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A Multi-axial Cyclic Viscoplasticity Model for High Temperature Fatigue of P91 Steel --Manuscript Draft--

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A Multi-axial Cyclic Viscoplasticity Model for High Temperature Fatigue of P91 Steel

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Abstract: This paper presents a novel multi-axial, cyclic viscoplasticity material model for high temperature low cycle fatigue of P91 power plant steel. The model incorporates mechanisms-based variable strain-rate sensitivity and the key high temperature cyclic deformation phenomena of cyclic softening and non-linear kinematic hardening. The model has been calibrated to accurately represent the cyclic high temperature constitutive behaviour of 'as received' P91 steel. Details on the material Jacobian, with the consistent tangent stiffness for finite element implementation, are presented. The multi-axial implementation is applied to (i) a notched specimen under strain-controlled loading at 600 °C and (ii) a thin-walled pipe under representative pressurised thermo-mechanical fatigue loading conditions. It is shown that the model for variable strain-rate sensitivity of the present paper predicts significantly different Coffin-Manson notch fatigue life compared to the Chaboche power-law model. Ratchetting is shown to be a key candidate failure mechanism for next generation thermo-mechanical power plant loading conditions, for thin-walled pressurised pipes.

1. Introduction

The changing landscape of power generation is such that plants operate with increased flexibility resulting in increased thermo-mechanical fatigue (TMF) of plant components. As modern plant are designed for high creep loading, there is a requirement to characterise the current generation of plant materials under flexible loading. In addition, a move to an ultra-supercritical (USC) cycle results in improved plant efficiency and reduced CO₂ emissions, although there is a rise in steam pressure and temperature. Thus, modern plant are required to deal with harsher loading conditions, leading to a reduction in component life.

A newly developed material model [1, 2] for cyclic viscoplasticity and variable strain-rate sensitivity is applied to P91 steel, under multi-axial loading conditions. P91 steel is a 9Cr martensitic steel typically used for plant header and piping systems due to its high creep strength [3]. A number of material models capable of simulating the constitutive behaviour of candidate materials already exist, including the unified Chaboche model [4-7] and the two-layer viscoplasticity model [8, 9]. However, a hyperbolic sine material model has been chosen as it is more representative of the key mechanisms of deformation [10], enabling strain-rate independence of the material parameters and offers greater accuracy through the full stress regime [1, 2, 11]. Initial validation of the performance of the material model is conducted in Barrett et al. [1, 2] and the current application concerns the assessment of the multi-axial performance of the model. To this effect, a notched specimen model has been designed to cover a specific stress regime and the multi-axial capability of the hyperbolic sine material model is assessed by means of analysis of the trends observed in finite element (FE) simulations and comparison with similar work conducted in this field [12-14]. The Coffin-Manson (C-M) [15] relationship is used to predict failure of the notched specimen and to

compare the life of a notched specimen with that of a smooth test specimen. A comparison with the life predicted by a power law model, for both the smooth and notched specimens, is also presented.

The more realistic application of TMF loading of an axisymmetric pipe is also investigated within the present study. The thin-walled pipe is subjected to pressurised TMF cycles at a temperature rate representative of realistic plant [8, 9] and a temperature regime within the range of 400 °C to 600 °C, for which the material model has been calibrated. The importance of ratchetting as a failure mechanism is illustrated vis-a-vis increase in operating pressure, representative of future USC conditions.

2. Material Model

The material model, described briefly here, is based on a hyperbolic sine constitutive equation for the effective accumulated plastic strain-rate [1]:

$$\dot{\rho} = \alpha \sinh \beta f \quad (1)$$

where α and β are temperature dependent cyclic viscoplasticity material parameters and the function f defines the elastic domain if $f \leq 0$ and the viscous stress if $f > 0$. The increment in stress, $\Delta \sigma$, is evaluated using Hooke's law:

$$\Delta \sigma = \Lambda : \Delta \epsilon^{\text{el}} = \Lambda : (\Delta \epsilon - \Delta \epsilon^{\text{pl}} - \Delta \epsilon^{\text{th}}) \quad (2)$$

