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# Re-reading Jevons's Principles of Science – Induction Redux\*

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<sup>\*</sup>The origins of the thoughts in this essay go back to my graduate lectures on the history of economic thought and philosophy at UCLA in the early 1990s. I tried, then, to structure my lectures unconventionally, by focusing on the 'philosophy of science' contributions of Smith ([16]), Jevons ([6]) and Keynes ([10]), but also on their contributions as representatives of classical, neoclassical and modern economic theory. This mode and focus gave me a fulcrum around which I was able to tell the story of the development of economic thought against a backdrop of the evolution of the philosophy of science, mathematical logic and mathematics. However, the decisive impulse to embark on this study emerged after reading Bob Clower's Presidential Address to the Southern Economic Association ([3]) in early 1993. That inspired my 'inductive' contribution to the Clower Festschrift ([20]) and, eventually, to the journey towards this Jevonian odyssey. I am greatly indebted to Bob Clower, Francesco Luna, John McCall and Stefano Zambelli for the early inspirations and consistent encouragements, over many intervening years. They, however, are absolved from all responsibilities for the remaining errors and infelicities.

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#### Abstract

In this paper I try to substantiate the thesis that Jevons may have been too harsh on the vices of induction and generously optimistic about the virtues of deduction, as discussed, primarily, in his magnum opus, **The Principles of Science** [6]. With this aim in mind the paper attempts to suggest (modern), recursion theoretic, theoretical technologies that could reduce and, under conditions that I claim would be acceptable to Jevons, even eliminate the inductive indeterminacies that he had emphasised.

Key Words: Jevons, Induction, Inductive Inference, Bayes' Rule JEL Classifications: B16, B31, C11, C63

#### 1 Introduction

"Doubtless there is in nature some invariably acting mechanism, such that from some fixed conditions an invariable result always emerges. But we, with our finite minds and short experience, can never penetrate the mystery of these existences .... . We are in the position of spectators who witness the production of a complicated machine, but are not allowed to examine its structure. We learn what does happen and what does appear, but if we ask for the reason, the answer would involve an infinite depth of mystery."

[6], p.222; italics added.

William Stanley Jevons, a pioneer of neoclassical economics was implacably opposed to the inductive method. His methodological precepts against the inductive method were cogently presented in his monumental treatise on The Principles of  $Science^1$  (ibid, henceforth referred to as TPOS). However, a close reading of its almost 800 pages, against the backdrop of some knowledge of the principles underpinning the MDL principle has convinced me that the Jevonian opposition to the inductive method is untenable. In this paper I attempt a reinterpretation of TPOS as a treatise supporting what I have in earlier writings called The Modern Theory of Induction ([21], Chapter 5).

TPOS, a book of almost 800 dense pages refers to almost every known Western natural philosopher without, however, a single mention of William of Ockham, Occam's Razor or Ockham's Principle<sup>2</sup>! The closest he gets to anything like a (dismissive) mention of Occam's Razor is when he rejects Newton's Rule 1 for Natural Philosophy in the Principia as irrelevant for any inductive purpose, let alone for acting as an anchor to eliminate inductive indeterminacy:

"It is by false generalisation, again, that the laws of nature have been supposed to possess that perfection which we attribute to *simple* 

<sup>&</sup>lt;sup>1</sup>It is a striking fact that Adam Smith, Stanley Jevons and Maynard Keynes, who each developed a distinct framework of economic analysis, were also codifiers and innovators in the philosophy of science. Smith's History of Astronomy ([16] and, for a fine and comprehensive essay on this important work by Adam Smith, see also [12]) was, after all, an early attempt at codifying a framework to study the epistemology of scientific practice; Jevons's TPOS could even be read as a distant forerunner of Popper's Logik der Forschung, as emphasised, correctly I believe, by Mays ([13]), at least from a methodological point of view. As for Keynes, the Treatise on Probability (cf. [10]) and its methodological and epistemological place in the philosophy of science in general, and in the philosophy of economics in particular, has been so well mined that any comment at this juncture would be totally superfluous. But in the context of this paper, it may be useful to point out that this influential book by Keynes was instrumental in the development of one strand of recursion theoretic induction, the theoretical technology which I harness in §3, below (cf. [17] and [18]).

