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An Introduction to System Dynamics

Everything we do as individuals, as an industry, or as a society is done in the context of an information-feedback system.

Jay W. Forrester, *Industrial Dynamics* (1961), p.15.

Abstract This chapter presents the important concepts underlying the system dynamics modeling method. Following an initial definition of the term model, a summary of a successful system dynamics intervention is described. The key concepts of system dynamics – *stocks and flows* – are explained. The process for simulating stock and flow models – *integral calculus* – is described, with an example of a company's customer base used to illustrate how stocks change, through their flows, over time. A summary of dimensional analysis for stock and flow equations is provided before the second feature of system dynamics modeling – *feedback* – is presented. The chapter concludes by summarizing the system dynamics methodology, which is a five-stage iterative process that guides model design, development, test and policy design.

Keywords: Models - Stocks – Flows – Feedback – Integration

Models

Pidd (1996, p.15) defines a model as:

an external and explicit representation of part of reality as seen by the people who wish to use that model to understand, to change, to manage and to control that part of reality.

This is an insightful definition that also applies to system dynamics. The model building process focuses on a *part of reality* that needs to be understood and managed, and creates an *external and explicit representation*, in the form of a model, of this reality. This reality could be an organization faced with declining market share, a public health agency confronted by an infectious disease outbreak, or governments challenged by increased levels of carbon in the atmosphere, with the resulting rise in mean global temperatures. In these scenarios, decision makers are faced with a complex, and highly interacting, social system. Models provide a basis for decision makers to understand their world as an interconnected system, and to test out the impact of policy interventions *in silico*. Understanding leads to insight, and an opportunity to change, manage and control the system of interest.

In order for a model to be useful to decision makers, it must provide some view on future behavior, and Meadows et al. (1974) provide a valuable classification of the types of outputs models can provide:

- *Absolute, precise predictions*, for example, when and where will the next solar eclipse be observable?
- *Conditional, precise predictions*, for example, if a cooling systems fails in a nuclear power plant, what will be the maximum pressure exerted on the reactor's containment vessel?
- *Conditional, imprecise projections of dynamic behavior*, for example, if an infectious disease spreads through a population, what is the likely future burden of demand on intensive care facilities one month from the outbreak date?

Because system dynamics is primarily a technique for business and policy simulation modeling (Homer 2012), its primary focus is on the third class of model: those simulation models that provide conditional, imprecise projections of dynamic behavior. This is because social and business systems are by their nature unpredictable in the absolute sense (Meadows et al. 1974). So while *all models are wrong* (Box 1976), as they cannot generate precise point-predictions of future events in social systems, the challenge is to create *models that are useful* through extensive testing, benchmarking against available data, and continual iteration between experiments with the virtual world of simulation and the real world (Sterman 2002). System dynamics has a rich tradition of creating useful models across many disciplines, and, to illustrate this, an application of system dynamics to public health policy is presented.

System Dynamics in Action: Population Health Policy

In their paper - *Using system dynamics to develop policies that matter: global management of poliomyelitis and beyond* – Thompson and Tebbens (2008) document their award-winning research which demonstrates how system dynamics impacted global health policy analysis. This supported the Global Polio Eradication Initiative (GPEI) to eradicate wild polioviruses, which aimed to replicate the success of the eradication of smallpox in 1979 (Breman & Arita 1980). Initial results of this initiative, based on an intensive vaccination campaign, led to a reduction from 350,000 global annual cases to 1,000 cases per year.

However, the eradication project faced funding shortfalls during 2002-3, and the allocation of vaccination resources prioritized endemic countries where the virus circulated, leaving other countries vulnerable. This containment policy inevitably led to further outbreaks. While additional investment was made to regain lost ground, a new policy debate started that questioned the feasibility of eradication, and suggested that the guiding policy should switch to one of control, as this could save resources while maintaining outbreak cases at manageable low levels.

The authors proceeded to evaluate the impact of this proposed policy change, and assess the economic impact, and potential disease burden, of these two distinct policy options. Core to the analysis was a system dynamics disease outbreak model. This model represented the population as a set of stocks and flows, where people were classified as being susceptible to, infected with or recovered from the wild poliovirus. The stock and flow model distinguished between 25 different age groups. This model was also informed by their prior studies related to risk management, including a cost-based analyses of tradeoffs associated with outbreak response.

As a result of the model building process, the authors highlighted the impact of wavering. This describes a scenario whereby in the context of successful vaccinations comes the perception that a high level of continued investment in vaccine administration is not required. This view was represented in the system dynamics model. Focusing on two Northern Indian states, two policy options were evaluated. The first was to vaccinate extensively until disease eradication, the second was to vaccinate only if the costs per incidence remained outside an acceptable level. The simulation demonstrated the impact of these two policies, and showed that the containment option leads to more cases, and costs, over a 20-year time horizon. Therefore, the model provided evidence to support the policy of eradication.

The next stage of the process involved the authors presenting their model and results at a stakeholder consultation, convened by the WHO Director-General Dr. Margaret Chan. The goal of this meeting was to consider the option of switching from eradication to control. Their author's system dynamics model, with its quantitative approach, long time horizon, and its analysis of the impact of wavering commitment, supported the case to continue the eradication policy, and this subsequently led to further resources to implement the eradication policy.

There are three modeling insights from this case study. First, it shows how system dynamics can be successfully applied to real-world problems, and achieve an impact in terms of policy analysis and implementation. Second, it demonstrates the importance of model purpose, where the modeling activity was focused on the core issue of whether to eradicate or control a disease. Third, the paper provides an excellent sense of the interdisciplinary skills required to build system dynamics models. The authors invested considerable time to understand the specific problem, and were well-positioned to defend their work to national and international policymakers, financial donors, fellow modelers, economists and epidemiologists. Furthermore, reflecting on Pidd's (1996) earlier definition of a model, it is evident that this work has the following characteristics:

- It was an *external and explicit representation* of a problem, in that the model was a system dynamics representation of infectious disease transmission, based

on the classic Susceptible-Infected-Recovered (SIR) model, which is covered in detail in chapter 5.

- It focused on the *part of reality* that was important to the *people who wished to use the model* (key stakeholders), namely how best to improve overall population health in vulnerable countries through implementing the most appropriate disease containment policy.
- It provided a basis *to understand, change, manage and control* that part of reality by demonstrating the detrimental impact of wavering on future outbreaks, and so provided evidence to support the continuation of the eradication policy.