where Λ is the elasticity tensor, $\Delta \epsilon^{\text{el}}$ is the increment in elastic strain tensor, $\Delta \epsilon$ is the increment in strain tensor, $\Delta \epsilon^{\text{pl}}$ is the increment in plastic strain tensor and $\Delta \epsilon^{\text{th}}$ is the thermal strain tensor increment. The plastic strain tensor increment, $\Delta \epsilon^{\text{pl}}$, is obtained using a flow rule defined from a thermodynamic framework [1, 16] and described as:

$$\dot{\epsilon}^{\text{pl}} = \frac{\partial \Omega(f)}{\partial f} \frac{\partial f}{\partial \sigma} = \dot{\rho} \mathbf{n} \quad (3)$$

In equation (3), $\Omega(f)$ is the dissipation potential and $\mathbf{n}$ is the tensor normal. For the hyperbolic sine material model presented in this study, the dissipation potential is:

$$\Omega(f) = \frac{\alpha}{\beta} \cosh(\beta f) \text{sgn}(f) \quad (4)$$

and the function f is defined as:

$$f = J(\sigma - \chi) - R - k \quad (5)$$

where $J(\sigma - \chi)$ is the equivalent stress, σ is the stress tensor, χ is the back stress tensor which corresponds to the centre of the elastic domain in 3D stress space, R is the isotropic hardening variable which accounts for expansion/contraction of the elastic domain in 3D stress space and k is the temperature dependent initial yield stress. The following equations define the evolutions of isotropic and kinematic hardening respectively [17, 18]:

$$\dot{R} = b(Q - R) \dot{\rho} + \left(\frac{1}{b} \frac{\partial b}{\partial T} + \frac{1}{Q} \frac{\partial Q}{\partial T} \right) R \dot{T} \quad (6)$$

$$\dot{\chi} = \chi_1 + \chi_2 \quad (7)$$

$$\dot{\chi}_i = C_i \dot{\epsilon}^{pl} - \gamma_i \chi_i \dot{p} + \frac{1}{C_i} \frac{\partial C_i}{\partial T} \chi_i \dot{T} \quad (8)$$

where b , Q , C_i and γ_i are temperature dependent material parameters and $i=1, 2$. The material parameters for the temperatures modelled are calibrated in [1, 7] and presented in Table 1. Fig. 1 illustrates a typical uniaxial validation result produced by the material model outside the strain-rate, strain-range and temperature of calibration, with a more extensive validation of the material model conducted in [1, 2, 19] and Fig. 2 highlights the considerable cyclic softening observed during high temperature low cycle fatigue (HTLCF) testing at 600 °C.

The material model is implemented in multi-axial form in a UMAT user material subroutine. Viscoplastic behaviour occurs if $\Omega(f) > 0$ and $(\partial\Omega(f)/\partial\sigma):\dot{\sigma} > 0$ and the increment in plastic strain is obtained using an implicit integration scheme [1]. To ensure efficient simulations, the material Jacobian is provided in the form of the consistent tangent stiffness. The derivation of the consistent tangent stiffness (CTS) can be found elsewhere [19] and is defined as:

$$\begin{aligned} \delta\sigma = & Z_1 \delta\epsilon + \mathbf{I} Z_2 \mathbf{I} : \delta\epsilon + \mathbf{n} Z_3 \mathbf{n} : \delta\epsilon - \mathbf{n} Z_4 n_{tr} \mathbf{I} : \delta\epsilon \\ & - \chi_i Z_5 \mathbf{n} : \delta\epsilon + \chi_i Z_5 n_{tr} \mathbf{I} : \delta\epsilon + (\mathbf{s}^{tr} - \mathbf{x}_t) (Z_7 \mathbf{n} : \delta\epsilon - Z_8 n_{tr} \mathbf{I} : \delta\epsilon) \end{aligned} \quad (9)$$

where Z_i are material parameters (see Table A1 in Appendix) related to the parameters in equations (6) to (8), $\mathbf{I}$ is the identity matrix, n_{tr} is $\mathbf{n}:\mathbf{I}$, $\mathbf{s}^{tr}$ is the deviatoric trial stress tensor and $\mathbf{x}_t$ is the deviatoric back stress tensor from the previous increment. The UMAT implementation and the terms of the CTS are described in more detail in the Appendix.