<sup>&</sup>lt;sup>2</sup>The general literature seems to refer to William of Ockham but *Occam's Razor*; hence I retain this schizophrenia in my own spelling. Furthermore, Ockham's own most often stated version of the principle named after him seems to have been: 'Pluralitas non est ponenda sine necessitae' – 'plurality is not to be posited without necessity'. The more commonly attributed version: 'Entia non sunt multiplicanda sine necessitate' – 'entities must not be multiplied without necessity' – appears not to have been used by him (cf. [2], p.xxi).

forms and relations. ... Newton seemed to adopt the questionable axiom that nature always proceeds in the *simplest* way; in stating his first rule of philosophising, he adds: 'To this purpose the philosophers say, that nature does nothing in vain, when less will serve; for nature is pleased with *simplicity*, and affects not the pomp of superfluous causes.'.... *Simplicity* is naturally agreeable to a mind of limited powers, but to an infinite mind all things are simple."

TPOS, p.625; italics added.

Is Jevons suggesting, in the context of his times, beliefs and traditions, that the omnipotence and omniscience of the architect of the laws of nature – the designer of the 'complicated machine' – are such that we are as likely to witness the 'productions of a complicated machine' as to a simple one<sup>3</sup>. Jevons may have been trying to make the point that Newton's was a metaphysical assumption and that we have no grounds for assuming anything about structure in the absence of empirical evidence to the contrary<sup>4</sup>. However, Jevons, who was almost as obsessed with consistency, as he was with deduction<sup>5</sup>, did not obey his own precepts when it came to choosing the order and degree of equations to fit observed data. In such an example he argues clearly in favour of choosing the simplest hypothesis, at least in the first instance:

"It is a general rule in quantitative investigation that we commence by discovering linear, and afterwards proceed to elliptic or more complicated laws of variation."

TPOS, p.474; italics added.

Perhaps, given the times and context, one can be generous to Jevons – more generous than he was to Newton and more, also, than Marshall was to Jevons – and suggest that he was doubtful about any reliance on Occam's Razor because he did not feel it possible to give a rigorous, invariant, analytical definition of simplicity. I think, therefore, it may be reasonable to assume, counterfactually, that Jevons would have accepted the use of Occam's Razor in hypothesis selection and inductive inference, had it been possible to demonstrate that it was possible to define, rigorously, the notion of simplicity.

After developments in the recursion-theoretic approach to induction, rereading Jevons and substantiating a rigorous method of inductive inference is

<sup>&</sup>lt;sup>3</sup>I cannot but reflect on Einstein's wise maxim when faced with the *Great Scorer's* devises, 'Subtle is the Lord, but malicious he is not' – *Raffiniert ist der Herrgott aber botshaft ist er nicht* – and wish Jevons had shown some humility in the face of Einstein's undisputed predecessor's, i.e., Newton's, own methodological maxims.

<sup>&</sup>lt;sup>4</sup>The Einsteinian example of the way he reasoned his way towards the general theory of relativity from the special theory is clearly described and discussed by Kemeny, [9]. This example is paradigmatic, of the role of *simplicity* in hypothesis selection and formation, in the logic of scientific practice.

<sup>&</sup>lt;sup>5</sup>He would, surely, find it uncomfortable to live in a post-Gödelian world where consistency – in the Hilbertian sense – has been dethroned from its crowning place in the deductive enterprise!

not the most difficult task for a philosophy of science. This will be attempted, in §3, after first summarising the Jevonian vision of inductive indeterminacy in the next section. The concluding section tries to draw the threads together and suggest a general interpretation of why Jevons may have been less critical of the possibilities for formalising, rigorously, an inductive logic.