In summary, this highlights the potential of system dynamics modeling to make a difference to society. Within the field there are many documented examples of how this simulation method has been successfully applied across a range of disciplines, including business systems, project management, energy policy and health care. A further advantage of the system dynamics method is that it is grounded in the theory of dynamic systems, and in particular, it uses calculus – and the ideas of stocks and flows – to generate quantitative projections of a system’s behavior over time.

Stocks and Flows

A stock is the foundation of any system (Meadows 2008), and stocks and flows are the building blocks of system dynamics models. They characterize the state of the system under study, as well as providing the information upon which decisions and actions are based (Sterman 2000). Stocks can only change through their flows, which are the quantities added to (inflow), or subtracted from (outflow), a stock over time. Stocks are present in many business and social systems, and examples include:

- Warehouse inventory (stock keeping units), which is the amount of stock within the four walls of a warehouse at a given point in time. Inflows include inventory arriving from suppliers (stock keeping units/week), and returns sent back by customers (stock keeping units/week). Outflows are goods shipped to customers (stock keeping units/week).
- Employees in an organization (people) across all processes and functions. Inflows include new hires (employees/month). Examples of outflows from this stock include employee retirements (people/month), employee attrition (people/month) and employee redundancies (people/month).
- The number of people suffering from an illness in the population. Inflows are the number of people becoming ill during a time period (people/week). Outflows are the number of people that recover over each time period (people/week).

Note that in all cases the units of the flows are the *units of the stock* divided by the *time period*. This time period is determined by the system under study, and can vary from seconds to years, depending on the problem's time horizon. In order to explore the stock and flow concept, an example from public health is presented, where the focus is on the presence of illness in a population. The stock and flow model for this is shown in figure 1.1.

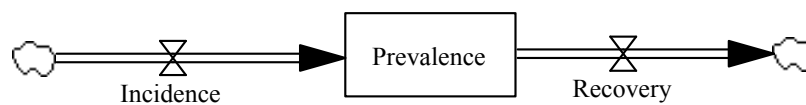


Figure 1.1: A stock and flow model of illness in a population

The model visualization shows the stocks as *containers*, and the flows as *pipes* filling and draining containers, where the flow rates are controlled by valves. The variable names used for this initial model is informed by public health professionals, and it is usually good practice to build a model that practitioners can identify with. Therefore the following definitions are used (Giesecke 2002).

- *Prevalence* is defined as the number of people who have that disease at a specific time, and this is a stock. For example if the model captured the dynamics of seasonal influenza, this would be the number of people infected with influenza.
- *Incidence* is defined as the number of people who become ill with a certain disease during a defined time period. For seasonal influenza, this is usually measured each week, and the units are therefore (people/week). Incidence is a flow.
- *Recovery* is the number of people removed from the ill population per time period. Recovery is a flow, and its units are (people/week).

A feature of this one-stock model is that it can be used to highlight three principles of stock and flow systems. These ideas relate the behavior of the stock to the values of the net flow, where the net flow is the difference between all inflows and all outflows. For example, in a given week, if 1000 people contract influenza and 800 people recover from their bout of the virus, the net flow for the week is +200, which is the difference of the two flows. Because the difference is greater than zero, the prevalence will rise over this time period. Therefore, in the general case of any stock and flow system, the following conditions hold true:

- When the total sum of all inflows to a stock *is greater than* the total sum of all outflows, the stock will rise.
- When the total sum of all inflows to a stock *is less than* the total sum of all outflows, the stock will fall.
- When the total sum of inflows to a stock *equals* the total sum of outflows, the stock will remain unchanged. This is an interesting and often desired state of many systems, and is known as *dynamic equilibrium*.

These three principles can be applied to any system that changes over time, including challenges related to global warming, economics, and population planning. For example, figure 1.2 shows a model of carbon in the earth's atmosphere (a stock). This stock is increased by emissions, and reduced by absorptions. As the earth's carbon absorption rate is currently less than the carbon emissions rate, the amount of carbon in the atmosphere is increasing, and this is now shown to impact global temperatures. The second model describes, at a highest level of aggregation, the population of a country, with inflows of births and immigration, and outflows of deaths and emigration.

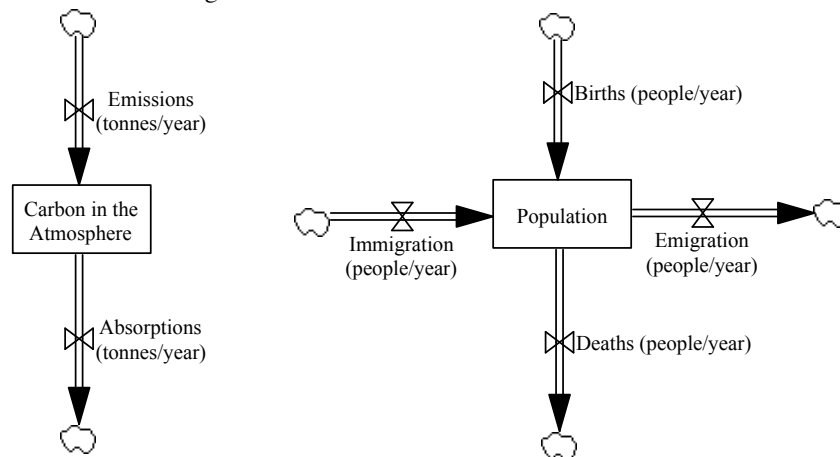


Figure 1.2 Further examples of stock and flow systems

These two models are high-level, and represent the system of interest by a single stock. However, stock models can also be disaggregated to reveal finer-grained dynamics. Disaggregation is an important part of system dynamics modeling, and it is necessary when there are sufficient differences in subsets of a variable, for example, cohorts in a population. This is shown in figure 1.3, where a country's population is broken down into age cohorts, and the stocks are cascaded in order to capture the dynamics of aging. Disaggregated population model structures such as this are particularly useful when exploring long-term dynamics of health systems, where age is an important determinant of health. To simplify the model, migrations are excluded, with the main focus on how the age profile of the population changes over time.

