamical system

\[ \dot{h} = c(1-h) \left[ \lambda(m(1-h) - 1) + \left( \frac{\alpha \nu - \beta \theta \cdot z^2}{(\alpha + \beta \cdot z)(\nu + \theta \cdot z)} \right) \cdot \frac{1}{z} \right] \]

\[ \dot{z} = a \left[ I(h,z) - s \cdot h \cdot z \right] \quad a,c > 0 \text{ are speed of adjustment parameters.} \]

Let us analyze this system:

0: In this system \([I_s: (h = 1, z)]\) is an Invariant subspace, i.e. any orbit belongs to \(I_s\) remains in \(I_s\). And no trajectory can cross \(I_s\).

1: There are two fixed points to the coupled dynamical system, an economically trivial one with zero wage share at \(h_1 = 1\) and the other at \(h_2 = 1 - \frac{1}{m}\), where \(m > 1\) by definition. That is, (i) \(\dot{z} = 0, h = 1\)

(ii) \(\dot{z} = 0, h = 1 - \frac{1}{m}\)

** Condition for the existence of the trivial fixed point at \((h_1 = 1, z_1 = z_1^*)\)

i.e., \(\dot{h} = 0, \dot{z} = 0\) at \((h_1 = 1, z_1 = z_1^*)\) is \(I(I_s, z_1^*) = s \cdot z_1^*\)

** Condition for the existence of fixed point at \((h_2 = (1 - \frac{1}{m}), z_2 = z_2^*)\) is:

at \(h_2 = 1 - \frac{1}{m}\), for \(\dot{z} = 0\) we need \(I(z_2^*, 1 - \frac{1}{m}) = s \cdot (1 - \frac{1}{m}) \cdot z_2^*\)

If we assume \(I = i \cdot h + j \cdot z\), here \(i\) and \(j\) are positive constants, then we have,

\(s \cdot (1 - \frac{1}{m}) \cdot z_2^* = i \cdot (1 - \frac{1}{m}) + j \cdot z\)

\(\Rightarrow z_2^* \left[ j - s \cdot (1 - \frac{1}{m}) \right] = a \left( \frac{1}{m} - 1 \right)\)

Then for \(z_2^* > 0\) the condition is \(j < s \cdot (1 - \frac{1}{m})\) \(\ldots\) (III.16)
2: The Jacobian matrix of partial derivatives, evaluated at trivial equilibrium, \( h=1 \), is given as,

\[
J \bigg|_{h=1,z=z^*} = \begin{bmatrix}
 a(I_z - s.h) & a(I_h - s.z) \\
 0 & \lambda_c
\end{bmatrix}
\]

Where \( \text{Trace} \), \( T : a(I_z - s.h) + \lambda_c \) and \\
\( \text{Deter} \), \( D = a.c.\lambda.(I_z - s.h) \)

Here, if we assume \((I_z - s.h) < 0\), the usual stability criteria for one variable Keynesian model, then for trace \( T \) to be negative we should have \( \lambda \gg c > 0 \), which is plausible in many circumstances. Note here that the determinant \( D < 0 \). We shall discuss the nature of instability of this fixed point when we formally state the properties of the system.

3: The Jacobian matrix of partial derivatives evaluated at the other equilibrium \( h_2 = 1 - \frac{1}{m} \) is given as,

\[
J \bigg|_{h_2=1-\frac{1}{m},z_2=z_2^*} = \begin{bmatrix}
 a(I_z - s.h) & a(I_h - s.z) \\
 c(1-h).g(z).(I_z - s.h) & c(1-h)[\lambda_c.m + g(z).(I_h - s.z)]
\end{bmatrix}
\]

\[
T : a(I_z - s.h) - c.\lambda_c.\left[1 + \frac{h}{\lambda}g(z).(I_h - s.z)\right] + c.g(z).(I_h - s.z) \quad (III.17)
\]

\[
D : -a.c.\lambda_c.(I_z - s.h) \quad (III.18)
\]

Here there are two possibilities exist depending on whether investment responds more or less strongly than saving with respect to changes in profit share.

Consider the Stagnationist case, where \((I_h - s.z) < 0\), i.e. investment responds relatively weakly compared to saving to changes in profit share. In this case, there is no ambiguity about the local stability of the system as the trace is negative (see condition III.17) and
determinant is positive (condition III.18) and, therefore, the system is locally asymptotically stable. But in the Exhilarationist case, where investment responds relatively more strongly than saving to changes in profit share, i.e. \( (I_h - s.z) > 0 \) the local stability analysis suggests wider possibilities with richer dynamics, which is formalized in the following theorem.