In the rest of this introductory section, I outline some general thoughts on Jevonian mathematical and logical methodologies, just to set the themes in the paper in some context.

Collison Black has persuasively argued that Jevons was a 'transitional figure in both economic theory and economic policy'. (cf., [4], in particular, p. 163, ff). It is my thesis that he was also a transitional figure in the two fields that formed the leitmotifs of TPOS. His central theme in this massive work, the logic of the scientific method, was very much a transitional subject for almost the whole of the period during which Jevons was involved in it; his secondary theme, the role of applied mathematics<sup>6</sup> as hilfenkonstruktion, to set in its paces the main theme, was also a transitional subject – in the sense in which it was used by Collison Black – but, perhaps, less intensively so in Jevons's own lifetime.

In the former case, Jevons was, in his own opinion - substantiated by extensive and impressive work<sup>7</sup> – a follower of the revolutionary movements in logic started by Boole and De Morgan. But, unfortunately, the period between 1847 - the year that Boole and De Morgan published their foundational works - and 1979, the year that saw the publication of Frege's *Begriffsschrift*, was a classic transition period. Even the second edition of TPOS – published in 1877 – came too early to be influenced by the currents that were set in motion by Frege.

Similarly, in applied and pure mathematics, the arithmetization of analysis and the various foundational axiomatizations, as well as emerging developments in applied mathematics – particularly in the theory of differential equations at the hands of Peano, Poincaré and Rayleigh<sup>8</sup> – were to make the Jevonian analytic apparatus obsolete even as they were being fashioned. For example, Jevons's almost monomaniacal faith in the linear superposition principle to understand forced vibrations between two (implicitly) coupled linear oscillators led him to absurdities in theorising about the so-called decennial cycle in economic activity. This naive theory came to be known as the 'sunspot theory of the cycle', although the Jevonian basis for it was abandoned in formal business cycle theory (and a Pigovian underpinning came to be substituted). Almost simultaneously as Jevons was trying to extract impossible dynamics from coupled and forced linear oscillators, Lord Rayleigh was changing the landscape of the theory of oscillations by beginning the imaginative experimental study of forced nonlinear oscillators. The point to be emphasised, in a Jevonian sense, is that oscillators of the Rayleigh-type show that the period of the forcing term need

<sup>&</sup>lt;sup>6</sup>In particular, probability theory, the techniques of permutations and combinations and the theory of differential equations.

<sup>&</sup>lt;sup>7</sup>See for example the extremely interesting essays in [7], particularly Part I.

<sup>&</sup>lt;sup>8</sup> Marshall, for example, was more tuned to some of these developments, and so was Walras. The former had been Second Wrangler at the Mathematical Tripos in Cambridge to Rayleigh; the latter had considerable knowledge of Picard's work in differential equations.

not bear any meaningful analytical or discoverable relation with the period or amplitude of the forced oscillator.

These contextual observations substantiate my main point that Jevons was, unfortunately, a transitional figure – not only in economic theory and economic policy. I suppose, however, all pioneers are inevitably transitional figures because they initiate currents that are tamed and channelled by followers who are able to see further because they were 'standing on the shoulders of giants', as Newton famously and humbly 'confessed'. The issues that Jevons considered crucial for the formalization of inductive inference remain current; their resolutions came about by followers of the pioneers who saw further by standing on the shoulders of the pioneers.

## 2 The Jevonian Vision on Induction and its Indeterminacy

"What especially characterised Jevons's view of logical method was the prominence he attached to the combination of formal and empirical principles through the inverse application of the theory of probability"

[8], p.638; italics added

I shall summarise, rather telegraphically – but with exact references to the source in TPOS – Jevons's precepts on inductive inference. This will, then, enable me to refer to them conveniently in the next section when a simple case is made to encapsulate the Jevonian vision in the modern inductive fold.

The following discussion summarises, however audacious the task of encapsulating summarily a sustained criticism of the inductive method, spread over a discursive book of almost 800 pages (all quotes in this list are from TPOS), the Jevonian vision of induction.