3. Results and Discussion

3.1 Notched Specimen

The notched specimen (NS) model within the current study is based on a fatigue test specimen with a notch of radius 0.5 mm at the midpoint of the gauge length (see Fig. 3). The notched specimen has an elastic stress concentration factor of 2.9, defined in terms of the axial stress, where the nominal stress is based on the stress remote from the notch root. Fig. 4 depicts the stress ranges covered by the uniaxial test data [20, 21] and the local notch stress regime at 600 °C along with the correlation achieved by the model to the experimental data of [20, 21]. The comparison of the material model presented in equation (1) with minimum creep-rate data of [20, 21], which have been obtained via constant load tests and presented in Fig. 4, illustrates the ability of the model to operate across a broad range of strain-rates. This result enables reliable extrapolation from accelerated laboratory test conditions to the strain-rates typically observed in modern plant [8, 9, 22]. Fig. 5 depicts a comparison of the constitutive behaviour of P91 steel for the initial and 50th cycles for the smooth specimen (SS) and the stress-strain response of the notched specimen at the central axis and notch root, at a temperature of 600 °C with a nominal strain-rate of 0.1 %/s and nominal strain-range of ± 0.5 % (see Fig. 6), illustrating the effect of the notch on the stress-strain ranges produced. The nominal strain here is the average strain applied to the gauge length of the specimen of Fig 3, which corresponds to the applied displacement divided by the gauge length. The nominal stress is the total load (in response to the applied displacement) as measured at the

unnotched section of the gage length, i.e. remote from the notch section of Fig 3, e.g. section A-A.

In multi-axial creep and plastic failure, triaxiality, and hence the hydrostatic stress, represent key parameters for failure [23]. Fig. 7 illustrates the predicted distributions of hydrostatic stress and axial plastic strain range with radial position from the central axis to the notch root. The hydrostatic stress is dominated by the axial stress and in the initial elastic regime ($\sigma < \sigma_y$), the highest stress is observed at the notch root. With viscoplastic behaviour, yielding and the maximum plastic strain occur at the notch root. As the loading increases, the maximum hydrostatic stress redistributes away from the notch root. The hydrostatic stress reduces to an almost constant value towards the centre of the specimen. This result is consistent with the work of [12, 14]. As the cycles increase, the stress from an initial peak distribution ($N=1$), reduces due to the cyclic softening behaviour of P91 steel [7, 24], as illustrated in Figs. 5 and 7. The high localisation of the stress relatively close to the notch root (see Fig. 7a) is concomitant with a high localisation of plastic strain in the zone surrounding the notch root as highlighted in Fig. 7b. However, away from the notch root, a significant portion of the specimen has a strain value slightly lower than the nominal applied strain of ± 0.5 %. Initially the maximum von Mises stress is predicted at the notch root, but with increased time is predicted to redistribute circumferentially around the notch to a region labelled 'A' in Fig. 8. Fig. 8 also illustrates the predicted redistribution of von Mises stress in terms of the contour plots produced by the FE model at times of 2 s and 505 s respectively.

As the FE predictions illustrate, the inclusion of a stress concentration in the form of a notch has a considerable effect on the performance of P91 steel. This is predicted to cause a significant reduction in life based on a C-M prediction, with the C-M relationship given as [15]:

$$\frac{\Delta \varepsilon^{pl}}{2} = \varepsilon_f' (2N_f)^c \quad (10)$$

where N_f is the number of cycles to failure, ε_f' is the fatigue ductility coefficient and c is the fatigue ductility exponent. From the HTLCF experiments conducted by [7, 25, 26] and subsequent identification of the Coffin-Manson constants in [26], ε_f' has a value of 0.4896 and c is -0.5927 at 600 °C. Table 2 contains the C-M predictions for the SS and NS cases, illustrating the influence of a stress concentration on the predicted life in both cases, with excellent agreement achieved for the SS case via comparison with the experimental fatigue life of [26]. The result of the C-M prediction is based on the value of highest axial plastic strain range and future work concerns the execution of a critical plane approach to identify the plane of maximum plastic strain range and account for the multi-axial effects of the inclusion of a notch.

