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Nonmonotonic Reasoning and the Foundations of Rational Choice

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Abstract

The paper applies a technique developed in artificial intelligence to frame and analyze on the foundational level A. Sen's critique of internal consistency conditions in various choice situations.

Keywords: internal consistency conditions, nonmonotonic reasoning

JEL Classification: D24, D71

1. Introduction

The assumption of rational behaviour on the part of individual is pervasive not only in economics but also in political science (the so called 'rational actor model'). However, Sen (1993, 1995) has argued persuasively against a priori imposition of internal consistency conditions in various areas of choice analysis.

In this paper we will argue that the techniques recently developed within the field of nonmonotonic logic may provide an appropriate framework to analyze the difficulties pointed out by A. Sen in his critique of consistency conditions. The study of nonmonotonic logic was initiated recently by researchers in artificial intelligence who were concerned that the standard logical theories could not account for the kind of defeasible generalizations that constitute so much of our commonsense knowledge. Since Sen is insistent in going beyond internal consistency and spelling out external context and in particular social or ethical norms, it is perhaps reasonable to find some connections between the fields. After all, much of our ethical or legal reasoning carries the kind of defeasible qualities that motivated the development of nonmonotonic logic.

The plan of this paper is as follows. Section 2 introduces some internal consistency conditions and presents Sen's critique of them. Section 3 introduces the new technique of nonmonotonic logic and applies it to Sen's critical examples. Section 4 concludes with some brief remarks.

2. Choice and internal consistency conditions

We will examine the problems with the use of internal consistency conditions of the rational choice on the foundational level. Sen (1993) points out that on this level, some difficulties may arise from the implicit assumption that acts of choice, on their own, like statements can be consistent with each other or can contradict each other. For example, statements A and ¬A (not-A) are contradictory, while choosing x from {x, y} and choosing y from {x, y, z} may not be.

In what follows we will assume that the choice function C(S) specifies for any finite non-empty set S of alternatives, a non-empty subset C(S), called the choice set of S. A variety of different choice-consistency conditions are imposed on C, such as Samuelson's weak axiom of revealed preference (WARP), Arrow's axiom (AA), Chernoff's basic contraction consistency condition (or pr operty α) among many others. For example, according to WARP, if an agent chooses x and rejects y, then s/he should not choose y on another occasion and reject x when x is still available. Chernoff's condition simply says that if some x is chosen from a set S that contains another set T (and x belongs to T), then x must be also chosen from the subset T (for a precise formulation of numerous choice consistency properties see Sen (1986) or Suzumura (1983)).

Can a set of choices be consistent or inconsistent on purely internal grounds without bringing in something external to choice, such as underlying objectives, norms and so on? For example, following Sen (1993, 1995), suppose that a person has the following two choices:

$$C(\{x, y\}) = \{x\} ; \quad (1)$$

$$C(\{x, y, z\}) = \{y\}. \quad (2)$$

This pair of choices clearly violates WARP (but it also violates a weaker Chernoff's condition). However, Sen (1993, 1995, 1997) argues that this presumption of inconsistency or irrationality can be disputed by clearly specifying the external contexts. For example, suppose that a person has a choice between a single apple in the fruit basket (y) and having nothing (x). Assuming that this person behaves decently and picks (x), rather than choosing (y) would not constitute an outrageous assumption. However, if the basket had contained two apples instead and the person had the choice between having nothing (x), having one nice apple (y) and having another nice apple (z), she could rationally choose one (y), without violating any norms of decent behavior.

Another example from Sen (1993) illustrates an epistemic value of the menu. Suppose that an agent given the choice between having tea at a distant acquaintance's home (x), and staying home and not going there (y), may choose (x). However, the same person who chooses to have tea (x), may prefer to stay home (y) if offered instead a larger menu consisting of having tea (x), staying home (y) and having cocaine (z).