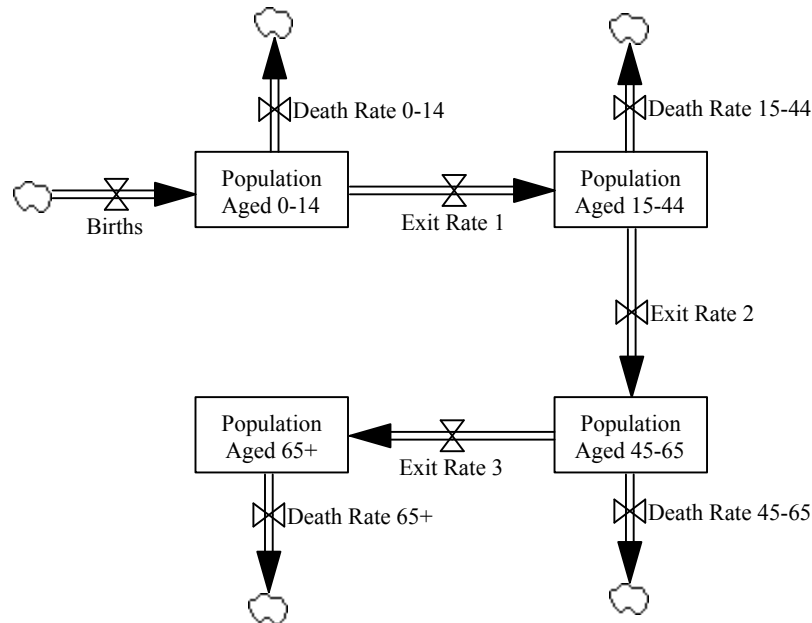


Figure 1.3 A disaggregated model of a population, excluding migration.

While these stock and flow models may appear straightforward – which is beneficial from a model building viewpoint – an important challenge is to formulate the inflows and outflows. For example the following questions must be addressed:

- How are delays in a system modeled, where items stay in a stock for a period of time and then progress?
- How are rate variables such as the number of births modeled, particularly when variables may depend on other system stocks?
- How are decisions in a system modeled, where a manager decides, for example, how many new hires to take on in order to replenish the employee stock, and so maintain a company's resource base, and capacity to deliver services to customers?

Flows structures such as *fractional increase* and *fractional decrease* are explored later in this chapter (Duggan 2016b), and chapter 4 (Duggan 2016e) will describe formulating delays, and how to model management decisions such as stock replenishment. In summary, stocks are present in many social systems. They represent accumulations, and can only change through their inflows and outflows. Stocks are solved using the mathematical process known as *integration*, and this is how system dynamics models are simulated.

Integration

Integration is the mathematical process of calculating the area under the net flow curve, between initial and final times. There are two main methods for integrating. The first method is analytical, where an integral is expressed as an equation that can be used to determine the stock's value at any future point in time. The second approach is numerical, which is commonly used for more complex higher-order (i.e. many stocks) systems, and is the method that will be used throughout this text.

The two methods are now explored, using a linear net flow equation where $f(t) = 2t$. Therefore the net flow starts at 0, and climbs to 20 after 10 time units. A quick visual inspection, using the formula for calculating the area of a triangle, will show that the integral after 20 time units is $0.5 * 20 * 10 = 100$.

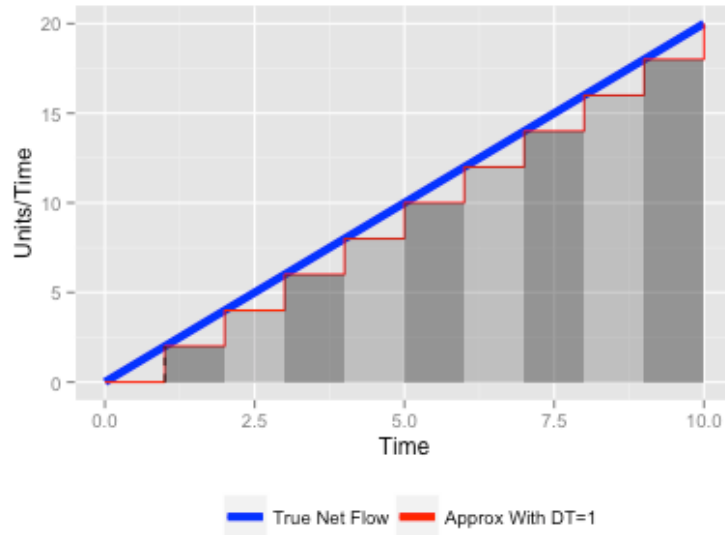


Figure 1.4: Representation of an integration problem

In order to solve this analytically, the standard integration method can be used (1-1). To achieve this, the net flow is represented as a derivative (1-2), with a corresponding indefinite integral solution (1-3) is found through applying (1-1). However, in this case the time interval is known, and therefore the area between two specific points can be evaluated as the difference in the indefinite integral solution over the time interval, and this is shown to be 100 in (1-4).

$$\int t^n dt = \frac{1}{n+1} t^{n+1} + c \quad (1-1)$$

$$\frac{dy}{dt} = 2t \quad (1-2)$$

$$y = \int 2t \, dt = t^2 + c \quad (1-3)$$

$$y_{t=10} = \int_0^{10} t^2 = 10^2 - 0^2 = 100 \quad (1-4)$$

The analytical solution shown in (1-4) can be used to calculate the stock's value at any future time interval. However, as already discussed, exact analytical solutions may not be feasible for higher-order, non-linear stock and flow systems. Approximate solutions can be calculated, and a widely-used numerical algorithm is known as *Euler's method*.

Euler's approach involves estimating the area under the net flow curve through a sequence of rectangles of identical width. The rectangle height is *the opening value of the net flow applied over the interval DT*, where DT is also known as the time step. As the time step gets smaller, the overall numerical solution becomes more accurate. Euler's equation accumulates the successive areas of these rectangles (1-5) by assuming that the net flow is constant over each time interval (the opening value of the net flow is taken).

$$Stock_t = Stock_{t-dt} + (Inflow_{t-dt} - Outflow_{t-dt}) * DT \quad (1-5)$$

Figure 1.4 uses a time step of 1 (normally this would be too large a value to use for an accurate simulation). From the time series plot, the sequence of successive rectangles is shown, and the stock's value is simply the summation of these rectangle areas, based on (1-5). The solution process is summarized in table 1.1, which also shows the error term (the difference between the approximate integration and the true integration). In this example, the error term is the sum of the small triangle areas between the blue and red lines. This error term can be reduced by selecting a smaller time step, usually for social simulations a time step value of 1/8 or 1/16 is used.

Time	Stock _t	Net Flow	Stock _A = t^2	Error
0	0	0	0	0
1	0+0=0	2	1	1
2	0+2=2	4	4	2
3	2+4=6	6	9	3
4	6+6=12	8	16	4
5	12+8=20	10	25	5
6	20+10=30	12	36	6
7	30+12=42	14	49	7
8	42+14=56	16	64	8
9	56+16=72	18	81	9
10	72+18=90	20	100	10

Table 1.1 Approximate values of the integral of $dy/dt = 2t$, $dt = 1$, Euler's method

In summary, integration is the basis for all system dynamics simulation runs. Once a model is expressed in terms of stocks and flows, the integration process is applied to every stock, for each time step. Therefore when all the initial stock values are known, and each flow has a defined equation, the integration process will simulate the behavior of all model variables.