The current study also presents a comparison between the hyperbolic sine model [1] and the more widely used power law model [4, 5]. The constitutive equation in the power law model is defined as:

$$\dot{p} = \left\langle \frac{f}{Z} \right\rangle^n \quad (11)$$

where Z and n are the power law cyclic viscoplastic material parameters, obtained from the literature [7] and f is defined by equation (5). In Fig. 9a, the stress-strain response produced by both models for the initial loop at the position labelled 'axis' in Fig. 3 are presented, where almost identical results are obtained. At this position, the material response is close to the nominal loading conditions and thus, as this is close to the regime of calibration in both cases, the excellent comparison in Fig. 9a is expected. However, Fig. 9b presents the response at the notch root, where the effects of a stress concentration are observed and a considerable difference between both models is predicted. The power law model predicts a significantly lower plastic strain-range, leading to a 28 % higher predicted life, as shown in Table 2. Referring to Fig. 4, the hyperbolic sine model is correlated with experimental data across the full range of test data, giving confidence for extrapolation. The power law model is a linear approximation to a relatively small range of test data and should only be used within that calibrated range of strain and strain-rate. Hence, for this application to the notch root or similar stress (and hence strain-rate) concentrations in realistic applications, it is argued that the hyperbolic sine model is more suitable. A closely related issue is that of extrapolation from common laboratory test conditions (typically comparatively high strain-rate) to realistic (much slower strain-rate) conditions. It is likewise argued here, based on Fig. 4 and the results presented, that the present (hyperbolic sine) model is more suitable and reliable for this purpose.

3.2 Thin-Walled Pipe

The material model is applied to the case of a thin-walled pipe with uniform temperature across the pipe wall, with geometry as in Fig. 10 and pressurised TMF loading conditions as per Fig. 11. Two different cyclic ($R=0$) operating pressures are considered, a low pressure (LP) case of 17 MPa and a high pressure (HP) case of 25 MPa. The former is representative of current (subcritical) plant operating conditions and the latter of future USC conditions. The calculated hoop stress-strain relationships for the two maximum cycling pressures are illustrated in Fig. 12. In both cases, ratchetting is predicted initially, with significantly more severe values of ratchet strain predicted for the HP case. For the LP case, it is predicted that the effect of ratchetting effectively reduces to plastic shakedown as the cycles increase. However, as depicted in Fig. 13, the effect of ratchetting increases with the number of cycles for the HP case and this illustrates the need to account for the potential of ratchetting as a failure mechanism in thin-walled tubes as plant operating pressures increase. The number of cycles to ratchetting failure, N_r , may be estimated by the following equation [27, 28]:

$$N_r = \frac{\varepsilon_c}{\sum_{i=1}^{N_r} \Delta \varepsilon_r} = \frac{\varepsilon_c}{\sum_{i=1}^{N_r} \sqrt{\frac{2}{3} \Delta \varepsilon_{ij}^r \Delta \varepsilon_{ij}^r}} \quad (12)$$

where ε_c is a critical strain, typically taken to be the material ductility, $\Delta \varepsilon_r$ is an equivalent ratchet strain and $\Delta \varepsilon_{ij}^r$ are the increments in ratchet strain components. Assuming a material ductility of 4 % [29], application of equation (12) predicts 240 cycles (~4 yrs) to failure by ratchetting for the HP case. These results highlight possible issues in moving to higher pressures, e.g. USC loading conditions, and the need to conduct life analysis of plant components under increased pressure loading and realistic temperature-pressure histories [8,

9]. It is also noted that future work should assess the ability of alternative non-linear kinematic hardening (NLKH) models to predict the effects of ratchetting accurately, as it is widely accepted that the Armstrong-Frederick NLKH model tends to over-predict ratchet strain [30].

