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# **On Growth and Saving**

Joan O'Connell\*

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Department of Economics,  
National University of Ireland, Galway

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• Contact address: Department of Economics, National University of Ireland, Galway.  
E-mail: [joan.oconnell@nuigalway.ie](mailto:joan.oconnell@nuigalway.ie), Telephone: +353-91-524411 (X 3140), Fax: +353-91-524130

### **Abstract**

This paper is concerned with the relevance of the macroeconomic theory of income distribution to the ‘new’ growth theory. Specifically, it shows that the Cambridge equation, originally outlined in the context of the Harrod-Domar growth model, and then extended to Solow’s (1956) neoclassical model, may also be derived in the case of Jones’s (1995, 2002) semi-endogenous growth theory.

**Keywords:** Distribution, Growth, Interest, Savings

**JEL Classification:** D3, O4.

## 1. Introduction

In their recent paper, Carroll *et al.* (2000) suggest that the reason why national saving and investment rates are strongly positively correlated is that growth causes savings, and they cite habit formation on the part of consumers as a way of accounting for this. Their argument is outlined in the context of an AK model of economic growth which is subject to exogenous shifts in productivity. The present short paper is also concerned with the way growth generates savings in part of the new growth theory. The context, however, is that of the semi-endogenous growth model of Jones (1995, 2002), and the mechanism, the post-Keynesian one of changes in the distribution of income, which was originally proposed by Kaldor (1955-1956) and Pasinetti (1962) as a way of aligning the warranted rate of growth of the Harrod-Domar model with the natural rate. In post-Keynesian theory, the natural rate of growth, together with the propensities to save out of wages and profits (Kaldor, 1955-1956) or of capitalists and wage-earners (Pasinetti, 1962) are all exogenously determined constants. The aggregate savings ratio, on the other hand, is endogenous, and adjusts to the value required by the model through changes in the division of income between components (Kaldor, 1955-1956) or classes (Pasinetti, 1962).

Features of Jones's model are introduced in section 2 below. The Cambridge equation (Pasinetti, 1974), gives the rate of return on capital as the natural rate of economic growth, divided by the propensity to save of the capitalist class. Subsequent to the development of the Harrod-Domar model, this result constituted the central proposition of the post-Keynesian theory of growth and distribution. Samuelson and Modigliani (1966) demonstrated that the Cambridge equation could apply to the neoclassical model of Solow (1956). In the case of the more recent AK model, Bertola (1994) showed that the return on capital could be presented in terms of the rate of economic growth and the fraction of capital income saved, but since both the savings propensity and the growth rate are endogenous in the model, this result is distinct from the Cambridge equation.

It is demonstrated in section 3 of the following that the Cambridge equation holds in Jones's model. This is despite the fact that Jones includes endogenous technical change and capital gains in his work, both of which features are absent from the post-Keynesian and Solow (1956) models. Conclusions and some discussion are contained in the final section.

## 2. Semi-endogenous Growth

In Jones's model, (1995, 2002), the labour force  $L$  grows at a constant, proportional rate  $n$ . The model comprises three sectors, a final output sector, a research sector and an intermediate goods sector. Labour is used in research ( $L_A$ ) and in the production of final output ( $L_Y$ ), and is fully employed ( $L = L_A + L_Y$ ). The analysis naturally throws up two measures of national income along the lines proposed by Eisner (1988), although this is not made explicit by Jones. These are, first, final output  $Y$  and, secondly, an extended measure of output  $Q$ , which here comprises the value of research as well as  $Y$ .

Final output  $Y$  is produced by intermediate goods and labour, and satisfies aggregate consumption demand  $C$  and, where  $K$  is the real capital stock, investment in physical capital  $\dot{K}$ . Intermediate goods are produced by physical capital on a one-to-one basis. Assuming a symmetric equilibrium, the final output sector uses all intermediate goods in the same quantity. Therefore, where  $A$  denotes the number of intermediate products currently available, and  $x$  is the output of each,  $K$  may be presented as:

$$K = Ax, \quad (1)$$

and the production function for  $Y$  by

$$Y = K^\alpha (AL_Y)^{1-\alpha}, \quad 0 < \alpha < 1. \quad (2)$$

Designs for new intermediate products are produced by the research sector with the aid of existing knowledge and labour. They grow at the rate (Jones, 1999)

$$g_A = \frac{\dot{A}}{A} = \frac{\delta L_A}{A^{1-\phi}} = \frac{n}{1-\phi} \quad (3)$$

in which  $\phi$  ( $0 < \phi < 1$ ) is an exogenously given constant, and  $\delta > 0$ , also an exogenous constant, is a productivity parameter<sup>1</sup>.