A System Dynamics Model of Customers

In order to demonstrate how a system dynamics model is constructed, a one-stock model of an organization's customer base is modeled. Given that the customer base is an accumulation, it can be modeled as a stock. The inflow is recruits, and the outflow are losses, also known as the churn rate. The goal of organizations is to limit the losses and maximize the recruits, in order to maintain increasing customers levels, and therefore support company growth. The steps for building this model are:

- Identify the stock, provide an initial value, and decide on the flows that change the stock
- Formulate equations for the flows
- Decide on the time units, for example, is the simulation in days, months or years.
- Decide on the time interval, which is the start and finish time of the simulation run.

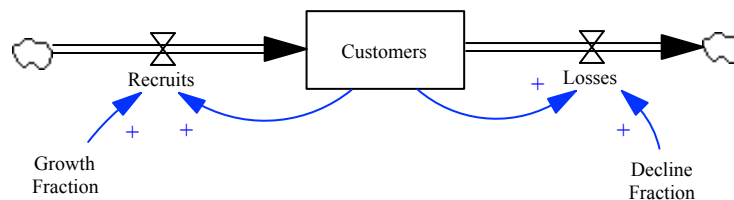


Figure 1.5: A stock and flow model of customers

The stock and flow model is shown in figure 1.5, and the information dependencies between equations are shown, along with the type of relationship. For example, the “+” sign at the end of a link indicates that the variables move in the same direction. These type of causal links will be described shortly, and are important when considering the feedback structures of system dynamics models. The stock is expressed as an integral function, where the arguments are the inflows less the

outflows, followed by the initial value. In effect, the structure of (1-6) is the similar to that shown earlier in (1-5). Stock equations are usually the most straightforward to formulate, as they can only change via their flows. The initial value of the stock for the simulation run is required, otherwise the integration process could not proceed.

$$Customers = INTEGRAL(Recruits - Losses, 10000) \quad (1-6)$$

Following the stock definition, all that remains is to formulate the inflow and outflow, and any auxiliary variables that they may depend on. An auxiliary variable is one that is not a stock or a flow, and is generally used to simplify flow equations. For most modelers, the most challenging task in system dynamics is the composition of flow and auxiliary equations (Dangerfield 2014). Conveniently, there are a number of pre-defined flow equation structures that can be used. In this case, two ideas will be used to formulate the inflow and outflow (Sterman 2000). These are:

- The *fractional increase rate*, where the inflow to a stock is proportional to the stock.
- The *fractional decrease rate*, where the outflow of a stock is proportional to the stock.

For the customer model, these are reasonable assumptions. For example, all companies have annual expansion goals, where they seek to increase their customer base by a target growth fraction. On the other hand, companies are faced with the challenge of retaining customers, and therefore will seek to minimize the churn rate, or the fraction of customers that are lost each year. The flow equations can be formulated to reflect this real-world scenario. The inflow (1-7) is the product of the customers and the growth fraction, and this is a commonly used structure in system dynamics models.

$$Recruits = Customers \times Growth\ Fraction \quad (1-7)$$

The multiplier of the inflow is the growth fraction (1-8), and, for this example, this value varies over time, through the use of the *STEP* function. The *STEP* function, which is available in all system dynamics software, has the form: *STEP(<amount>, <time>)*, and changes a variables value by <amount> at the specified simulation time <time>. In this case, the growth fraction starts at 0.07, drops to 0.03 at 2020, and drops by a further 1% to 0.02 in 2025. In a more complex model, this growth fraction could depend on other system variables, for example, the number of marketing resources, product quality, and the size of the potential market. When an auxiliary does not directly depend on another model variable it is termed an *exogenous variable*. This type of variable will be discussed in greater detail later in this chapter (Duggan 2016a, 2016b).

$$Growth\ Fraction = 0.07 - STEP(0.04, 2020) - STEP(0.01, 2025) \quad (1-8)$$

The losses are formulated as a fixed proportion of the customer stock, and this is shown in (1-9). The decline fraction is fixed at 3%, and this is captured in (1-10).

$$\text{Losses} = \text{Customers} \times \text{Decline Fraction} \quad (1-9)$$

$$\text{Decline Fraction} = 0.03 \quad (1-10)$$

This finalizes the model formulation, with five equations for the simulation model. The equations are complete, as all the variables shown in figure 1.5 are specified. There are no gaps, no ambiguities, just concrete equations that will simulate the customer model. All that remains is to decide on the simulation run settings, which are the time interval (2015-2030), the time step DT (0.25), and the time units (years). The model can then be simulated using a number of approaches, and in this case R's **deSolve** library was used. The simulation output is shown in figure 1.6.

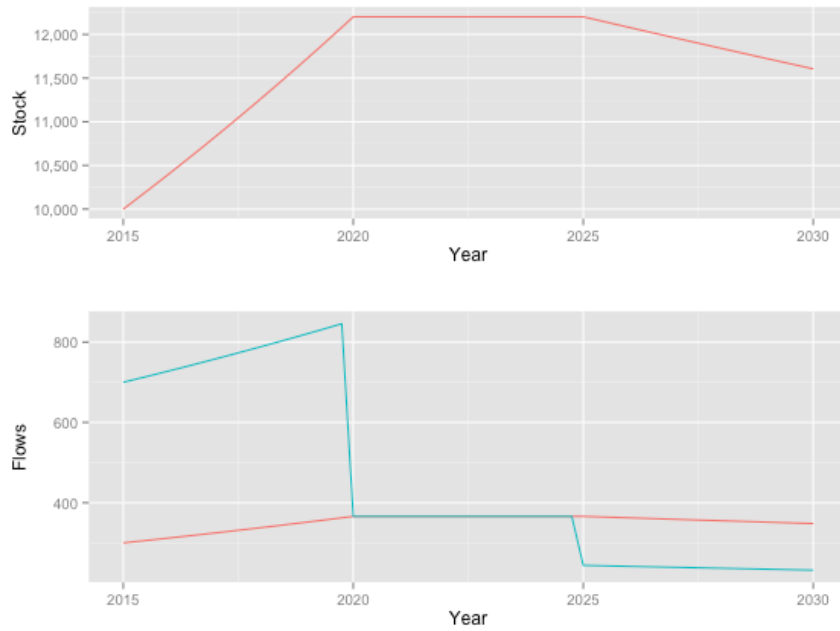


Figure 1.6: Simulation output from the customer model

It is worth reflecting on the simulation output in terms of how the stock behaves over time, which can be classified into three different phases.