4. Conclusions

A recently developed unified cyclic viscoplasticity material model for variable strain-rate sensitivity is presented for high temperature fatigue of P91 steel and applied to the multi-axial modelling of (i) a notched tension specimen and (ii) a thermo-mechanically pressurised thin-walled pipe. The study concludes that:

- The new material model presented here has the ability to operate across a large stress and strain-rate regime, as typically observed under realistic loading and geometry conditions, and with the ability to allow reliable extrapolation from high-rate laboratory test conditions to low-rate plant conditions.
- When applying material models to conditions outside the regime of calibration, care must be exercised to ensure reliable results. The present novel constitutive model predicted a notch (Coffin-Manson) fatigue life of 28 % less than the power-law model, for equal smooth specimen lives, thus demonstrating the need for a reliable extrapolation capability in constitutive modelling.
- Experimental testing of the present material under a multi-axial stress state, e.g. using a notch geometry similar to [13] or tension-torsion tests as presented in [31], to further validate the present material model under multi-axial loading conditions is the subject of ongoing work.
- For pressurised TMF loading of thin-walled pipes, ratchetting is predicted to be a candidate mode of failure, with increased pressure predicted to significantly increase the ratchet rate and strain, and hence reduce component life.

Appendix: UMAT implementation

The hyperbolic sine unified cyclic viscoplastic material model described above is implemented in a UMAT user subroutine for use with Abaqus. The increment in effective plastic strain is determined using an implicit integration scheme:

$$\Delta p = \Delta p + d\Delta p \quad (\text{A1})$$

and the iterative increment in effective plastic strain, $d\Delta p$, is defined as [1]:

$$d\Delta p = \frac{\left[\varphi - (\Delta p / \Delta t) - \mathbf{Zn} : \boldsymbol{\chi}_i \frac{1}{C_i} \frac{\partial C_i}{\partial T} dT - Z \left(\frac{1}{b} \frac{\partial b}{\partial T} + \frac{1}{Q} \frac{\partial Q}{\partial T} \right) R dT \right]}{\left[\frac{1}{\Delta t} + 3GZ + ZC_i - \mathbf{Zn} : \boldsymbol{\chi}_i \gamma_i + Zb(Q - R) \right]} \quad (\text{A2})$$

where $Z = \partial \varphi / \partial \sigma$ and:

$$\varphi(\Delta p, \boldsymbol{\chi}, R) = \alpha \sinh \beta \left(\sigma_e^{\text{tr}} - 3G\Delta p - R - k \right) \quad (\text{A3})$$

In equation (A3), G is the shear modulus and σ_e^{tr} is the equivalent trial stress, defined as:

$$\sigma_e^{\text{tr}} = \left[\frac{3}{2} (\mathbf{s}^{\text{tr}} - \mathbf{x}_t) : (\mathbf{s}^{\text{tr}} - \mathbf{x}_t) \right]^{1/2} \quad (\text{A4})$$

The material Jacobian, $\mathbf{D} = \partial \delta \boldsymbol{\sigma} / \partial \delta \boldsymbol{\varepsilon}$, must be provided for each increment that the UMAT is called. To ensure more efficient simulations, the material Jacobian is provided in the form of CTS matrix. The CTS is derived from the definition of the tensor normal [32]:

$$\mathbf{n} = \frac{3}{2} \frac{\mathbf{s}^{\text{tr}} - \mathbf{x}_t}{\sigma_e^{\text{tr}}} = \frac{3}{2} \frac{\mathbf{s} - \mathbf{x}}{\sigma_e} \quad (\text{A5})$$

Rearranging and applying the differential operator produces:

$$\delta \mathbf{s} = \frac{\sigma_e}{\sigma_e^{\text{tr}}} \delta \mathbf{s}^{\text{tr}} + \mathbf{s}^{\text{tr}} \delta \left(\frac{\sigma_e}{\sigma_e^{\text{tr}}} \right) - \mathbf{x}_t \delta \left(\frac{\sigma_e}{\sigma_e^{\text{tr}}} \right) + \delta \mathbf{x} \quad (\text{A6})$$