Basu (2000) suggests two possible responses to Sen's arguments against internal choice consistency conditions. The first response is to assume that any axiom that has an explanatory power must be falsifiable. Then, Sen's examples illustrate that choice-consistency conditions such as WARP or Chernoff's may not be valid in some situations. As long as we are willing to maintain that these examples represent only some marginal possibilities, then all these examples show that these consistency conditions can be falsifiable and the power of this argument may depend on the empirical question of how marginal the cases of violation of choice-consistency properties are likely to be. Basu's own guess is that in the domain of political economy they will not be marginal at all.

Basu's second response is to accept Sen's argument and to show that it actually calls into question the existence of a choice function. Indeed take again Sen's example with an invitation to tea, only this time assume that making this invitation, the acquaintance second time will rummage through his bag taking out the cocaine pack. Hence, it is reasonable to assume that while an agent may choose x first time, he may prefer y second time, that is:

$$C(\{x, y\}) = \{x\}; \tag{3}$$

$$C(\{x, y\}) = (y). \tag{4}$$

Of course, this means that on a given domain, this individual will not have a well-defined choice.

Bossert (2000a, 2000b) has suggested yet another interpretation of Sen's example. According to him, the objects that appear as menu items are different from the objects that the agent ultimately cares about, namely the consequences of his choices. Therefore, it seems natural to think of the consequences 'having tea at a place where cocaine is consumed' and 'having tea at a cocaine-free place' as being different alternatives. When offered 'tea', the agent does not know whether the consequence of the choice will be 'having tea at a cocaine-free place' or 'having tea at a place where cocaine is consumed'. However, if cocaine' is offered as an additional choice on a menu, the agent will be certain that choosing ‘tea’ will lead to the second possibility.

3. Nonmonotonicity and Consistency

We will try to provide an alternative interpretation of Sen’s examples on the foundational level. We will utilize for this purpose the logical apparatus. Specifically, as Sen himself points out, we assume that the agent evaluates choices relative to some underlying objections, social or ethical norms and so on. Let t stand for ‘offering tea at a distant acquaintance home’, c denotes ‘offering cocaine at a distant acquaintance home’, p stands for ‘accepting an invitation to a tea party’, s stands for ‘not going or staying home’. Then, behavioural standard or ethical norm of the agent could be described by the following set of sentences:

$$\Delta = \{(t \to p), (t \wedge c \to s)\}$$

Of course, we also have the following two sentences: $s \to \neg p$ and $p \to \neg s$, where $\to$ is a material implication which reads as ‘if _, then _’. Hence the behavioral standard of the agent indicates that he will accept an invitation, if tea is offered, but will stay home if in addition to tea, cocaine is offered too. Given the classical consequence relation |-, we can write $t \vdash_{\Delta} p$, meaning that given the behavioral standard Δ, t classically implies p. Of course, since |- is a monotonic consequence relation, we also have $t \wedge c \vdash_{\Delta} p$. But we also have $t \wedge c \vdash_{\Delta} s$ which leads to a contradiction because $s \to \neg p$ and hence we also have $t \wedge c \vdash_{\Delta} \neg p$ (we use ‘$\neg$’ as a negation and ‘$\wedge$’ as a conjunction of classical propositional language).

The crucial step in deriving a contradiction was our use of monotonicity of classical consequence operator. Perhaps, we should reject the monotonicity property which is satisfied by all methods based on classical logic. Indeed, researchers in Artificial Intelligence (AI) in 1980s noticed that humans would often derive some sensible conclusions on the basis of what they would know and then, when new information would be available, they would take back previous conclusions. For example, we may hold the assumption that normally (most) birds fly, but that penguins are birds that do not fly. From the fact that Tweety is a bird, we can sensibly infer that it flies, but learning that Tweety is also a penguin would make us abandon our previous conclusion.