Jevons acknowledges, explicitly, that the theory of inductive inference in TPOS was 'suggested by [his] study of the Inverse Method of Probability' (p.265) and that, therefore, his working hypothesis is that induction is, formally, the *inverse* operation of deduction (p.121). Furthermore, he defines *perfect induction* as follows (pp. 146-7):

**Definition 1** If all possible instances of the phenomenon under consideration is **enumerable**, then induction is defined to be perfect; it is defined to be imperfect if enumeration is infeasible

**Remark 2** I shall assume that enumerability here refers to the formal mathematical notion; i.e, at most denumerably infinite<sup>9</sup>.

<sup>&</sup>lt;sup>9</sup>Thus the number of instances of any inductive phenomenon is, at most, denumerably infinite; and the number of alternative hypotheses that may be entertained to account for any given inductive phenomenon is also, at most, denumerably infinite.

These definitions imply, for Jevons, that the results of imperfect induction are never more than *probable*:

"Only in proportion as our induction approximates to the character of *perfect induction*, does it approximate to certainty. The amount of uncertainty corresponds to the probability that other objects than those examined may exist and *falsify our inferences*; ...".

(p.229; italics added)

Next, Jevons suggests the deductive disciplining criteria for inductive processes. First of all, inductive processes are those, and only those, that generate general laws such that the hypothesis underlying them 'yield deductive results in accordance with experience' (pp. 226-8):

"That process only can be called induction which gives general laws, and it is by the subsequent employment of deduction that we anticipate particular events. .... I hold that in all cases of inductive inference we must invent hypotheses, until we fall upon some hypotheses which yields deductive results in accordance with experience."

Secondly, the extraction - i.e., *inference* - of such *general laws*, from a denumerably infinite set of plausible hypotheses, proceeds by way of applying the 'inverse method of probability' (i.e., using *Bayes's Rule*):

"[I]n all cases ... of inductive inference where we seem to pass from some particular instances to a new instance, we ... form an hypothesis as to the logical conditions under which the given instances might occur; we calculate inversely the probability of that hypothesis and compounding this with the probability that a new instance would proceed from the same conditions, we gain the absolute probability of occurrence of the new instance in virtue of this hypothesis. But as several, or many, or even an infinite number of mutually inconsistent hypothesis may be possible, we must repeat the calculation for each such conceivable hypothesis, and then the complete probability of the future instances will be the sum of the separate probabilities."

It is clear, therefore, that Jevons's inductive method, despite its rhetoric about being simply 'the inverse of deduction', is nothing other than a simple Bayesian procedure.

Up to this point there is nothing controversial; Jevons does not even try to suggest that induction or inductive inference is formally impossible. Next come the two assertions that seem to have stamped the Jevonian authority on the impossibility – or the indeterminacy – of formal, i.e., mechanical, induction or inductive inference.

First of all, he observes (confidently) that there is no rule or uniform principle on the basis of which it is possible to assign priors to implement 'the inverse method of probability' in the mechanical way in which deductive rules can be applied:

"To assign the antecedent probability of any proposition, may be a matter of difficulty or impossibility, and one with which logic and the theory of probability have little concern."

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(p.211-2)
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Next, there is one of his famous impossibility of induction propositions:

"All logical inference involves classification [and it] is not really distinct from the process of perfect induction. [But] there will be no royal road to the discovery of the best system [of logical inference] and it will even be impossible to lay down the rules of procedure to assist those who are in search of good arrangement."

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(pp.673-90; italics added)
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The Ramean Tree (pp.702-3), is an encapsulation of the 'exhaustive method of classification'.

## 3 Disciplining Jevonian Inductive Indeterminacies with Computable Induction<sup>10</sup>

"[T]he most probable cause of an event which has happened is that which would most probably lead to the even supposing the cause to exist."

TPOS, p.243; italics added.