The differential kinematic hardening term, $\delta \mathbf{x}$, is defined in a similar manner to equation (8) and requires the differential increment in effective plastic strain, $\delta \Delta p$:

$$\delta \Delta p = \frac{\mathbf{Yn} : \delta \boldsymbol{\sigma}^{\text{tr}}}{\left[\frac{1}{\Delta t} + 3GY + YC_i - \mathbf{Yn} : \boldsymbol{\chi}_i \boldsymbol{\gamma}_i + Yb(Q - R) \right]} = \frac{Y}{D} \mathbf{n} : \delta \boldsymbol{\sigma}^{\text{tr}} \quad (\text{A7})$$

where Y is defined in a similar manner to Z . Thus, equation (A6) may be redefined in conjunction with equations (8) and (A7) to produce the CTS:

$$\begin{aligned} \delta \boldsymbol{\sigma} = & Z_1 \delta \boldsymbol{\varepsilon} + \mathbf{I} Z_2 \mathbf{I} : \delta \boldsymbol{\varepsilon} + \mathbf{n} Z_3 \mathbf{n} : \delta \boldsymbol{\varepsilon} - \mathbf{n} Z_4 n_{\text{tr}} \mathbf{I} : \delta \boldsymbol{\varepsilon} \\ & - \boldsymbol{\chi}_i Z_5 \mathbf{n} : \delta \boldsymbol{\varepsilon} + \boldsymbol{\chi}_i Z_5 n_{\text{tr}} \mathbf{I} : \delta \boldsymbol{\varepsilon} + (\mathbf{s}^{\text{tr}} - \mathbf{x}_t) (Z_7 \mathbf{n} : \delta \boldsymbol{\varepsilon} - Z_8 n_{\text{tr}} \mathbf{I} : \delta \boldsymbol{\varepsilon}) \end{aligned} \quad (\text{A8})$$

where the Z_i terms are defined in Table A1.

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Table 1: Hyperbolic sine material model parameters.

	400 °C	500 °C	600 °C
E (GPa) ^[1]	184	180	142
α_{COE} (/°C) ^[33]	12.95×10^{-6}	13.31×10^{-6}	13.59×10^{-6}
ν	0.3	0.3	0.3
Q (MPa) ^[7]	-55.0	-60.0	-75.4
b ^[7]	0.45	0.60	1.00
C_1 (MPa) ^[7]	352500	215873	106860
γ_1 ^[7]	2350	2192	2055
C_2 (MPa) ^[7]	48600	48235	31160
γ_2 ^[7]	405	461	463
k (MPa) ^[7]	96	90	43
α (/s) ^[1]	1.4×10^{-6}	8.0×10^{-7}	1.0×10^{-7}
β (/MPa) ^[1]	0.07	0.064	0.055

Table 2: C-M life predictions at 600 °C for the smooth and notched specimens.

Model	Test Case	
	Smooth specimen	Notched specimen
Present sinh	2231	83
Power law	2272	116

Table A1: Definition of the terms in the CTS.

$Z_1 = 2G \frac{\sigma_e}{\sigma_e^{\text{tr}}}$	
$Z_2 = K - \frac{1}{3} Z_1$	$K = \text{bulk modulus}$
$Z_3 = 2G \frac{2}{3} C_i \frac{Y}{D}$	
$Z_4 = \frac{1}{3} Z_3$	
$Z_5 = 2G \gamma_i \frac{Y}{D}$	
$Z_6 = \frac{1}{3} Z_5$	
$Z_7 = 2G \left(\frac{1}{\sigma_e^{\text{tr}}} - \frac{1}{\sigma_e^{\text{tr}}} 3G \frac{Y}{D} + \mathbf{n} : \boldsymbol{\chi}_i \frac{1}{\sigma_e^{\text{tr}}} \gamma_i \frac{Y}{D} - \frac{\sigma_e}{\sigma_e^{\text{tr}2}} \frac{Y}{D} \right)$	
$Z_8 = \frac{1}{3} Z_7$

Figure 1: Comparison of the stress-strain response predicted by the FE model with the experimental data of [6] for (a) the initial cycle and (b) the 103rd cycle at 550 °C and a strain rate of 0.01 %/s.

Figure 2: FE predicted stress-strain response for the initial and 300th cycles at 600 °C and a strain-rate of 0.1 %/s, illustrating the cyclic softening behaviour in 9-12Cr steels.

Figure 3: The notch specimen gauge length, axisymmetric FE mesh and symmetrical boundary conditions.