Various formalisms were proposed to perform such nonmonotonic inferences (see Makinson (1993) for a survey of the field of nonmonotonic reasoning). We will propose a somewhat different formalism that will help us to explain Sen's examples. Specifically, we want to account for the fact that in Sen's examples the agent will normally (or typically) take an apple from the basket (provided that it is not the last one), or the agent normally will accept an invitation to a tea party by a distant acquaintance (provided that this acquaintance is not a drug addict or a drug dealer). For example, given the behavioural standard of the agent and a cocaine-free environment, we want to conclude that he would accept an invitation to a tea party. However, the change in the environment would force the agent to abandon this conclusion. Let Δ be a consistent set of formulas in a classical propositional language representing the behavioral standard of the agent, and let Γ be also a consistent set of formulas in the same language representing the environment. We will introduce a binary relation $\triangleright$ between a set of formulas, say Γ, and a formula α, $\Gamma \triangleright \alpha$. We read this expression as 'Γ supports α'. Then we can define the nonmonotonic consequence relation as follows:

$$\alpha \,\|\!\!= \beta \Leftrightarrow \Gamma \mid\!-_{\Delta} \beta \text{ for some } \Gamma \text{ such that } \Gamma \vartriangleright \alpha$$

We read $\alpha \,\|\!\!= \beta$ as 'normally if α is accepted, then β should also be accepted.' Following Gardenfors and Makinson (1994), we say that $\|\!\!=$ is an <u>inference relation</u> if and only if it satisfies the following four conditions:

$\alpha \,\|\!\!= \alpha$	(Reflexivity),
If $\mid\!-_{\Delta}\alpha \leftrightarrow \beta$ and $\alpha \,\|\!\!= \gamma$, then $\beta \,\|\!\!= \gamma$,	(Left Logical Equivalence),
If $\mid\!-_{\Delta} \beta$ and $\gamma \,\|\!\!= \alpha$, then $\gamma \,\|\!\!= \beta$	(Right Weakening),
If $\alpha \,\|\!\!= \beta$ and $\alpha \,\|\!\!= \gamma$, then $\alpha \,\|\!\!= \beta \wedge \gamma$	(And).

In order to justify these conditions we will impose the following conditions on the relation $\vartriangleright$:

If $\alpha \in \Gamma$, then $\Gamma \vartriangleright \alpha$	($\vartriangleright$-Reflexivity) $\vartriangleright$-Ref
If $\mid\!- \alpha \leftrightarrow \beta$ and $\Gamma \vartriangleright \alpha$, then $\Gamma \vartriangleright \beta$	($\vartriangleright$-Equivalence) $\vartriangleright$-Eq
If $\Gamma \vartriangleright \alpha$ and $\Sigma \vartriangleright \alpha$, then $\Gamma \cup \Sigma \vartriangleright \alpha$	($\vartriangleright$-And).

We can establish the following result:

<u>Theorem 1.</u>

Suppose $\vartriangleright$ satisfies $\vartriangleright$-Ref, $\vartriangleright$-Eq and $\vartriangleright$-And. Then, $\|\!\!=$ is an inference relation.

<u>Proof.</u>

Suppose $\rhd$ is a relation satisfying $\rhd$-Ref, $\rhd$-Eq and $\rhd$-And. We will show that $\|\!=$ is an inference relation. Since $\rhd$ satisfies $\rhd$-Ref, we can easily establish that $\|\!=$ satisfies Reflexivity. To establish that $\|\!=$ satisfies Left Logical Equivalence suppose $\vdash_\Delta \alpha \leftrightarrow \beta$ and $\alpha \|\!= \gamma$. Then, we have $\Gamma \vdash_\Delta \gamma$ for some $\Gamma \rhd \alpha$. Applying $\rhd$-Eq, we have $\Gamma \rhd \beta$ and therefore $\beta \|\!= \gamma$. To establish Right Weakening, suppose $\alpha \vdash_\Delta \beta$ and $\gamma \|\!= \alpha$. Hence $\Gamma \vdash_\Delta \alpha$ for some $\Gamma \rhd \gamma$. By transitivity of $\vdash_\Delta$, we have $\Gamma \vdash_\Delta \beta$ and therefore, $\gamma \|\!= \beta$. Finally, to prove that $\|\!=$ satisfies And suppose $\alpha \|\!= \beta$ and $\alpha \|\!= \gamma$. Then, there is Γ such that $\Gamma \vdash_\Delta \beta$ and $\Gamma \rhd \alpha$, and there is Σ such that $\Sigma \vdash_\Delta \gamma$ and $\Sigma \rhd \alpha$. By $\rhd$-And, we have $\Gamma \cup \Sigma \rhd \alpha$ and we also have $\Gamma \cup \Sigma \vdash_\Delta \beta \wedge \gamma$. Hence, $\alpha \|\!= \beta \wedge \gamma$. QED