I claim that 'most probable', in the above Jevonian sense of being encapsulated within the inverse probability framework, is equivalent to the precise recursion theoretic inductive inference concept of simplest and it removes, effectively, the much vaunted indeterminacy of induction. The fundamental notion of the modern theory or recursion theoretic induction can be stated as the following proposition:

**Proposition 3** An event with the highest probability of occurring is also that which has the simplest description

Let me give a brief and elementary sketch of the kind of analysis that makes such an equivalence possible – i.e., to be able to use *Occam's Razor* to eliminate

<sup>&</sup>lt;sup>10</sup> For the technical literature on the theoretical technology I am using in this section, see [15], [24] and [1]. The first is the original source; the second an excellent pedagogical exposition and the third updates developments and brings the reader to the frontiers of the subject.

the indeterminacy in the 'inverse probability' method, correctly identified by Jevons. Consider a standard version of Bayes's rule:

$$P(H_i|E) = \frac{P(E|H_i) P(H_i)}{\sum_{i} P(E|H_i) P(H_i)}$$
(1)

Where, apart from absolutely standard, textbook interpretations of all variables and notations, the only implicit novelty – for a Jevonian vision – is the assumption of a denumerable infinity of hypotheses. This, in a standard inverse probability exercise, E, the class of 'observed' events, and  $P(H_i)$  are given; Jevons's inductive inference problem is, then, to find the 'most probable'  $H_i$  that would 'most probably' lead to the observed event of relevance. To get the perspective I want, rewrite (1) as:

$$-\log P(H_i|E) = -\log P(E|H_i) - \log P(H_i) + \log P(E)$$
(2)

where the last term on the r.h.s of (2) is a shorthand expression for the denominator in (1) which, in turn, is the normalising factor in such inverse probability exercises.

Now, finding the Jevonian 'most probable hypothesis' is equivalent to determining that  $H_i$ , w.r.t which (2) is *minimised*. However, in (2),  $\log P(E)$  is invariant w.r.t  $H_i$ ; hence the problem is to minimise (w.r.t.,  $H_i$ ):

$$-\log P\left(E|H_i\right) - \log P\left(H_i\right) \tag{3}$$

However, it is clear that the problem of indeterminacy remains so long as we do not have a principle on the basis of which the *prior* cannot be assigned universally.

Recall, now, that the Jevonian inductive enterprise is supposed to interpret a class of observations, events, data, etc., – 'the production of a complicated machine' – in terms of a denumerable infinity of hypotheses, in such a way that a general law is formalised from which, by deductive processes, the outcomes with which one began are generated. These entities are formalised – in pre-set theoretic days – in terms of logical and mathematical formulas. As far as the requirements of the logic of the inductive method recommended in *TPOS* is concerned, we need only formalise, at most, a denumerable infinity of outcomes in an observation space, and there is a similar quantitative upper bound for the number of hypotheses. Thus the space of computable numbers is sufficient for this formalisation exercise.

Suppose, now, that every element in the outcome space and every potential hypothesis – being denumerably infinite – is associated with a positive integer, perhaps ordered lexicographically. In *TPOS* every outcome and every hypothesis is framed as a logical proposition. Every such proposition can, therefore, be assigned one of the computable numbers and they, in turn, can be processed, say, by a Turing Machine. Next, the *binary codes* for the assigned computable numbers can be constructed, and thereby they can also be given a precise quantitative measure in terms of their counts in *bits*. Thus the basic result of modern

recursion theoretic inductive inference, summarised in the above proposition, results from the following Rissanen Rule of Minimum Description Length (MDL) Inductive Inference:

#### **Proposition 4** Rule of Induction<sup>11</sup>

The 'best theory' is that which minimizes the sum of:

- (a). The length, in bits, of the number theoretic representation of the denumerable infinity of hypothesis;
- (b). The length, in bits, of the elements of the space of outcomes (also, by assumption, at most, denumerably infinite);

The conceptual justification for this 'rule' as the underpinning for Proposition 4 is something like the following reasoning> if the elements of the observation space (E) have any patterns or regularities, then they can be encapsulated in a law, on the basis of some hypothesis. The idea that the best law is that which can extract and summarise the maximum amount of regularities or patterns in E and represent them most concisely captures the workings of Occam's Razor in an inductive exercise. In homely terms: if two hypotheses can encapsulate the patterns in the data, then choose the more concise one.