Figure 4: Comparison of the hyperbolic sine material model with the experimental data of [20, 21], illustrating the uniaxial test range and the simulated NS range.

Figure 5: Axial stress-strain response for (a) the initial cycle and (b) the 50th cycle at a temperature of 600 °C and a nominal strain range of $\pm 0.5\%$ and nominal strain rate of 0.1 %/s.

Figure 6: Strain-controlled loading conditions for a nominal strain-rate of 0.1 %/s and nominal strain-range of ± 0.5 %.

(a)

Figure 7: FE predicted variation of (a) the hydrostatic stress and (b) the axial plastic strain range with radial position for various cycles, illustrating the effect of stress redistribution and cyclic softening of P91 steel.

Figure 8: FE predicted von Mises contour plots produced by the hyperbolic sine material model at 2 s ($N < 1$) and 505 s ($N = 25$), under strain-controlled HTLCF loading conditions.

Figure 9: FE predicted stress-strain response produced by both the hyperbolic sine and power law material models at (a) the axis of symmetry for the initial loop and (b) at the notch root for the 50th cycle.

Figure 10: FE model setup for the axisymmetric thin-walled pipe geometry.

Figure 11: TMF-OP loading conditions for the pressure and temperature cycles corresponding to a temperature rate of 0.33 °C/s.

Figure 12: FE predicted hoop stress-strain response for TMF-OP loading in the 400-600 °C temperature range and cycling pressure for (a) HP and (b) LP loading conditions.

Figure 13: FE predicted (a) hoop mechanical strain and (b) hoop ratchet strain, plotted as a function of the number of cycles illustrating the effect of ratchetting observed for the conditions outlined in Fig. 12.

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Reviewer Response

The Authors thank the reviewer for the time and effort which was obviously invested in reading, in detail, our work. This document summarises changes made in response to the reviewer comments. Each reviewer comment is addressed separately. The changes made are highlighted in yellow in the reviewed manuscript. We believe that the changes made, in light of the reviewers' comments, have enhanced the quality of the submitted manuscript.

Reviewer #1:

The multi-axial cyclic viscoplasticity model is an interesting important topic. Unfortunately, the paper's lack of experimental data to prove its great potential. To ask for more experimental work from a group mainly working with modelling is however too much to ask for. But I have some comments that I would like that the authors consider before publication:

1. To clarify to the readers, the authors should give more details regarding the test conditions for the experimental data taken from ref [20] which is used in Fig4. The data you have used are most likely steady-state creep rates from constant load tests I guess. If so this must be obvious to the readers.

This section has been updated to highlight that constant load creep tests were conducted to achieve this minimum creep-rate data.

2. Reference [20] is nothing but a literature review in my opinion, and thus it does not contain any original work. Anyhow the authors use this reference as the source for some of their experimental data. It is much better to give the original reference and give credit to the researchers who deserves it!

The original references have now been provided.

3. The experimental data in Fig. 4 is not identical to the data that can be found in reference [20]. In my opinion there is a shift by a factor of 10 and I have not figured out why? Perhaps I miss something but then there must be some kind of manipulation of the data that is not really clear to the readers.

The authors thank the reviewer for this observation. In fact, there was a discrepancy with respect to the original published data. This has now been rectified, but has not resulted in any changes to the identified material parameter, β , for the cyclic viscoplasticity model.

4. The definition of nominal stress and especially nominal strain is not clear. Nominal strain is not defined at all in the paper. This must be addressed in revised version of the paper.

The text has been updated to clarify the definitions of both nominal stress and nominal strain.

5. As already mentioned above. I think the paper lack of experimental data to verify that the model is suitable for multi-axial purposes. I do not ask to do more testing but I think you must mention that there is a need for more experimental work to be carried out to verify this type of modelling.

A conclusion has been added pointing out the need for multi-axial experimental validation tests for the present material and thus material model. This is the subject of ongoing work by the authors.

Further change to manuscript:

A further change in the text has been included in relation to the choice of Coffin-Manson parameters. The text has been updated to reflect the recent publication of the number of cycles to failure and subsequent C-M constants for the particular 'as-received' material used in this study.