If we want to extend our set of postulates, for example, by including OR

If $\alpha \|\!= \gamma$ and $\beta \|\!= \gamma$, then, $\alpha \vee \beta \|\!= \gamma$,

then we have to impose the following condition on $\rhd$:

If $\Gamma \rhd \alpha$ and $\Sigma \rhd \beta$, then, either $\Gamma \rhd \alpha \vee \beta$ or $\Sigma \rhd \alpha \vee \beta$ ($\rhd$-OR).

However, we don't want to impose the following strong condition on $\rhd$ that will justify both transitivity and monotonicity of $\|\!=$.

If $\Gamma \rhd \alpha$ and $\Gamma \subseteq \Sigma$, then, $\Sigma \rhd \alpha$ ($\rhd$-Mon).

Theorem 2.

Suppose $\rhd$ satisfies $\rhd$-Mon. Then, $\|\!=$ satisfies the following two conditions:

If $\alpha \|\!= \beta$ and $\beta \|\!= \gamma$, then $\alpha \|\!= \gamma$ (Transitivity)

If $\alpha \mid\!-_{\Delta} \beta$ and $\beta \|\!= \gamma$, then $\alpha \|\!= \gamma$ (Monotonicity).

Proof.

To establish that $\|\!=$ satisfies Transitivity, suppose that $\alpha \|\!= \beta$ and $\beta \|\!= \gamma$. Then, there is Γ such that $\Gamma \rhd \alpha$ and $\Gamma \mid\!-_{\Delta} \beta$ and there exists $\sum$ such that $\sum \rhd \beta$ and $\Sigma \mid\!-_{\Delta} \gamma$. Applying $\rhd$-Mon, we can conclude that $\Gamma \cup \Sigma \rhd \alpha$ and $\Gamma \cup \Sigma \mid\!-_{\Delta} \gamma$. Hence, $\alpha \|\!= \gamma$. Of course, Transitivity trivially implies Monotonicity. For suppose $\alpha \mid\!-_{\Delta} \beta$ and $\beta \|\!= \gamma$.

Using Reflexivity, we have $\alpha \|\!= \alpha$ and by Right Weakening we can conclude that $\alpha \|\!= \beta$. By Transitivity then $\alpha \|\!= \gamma$. QED

Let's apply this formalism to Sen's example. As before the behavioral standard of the agent is described by the same set of sentences:

$$\Delta = \{(t \to p), (t \wedge c \to s)\}$$

We have two different sets corresponding to two different environments – one is a cocaine-free environment $\Gamma = \{t\}$, and another is an environment where cocaine is available along with tea $\Sigma = \{t \wedge c\}$. Then, we can consistently maintain that $t \|\!= p$ on the one hand, while $t \wedge c \|\!= s$, that is, if tea is offered, then the agent will accept it, while he will stay home if tea and cocaine will be offered. Notice that this time we can not derive $t \wedge c \|\!= p$ from $t \|\!= p$ because $\|\!=$ is not monotonic consequence operator.

4. Concluding remarks

Inspired by A. Sen's critique of internal consistency properties, we have developed a new framework to analyze the various problems of choice situations on the foundational level. The distinct feature of this new formalism is an introduction of a new binary relation $\rhd$ between the consistent set of formulas Γ and a formula α. By imposing only Reflexivity on $\rhd$, we can get a basic inference operator. Various other extensions of this basic inference operator could be generated by imposing additional conditions on $\rhd$. In fact, it would be interesting to investigate how much of the known nonmonotonic systems we can generate by imposing some reasonable conditions on our binary relation $\rhd$. However, we have decided to postpone this investigation until next occasion.

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