The final link in this inductive saga is a universal formula for the prior in the inverse probability exercise.

**Proposition 5**  $\exists$  a probability measure m(.) that is universal (in the sense of being invariant except for an inessential additive constant) such that:

$$\log_2 m\left(.\right) \approx K\left(.\right) \tag{4}$$

where, K(.): the Kolmogorov complexity of the best theory generated in the implementation of the rule of induction.

I think this closes the circle consistently with the aims set forth in TPOS for an inductive exercise. Thus, I rest my case for Jevons as an inductivist.

## 4 Concluding Notes

"[W]e may think of a system of inductive logic as a design for a 'learning machine': that is to say, a design for a computing machine that can extrapolate certain kinds of empirical regularities from the data with which it is supplied."

[14], 1975 (1963), p.297.

I believe Jevons would have risen to the challenge of designing such a machine. He would have seen, quite immediately, that inductive logic is not simply

<sup>&</sup>lt;sup>11</sup>The problem of summing an infinite sum has to be resolved by some kind of standard normalization procedure in the case, as here, of denumerable infinity of hypotheses. I shall ignore this detail here.

the inverse of deductive logic - just as it is no longer quite straightforward to say that integration is simply the inverse of differentiation.

He would also have seen that his harsh strictures against the prescience of Boole on the importance of the exclusive - or, particularly for induction (but also for deduction), were slightly misplaced and unfair. This is only partly because every proposition (in the  $predicate\ calculus$ ) can be expressed as a string made up of the negation of the exclusive - or:

$$X \ OR \ ELSE \ Y \equiv (X \lor Y) \land [\sim (X \land Y)] \equiv (X \land \sim Y) \lor (Y \land \sim X) \tag{5}$$

The negation of (5), i.e,  $\sim [(X \land \sim Y) \lor (Y \land \sim X)]$ , is a universal proposition – it can express every proposition (in the predicate calculus) as strings of itself – in the same sense in which m(.) is a universal prior 12. Even more importantly, every learning machine must include a component, in its architecture, to enable it to identify such a string, so long as the proposition is non-trivial. That Jevons did not realize the significance of the exclusive – or implied, naturally, that he could not even begin to think of a 'design for a learning machine', and thereby could not harness his considerable experience in constructing deductive logic machines to implement a mechanical design for any system of inductive logic.

On the other hand, Jevons had the insights of the pioneer to see the importance of the 'method of classification' (see §2, above) – i.e., the construction of the  $Tree\ of\ Poryphry$  or the  $Ramean\ Tree\ (TPOS, pp.\ 702-4)$  – in both deductive and inductive exercises. The standard method for constructing a computable approximation of K (.) completely vindicates Jevons's dim but clear perceptions on the  $combinatorial\ complexities$  of constructing such trees. He observed, quoting Jeremy Bentham's results, that there are 'two objections to the extensive use of the method of "bifurcate classification" [i.e., constructing the  $Ramean\ Tree$ ]':

- "(1). [T]hey soon become impracticably extensive and unwieldy, and
- (2) [T]hey are uneconomical."

TPOS, p. 704.

This observation is given precise quantitative confirmation within the framework of the modern theory of induction<sup>13</sup>. These are the two most important reasons for the practical, i.e., empirical, infeasibility – intractability – of perfect

 $<sup>^{12}\,\</sup>mathrm{Or},$  in the same sense in which there are Universal Turing Machines or Universal Dynamical Systems.