**Theorem:**

*Limit Cycles exist in an Exhilarationist regime*

**Proof:**

The coupled dynamical system in capacity utilization (\( z \)) and profit share (\( h \)),

\[
\dot{z} = a \left[ I(h,z) - s.h.z \right]
\]

\[
\dot{h} = c (1 - h) \left[ \lambda \cdot \left( m \cdot (1 - h) - 1 \right) + \frac{a \cdot \nu - \beta \cdot \theta \cdot z^2}{(\alpha + \beta \cdot z)(\nu + \theta \cdot z)} \right] \frac{1}{z} \cdot \dot{z}
\]

has the following properties.

- \( I(.) \) and \( S(.) \) are continuously differentiable in the non-negative orthant \( R \), with \( I_h, I_z, S_h, S_z > 0 \).
- There exists a finite \( \bar{z} \), such that \( \forall \ z < \bar{z}, \ I(h,z) < S(h,z) \). Furthermore \( \dot{z} \) need not be monotonically decreasing throughout in \( (h,z) \) space.
- \( I_s; (h=1,z) \) is an invariant subspace. Since any orbit which belongs to \( I_s \) remains in \( I_s \) and also no trajectory can cross \( I_s \), which implies that \( h=1 \) line is a natural boundary of this system.

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\[24\] This functional form is the one Kaldor assumed to prove the existence of cycles in his model. See Kaldor (1940, p.85). The assumption underlying this functional form is that for low values of \( z \), \( \dot{z} \) is negative i.e. the response of investment is lower than the response of saving for unit changes in capacity utilization. However, note that this is only one of the plausible functional forms. See Appendix B in Bhaduri and Marglin (1990), for further discussion.
There exists two fixed points \((h_1^*, z_1^*)\) and \((h_2^*, z_2^*)\) in the positive orthant and their stability properties are given by:

**For the Fixed point, say A** (see Figure 1, below), at \(h_1 = 1, z_1 = z_1^*\). From the Jacobian matrix of partial derivatives, evaluated around this fixed point, it is clear that \(\frac{\partial z}{\partial z} < 0\) (given the assumption \((1 - s - h) < 0\) and \(\frac{\partial h}{\partial h} > 0\) (with both \(c\) and \(\lambda\) being positive)

This implies that this fixed point on the invariable subspace has one stable arm with respect to \(z\)-axis and a transverse unstable arm with respect to \(h\)-axis (see Fig. 1).

**For the fixed point, say B** (see Figure 1, below), at \(h_2 = (1 - \frac{1}{m}), z_2 = z_2^*\).

With \(D > 0\), the negativity of the trace is ensured here only if \(a >>> c > \theta\) (see condition III.17). However this condition could be violated with trace \((T)\) being positive in this regime, when
\[
a |(1 - s - h) + c(\lambda + (1 - h)g(z)(1_h - s.z))| > 0
\]
or \(a |(1 - s - h) + c(\lambda + (1 - h)g(z)(1_h - s.z))| > 0\)

\[
\Rightarrow \quad |c(\lambda + (1 - h)g(z)(1_h - s.z))| > a |(1 - s - h)|
\]

or \(\frac{c}{a} > \frac{|(1 - s - h)|}{\lambda + (1 - h)g(z)(1_h - s.z)}\) \((\text{III.19})\)

The stability of the fixed point B is ensured only if \(a >>> c > \theta\), i.e. only if the speed of adjustment of the level of output or capacity utilization \((z)\) is faster than the speed of adjustment of distribution of income or profit share \((h)\), which is plausible in many real economic circumstances. However, condition (III.19) shows the possibility that the speed of adjustment of distribution of income or profit share \((c)\) is faster than the speed of adjustment of the level of output or
capacity utilization (a), which shows the ambiguity of the sign of trace in the exhilarationist case.\textsuperscript{25} In order to establish the existence of limit cycle one needs to show that the system (III.12)-(III.15) has a compact invariant set in the positive orthant. Given that the invariant subspace (I, h=1) serves as a natural boundary to the system and the fact that the capacity utilization ratio (z) is bounded, the set $F = \{h, z|0,1\times[0,1]\}$ is a compact invariant subset in the positive orthant. Since the fixed point $B \left( h_2 = \frac{1}{m}, z_2 = z_2^* \right)$ lies within the set $F$ and is unstable, by Poincare-Bendixson theorem, a trajectory starting anywhere within the invariant set (except in the equilibrium point itself) will converge to a closed orbit.\textsuperscript{26} This proves the existence of limit cycles in the case of an Exhilarationist regime.