- *Phase 1*, from 2015-2020, where the stock increases, as the net growth fraction is $0.07 - 0.03 = 0.04$. While this growth may look linear, it is in fact exponential, similar to how compound interest is calculated for a bank savings account. For example, it can be shown that solving the differential equation $dy/dx = gY$,

where g is the fractional increase rate, yields the resulting integral equation solution $Y_t = Y_0 e^{gt}$, which confirms that in the stock growth is exponential.

- *Phase 2*, from 2020-2025, where the stock remains constant (dynamic equilibrium), given that the growth and decline fractions are equal, and cancel one another out. In this case the model is in dynamic equilibrium.
- *Phase 3*, from 2025-2030, where the decline fraction exceeds the growth fraction, and this results in a declining stock over time, as the net flow is negative. This decline in the stock is exponential, as it can be shown that solving the differential equation $dy/dx = -rY$, where r is the fractional decrease rate, yields the resulting integral equation solution $Y_t = Y_0 e^{-rt}$, where confirms that the stock decline follows an exponential decay pattern.

What is noteworthy about the three phases is that they confirm the fundamentals of stock and flow systems. If the inflow exceeds the outflow (i.e. time interval 2015-2020), the stock rises; if the inflow equals the outflow (i.e. time interval 2020-2025), the stock remains in equilibrium; and, if the outflow exceeds the inflow (time interval 2025-2030), the stock falls. While this is a simple model, these concepts are relevant to any system dynamics model, and can be applied to more complex models to support policy analysis and design. For example, the comparison of inflows to outflows forms part of epidemic threshold calculations, and this will be presented in chapter 5 (Duggan 2016f).

Dimensional Analysis for Stock and Flow Equations

In the physical sciences and engineering, any equation representing a real-world process needs to have the units (i.e. dimensions) balanced on each side of the '=' sign (Dangerfield 2014). This checking – also known as dimensional analysis – is also an important activity in system dynamics, as it provides an excellent validation mechanism for any simulation model. As a starting point, the units for system stocks are identified. For example, stock units from a range of modeling challenges in business, health and the environment are shown in table 1.2.

Application Area	Stock	Units
Business	Inventory	Stock Keeping Unit (SKU)
Financial Planning	Cash	€, \$
Education Planning	Students	People
Epidemiology	Infected	People
Demographics	Population	People
Climate Change	Carbon in the Atmosphere	Metric Tons

Table 1.2 Sample stock variables along with indicative values for units

Stocks change through their flows, and therefore, in order to maintain dimensional consistency, a flow must have units of the stock it feeds, *divided by the units in which time is measured* (Coyle 1996). The selection of time unit depends on the problem being explored, for example, planning in a higher education context has annual student intake, therefore the most suitable time unit would be *year*. However, measuring the spread of an infectious disease such as influenza is typically performed on a weekly basis. Societal challenges such as global warming and efforts to controlling the amount of carbon in the atmosphere can have a time horizon measured in decades, and even centuries. An indication of flows, and their units is provided in table 1.3, covering a wide range of applications areas.

Stock	Inflow	Outflow	Flow Units
Inventory	Arrivals	Shipments	SKU/week
Cash	Deposits	Withdrawals	€/day, \$/day
Student	Registrations	Graduations	People/year
Infected	Incidence	Recovery	People/day
Population	Births	Deaths	People/year
Carbon in the Atmosphere	Emissions	Absorptions	Metric Tons/year

Table 1.3 Sample flow variables along with indicative values for units

Once the units for stocks and flows are identified, dimensional analysis can be performed, where both sides of an equation are simplified to their basic units. If the two sides of the dimensional equation are equal, then the equation is dimensionally consistent. The customer model from figure 1.5 is used, and the integral equation, similar to the format shown in (1-5), is shown in (1-11).

$$\begin{aligned}
 \text{Customers}_{t_i} &= \text{Customers}_{t-dt} + (\text{Recruits} - \text{Losses}) * DT & (1-11) \\
 \text{people} &= \text{people} + (\text{people/year} - \text{people/year}) * \text{year}
 \end{aligned}$$

This equation is dimensionally consistent, as the inflow and outflow denominator (year) cancels with the dimensions of DT (year) to arrive at the dimension (people). This process also applies to flows in system dynamics models. Once the units of the flow multiplied by the time units equal the stock units, the stock equations will be dimensionally consistent. However, it is not sufficient just to have stock equations dimensionally accurate, *all model variables* should have their units checked and validated. For example, the equation for *recruits* (1-7) and *losses* (1-9) also need to be checked for dimensional consistency.

$$\begin{aligned}
 \text{Recruits} &= \text{Customers} * \text{Growth Fraction} & (1-12) \\
 \text{people/year} &= \text{people} * (\text{people/year})/\text{person}
 \end{aligned}$$

$$\begin{aligned}
 \text{Losses} &= \text{Population} * \text{Decline Fraction} & (1-13) \\
 \text{people/year} &= \text{people} * (\text{people/year})/\text{person}
 \end{aligned}$$

In (1-12) and (1-13), *recruits* and *losses* are flows, and therefore their respective units are (people/year). The units of the growth and decline fractions are (1/year), as these values are based on the number of people added/removed each year, divided by the number there to start with, which yields dimensions of (person/year)/person, or (1/year) (Dangerfield 2014). Therefore, the two flow equations are dimensionally consistent, and the customer model passes its dimensionality test. Software packages for system dynamics support dimensional checking, so adding in units at an early stage can improve the model building process. Later in chapter 6 (Duggan 2016g), additional methods for validating system dynamics models are explored, where the benefit is to improve the model quality, and enhance client confidence. Before that, the second foundational concept of system dynamics is presented. This idea provides valuable insight to guide decision making in complex systems, and is known as *feedback*.

Feedback

Feedback is a defining element of system dynamics (Lane 2006), and identifying feedback loops in social systems is an important part of modeling building. Meadows (2008) describes a feedback loop as:

A closed chain of causal connections from a stock, through a set of decisions or rules or physical laws or actions that are dependent on the level of the stock, and back again through a flow to change the stock.