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<sup>1</sup> Romer (1990) outlined the production function for designs as  $\dot{A} = \delta L_A A$ , which shows the output of patents as dependent on the number of researchers and the existing stock of ideas. However, this production function yields  $g_A = \delta L_A$ , that is, the counterfactual result that when the fraction of the labour force engaged in research is constant, the growth rate of designs is proportional to the population. For this reason, Jones (1995, 2002) writes  $\dot{A} = \delta L_A^\lambda A^\phi$  where  $0 < \lambda < 1$  and  $0 < \phi < 1$ . As long as  $\phi > 0$ , existing knowledge generates positive externalities in the production of new ideas. The assumption that  $\lambda < 1$  allows for the possibility that the more researchers there are, the more likely it becomes that research effort is duplicated.

In the foregoing text, Jones's (1999) simplification that  $\lambda = 1$  is introduced. This simplification makes no difference to the results outlined in this paper.

Designs are patented. Measured in terms of final output, their aggregate value is equal to  $P_A A$  where  $P_A$  is the price of a patent expressed in units of  $Y$ . Broad output  $Q$  is equal to the value of research and final output and, also, to the sum of consumption expenditure and expenditure on investment in both intellectual and real capital.  $Q$ , therefore, may be given by:

$$Q = Y + P_A \dot{A} = C + \dot{K} + P_A \dot{A}. \quad (4)$$

In balanced growth, the two components of the labour force,  $L_Y$  and  $L_A$ , each separately grows at the rate  $n$ . In addition, the variables  $Y$ ,  $K$ ,  $C$  and  $P_A A$  all increase at a constant and equal rate. When combined with equations (1)-(3), these conditions show that  $\dot{x}/x = \dot{P}_A/P_A = n$ , and that the common rate of growth of  $Y$ ,  $C$ ,  $K$  and  $P_A A$  equals

$$g = g_A + n. \quad (5)$$

When disaggregated into income components, the extended measure of output  $Q$  may be alternatively represented as

$$Q = wL + rK + \pi A. \quad (6)$$

In this equation,  $w$  is the real wage,  $r$  the rate of interest on real capital and  $\pi$  the profit on each intermediate product. The labour market is competitive, and the wage rate  $w$  is equal to the marginal product of labour in the final goods sector. Aggregate wages  $wL$  equal  $(1 - \alpha)Y + P_A \dot{A}$ . Total income from capital, the surplus of broad output  $Q$  over wages  $wL$ , which is equal to  $\alpha Y$ , is divided between interest on real capital and the profits of patent-holders. In the model, profits  $\pi A = \alpha(1 - \alpha)Y$  and the rate of interest  $r = \alpha^2 Y/K$ . Because  $K$  and  $Y$  grow at the uniform rate  $g$ ,  $r$  is constant over time. Profit on each intermediate product  $\pi$  grows at the same proportional rate as the output of the product  $x$ , that is, at the rate  $n$ . Therefore, the rate of profit, which is given by  $\pi/P_A$ , is constant, as is profit on each unit of all  $x$ 's produced.

Returns on patents include capital gains as well as current profits. Patents and real capital represent alternative investments. Thus, arbitrage requires the total returns on patents to equal the interest payable on real capital, or,

$$\pi A + nP_A A = rP_A A. \quad (7)$$

The left-hand-side of this condition shows the dividends ( $\pi A$ ) plus the capital gains ( $nP_A A$ ) yielded by patents, and the right, the interest forgone on their acquisition.

### 3. The Cambridge equation

The steady-state growth rates,  $g_A$  and  $g$ , outlined in equations (3) and (5), respectively, are independent of demand. In this respect, the model is in the tradition of the older exogenous growth theory rather than in that of the newer endogenous models (Eicher and Turnovsky, 1999). Demand is now added to the model, and for this purpose, aspects of the post-Keynesian theory and the traditional neoclassical theory of economic growth with two assets are combined.<sup>2</sup> In the post-Keynesian theory, savings may be constant fractions of wages and profits (Kaldor, 1955-56). Alternatively, savings may be constant fractions of the incomes of wage-earners and capitalists, where capitalists are defined as a class whose members receive only unearned income (Pasinetti, 1962). Here it is assumed that wage-earners do not save. Under this condition all profits accrue to the capitalists, and the two post-Keynesian versions of savings behaviour coincide.

As in the theory of monetary growth, the present model is characterized by two assets between which the relative price is variable. Following the suggestion of the monetary theory (e.g. Solow, 1970), it is assumed that in deciding how much to consume and save, wealth-owners treat capital gains on patents in the same way as non-wage income.