<sup>&</sup>lt;sup>13</sup>Constructing an absolutely minimum Ramean Tree is, in the technical terms of modern applied recursion theory – in particular computational complexity theory – an NP-Complete Problem (cf [23]). But Jevons's perceptive insight was mixed, as almost as always, with his penchant for rushing into unwarranted technical conclusions. There is, in fact, an effective rule – in the strict formal sense of recursion theory, see, for example, [5], chapter 7, §7.11 – to construct such an absolute minimum; it is just that it is nor computationally tractable. Jeremy Benthan was exactly right; Jevons, again sadly, 'as usual', only almost right, bordering on being wrong!

induction. The construction of a Ramean Tree, or any formal object that is its analytical equivalent, is necessary if perfect induction is to be the starting point.

But, then, there was also his uncritical faith in the law of the excluded middle - tertium non datur - in all formal reasoning, whether constrained or unconstrained by untamable infinities (despite lip service to 'we, with our finite minds and short experience'). He could not have foreseen the havoc Brouwer was about to unleash on an unsuspecting community of deductivists in mathematical logic and a variety of formalists and logicists. I give this as an example of an infelicity in Jevons's faith in the power of mathematical logic for two reasons. Firstly, even although he was admirably aware of combinatorial complexities – we may, today, be more used to calling it 'the curse of dimensionality' – he did not pay due care to the paradoxes of extrapolating from the finite to the infinite in logic and mathematics. Secondly, the tertium non datur was one of the three foundations on which the laws of thought were constructed. But it was never made clear – in TPOS – whether the laws of thought are subject to the laws of nature, or vice versa<sup>14</sup>. These may well be the reasons he never realised, in any of his logical works, that logic had appropriated reasoning processes from mathematics, in an era before Boole and De Morgan tried to go in the other direction. Once again, it was left to the post-Boole and post-De Morgan generation to make this explicit – but that was too late for Jevons.

I remain puzzled that Ockham does not even receive a passing nod in TPOS. But I also think he would have found that Occam's Razor provides an anchor to determine, uniquely, the inductive hypothesis. Above all – in classic 'counterfactual' vein – I surmise that he would have been an enthusiast of modern recursion theory and, hence, would have seen the possibilities of formalising, exactly, the processes of inductive inference. I attach some metaphysical significance to the fact that both Jevons and Turing – tragically robbed of long-lives, both of them – were, for some significant part of their respective lives, 'Manchester men'! I have often speculated on the nature and content of an imaginary dialogue between the author of "On a General System of Numerically Definite Reasoning' and the author, 66 years later of, 'On Computable Numbers, with an Application to the Entscheidungsproblem' ([19]) – Turing as a colleague in Jevons's time, context and traditions, and vice versa! So far as I know, there is no record of turing having been inspired by the various Jevonian logic machines, or that he even knew of its existence.

In his contributions to business cycle theory, Jevons's reliance on the (linear) superposition principle and, hence, over-enthusiasm for the explanatory power of 'forced oscillations' (in linear dynamical systems) to explain and describe the nascent facts of the decennial cycle of industrial activity, was one of his most serious infelicities. This was a further manifestation of rushed conclusions, based on imperfect perceptions and understandings of general laws and constraints, mistaking particular mathematical and logical laws for permanent truths. Against

<sup>&</sup>lt;sup>14</sup>I have discussed some aspects of this problem in my recent attempt to contrast what I call the *limits of science* with *limitless thought* (cf. [22]).

<sup>&</sup>lt;sup>15</sup>Or – and – of 'On the Mechanical Performance of Logical Inference' (cf., [7], for these two references)

the backdrop provided by the discussion in this paper,I believe Keynes's pithy characterization of TPOS is easy to concur with:

"Combining insight and error, he spoilt brilliant suggestions by erratic and atrocious arguments. His application of inverse probability to the inductive problem is crude and fallacious, but the idea which underlies it is substantially good. ... There are few books, so superficial in argument yet suggesting so much as Jevons's *Principles of Science*."

Keynes, [11], p.204

In the field of mathematical logic and applied mathematics – even in real analysis – no era could have been worse for a man of insight and conviction, on the importance of these fields for epistemology and methodology – than the precise one in which he studied and tried to develop the subjects for application.

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