\textbf{Figure 1: Dynamics of profit share and capacity utilization}

\textsuperscript{25} By Bendixson’s Negative Criteria this condition hints at the possibility of cyclical fluctuations. Cf Cesari (1971)
\textsuperscript{26} Cf: Hirsch and Smale (1974, p. 248-50)
Discussion:

We may elaborate the economic description of the cycle along the lines discussed above.

At a low level of capacity utilization, i.e. $z < \sqrt{\frac{\nu \alpha}{\theta \beta}}$, the unit variable cost $(\frac{w}{w_x})$ falls. With prices remaining fixed, i.e. the actual profit share is equal to the target profit share targeted at the give level of markup, the falling unit variable cost implies a rise in the share of profits. In an exhilarationist regime with a sufficiently strong effect of profitability on investment, i.e. investment responding more than saving to changes in profit share, $(I_h - s.z) > 0$, investment increases unambiguously as both the profitability effect and accelerationist effect are positive. As a result, output expands in zone I and drives the cycle towards to zone II. With the speed of adjustment of profit share much faster than the speed of adjustment of capacity utilization, the mechanics of the cycle drives the cycle up towards zone II, where the economy moves into high profit share, high capacity utilization zone. Here in this zone, as it can be inferred from the Figure 1, the rate of change in profit share is positive while the rate of change in capacity utilization is negative signaling the problem of effective demand, which limits a further rise in the level of output or capacity utilization. The problem of effective demand, which limits a further rise in the level of output or capacity utilization is negative signaling the problem of effective demand, which limits a further rise in the level of output or capacity utilization (z). In other words, what we get here is a kind of negative multiplier effect where, the negative effect of consumption, due to adverse distribution of income between classes, more than offsets the positive effect of investment on the level of output. Any further increase in the level of output or capacity utilization would drive it to its critical limit $z > \sqrt{\frac{\nu \alpha}{\theta \beta}}$, where the unit variable cost rises to exert downward pressure on profit share (h), from the cost side. This can also be seen from the point of changes in price in response to changes in unit variable cost from condition (III.19).

For the right hand side in condition (III.19) to be positive, we require

$$[- \lambda + (1 - h)g(z)(I_h - s.z)] > 0$$

which implies $\lambda < (1 - h)g(z)(I_h - s.z)$

(III.20)
where, as explained earlier, $\lambda$ is the speed of adjustment of price in response to unit variable cost (see Eqn. III.10). Since $\lambda < 1$ in (III.20), the adjustment in price is slower in response to adjustment in unit variable cost. In other words, in zone III, rising unit variable cost is not adequately compensated by a corresponding rise in price (percentage terms) implying a fall in the profit share.\(^{27}\) Since the regime is exhilarationist by assumption, the negative impact of profit share on investment is stronger than that of the positive effect of high capacity utilization to effect the turn around in the cycle. In terms of the mechanics of the system, the fall in the share of profits feeds on to the output equation (see $z$ equation of the system) resulting a fall in the latter, which is due to a more than offsetting fall in investment over any rise in consumption.\(^{28}\) Hence at the peak of activity i.e. $z > \frac{\sqrt{v \alpha}}{\theta \beta}$, the negative impact of profitability on investment outweighs the positive accelerationist impact of capacity utilization to bring the expansion in investment to an end and consequently capacity utilization and output also fall. Both profit share and capacity utilization fall to lead the cycle to zone IV. Consequently the expansion in both investment and capacity utilization comes to an end, until capacity utilization is again low enough ($z < \frac{\sqrt{v \alpha}}{\theta \beta}$) to make profit share rise and repeat this process of dynamic oscillation of the economy.

\(^{27}\) This is an interesting point about our result: the rise in the unit variable cost is not adequately compensated by a rise in price (in percentage terms) signifies that our model is able to explain the change in profit share in two different market structures, i.e. in a Monopsonistic labour market, where the rate of change in money wages is greater than the rate of change in productivity or in the context of a Competitive product market, where the productivity gains are passed on to the consumers.