Adding this condition to the usual post-Keynesian assumptions yields savings as a constant fraction  $s_p$  ( $0 < s_p < 1$ ) of non-wage income and capital gains. The remainder of non-wage income and capital gains, plus wages  $wL$ , make up aggregate consumption  $C$ . Substituting accordingly for  $C$  in equation (4) gives

$$\dot{Q} = \dot{K} + P_A \dot{A} + wL + (1-s_p)(rK + \pi A + nP_A A). \quad (8)$$

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<sup>2</sup> Jones (1995) introduced optimizing consumers. In his 1999 paper, Jones adopted a constant savings ratio.

Combining this equation with (6) yields

$$s_p(rK + \pi_A + nP_A A) = \dot{K} + P_A \dot{A} + nP_A A, \quad (9)$$

which shows savings equal to aggregate current investment, plus capital gains. The left-hand-side of this equation, in conjunction with (7), can be written as  $s_p r(K + P_A A)$  while conditions (3) and (5) can be used to show that the right-hand side is equal to  $g(K + P_A A)$ . Hence, equation (9) can be presented as

$$s_p r(K + P_A A) = g(K + P_A A). \quad (10)$$

Whereas in the preceding section,  $r$  was given as  $\alpha^2 Y/K$ , equation (10) now provides the following alternative expression for the rate of interest

$$r = \frac{g}{s_p}, \quad (11)$$

in which, by assumption,  $s_p$  is exogenous and constant.

Jones refers to his model as ‘semi-endogenous’ because while he follows the Harrod-Domar and subsequent ‘older’ growth theory in assuming that the labour force grows at a constant, exogenous rate, at the same time his model is also ‘new’ in that the of technical progress is retained (1995, p.761). The rate of growth of income and assets, represented here by  $g$ , depends only on the two exogenous constants  $n$  and  $\phi$  and, consequently, is itself constant. Equation (11), therefore, is the Cambridge equation. In this model, savings adjust to growth and investment through changes in non-wage income and capital gains. If, for example,  $n$  and, therefore,  $g$ , were higher, then  $r$ , as well as asset stocks, non-wage income and capital gains would all also be greater, thus ensuring additional savings to match the increased accumulation of wealth.



## 4. Conclusions

The purpose of this paper has been to show that the Cambridge equation, together with the post-Keynesian adjustment of savings to growth, holds in Jones's (1995, 2002) theory of 'semi-endogenous' growth. This is despite the fact that, unlike the post-Keynesian models of Kaldor (1955-1956) and Pasinetti (1962), Jones's theory incorporates endogenous technical progress and capital gains.

In Pasinetti's (1962) derivation of the Cambridge equation, in which wealth comprises only a single asset, real capital, capitalists' savings are given by a fraction of their wealth equal to their savings ratio, times the rate of return, while capitalists' capital grows at the same rate as the economy. Thus, the condition that capitalists' savings equal capitalists' investment yields the Cambridge equation. In the case of Jones's (1995, 2002) model, wealth includes intellectual as well as physical capital. As depicted in equation (10), in these circumstances, capitalists' savings equal the fraction  $s_p r$  of wealth, while wealth expands at the rate  $g$  of the economy. Thus, again, the equilibrium condition equating capitalists' savings and capitalists' investment which now, however, make up national savings and investment, respectively, yields the Cambridge equation.

The post-Keynesian model relies on differential savings behaviour. The assumption introduced in section 3, that wage earners do not save, may be viewed as a special case of either Pasinetti's (1962) condition that capitalists' propensity to save exceeds that of wage-earners or of Kaldor's (1955-1956) assumption that a greater fraction of profits is saved than of wages. Kaldor (1966) justified his assumption by reference to the fact that it is mainly corporations that save. Pasinetti (1966), on the other hand, adopted the customary procedure of ignoring institutions, thereby implying that the actions of corporations are transparent to the share-holders. Under these conditions, the differential savings assumption may be supported by evidence that, among households, it is the rich that save (Browning and Lusardi, 1996). More controversial, however, is the assumption underlying class-based savings that there exists a class of capitalists who do no work and rely entirely on returns to capital (e.g. Pasinetti, 1966).

Is there such a class? There is, indeed, according to Updike (1988) in his recent novel *Towards the End of Time*. Updike characterizes members of the class as follows:

Some of these men have never held a job. Their life stages have been marked by a succession of games: the child, introduced by his nurse-maids to croquet and badminton and then given tennis and sailing and equestrian lessons; the boarding-school boy, hardened at soccer and ice hockey and lacrosse; the college man, persuaded to risk his bones in the football line and test his eyes and nerves on the baseball team, ..., ... the suburban husband, partnered with his wife at paddle tennis and matched against his old college room-mate at squash; the country squire, ten pounds heavier and rosier in the face, caught up in the physically lighter but financially heavier exertions of polo and yachting; the paunchy man of distinctly mature years, passionate for the pederstrian challenge of golf ... the stoop-shouldered dotard, ... extracting competitive thrills from billiards, bridge, backgammon, and, yes, croquet again, in a more formal, while-clad version.

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