A feedback loop is a chain of circular causal links, where the level of a stock influences a flow, which in turn will change the stock. The stock can influence the flow directly, or that influence could be determined through a series of intermediate auxiliary variables. Feedback processes are present in many systems. Earlier, when discussing stocks and flows, a warehouse example was presented. This can be examined in more detail to uncover a feedback process in operation.

In the warehouse, there is a quantity of products on shelves, and this accumulation is a stock. The company would have a target quantity of product to store, to ensure that stockouts would not happen, and to maintain high levels of customer satisfaction. For example, this target value could be two weeks of expected demand. At regular intervals (perhaps once per week), the warehouse manager would note the current level of the stock, and compare this to the target value. If more stock was needed, orders would be made from suppliers. These orders would then arrive at the warehouse, and their arrival would be modeled as an inflow to the stock. This inflow increases the stock, and so completes the feedback process that connects the stock to the inflow.

Consider a home heating system, and how its feedback process operates (figure 1.7). The occupant sets the desired room temperature. A heat sensor records the actual room temperature, and this is relayed to a controller. The controller logic determines if the temperature is lower than the desired. If it is the heater is activated, and the generated heat raises the room temperature. As the room temperature rises, the sensor detects monitors towards the desired value, and once this value is reached, the heater is switched off.

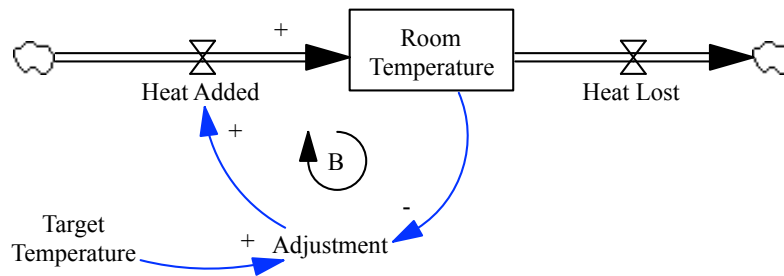


Figure 1.7 Temperature control: an example of negative feedback

This is a further example of feedback, where the level of a stock (heat in the room) determines the amount of heat added (the flow) which in turn changes the heat in the room (the stock). It is an example of a *goal seeking system*, in that once a target is established the system is continually moved towards that target. These are known as *negative feedback loops*, and are annotated using the balancing (B) icon on the stock and flow model.

Loop polarity can be evaluated for any feedback loop, by examining the individual links contained in that loop. A link captures a cause and effect relationship between two variables (e.g. x and y), and an individual link can be either positive or negative. A positive link occurs when, all else being equal, the cause x increases, the effect y increases above what it would have been. A negative link means that as the cause x increases, then the effect y decreases below what it would have been (Sterman 2000). In the room temperature model, the feedback loop contains positive and negative links. A positive link occurs when the cause and effect move in the *same direction*, for example, as the adjustment increases, so to does the amount of heat added. A negative link implies that the cause and effect move in *opposite directions*, for instance, as the temperature rises, the adjustment falls.

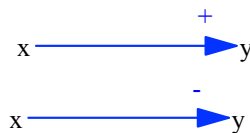


Figure 1.8 Causal links and their polarity

Calculating loop polarity is a straightforward task. The loop is broken down into a set of the causal links, and the impact of a change in one variable is traced through the causal chain, and back to the original variable. In this example, the loop contains three variables: *Room Temperature*, *Adjustment*, and *Heat Added*.

Room Temperature	↓	Adjustment	↑
Adjustment	↑	Heat Added	↑
Heat Added	↑	Room Temperature	↑

Table 1.4 Tracing the changes of variables through the room temperature loop.

Table 1.4 shows the impact of a change in room temperature, where a variable in the loop can either rise (↑) or fall (↓). Assuming that the room temperature is falling due to heat loss (due to the stock outflow), the impact of this change through the feedback loop is as follows:

- As the *room temperature* decreases, the *adjustment* (which is the difference between desired and actual temperature) increases, as it is a negative link so the two values move in opposite directions.
- With an increase in *adjustment*, the amount of *heat added* also increases, as this is a positive link where the cause and effect move in the same direction.
- An increase in *heat added* then leads to an increase in *room temperature*, as this is also a positive link.

The individual link polarities combine to determine the overall loop polarity. With one iteration through the loop, the direction of the original variable has been impacted. In this case, at the outset the room temperature was falling, and following the sequence of circular causal links, the temperature rises. Room temperature has moved in the *opposite direction* after one iteration through the loop. This is an example of a regulating system, or more generally, *negative feedback*. A negative feedback loop also has an odd number of negative links (in this case 1), and this heuristic can be used to quickly calculate loop polarity.

The loop polarity calculation can be applied to a different model, involving the interplay between capital and output, often termed the engine of economic growth (Meadows 2008).

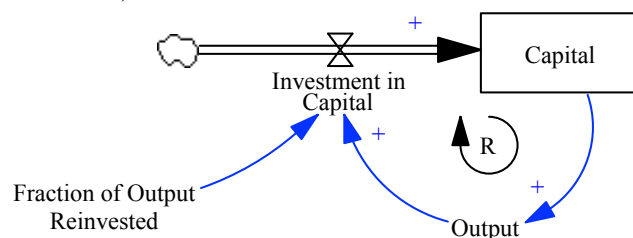


Figure 1.9 Capital growth: an example of positive feedback

The more machines and factories (capital) there are, the more goods and services (output) that can be produced. This model can be viewed as a set of circular causal links, as the loop contains three variables: *Capital*, *Output*, and *Investment in Capital*.

Capital	↑	Output	↑
Output	↑	Investment in Capital	↑
Investment in Capital	↑	Capital	↑

Table 1.5 Tracing the changes of variables through the capital investment loop.

Table 1.5 traces the behavior of the loop's variables, from an initial starting point where we assume that the capital is increasing. The causal links are follows:

- As the *capital* increases, so to does *output*, and this is a positive link, as the variables move in the same direction.
- With an increase in *output*, the inflow *investment in capital* will increase. Again, this is a positive link.
- As *investment in capital* increases, the amount of capital (i.e. the stock) also increases, and this final link in the feedback loop is also positive.

In contrast to the room temperature example, the direction of change of capital has been *reinforced* or amplified as a consequence of the loop. Increased capital, through a cycle of reinvestment, leads to more capital. This is a classic example of *positive feedback*, which drives exponential growth, and terms such as *virtuous cycle* and *success to the successful* are often used (where the effect is desirable). On the other hand, positive feedback can also have detrimental effects (e.g. a run on a bank), where a value spirals out of control, and in this case the term *vicious cycle* is used instead. A positive feedback loop will always have an even (including zero) number of negative links, and this can be a useful shortcut taken in order to calculate loop polarity.