\(^{28}\) The shift within the wage distribution to upper income groups may also hamper the growth in the consumption demand. For instance, it has been pointed out that the growth of managerial (overhead labour or non-operative labour $M$, in our model) worker’s share in the total wage share has been higher than that of production and nonsupervisory (operative labour $N$, in our model) worker’s share during 1979-1999. See Palley (2002, p.12).
Conclusion

In the decade of the nineties the U.S. economy registered the longest expansion in its post-war era. However, when the economy entered into a recession in 2001, the limit to such a process, despite a worsened income distribution scenario of the decade, was articulated in terms of the presence of excess capacity in the manufacturing industry. Moreover, it was argued that the demand side effects of the deteriorated income distribution were masked by many margins of compensation provided by the modern financialized economy and investment exhilarationism of the decade was considered as one among them. In this paper, we have rigorously theorized the empirics of the U.S economy and given a plausible explanation for the recession of 2001 through the decreased intensity of endogenous compensation mechanisms, such as changes in income distribution and its impact on the components of aggregate demand. In essence we are addressing the question - Is there a limit to exhilarationism? We examine this problem purely from a functional distribution point of view without relying on exogenous mechanisms such as ‘ceiling/floor’ capacity utilization ratios along the lines of Kalecki, Kaldor and Goodwin to understand the limits of such a process. We approach this problem by proposing a model of endogenous interaction between distributive shares and the level of output/capacity utilization, where labour productivity owing to scale economies affects the share of profits, which, in turn, affects the level of output through the investment function. The result of such an interaction generates a similar kind of dynamics posited by Kaldor and Goodwin, stressing the role of effective demand in the context of an economy with scale economies. In other words, we show that the problem

\[ z > \frac{\sqrt{\alpha}}{\theta \beta} \]

would drive up the unit variable cost, which, in turn, exerts a downward pressure on the actual profit share due to the wage-price dynamics (see eq. III.20).

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29 Elsewhere, on the empirical side it was observed that the virtuous cycle between demand growth and productivity growth favorable to employment dynamics that characterized the sixties and the seventies, in Europe, seemed to have died down in the last fifteen years. See Piacentini and Pini (1988), Boyer and Petit (1988).

30 See Velupillai (1998, 2004) for an analytical history of the evolution of the Theory of Macrodynamics along the lines of Kalecki, Hicks, Goodwin and Kaldor.

31 Here it may instructive to elucidate the results of our model from the point of view of Goodwin’s (1967) model. In his model, employment increases when profitability is high and that profitability suffers in periods of high employment. Analogously in our model, profit share increases with capacity utilization due to scale economies, which, in turn, suffers at the higher levels of capacity utilization. This is due to the following effects: First, due to the adverse impact of higher profit share on consumption demand more than outweighs its positive impact on investment demand to diminish the rate of growth of output to limit the possibility of realizing intended profit share set at a given level of markup. Secondly, higher level of capacity utilization, i.e. 

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of lack of effective demand, due to the ambiguous effect of distribution of income on both the components of aggregate demand, might still operate as a constraint and could limit the process of exhilarationist expansion. However, this model also suffers from the same limitation as in Kaldor, Goodwin models, namely, the neglect of labour market. Assuming significant unemployment and exogenous labour supply, the problem of interrelation between distribution of income and its level is built around the product market. The second interesting observation to be noted here is that the fall in profit share at the peak of activity in this regime stands, somewhat, in contrast to the widely observed empirical fact of pro-cyclical behaviour of profit share. This requires the percentage rise in money wages to be strong enough to outweigh the advantages of higher labour productivity at higher capacity utilization. But, the reason why higher wage share couldn’t translate itself into a higher level of aggregate demand may well be the case of wage shift, a phenomenon that has been observed in the context of the U.S economy where the share of wages of technical or overhead labour out of total wages seem to be increasing in the last decade. Finally the fact that the problem of effective demand resurfaces in a model of increasing returns stresses the need to focus on the question of the distribution of income for sustainable aggregate demand generation, even in the New Economy contexts.

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32 In a sense supply-side assumptions forming a basis for this demand-driven model. This limitation could be overcome by relating the changes in the employment rate to the growth of output. Here again, we have not considered the rate of growth in output and its relation to the distribution of income. I am greatful to the referee for pointing out this lacuna in our model. See Skott (1989), for such a model, where he integrates Keynesian ideas on effective demand with a Marxian emphasis on class struggle and reserve army of labour. Also see Flaschel and Skott (2004) for integrating labour market and analyzing the Marxian reserve-army mechanism in Stenidl’s 1952 model.
Reference:


Hall, R. E et al. (2003). *NBER Business Cycle Dating Committee Report,* NBER.