In summary, a complex system is an interlocking structure of feedback loops, and this loop structure is found many real-world processes (Forrester 1969). In particular:

- A feedback loop is a closed chain of causal links from a stock, through a flow, and back to the original stock again.
- There are two classes of feedback loops. Negative feedback counteracts the direction of change, whereas positive feedback amplifies change and drives exponential growth.
- Loop polarity is calculated by examining the individual link polarities in a circular causal chain. If there are an odd number of negative links, the loop polarity is negative, otherwise the loop polarity is positive.

Modeling Feedback

Creating feedback models in system dynamics is challenging. It requires domain knowledge, and the skill to see the interrelationships between different system elements. The goal is to identify those feedbacks that influence overall system behavior. Forrester (1968) defines an important principle, centered on the idea of a system boundary:

In concept a feedback system is a closed system. Its dynamic behavior arises within its internal structure. Any interaction which is essential to the behavior mode must be included inside the system boundary.

This definition provides a valuable context for identifying feedback structures. The challenge is to build on an initial model, and map the additional stocks, flows and feedbacks that influence the systems behavior. This is done through collaborating with multiple stakeholders, who provide different perspectives and knowledge on the problem being addressed. As part of this interaction, people share their understanding of what variables need to be included inside the model boundary. At some point in the model building process, there should be consensus that all the relevant stocks, flows and feedbacks have been included within the model boundary. This structure is then the closed system representation of the problem.

Richardson (2011) builds on this idea, and emphasizes the importance of the system boundary by discussing what is known as the *endogenous point of view*:

The most salient aspect of the system dynamics approach are undoubtedly stocks and flows and feedback loops. These visible elements stand out and demand our attention. But it is worth noting that feedback loops are really a consequence of the endogenous point of view.

Endogenous refers to the idea that actions are caused by factors from *inside of the system*. With the endogenous viewpoint behavior can be explained through the system's feedback structure, and not through the actions of an external, uncontrollable, exogenous source. Sterman (2002) writes that system dynamics practitioners are trained to be suspicious of exogenous variables, and they must challenge model constants in order to see whether they could be part of the feedback structure. This process of *challenging the constants* is central to the endogenous perspective, and can be used to discover important feedback loops.

In order to provide an example of how the endogenous point of view can be used to identify feedback structures, a one-stock model is presented, as shown on the left in Figure 1.10. This stock increases based on the growth fraction, and is structurally similar to the capital growth model shown earlier in figure 1.9. The growth fraction is a constant, and is exogenous, as the value has its source *outside of the system*. In other words, this exogenous variable is not influenced by any other model variable. With a constant growth rate, the system stock will grow exponen-

tially, with no physical limits. However, growth without limits is unrealistic, as for any system, there are always factors that limit growth. Therefore, the flawed assumption of this initial model is that the growth fraction never changes. With the endogenous, this assumption can be challenged.

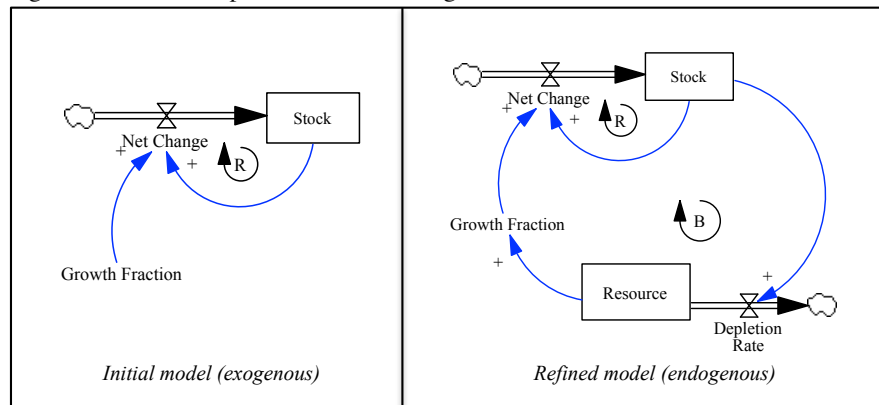


Figure 1.10 Evolving an endogenous feedback perspective

The model boundary is expanded to include other stocks that may impact the systems behavior. The target of enquiry now becomes the constant (exogenous) variable *growth fraction*. In this case, the following question can be asked: *what is the growth fraction dependent on?* In this generic model, it is assumed that the growth fraction depends on the availability of a non-renewable resource. There are well-documented cases, such as the population growth and decline on Easter Island (Brandt and Merico 2015), where stocks have grown based on the availability of non-renewable resources, only to decline once those resources were consumed. From this we can extend the model in three ways:

- The growth fraction depends on the resource availability, where resources are a stock. This is a positive link. More resources lead to a higher growth rate.
- The resource depletion rate depends on the level of the stock. This is positive link. The higher the stock, the greater the depletion rate.
- The resource is reduced by the depletion rate. This is a negative link, as a higher depletion rate leads to a reduction in stock. In this model, the resource is assumed to be non-renewable, as there is no inflow to replenish lost resources.

This is an example of extending the model boundary, has revealed a new, and significant, feedback structure. What was previously an exogenous variable (growth fraction) is now endogenous. As a result, there is now a more realistic model that links, via feedback, two system stocks that are clearly interdependent. Based on this endogenous feedback model, we can also determine the polarity of the new feedback loop by taking a variable of interest and tracing the impact of its increase through each feedback loop.

Stock	↑	Net Change	↑
Net Change	↑	Stock	↑

Table 1.6 Calculating the polarity for first feedback loop

The first feedback loop is summarized in table 1.6. As the stock's change is reinforced after a single iteration, it is a positive feedback loop, and so will exhibit exponential growth or decline. The second feedback loop, which emerged as a result of focusing on the exogenous variable *growth fraction*, is summarized in table 1.7. This shows that the direction of change for the variable of interest (Stock) is reversed following a cycle through the loop. Therefore, this is a negative feedback loop that acts as a limiting factor to the stocks growth.

Stock	↑	Depletion Rate	↑
Depletion Rate	↑	Resource	↓
Resource	↓	Growth Fraction	↓
Growth Fraction	↓	Net Change	↓
Net Change	↓	Stock	↓

Table 1.7 Calculating the polarity for second feedback loop

This example highlights the process for identifying model boundaries, which can then ensure that important feedbacks are considered throughout the modeling process. The limits to growth model is explored in further detail in chapter 3 (Duggan 2016d), where a stock grows rapidly based on a resource, but as the resource diminishes, the stock enters a period of rapid decline. Furthermore, a healthcare model is formulated in chapter 4 (Duggan 2016e), and feedbacks identified between the different model sectors.

It is also worth reiterating that modeling feedback in system dynamics is challenging, and the interested reader is recommended to follow up with excellent examples of feedback thinking from the system dynamics literature. These include:

- How system dynamics models can help the public policy process (Ghaffarzadegan et al. 2011).
- Identifying feedback structures in the project management process (Lyneis and Ford 2007), and,
- System dynamics models applied to understand population health outcomes (Homer 1993).

The Model Building Process

The starting point for system dynamics is that the model must be created for a specific purpose (Forrester 1969). For example, consider the following three scenarios:

- In the food industry, a multinational company might want to find ways to improve its supply chain performance, and a system dynamics model could provide ways to evaluate different production and distribution strategies.
- In the public health domain, stakeholders may wish to assess the impact of mass vaccination to protect citizens against an outbreak of a virulent influenza virus.
- In the airline industry, a company could face a challenge of declining passenger numbers, and look for ways to assess the impact of further capacity investment.

The common thread across these examples is that each model addresses a clear problem, and therefore each model has a definite purpose. With a clearly defined goal, a valuable strength of system dynamics is that it is supported by an iterative five-stage methodology. This can be used to structure projects and ensure a process for engaging with clients and problem owners in order to address problems.

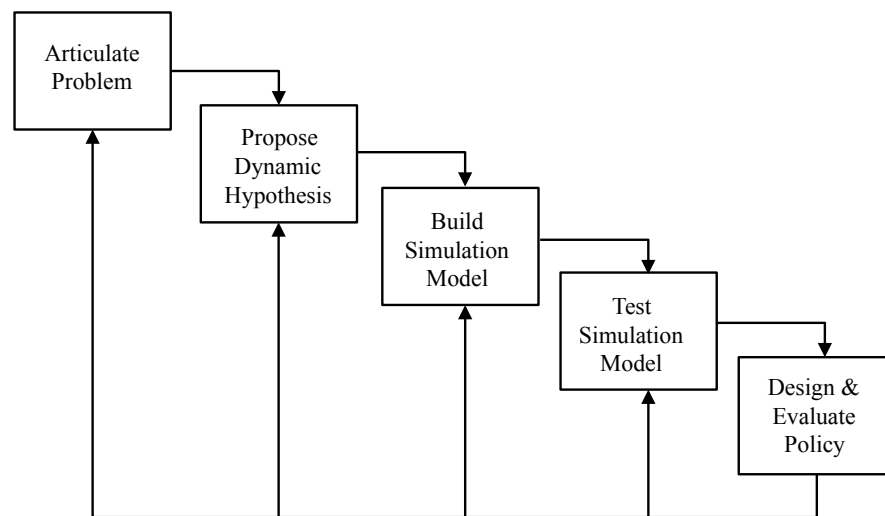


Figure 1.11 The system dynamics modeling process

The model building process comprises five inter-related activities, shown in figure 1.11, and based on Morecroft (2007).

1. *Articulate Problem.* This involves identifying the specific problem with clients, and exploring the reasons why it is a problem worth addressing. Important variables are selected, and the appropriate time horizon identified. Historical data is gathered to support this initial analysis, and behavior over time graphs can provide valuable input into this problem definition stage.
2. *Propose Dynamic Hypothesis.* Following on from this initial stage, a *dynamic hypothesis* is proposed where the aim is to identify the stock, flow and feedback structures can best explain the problematic behavior. The problem is mapped using tools such as causal loop diagrams, stock and flow maps, and other appropriate facilitation tools.
3. *Build Simulation Model.* With a mapping structure and feedbacks identified, the *simulation model* can be formulated, with the stock and flow structure and decision rules. Tasks such as parameter estimation - covered in chapter 7 (Duggan 2016h) - and initial tests can be performed, and feedback gained from clients.
4. *Test Simulation Model.* The fourth stage is *testing*, where the model behavior is compared to the known reference models, and its robustness is tested under extreme conditions. Chapter 6 (Duggan 2016g) shows how extreme condition tests can be designed and implemented. Sensitivity testing can also be used to evaluate the impact of uncertainty in model parameters on overall outcomes.
5. *Design and Evaluate Policy.* The fifth stage is *policy design and evaluation*, which requires that the model is robust and has passed a suite of rigorous tests. In this activity, new decision rules, strategies and structures that could be implemented in the real-world are evaluated. The simulation model can be used to perform *what-if analysis* to observe the potential impact of policies. Following this, improvement actions can be agreed with clients, and the implementation of system changes can follow.

While the process flow would indicate a linear sequence from problem definition to policy design, the reality is that building system dynamics models is a highly iterative process. Smaller, high-level, models may be initially mapped and implemented, and the revised based on feedback from stakeholders. The idea of having a final unchanging model is unrealistic, as models are always in a continuous state of evolution, where “each question, each reaction, each new input of information, and each difficulty in explaining the model leads to modification, clarification, and extension” (Forrester 1985). Further insights into the model building process is provided by Vennix (1996), who offers excellent guidance on how to perform group model building, and so manage a system dynamics intervention with multiple stakeholders.

Summary

This chapter provided an introduction to system dynamics. This simulation method is based on finding stocks, flows and feedbacks that are relevant to the problem of interest. The technical solution process used is integration, where stocks accumulate their inflows, less any outflows. The process of finding feedback by exploring the system boundary was discussed, as was the overall five-stage problem solving process. System dynamics equations can be solved using special purpose simulation tools. In this text the R framework is used to solve equations, and an introduction to R is presented in chapter 2 (Duggan 2016c).

Exercises

1. The net flow for a population is given $dP/dt = rP$, where r is the fractional growth rate. From this, show that the integral is given by $P_t = P_0 e^{rt}$ where P_0 is the initial value of the population.
2. Create a two stock system for a University. One stock is models students, the other staff. Identify inflows and outflows for each stock. Add an additional variable to the model called *student staff ratio*. Higher values of this ratio make the University less attractive for students, and also result in the University hiring more staff. Show any feedback loops, and calculate the loop polarities using two methods.
3. Consider the net flow $dy/dt = 4t$. Assuming the stock y is initially zero, solve analytically for the value of y after 10 time units. Use Euler's equation, with $DT=0.5$, to solve for y . Calculate the error term, and plot both solutions using Excel.

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